Power Loss Model for Archimedes Screw Turbines

by

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A Thesis Presented to

The Faculty of Graduate Studies

of

The University of Guelph

In partial fulfillment of requirements

for the degree of

Master of Applied Sciences

in

Engineering

Guelph, Ontario, Canada

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Abstract

Power Loss Model for Archimedes Screw Turbines

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This thesis presents a complete power loss model for an Archimedes screw turbine (AST), capable of predicting the mechanical power of an AST based on available energy head. This model amends a prior idealized frictionless AST performance model to include losses from inlet and outlet entrance effects, internal hydraulic friction and outlet submersion. Laboratory experiments on scale-model ASTs were conducted to determine variable relationships and validate power loss models. Gap leakage was experimentally measured in an AST, and results were compared to gap leakage models. The performance of a 7 kW grid-connected AST was measured and compared to model predictions. Finally, a delivery channel loss model is presented that predicts the needed supply reservoir head to provide an AST with a specified flow. The proposed AST power loss model improves the prior frictionless power model significantly and is generally capable of predicting the power for a real-world AST.
Acknowledgements

I owe many people so many things, so this section might be a little long...

I first need to give a mighty big thanks to my advisor William David Lubitz, possibly the best advisor on the planet (and I sincerely mean it, if any of you potential future graduate/Ph.d students seeking an advisor are reading this). Your guidance, patience, and generosity are unparalleled. I couldn't have asked for a better supervisor or project, so for that I sincerely thank you.

A huge thank you to the GreenBug Energy team. My project wouldn't even exist without you. Tony and Brian, your support has been absolutely amazing and I wish GreenBug nothing but success in the future, you guys really do great work. And Murray too, thanks for all your help along the way.

I need to give a huge acknowledgement and thank you to Dirk Nuernbergk. Though I've never met you, your incredible work is the foundation upon which this thesis rests.

I'd also like to give a special thanks to Doug Joy for supporting this project and myself for a long time. I've greatly appreciated all your help and even your courses over the years. Similarly, I need to thank Ramesh Rudra and Trevor Dickinson, who have always supported me.

I also need to give a huge thanks to Martin Williams, the greatest physics professor of all time! Your guidance over the years has help me dearly. I particularly appreciate all the help you gave me considering I'm just a lowly engineer. Your door has always been open to me and you've never hesitated to help when asked, and for that I can't thank you enough.

I'd like to thank all of Lubitz's researchers who contributed to my project: Alvaro, Shivani, the other Andrew, Brent, Kathleen, and Devon. Shivani, thanks so much for the room! And Devon, thank you for running the lab so fantastically, you were the best research assistant ever (even if your mind was gone in the summers...). You always came through no matter what the task. And Kasha, I wish you luck as you take over. And a special thanks to Scott for the SolidWorks wizardry!

And of course, the Legendary Jack Weiner, calculus just hasn't been the same without you. You've had a huge impact on our lives. We are smarter because of you, and for that we thank you tremendously. And Linda Allen too, the greatest counselor, you will be missed dearly.

Also, my entire office mates crew. King Felix, thanks so much for all the times you let me use your house as a crash pad! I'm sorry I never got you your proper party. Similarly, Jody and Peter, thank you so much for all the nights you also let me use your house as a crash pad! You two are truly the best. And my brother Eric...thank you for also letting me use your house as a crash pad (I see a pattern happening here...). I love you all.

And of course Jamie, you are the sharpest person on the hallway, and I'm so glad we shared our half of the room together. I appreciate all the help you've provided me and you always had the best
show recommendations! Sorry I was always behind on the plots. And hey, the Jays had a great run!

And, Sarah, honestly, sorry it took so long to get to you. You are the best engineer I know and a great friend. I could write an entire thesis on all the help you've given me over the years, but I'll try to keep it short. I owe you so much, I'm not even sure where to begin. From all our design projects, to all the course work help, proof reading and edits. My academic average is at least 10% higher than it should be because of you. I have scholarship money I probably shouldn't because of you. I've won awards I probably shouldn't because of you. And I've gotten jobs I probably shouldn't...because of you. The only reason I could dabble with physics for so long was because you always made sure my engineering was proper. I hope that Milo's not terrorizing you, and congratulations on Anna Joy! I wish you and Vaughan all the best...but you'll always be a Pacman to me.

And Kadeem the great! No one’s been closer to me these past years, you truly are family. I sincerely miss you. And I am so so so so sorry about Cairo. We've been through a lot; I'm laughing in my mind right now at all the things we've done that I shouldn't put in writing. You are the hardest working person I know, and I admire you greatly. This place just never was the same without you, and I truly mean that. You made us all so proud nailing that exam and promotion! And no, I don't want your first born! Come home safe.

Nikkie. Thank you for all your help over the years. I know, you're probably thinking I've helped you more, but that's not true. You never hesitated when asked, and you absolutely saved me when I was in Europe...seriously, I can't thank you enough for that! And the time you spent working with me in my lab was a great help to this project in particular. Tell Natasha I miss her. If your family asks, I'll remember, you stayed at my place last night. Don't worry, I won't run away without you, but if I do, you can always follow…

And speaking of being saved in Europe, Paula! Thank goodness you got my Super’s email! Paula, honestly, you were the best boss I’ve ever had, thanks so much for keeping me in the library so long! I wish you nothing but the best!

And now the family. Mom, I owe you everything. You are the Yoda of my life. I'm so sorry we were all such a handful, you deserved better than us. I love you more than I could ever express with words. Dad, you too, I love you very much, and I'm so happy to see us doing better these days. Ruben, my goodness, I miss you so much sometimes it hurts. I'm so sorry it is taking so long for me to get over there. We both did what we had to do, but I'm coming, I promise. It's been so many years since you left, I'm having trouble remembering what life was like when you were here. I feel like I missed so much. I love you baby j.

Kayla and Jaime, you two are the closest things to sisters I'll ever have. I wish you both nothing but happiness and success and no matter what you two do in life, you know we'll always have your backs. Thanks for being so good to my mom all these years, it means so very much to me.
Cousin Matt, get well and fight that thing off! I'm sorry we drifted at times. We've literally been connected since birth. I wish you and Lauren well and I'm sorry about the whole can't get out of Catholicism thing. We'll figure something out.

Pardeep, Tara, and Kiran, I haven't forgotten about you guys either. It’s insane when I think you’ve literally been in my life since the day I was born. We’re family, whether you like it or not!

And my whole team. Juan, just too many stories, I love you like a brother. Chris, stay out of trouble! Simon, you've been my best friend for so long, it's impossible to know where to start. I have so much to thank you for. We've been many places together, seen many things. We've been in a lot of trouble together. It still amazes me when I think back to all the things we did in our youth. My mom keeps asking to see Maliki, you know how she is, so you have to find time to bring him around. And of course Aaron. You remember how I never trusted you when we first met? Now look at us. I've never had a better friend. You helped me get through my lowest lows. And look how far we've both come? It’s a long way from Weston Road and Martha Eaton Way. I'm so proud of the man you have become. I miss Gulliver. And Dufferington. Late night runs to Commisso brothers. Fairbank hotel. Knocking out in the Cadi. Waking up on Juan’s floor and trying to get to school! Maple Leaf and Fabric Land (okay I don't miss those one bit). Blue night missions home. Hustlin’ jits. How can I forget Samron and his Samurai hat! Weston Collegiate, though none of us lasted. Central tech…though some of us lasted! Benjamin, Brian and James. Back of Frontlines, Bellevue, HJ, harbor front sessions, and the twin towers. And of course, the beats.

My little man Chad, keep basking!

Ravina! Stop being so hard on yourself. Do your best to keep good people around you and try to relax when you can. I will look out for my pirate always!

I miss you Jenny. I know it's been a long time but your friendship means the world to me. I still, to this day, look at the picture and read your message when I need it. And you too Daphne. I am the person I am because you came into my life. And for that I will always be grateful and love you.

Keanna, you're are just beginning, but you have so much promise. You’ve brightened my life immensely. I hope you always keep painting beautiful, sunsets and skies. Beautiful and golden? Check. Now just work on the third one. And yes, you can do it, believe in yourself. Be good to those around you, even when they aren’t being good to you. Be patient, open-minded and understanding, and go easy on the chairs! Be appreciative and thankful for what you have and never feel sorry for yourself. Always pick the flawed elephants because those ones are the most special. Live without regret, in the here and now. And don’t take things to heart, you only hurt yourself like that. Life happens when you least expect it. Above all, remember, everything is fixable. Nothing can touch you because I got you. Keep living in colour.

And last, but most especially never least, Sylvia. I don't have to say much Chabi because you already know. I couldn’t have done any of this without you.
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<th>Abbreviation</th>
<th>Definition</th>
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</thead>
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<tr>
<td>ASG</td>
<td>Archimedes Screw Generator, complete with turbine, electric motor and gearbox</td>
</tr>
<tr>
<td>AST</td>
<td>Archimedes Screw Turbine</td>
</tr>
<tr>
<td>ASP</td>
<td>Archimedes Screw Pump</td>
</tr>
<tr>
<td>Overflow Leakage</td>
<td>Leakage that occurs between buckets that occurs over the central shaft of the screw (beyond the 100% bucket filling point)</td>
</tr>
<tr>
<td>Gap Leakage</td>
<td>The leakage that occurs between buckets through the gap between the screw and the containing trough</td>
</tr>
<tr>
<td>Characteristic average flow area</td>
<td>The average area of flow in the screw that would occur without the helical plane surfaces</td>
</tr>
<tr>
<td>Delivery Channel</td>
<td>Inlet channel that supplies water to the AST</td>
</tr>
<tr>
<td>Outlet Channel</td>
<td>Channel that receives discharge flow from the AST</td>
</tr>
</tbody>
</table>

(Kibel P., 2007) Volume of water entrapped between to helical plane surfaces
Nomenclature

\( A_c \quad \text{Characteristic average flow area (m}^2\text{)} \)

\( D_i \quad \text{Inner screw diameter (m)} \)

\( D_o \quad \text{Outer screw diameter (m)} \)

\( \text{eff} \quad \text{Nominal power efficiency calculated by MATLAB model (-)} \)

\( f \quad \text{Bucket volume fill height (-)} \)

\( F_r \quad \text{Froude number (-)} \)

\( g \quad \text{Gravitational constant (9.81 m/s}^2\text{)} \)

\( G_w \quad \text{Gap width (m)} \)

\( h \quad \text{Total screw across screw (m)} \)

\( h_{\text{avg}} \quad \text{Idealized average water depth in a screw (m)} \)

\( h_d \quad \text{Water depth in delivery channel immediately upstream of screw (m)} \)

\( h_f \quad \text{Friction head loss due to AST trough (m)} \)

\( h_{\text{lw}} \quad \text{Lower basin water level above bottom of AST (m)} \)

\( h_O \quad \text{Head loss at water outlet (m)} \)

\( h_{\text{ue}} \quad \text{Water height above the maximum bucket elevation point (m)} \)

\( H_{\text{res}} \quad \text{Total reservoir head (m)} \)

\( L \quad \text{Screw length (m)} \)

\( N \quad \text{Number of helical plane surfaces (-)} \)

\( R_e \quad \text{Reynolds Number (-)} \)

\( n_b \quad \text{Number of buckets in a screw (-)} \)

\( P \quad \text{Power (W)} \)

\( P_{\text{bl}} \quad \text{Power loss due to bearing friction (W)} \)

\( P_{\text{loss, exit}} \quad \text{AST exit power loss (W)} \)

\( P_{\text{loss, plane friction}} \quad \text{Power loss due to shear friction on the helical planes (W)} \)

\( P_{\text{loss, wall friction}} \quad \text{Power loss due to shear friction on the AST trough walls and center shaft (W)} \)

\( p \quad \text{Hydrostatic pressure on the screw surfaces (Pa)} \)

\( P_{\text{avail}} \quad \text{Available power (W)} \)

\( P_w \quad \text{Wetted Perimeter (m)} \)

\( Q \quad \text{Total flow (m}^3\text{/s)} \)

\( Q_b \quad \text{Bucket water flow rate (m}^3\text{/s)} \)
\( Q_{GL} \)  
Gap leakage flow (m\(^3\)/s)

\( Q_o \)  
Overflow leakage (m\(^3\)/s)

\( R_h \)  
Hydraulic Radius (m)

\( S \)  
Screw pitch (m)

\( T \)  
Torque created by a single water bucket (N \( \cdot \) m)

\( T_{tot} \)  
Torque created by all water buckets in a screw (N \( \cdot \) m)

\( V \)  
Volume of water in a single bucket (m\(^3\))

\( V_{tot} \)  
Total volume of water in a screw (m\(^3\))

\( v_t \)  
Transport velocity (m/s)

\( \beta \)  
Screw angle of inclination (rad)

\( \omega \)  
Angular velocity of the screw (rad/s)

\( \rho \)  
Density of water (kg/m\(^3\))

\( \xi_i \)  
AST inlet head loss coefficient (-)

\( \xi_o \)  
AST outlet head loss coefficient (-)

\( \psi \)  
Relative height of water above the bottom point of AST (-)

\( \eta_T \)  
Torque loss efficiency coefficient (-)
Chapter 1: Introduction

Archimedes Screw Turbines (ASTs) are an emerging form of hydroelectric power generation intended for small scale hydro projects. ASTs offer a renewable source of energy that is efficient for low head sites and safer for wildlife and fish than traditional turbines (Kibel, 2007). While ASTs have only been in use since the 1990s, they are a modification to the ancient Archimedes Screw pump technology (Lashofer et al., 2012).

An Archimedes Screw consists of several helical planes attached to a center cylindrical shaft, contained in an enclosing inclined trough. A small gap exists between the trough and screw, allowing the screw to rotate freely. ASTs function by allowing water to transverse the screw from high to low elevation. As water passes through the screw, a torque is created on the helical plane surfaces causing the screw to rotate. Attaching a generator allows this mechanical rotation to be used to produce electricity. Figure 1-1 shows a typical Archimedes screw used in a turbine configuration.

![Figure 1-1. A typical Archimedes Screw Turbine (AST), side view (Lubitz, 2014)](image-url)
Currently, there are several existing models capable of predicting AST performance. However, these models have limitations that prevent them from being fully utilized as an engineering design tool capable of predicting AST power output for a given site. Currently, one of the most complete AST models was created by Lubitz et. al (2014). It is a completely physics-based power model. The Lubitz et al. (2014) model determines AST output power by calculating the hydrostatic pressure created by the water across the helical plane surfaces. However, this model does not incorporate any mechanical or hydraulic power losses that occur within ASTs.

It is the purpose of this thesis to extend the Lubitz et al. (2014) AST model to incorporate a complete AST loss model that includes the hydraulic and mechanical losses. The model will be adapted to compensate for hydraulic losses that occur both at the inlet and the outlet of an AST. At the inlet, flowing water transitioning from an inlet channel to the trough experiences sudden changes in velocity, pressure and geometry. These rapid changes are accompanied by energy losses through the transition region. Similar changes occur at the outlet as the flowing water merges with the discharge basin or stream. If the outlet of the AST is submerged, additional drag forces create losses in power. The Lubitz et al. (2014) model assumes that the water within a screw is in a quasi-static state, however, the water within an operating screw can be observed to be in motion, particularly at high rotational speeds. AST power losses increase with rotational speeds as the quasi-static assumption becomes less valid, and the mechanical friction in the bearings increases.

Each of these loss factors (inlet, outlet, internal hydraulic losses and mechanical friction) was analyzed experimentally using laboratory-scale ASTs. Data from these experiments was analyzed
with the goal of creating predictive mathematical models that can successfully amend the Lubitz et al. (2014) model to better predict AST output power for a wider range of operating conditions.

The Lubitz et al. (2014) model will also be extended to include an AST inlet channel model, capable of calculating the hydraulic losses and water levels in the AST delivery channel responsible for supplying water to the generator. From this, the required total energy head of the AST water supply can be determined. These effects were incorporated into a new model that extended the Lubitz et al. (2014) model into a complete power model capable of determining the total mechanical power of any AST, for any site or location, resulting in a more complete engineering design tool.
Chapter 2: Literature Review

2.1 History of Archimedes Screw Turbines (ASTs)

The Archimedes screw is an ancient hydro-technology whose origins date back nearly two millennia (Koetsier & Bluwendraat, 2004). The invention of the Archimedean screw is widely attributed to its namesake, Archimedes of Syracuse, the 3rd century (BC) Greek physicist, mathematician, and inventor. However, there exists evidence that the invention and use of the screw technology may date back to over three centuries prior to Archimedes (Dalley & Oleson, 2003). Historically, Archimedes screws have been used as water pumps (Koetsier & Bluwendraat, 2004), particularly for irrigation and de-watering purposes. Archimedes screws are robust pumps that are often unaffected by debris and obstructions and are, therefore, often found in modern-day wastewater treatment facilities (Brada, 1999).

Like most pumps, Archimedes Screws can be driven by a flowing fluid and used as turbines. Despite being a technology that dates back to antiquity, the use of Archimedes Screws as generators is relatively new. The first AST was implemented within the last two decades. Brada (1999) installed the first Archimedes Screw Turbine (AST) in the 1990s. Since then, several hundred ASTs have been installed to generate electricity (Lashofer et al., 2012). These ASTs have almost all been built in Europe to date. Currently there exists only one operational AST in North America, installed near Waterford, Ontario, Canada (Lubitz et al., 2014).
2.2 ASTs Relative to Other Hydropower Technology Options

Hydropower, in general, is one of the oldest forms of energy utilization. The earliest reference to the use of water as a source of energy dates back to 85 BC (Breeze, 2005) and it is widely believed that first forms of hydropower technology date back to 100 BC with the first water wheels (Lynch & Rowland, 2005). Water wheels, the earliest and simplest form of hydroturbine, were largely used to convert energy from flowing water directly to mechanical energy for grinding grain (Lynch & Rowland, 2005). Waterwheels can be oriented either vertically or horizontally, though vertical designs typically produce more power and were more commonly used. Horizontally oriented water wheels (with a vertical rotation axis) are primitive in design, and typically function by allowing the momentum from a jet of flowing water to exert a force on paddles connected to a central cylinder, causing it to rotate. Vertically oriented water wheels (which have horizontal rotation axes) are slightly more efficient, as they can allow small buckets of water to form between the paddles, allowing the weight of the water in the buckets, along with the water’s momentum, to drive rotation (Lynch & Rowland, 2005). Well-constructed water wheels could see efficiencies between 50-70% (Lynch & Rowland, 2005). Water wheels were widely used in the Roman Empire and in China by 100 AD and commonly found in Europe by 300 AD (Breeze, 2005).

Modern turbines can generally be classified into two categories: impulse and reaction turbines. Impulse turbines are momentum-driven devices, typically designed for sites with high head. High head conditions (potentially several hundred meters) can create extreme pressures that can create high velocity water jets when channelized through a nozzle. Impulse turbines direct this high momentum flow into paddles or buckets attached to a central cylinder causing it to rotate. Impulse turbines always operate in air as operating submerged creates drag forces leading to inefficiencies
The Pelton turbine is a commonly used impulse turbine suitable for high head, typically between 50 m and 1000 m (Williamson et al., 2014) and low flow sites. In Pelton turbines, a high-pressure jet of water is directed at bucket-shaped blades. The shape of the buckets allows for nearly all of the flow energy to be converted to rotational mechanical energy (Breeze, 2005). Another well-known impulse turbine is a Turgo turbine, which is a modification of a Pelton turbine. In Turgo turbines, the incoming water jet is directed at an angle of 20 degrees to the buckets allowing for a larger jet stream of water relative to the turbine diameter than a Pelton turbine. Therefore, for the same flow and head, a Turgo can generally produce more power or be constructed smaller, though at the expense of reduced overall efficiency (Subramanya, 2013). Turgo turbines are suitable for heads typically ranging from 50 m to 250 m (Subramanya, 2013).

Reaction turbines are generally suitable for sites with heads less than 450 m. Unlike impulse turbines, reaction turbines are designed to operate at peak efficiency when submerged. While impulse turbines are momentum based devices that harvest the kinetic energy of flowing water, reaction turbines are pressure-driven devices. The turbine blades in reaction turbines are designed to allow the pressure difference across the blades created by the weight and momentum of the water to cause turbine rotation (Breeze, 2005). Reaction turbines account for approximately 80% of all hydropower turbines in use today (Breeze, 2005).

The two most common reaction turbines are Francis and Kaplan turbines. In Francis turbines, a pressure difference across the blades is accomplished by changing flow direction. Flow enters the turbine radially but is redirected in a direction along the axial length of the turbine. The blades are shaped to maximize energy extraction and are typically designed for the specific flow conditions
A properly designed Francis turbine operating under the right conditions can be 90 to 95% efficient (Breeze, 2005). Francis turbines are the most commonly used turbines in large hydropower stations and deliver best performance for heads between 40 m to 600 m (Subramanya, 2013).

Kaplan Turbines are another form of reaction turbines, well suited for low heads, typically less than 50 m (Rama, 2002). Kaplan turbines are a form of propeller turbine that operates similar to boat propellers but in reverse. Instead of the blades pushing against the surrounding water to move a boat, flowing water pushing past the propeller blades creates a pressure difference, causing rotation. Propeller turbines are best suited for heads and flow less than impulse and Francis turbines and are often utilized in slow moving rivers and streams with modest elevation drops (Breeze, 2005). Kaplan turbines are a specific type of propeller turbine in which the propeller angle can vary to optimize energy extraction for a given flow rate, allowing better performance in unsteady flow conditions (Breeze, 2005). However, most propeller turbine efficiencies are reduced dramatically when the flow rates drop below 75% of the intended design flow, so it is common to see several propeller turbines in parallel, each with different design flows (Breeze, 2005).

While there exist many turbine types for a range of heads and flows, there has historically not been a hydro turbine type that functions efficiently in combined low head and low flow conditions. However, the emergence of ASTs in the past few decades has begun to fill this gap. ASTs are particularly advantageous over other turbines for sites with low flow (under 10 m$^3$/s) and low head (under 5 m) conditions (Lashofer et al., 2012). For low flow, low head sites, ASTs are proving advantageous over traditional hydroelectric power generation technologies. Williamson et al.
(2014) compared the performance of several micro hydro technologies and found that, unlike most turbines, AST efficiency remains high even as available head approaches zero. Typical AST overall power plant efficiency has been found to range between 60 and 80% (Lashofer et al., 2012). Figure 2-1 shows Williamson et al. (2014) recommended hydro turbine selection criteria, demonstrating the regions of discharge and head suitability for the most common turbine types. ASTs operate within a previously unfulfilled hydro turbine selection region, enabling previously overlooked small-scale hydropower resources to be utilized.

![Figure 2-1. Head and flow selection criteria for hydro turbines (Williamson et al., 2014)](image)

2.3 The Case for Small-Scale Hydro

2.3.1 Mitigation of Environmental Impacts

Utilizing ASTs for small-scale energy production provides several unique advantages over traditional hydropower technologies. Hydropower is often considered a zero carbon energy source, as an operating hydroelectric generator produces effectively no greenhouse gases (GHG). Even
when considering the entire life-cycle of a hydropower generating station, including construction, repairs and maintenance, and decommissioning, the GHG emissions from hydropower remain relatively low when compared to traditional fossil fuel power (Tester, 2005). All hydroelectric plants utilize the potential energy available from flowing water found in natural watercourses. This water is supplied by the hydrologic cycle, which is driven by solar energy, making hydropower a true renewable source of energy.

However, there have been recent studies examining the potential for GHG generation and emission of modern hydroelectric power plants based on a complete life-cycle analysis. Nearly all hydropower station GHG emissions occur during the construction of large dams and supporting facilities and from flooding due to reservoir creation (Gagnon & van de Vate, 1997). Hydroelectric dams can often be massive constructions, with some even being among the largest creations produced by humans. Materially, large dams are comprised mainly of concrete and/or earthen material. The construction and transportation of these materials can have large CO$_2$ emissions.

Additionally, the flooding caused by reservoir formation can potentially generate substantial GHG emissions. A fully formed reservoir can submerge biomass material, ranging from trees to soil microorganisms. Decomposition of these materials often releases GHGs over many years (St. Louis et al., 2000). Due to large surface areas, it can be difficult to measure the GHG emissions from large hydroelectric dam reservoirs, but GHG emissions have been shown to be highest in reservoirs located in tropical climates; in extreme cases, dams located in tropical areas can potentially produce GHG emissions equivalent to or exceeding those from traditional fossil-fueled generating stations, for a station of similar generating capacity (Aurelio dos Santos et al., 2006).
Furthermore, damming rivers for reservoir formation has many environmental and social impacts. These include mercury contamination of ecosystems, silt deposition within the reservoir, which can lead to downstream shoreline erosion and the disruption of delta formation, and the disturbance, even dislocation, of human communities (Tester, 2005).

In contrast, ASTs typically operate on a scale that mitigates or even eliminates the aforementioned environmental impacts. An interesting, and common AST application, is to retrofit an AST into a small-scale dam site not currently being utilized for hydroelectric power generation. Within the province of Ontario, Canada, there currently exist approximately 2600 mostly small dams which have been built on the province’s vast network of tributaries, streams, and rivers (Keyes & Pastinak, 2011). Most of these dams were constructed for the purpose of flood control and milling and it is estimated that approximately 70% of these dams were constructed prior to 1970 (Demal, 2012). The expected service life of a typical small dam ranges between 50 and 70 years (Demal, 2012). The structural quality and safety is increasingly becoming a major concern as many small dams age. Within the province of Ontario alone, 70% of small dams will require major restorations or structural repairs within 10 to 15 years (Demal, 2012). A similar situation can be found in other Canadian provinces as well as abroad. In the United States, the Army Corps of Engineers national inventory estimates there are 18,140 dams with heights under 15 feet that are in need of structural repair due to improper or deferred maintenance (Association of State Dam Safety Officials, 2009).

At many dam sites, installing an AST (and performing related dam maintenance) would not only improve structural stability of dam infrastructure but also enable new power generating potential, without many of the environmental impacts associated with large-scale hydropower generation.
This is because, for these dam sites, the dam construction has already been completed and the environmental impacts incurred; retrofitting these sites requires minimal additional construction allowing ASTs to be implemented without the GHG emissions associated with large-scale hydro projects. Furthermore, currently existing dams have already changed the existing watercourse, so few additional environmental impacts in terms of ecosystem degradation are introduced when implementing ASTs. Finally, the small-scale nature of ASTs means that their associated environmental impacts are far below that of traditional hydropower generation plants.

Additionally, AST technology is also attractive because of the limited impacts it has on wildlife and aquatic species. Unlike most traditional turbine technologies, ASTs tend to operate at slow rotational speeds and have large openings that allow for the safe passage of small objects. When used as a pump, this feature allows debris and other obstructions to easily pass through, making them suitable for use in wastewater treatment plants (Hauser, 1996) and for transporting fish upstream (Craig, 2016). When used as a turbine, these same features allow fish and aquatic species to often pass through safely with a minimum of morbidity or mortality. Research has shown, that if fish are able to pass by the leading edges of the screw's plane surfaces, they will transverse the length of the screw unharmed. Kibel et al. (2009) demonstrated that fish with masses smaller than 1 kg can safely pass through an AST, rotating at typical operating speeds, without being harmed. They also found that utilizing rubber bumpers on the leading edges can ensure that fish up to 4 kg in mass can safely pass through ASTs without injury. Kibel (2008) studied the passage of several fish species, including trout, eels, and salmonids, and found they usually pass safely through commercial AST units operating in the United Kingdom.
2.3.2 AST Power Generating Potential in Ontario, Canada

Currently, hydropower plants supply approximately 3% to 4% of total human energy usage, making hydropower the most utilized non-biomass renewable energy source (REN21, 2013). In contrast, approximately 80% of global energy consumption is supplied by fossil fuels. It should be noted that the energy derived from fossil fuels cannot be replaced by hydropower alone, including just the electricity generation component (Tester, 2005). With ASTs operating on the smaller-scale of the hydropower spectrum, ASTs will not likely be a major replacement for traditional fossil fuels. However, there exists unique, niche commercial applications for AST technology.

While it is difficult to quantify the total global power potential of ASTs, it is possible to estimate this total on smaller, regional scales. In the province of Ontario, Canada, for example, nearly one quarter of all energy production is provided through hydro resources, representing approximately 90% of all of the provinces renewable energy supplies (OEB, 2014). This amounts to generating capacity of approximately 8100 MW, generated from 240 dam sites, across 24 rivers systems (OPA, 2010). It has long been an aim to increase the provinces hydropower supply, however the focus of these increases are largely for sites with generating capacity of greater than 1 MW (Hatch Acres, 2005). Sites under 1 MW power generating potential are where ASTs are best utilized.

Furthermore, approximately 1000 of the 2600 existing dams in the province of Ontario are owned by private individuals and landowners; the remaining dams are owned by the Ministry of Natural Resources, the mining industry, Ontario Power Generation, conservation groups, and local municipalities (CVC, 2011). This suggests significant growth potential for ASTs within the private and commercial sectors, particularly targeting small land owners, farmers and agricultural centers,
and municipalities. ASTs could provide sought after energy independence on a local scale to smaller and rural communities.

Currently ASTs are mainly installed on sites with less than 200 kW power generating capacity and head less than 5 m (Lashofer et al., 2012) and sites with already constructed dams or hydraulic structures such as weirs. According to the Ontario Hydro and Ministry of Natural Resources Water Potential Site database, there are approximately 280 sites within Ontario that meet these requirements (OMNR, 2004). Utilizing this database and typical AST efficiency figures, the total power generating capacity of ASTs within Ontario is approximately 16 MW (Kozyn et al., 2015). While this range is small relative to the total provincial energy output of 8100 MW, it is not inconsequential and could provide an important energy supply component at the local community level. Figure 2-2a shows the current hydropower plants located in Ontario; Figure 2-2b shows all the potential power generating sites (red) and AST suitable sites (green) in Ontario.

2.4 Current AST Understanding
2.4.1 Geometric, Flow and Power Models

There have been many attempts throughout history to analyze the geometry of the Archimedes screw itself, including efforts made by Cardano, Galilei, Bernoulli, Hachette and Weisbach (Koetsier and Blauwendraat, 2004). The foundation of contemporary Archimedes Screw knowledge within the English scientific literature dates back to 1968 with Nagel’s Archimedean Screw Pump Handbook, which outlines many of the basic sizing calculations, construction and operation of Archimedes pumps. Due to the difficulty in finding analytical solutions to the screw geometry, all historical attempts to quantify the geometry were either limited or empirically-based (Cardano, Galilei, Bernoulli, Hachette, Weisbach and Nagel). With modern computing techniques, Rorres (2000) derived analytical and numerical relationships for the water levels, flow rates, and flow leakages based on actual Archimedes screw geometry.

Despite its long history, the use of Archimedes Screws in power-generating turbines is a relatively new application. Due to the short history of use as turbines, ASTs remain not well understood within the scientific literature, particularly the English literature. The first contemporary attempt to model the power output of ASTs from first principles was by Müller and Senior (2009). They created a model that simplified the turbine geometry by assuming the helical planes to be two-dimensional surfaces. Their model assumed that hydrostatic pressure due to the water trapped between adjacent planes, termed buckets, drove screw rotation. The weight of the entrapped water created a hydrostatic pressure force across the plane surfaces, generating a torque. Müller and Senior’s model assumed quasi-static, steady-state flow conditions, and neglected hydraulic energy losses. Furthermore, mechanical frictional losses from the rotational motion were also neglected. They did, however, attempt to account for losses that occur due to leakage flow between the screw
planes and the containing trough using Nagel’s (1968) empirically based leakage model. While the Müller and Senior model had limitations, it was the first model to predict AST performance, and seemed to have general agreement with the initial experiments conducted by Brada (1999).

The definitive work to date on ASTs is Nuernbergk’s (2012) Wasserkraftschnecken: Berechnung und optimaler Entwurf von archimedischen Schnecken als Wasserkraftmaschine (Hydropower screw: calculation and optimal design of Archimedean screw as a water engine). In this book, Nuernbergk details the complete screw and flow geometry, flow models (including primary as well as leakage flows), outlet conditions, hydraulic losses, power and efficiency, plant operations, and analysis of design flow and head. Much of this work lays out groundwork for the AST knowledge generally, for all future work. The publication language is German, posing difficulties for English speaking audiences, and making using the publication as a practical design tool in North America problematic. Furthermore, much of the analysis provided by Nuernbergk, particularly in terms of the hydraulic energy losses has yet to be vigorously tested and validated experimentally.

Lyons (2014) created a more complete performance model, based on the same hydrostatic operating principles as Müller and Senior but accounting for the full three-dimensional geometry of the rotating screw. Calculating the complicated three-dimensional geometries required numerically-based computer methods. Similar to Müller and Senior (2009), the Lyons model assumed quasi-static, steady-state flow conditions neglecting many of the same energy losses.
Lubitz et al. (2014) modified the Lyons model, using many of the same physics principles but cast using slightly different geometry and included a different leakage flow model. Lubitz et al. included a physics-based leakage model for both the gap leakage and the overflow leakage. Unlike Nagel’s empirical gap leakage model used by Müller and Senior, the gap leakage model used by Lubitz et al. assumes gap leakage is driven by static pressure created by the weight of the water column above the gap. This gap leakage model uses essential elements and principles first established by Muysken (1932) and later republished by Nuernbergk and Rorres (2012), but cast in a different variable framework. Lubitz et al. modeled overflow leakage, or flow over the central cylindrical shaft, also using a model first introduced by Muysken (1932) and reported by Nuernbergk and Rorres (2012). This overflow leakage model assumes the inclined cylindrical shaft acts like a static v-notch weir; when the water levels rise above the overflow point, the overflow is computed using standard empirically-based weir equations and coefficients (Nuernbergk and Rorres, 2012). To date, there is insufficient data to validate both the gap and overflow leakage models. Gap flow leakage is examined in greater detail in Chapter 3.

2.4.2 Power Loss Models

While both the Müller and Senior (2009) and Lubitz et al. (2014) AST models neglect hydraulic losses, there has been one attempt to quantify losses at the inlet of ASTs. In their paper "Analytical Model for Water Inflow of an Archimedes Screw Used in Hydropower Generation," Nuernbergk and Rorres (2012) analyzed the inlet geometry and flow conditions to predict the upstream head of the inlet channel to an AST. They assumed that under steady-state conditions, the energy of the flow could be described by the one-dimensional Bernoulli energy equation. It was assumed that the only energy loss at the inlet was due to the change in geometries between the inlet channel and
the AST circular trough. Nuernbergk and Rorres modeled this energy loss using the well-known Borda-Carnot energy loss equation.

The Borda-Carnot energy loss relationship was originally derived for fully turbulent, incompressible flow in a pipe and assumes the fluid undergoes a rapid expansion with only the pressure forces acting on the fluid through the transition (Massey & Ward-Smith, 1998). By combining the momentum and energy balance equations, the Borda-Carnot relationship describes an energy loss proportional to either the upstream or downstream head based solely on the upstream and downstream geometries. While derived for pressurized flows, the Borda-Carnot relationships have been found to have generally good agreement with open-channel flows similar to those occurring in ASTs (Montes, 1998). The Borda-Carnot energy losses are typically not applied to contractions which are common to most AST inlets because the relationships only hold when there is sufficient length downstream of the contraction to allow the flow to fully develop. Nuernbergk and Rorres (2012) compared the energy losses predicted by the Borda-Carnot relationships to Brada’s (1999) experiments and found general agreement. It should be noted that Nuernbergk and Rorres did not present the energy losses directly but rather examined comparisons of the predicted inlet head. The losses due to transitions are often minor losses and inaccuracies in the model may not appreciably affect the overall head prediction. Therefore, the inlet loss model, as presented in their paper, while agreeing with the experimental data, has yet to be robustly verified.

Beyond the leakage and inlet losses, there have been no attempts made to quantify many of the remaining energy losses including the exit losses at AST outlets. Interestingly, the Borda-Carnot
energy loss model could be better suited for AST outlets rather than inlets. The trough that contains the turbine for an AST resembles an open-channel culvert. Borda-Carnott type energy losses have been successfully applied to predicting the exit losses for culverts, and actually have been found to agree with outlet energy loss experiments better than standard culvert energy loss methods such as those used in common open-channel hydraulics software (Tullis & Robinson, 2008).

Many of the remaining AST power losses are neither well examined nor understood. Zeng et al. (2010) analyzed the torque or power losses for a typical hydro turbine and identified four losses characteristic to turbines: mechanical (friction), hydraulic power loss, impact power loss, and volume power loss. While the power volume loss corresponds to the leakage losses of an AST, which have been previously quantified (Nagel (1968), Rorres (2000), Lubitz et al. (2014)), the mechanical friction has yet to be incorporated into any AST model. Zeng (2010) postulates that the friction losses are speed dependent and should remain constant for any given rotational speed (Zeng, 2010), which is reasonable for most rotating systems. Typical treatments of mechanical friction in turbines are to scale the friction proportional to the angular velocity squared and flow rate (De Jaeger et al., 1994). For an AST, the frictional losses occur primarily in the bearings and would be dependent on the bearing material, size and manufacturing processes.

Similarly, the water impact losses, or the losses that occur when the water impacts the leading helical planes when entering the screw, have not been analyzed or quantified; it is not clear how significant impact energy losses are relative to the total energy losses. It is likely these losses in ASTs are negligible under normal operating conditions as the fluid velocity in ASTs is usually low.
Finally, losses due to the internal fluid friction caused by shear stress on the trough, central cylinder, and turbine blades (i.e., the helical planes) have yet to be explored and modeled. As with any fluid moving in an open-channel or pressurized pipe, there are energy losses associated with the friction along the channel or pipe walls. For typical ASTs, the enclosure trough is normally made of concrete or metal, which are non-smooth surfaces capable of creating shear stress on the fluid. Frictional energy losses along channels or pipes are generally modeled using the Darcy-Weisbach head loss factor (Larock et al., 1999). The Darcy-Weisbach head loss is proportional to the length of the channel or pipe and flow rate squared and inversely proportional to the cross-sectional area. However, quantification of the hydraulic friction energy losses has also yet to be applied to any of the AST models. A similar effect is likely to occur between the AST turbine planes and the water, but again, the magnitude of this effect is at this time unknown.

2.5 Literature Review Summary

In summary, currently there are several physics-based AST power models in existence capable of predicting AST output power. The most complete of these models are the Lubitz et. al (2014) model and the Nuernbergk (2012) model. While the Nuernbergk model is more complete, it is described in German and is difficult to use as an engineering design tool. The Lubitz et al. (2014) model lacks both mechanical and hydraulic energy losses which have yet to be quantified experimentally or incorporated into the power model. Additionally, the Lubitz et al. (2014) model examines an AST in isolation from its surroundings. This makes it difficult to design an AST for a particular site because it lacks a connection to the available head for a given site. This thesis intends to investigate these power losses and propose modifications to the Lubitz et al. (2014)
model to account for these energy losses. Additionally, this thesis will create a delivery channel model, in order to predict the head requirements for any given AST, allowing for a more complete engineering design model.
Chapter 3: Problem Formulation

3.1 Geometry and Coordinate System

This thesis amends the Lubitz et al. (2014) AST performance model to include previously omitted power losses, therefore outlining the key components and vocabulary is required. The Lubitz et al. (2014) model is actually a series of mathematical models that can be used to simulate both the expected flows and predicted power for a given screw geometry and flow regime. This quasi-static model calculates the volumes of water and torque on the screw by assuming the screw is not rotating and experiencing no internal water flows.

A typical AST is just a conventional Archimedes screw, comprised of a number of helical planes (usually 3 or 4), situated within a containing cylindrical trough. In some Archimedes Screws, the trough is fastened to the screw directly, but in most ASTs the screw is detached from the trough, allowing for free rotation of the screw relative to the trough. The central idea of the Lubitz et al. (2014) performance model is the idea of a water bucket. A water bucket is a volume of water entrapped between two adjacent helical plane surfaces. For an ideal screw operating under steady-state conditions (steady flow, constant rotational speed), all the buckets within an AST will have the same shape and volumetric size. Figure 3-1 shows the critical dimensions and parameters of an Archimedes screw. The shape and size of a bucket is determined entirely by the geometry of the screw (specifically the inner diameter \(D_i\), outer diameter \(D_o\), screw pitch \(S\) (distance along the screw axis for one complete helical plane turn), inclination angle of the screw \(\beta\), and fill height of the bucket \(f\). The number of helical planed surfaces \(N\) determines the amount of buckets. The Lubitz et al. (2014) model determines the forces and flows operating within a single bucket for an idealized infinitely long screw; it is assumed that all buckets within the screw
effectively function identically to this idealized bucket. Forces, torques, and power then can be scaled up based on the total length of the screw ($L$).

Figure 3-1. Archimedes Screw geometry and parameters

The bucket fill height $(f)$ is defined by Lubitz et al. (2014) as the ratio of the water depth ($z_{wl}$) inside the bucket to the maximum fill level that can occur before the water overflows the bucket:

$$f = \frac{z_{wl} - z_{min}}{z_{max} - z_{min}}$$  \hspace{1cm} (3-1)

The fill heights are measured from the bottom, or lowest point, on the helical plane downstream of the bucket to the free surface of the water bucket. The maximum bucket water level, referred to as the 100% fill level, is located just above the inner cylindrical shaft at the point where the shaft meets the downstream helical plane. Figure 3-2 illustrates a typical water bucket within the screw and with the minimum ($z_{min}$), maximum ($z_{max}$), and bucket water levels ($z_{wl}$) indicated.
Figure 3-2. A typical water 'bucket', shown with the minimum and maximum vertical elevations and the elevation of the water line

Within the Lubitz et al. (2014) model, general positions on the helical plane surfaces are described in cylindrical coordinates, where the 'w' axis is located down the central cylindrical shaft. Vertical depths are determined by projecting physical locations on the helical plane surfaces to a vertically oriented Cartesian axis 'z'. The relationship between the angular and radial positions within the screw are seen in Figure 3-3.

Figure 3-3. Lubitz et al. (2014) Archimedes Screw model coordinate system
3.2 Bucket water volume and flow calculations

The Lubitz et al. (2014) model assumes that the first leading helical plane edge is vertically oriented at the top of the screw. For any given position along the 'w' axis, the radial and angular positions on the leading plane are described by the geometry of a helicoid of pitch length S:

\[ r(w) = r \]

and

\[ \theta(w) = 2\pi \left( \frac{w}{S} \right) \]

At any point \((r, \theta)\), the vertical position \(z_1\), on the leading helical plane surface is then defined by:

\[ z_1 = r\cos(\theta)\cos(\beta) - \frac{S\theta}{2\pi} \sin(\beta) \]

The same point on the proceeding (upstream) helical plane, \(z_2\), is defined by:

\[ z_2 = r\cos(\theta)\cos(\beta) - \left( \frac{S\theta}{2\pi} - \frac{S}{N} \right) \sin(\beta) \]

where \(N\) is the number of planes. Using these point definitions, the minimum and maximum (100%) fill heights are given by \(\theta = \pi, r = D_o/2\) and \(\theta = 2\pi, r = D_i/2\) respectively:

\[ z_{\text{min}} = -\frac{D_o}{2} \cos(\beta) - \frac{S}{2} \sin(\beta) \]

\[ z_{\text{max}} = \frac{D_i}{2} \cos(\beta) - S \sin(\beta) \]

These definitions allow the vertical elevation of the bucket water surface \((z_{wl})\) to be defined in terms of its fill height \((f)\):
An infinitesimal, cylindrical volume element \((dV)\) can be defined parallel to the 'w' axis connecting adjacent points on the helical planes on the upstream and downstream of the bucket. If only the portion of this elemental volume that is submerged below the water line is considered part of the overall water bucket volume, the overall volume of a bucket can be determined as:

\[
V = \int_{r=D_o/2}^{r=D_i/2} \int_{\theta=0}^{\theta=2\pi} dV
\]

(3-9)

where

\[
daV = \begin{cases} 
0 & z_2 > z_{wl}, z_1 > z_{wl} \\
\frac{(z_{wl} - z_1)}{z_2 - z_1} \frac{S}{N} r dr d\theta & z_2 \geq z_{wl}, z_1 \\
\frac{S}{N} r dr d\theta & z_2 < z_{wl}, z_1 > z_{wl}
\end{cases}
\]

(3-10)

### 3.3 Torque and Power Calculations

The Lubitz et al. (2014) model calculates the torque created by the bucket water volume using the same point and coordinate definitions defined in section 3.2. It is the water pressure on the helical planes that generates the torque experienced by the screw. Assuming static conditions within the buckets, the Lubitz et al. (2014) model determines the hydrostatic pressure at any given point on the plane surfaces as \((p)\):

\[
p = \begin{cases} 
\rho g (z_{wl} - z) & z < z_{wl} \\
0 & z \geq z_{wl}
\end{cases}
\]

(3-11)
Considering that the net pressure at any point on the helical plane surfaces is the difference between the up and downstream water pressures, the net torque on an element area of the helical plane surface is then given by:

\[ dT = (p_1 - p_2) \frac{S}{2\pi} r dr d\theta \]  

(3-12)

where \( p_1 \) and \( p_2 \) are the pressures on either sides of the plane surface. The total torque (\( T \)) generated by a single bucket can be determined by integrating the elemental torques for the entire bucket:

\[ T = \int_{r=D_i/2}^{r=D_o/2} \int_{\theta=0}^{\theta=2\pi} dT \]  

(3-13)

The total torque experienced by the full length of the screw is then scaled proportionally to the total number of buckets along the entire screw length:

\[ T_{total} = T \left( \frac{NL}{S} \right) \]  

(3-14)

The total power (\( P_{out} \)) is then calculated as:

\[ P_{out} = \omega T_{total} \]  

(3-15)

### 3.4 Flow and Leakage Models

#### 3.4.1 Bucket Flow

The bucket flow (\( Q_b \)), or the flow that occurs within the portion of the screw contained between the helical planes, can be determined from the water bucket volume and rotational speed, or screw angular velocity (\( \omega \)). The screw’s rotational speed causes the buckets of known volume to translate through space, yielding a flow rate of:
27

\[ Q_b = \frac{NV\omega}{2\pi} \] (3-16)

### 3.4.2 Gap Leakage Flow

For most ASTs, the screw is not directly attached to the containing trough, allowing for free rotation of the screw. Therefore, a small gap exists between the screw and the trough that water is able to leak through. This gap leakage flow \( Q_{gl} \) can be considered a power loss because any flow that passes through the gap is not contained within a bucket, and therefore contributes nothing to the torque generated on the screw. There are several gap leakage models in existence that attempt to model and quantify this flow. Nagel’s 1968 Archimedean Screw Pump Handbook includes one of the oldest presented leakage models. It is an empirically based model for use with Archimedes Screw pumps, not turbines, operating full \((f = 1)\) at normal rotational speeds. This Nagel model quantifies gap leakage as proportional to the gap width between the screw and the trough \((G_w)\) and the screw outer diameter raised to the power of 1.5:

\[ Q_{gl, Nagel} = 2.5G_wD_o^{1.5} \] (3-17)

where the units of \( G_w \) and \( D_o \) are in metres. The Nagel leakage model completely neglects all physical and dynamic properties of the actual flow regime experienced by the screw. There are no parameters that take into account the rotational mechanics of the screw or the fluid mechanics of the flow itself. The Nagel model will predict the same gap leakage flow regardless of the screw orientation, rotational speed and the supplied flow rate.

A more physics-based model was presented by Lubitz et al. (2014), which begins to incorporate some of the governing fluid mechanics. The model used by Lubitz et al. (2014) is a pressure driven
model that assumes the leakage through the gap is driven entirely by the static pressure difference across the gap, due to the water height difference between adjacent buckets. According to this gap leakage model, the leakage flow can be calculated as:

\[
Q_{gl, Lubitz} = CG_w \left( l_w + \frac{l_e}{1.5} \right) \sqrt{\frac{2gS}{N} \sin \beta}
\]  

(3-18)

where g is the gravitational constant, \( l_w \) is wetted perimeter of the gap portion submerged on both sides, \( l_e \) is the wetted perimeter of the gap portion submerged only on the upstream side, and \( C \) is a flow coefficient. The flow coefficient \( C \) should be equal to 1.0 if there are no minor losses associated with the flow through the gap; the authors used a \( C \) value of 0.9, which was determined experimentally on scale model ASTs under stationary conditions. Figure 3-4 graphically shows the Lubitz et al. (2014) leakage model variables relative to the screw geometry.
It should be noted that the Lubitz et al. (2014) leakage model is functionally equivalent to the leakage model originally attributed to Muysken (1932) and utilized recently by Nuernbergk & Rorres (2012), but cast in different geometric variables.

### 3.4.3 Overflow Leakage

Overflow leakage occurs when the buckets are filled passed the 100% fill level for the bucket geometry (i.e., when \( f > 1.0 \)). When the water rises above the center shaft, a secondary flow is created that allows water to pour into the downstream bucket. Currently, as far as the author can determine, the literature contains only one model for this type of leakage, expressed by Aigner (2008) and used by both Neurenbergk and Rorres (2012) and Lubitz et al. (2014). In this leakage model, the flow is essentially treated as typical weir flow. The weir type is considered to be a simple angled, V-notch weir since this is approximately the shape that the central shaft and the planes make at the overflow point. Effectively, the flow over the shaft is proportional to the height of the water above the shaft to the power of 5/2, similar to many V-notch weir flow models. Additionally, the flow is also proportional to the shape of the created V-notch which is determined by the orientation of the screw, namely the angle of inclination. Therefore, the Aigner (2008) overflow leakage \( (Q_o) \) model proposes that the overflow leakage can be determined as:

\[
Q_o = \frac{4}{15} \mu \sqrt{2g} \left( \frac{1}{\tan \beta} + \tan \beta \right) (z_{wl} - z_{max})^{5/2}
\]  

\( (3-19) \)
where \( \mu \) is a discharge coefficient. For Archimedes Screws, it is assumed that the flow area is approximately triangular, therefore the discharge coefficient is typically assumed to be approximately 0.537 (Nuernbergk & Rorres, 2012).

### 3.4.4 Total Flow Rate

With the bucket flow, gap leakage flow and overflow leakage known, the total flow through the AST \( (Q) \) can be determined as the total sum of all the flow pathways:

\[
Q = Q_b + Q_{gl} + Q_o
\]  

(3-20)

### 3.5 Lubitz el al. (2014) MATLAB Model

#### 3.5.1 Basic MATLAB Model

Due to the complex geometry of the screw, the models outlined in Chapters 3.1 through 3.4 do not lend themselves to easy analytical solutions (Rorres, 2000). Therefore, each of these models were implemented into a computer model written in MATLAB which numerically solves each model. It is this computerized model that serves as the foundation for the predictions of the Lubitz et al. (2014) model used throughout this thesis. This MATLAB model takes in the following inputs:

<table>
<thead>
<tr>
<th>Parameter in Thesis</th>
<th>MATLAB Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>B</td>
<td>Inclination angle (rad)</td>
</tr>
<tr>
<td>S</td>
<td>S</td>
<td>Pitch (m)</td>
</tr>
<tr>
<td>N</td>
<td>N</td>
<td>Number of helical planes</td>
</tr>
<tr>
<td>Di</td>
<td>di</td>
<td>Inner Diameter (m)</td>
</tr>
<tr>
<td>Do</td>
<td>do</td>
<td>Outer Diameter (m)</td>
</tr>
<tr>
<td>L</td>
<td>l</td>
<td>Screw length (m)</td>
</tr>
</tbody>
</table>

*Table 3-1. Lubitz et al. (2014) MATLAB model input parameters*
and yields the following outputs:

<table>
<thead>
<tr>
<th>Parameter in Thesis</th>
<th>MATLAB Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V$</td>
<td>$V$</td>
<td>Volume of a single bucket (m$^3$)</td>
</tr>
<tr>
<td>$T_{tot}$</td>
<td>$T_{tot}$</td>
<td>Total screw Torque (Nm)</td>
</tr>
<tr>
<td>$P$</td>
<td>$P$</td>
<td>Total mechanical power (W)</td>
</tr>
<tr>
<td>$P_{avail}$</td>
<td>$p_{avail}$</td>
<td>Total power available in flow (W)</td>
</tr>
<tr>
<td>$Q_b$</td>
<td>$Q_b$</td>
<td>Bucket flow (m$^3$/s)</td>
</tr>
<tr>
<td>$n_b$</td>
<td>$n_{buckets}$</td>
<td>Total number of buckets in the screw</td>
</tr>
<tr>
<td>$T$</td>
<td>$T$</td>
<td>Torque from a single bucket (Nm)</td>
</tr>
<tr>
<td>$P_{avg}$</td>
<td>$p_{avg}$</td>
<td>Average wetted perimeter pressure (Pa$^{1/2}$)</td>
</tr>
<tr>
<td>$l_w$</td>
<td>$l_w$</td>
<td>Gap wetted perimeter (m)</td>
</tr>
<tr>
<td>$L_e$</td>
<td>$l_e$</td>
<td>Gap perimeter wetted only on upstream side (m)</td>
</tr>
<tr>
<td>$h_u_e$</td>
<td>$h_u_e$</td>
<td>Height of water above 100% fill level (m)</td>
</tr>
<tr>
<td>$Q_l$</td>
<td>$Q_l$</td>
<td>Gap leakage flow (m$^3$/s)</td>
</tr>
<tr>
<td>$Q_o$</td>
<td>$Q_o$</td>
<td>Overflow leakage flow (m$^3$/s)</td>
</tr>
<tr>
<td>$Q$</td>
<td>$Q$</td>
<td>Total flow (m$^3$/s)</td>
</tr>
</tbody>
</table>

It should be noted, the Lubitz et al. (2014) model utilizes the fill level and rotation speed of the screw as input parameters and one of the outputs is volume flow rate of water through the screw. However, for many of the experiments conducted in this thesis, the fill level measurement could not be measured directly and instead the total screw flow rate ($Q$) was measured. Therefore, for these experimental runs, the Lubitz et al. (2014) MATLAB model was adapted to replace the input
fill level measurement with an input total flow measurement. For this implementation of the MATLAB model, the model was iterated over various fill height levels ($f$) using a bisection method until the total flow ($Q = Q_b + Q_o + Q_l$) matched the observed flow.

Running the model in this manner introduces several additional uncertainties. Specifically, running the model this way assumes that the gap and overflow leakage models correctly predict their respective leakage flow values. Any deviations in these flow values from their actual values have the effect of altering the bucket volumes, and therefore the torque and power predictions.

### 3.5.2 MATLAB model uncertainty and sensitivity analysis

#### 3.5.2.1 Output parameter uncertainties

Since the MATLAB implementation of the Lubitz et al. (2014) model is extensively used throughout this thesis, an uncertainty analysis was performed on the model. This was done to determine which parameters the model is most sensitive to, and allow for an uncertainty propagation. Since the MATLAB version of the model is essentially a multi-variable function with multiple inputs and outputs, the uncertainty of a general model output variable $x$ can be determined as (Taylor, 1996):

$$
\delta_x = \sqrt{\left(\frac{\partial x}{\partial \delta_1}\right)^2 + \left(\frac{\partial x}{\partial \delta_2}\right)^2 + \left(\frac{\partial x}{\partial \delta_3}\right)^2 + \left(\frac{\partial x}{\partial \delta_4}\right)^2 + \left(\frac{\partial x}{\partial \delta_5}\right)^2 + \left(\frac{\partial x}{\partial \delta_6}\right)^2 + \left(\frac{\partial x}{\partial \delta_7}\right)^2 + \left(\frac{\partial x}{\partial \delta_8}\right)^2 + \left(\frac{\partial x}{\partial \delta_9}\right)^2} \quad (3-21)
$$
Of primary concern are the uncertainties in the power and volumetric flow rate outputs. Therefore, in place of the generic variable \( x \), the uncertainties for each of the power \( (P) \), total flow \( (Q) \), bucket flow \( (Q_b) \), gap leakage flow \( (Q_{gl}) \) and overflow leakage \( (Q_o) \) were determined according to Equation 3-21. To accomplish this, the MATLAB model was run varying a single input parameter at a time while holding the remaining variables constant. Polynomial functions were fit between model outputs \( (P, Q, Q_b, Q_{gl}, Q_o) \) and the input parameter that was varied. The partial derivatives of these polynomial functions were then used in Equation 3-21, yielding the uncertainty for each parameter. Sixth order polynomials were chosen for each of the polynomial fit to ensure a proper curve fits for all required functions. Sixth order polynomials were chosen to ensure all of the derived relationships were adequately fit and allow for proper automation of this procedure using computer methods; relationships that did not require higher terms had those terms coefficients suppressed to zero. Since high order polynomials were determined through regression, the uncertainties are only valid in the range tested. Beyond these ranges, the uncertainty functions cannot be considered reliable. Since the MATLAB model has 10 physical input parameters, the uncertainties of each of the model output parameters requires 10 polynomial functions, one for each input parameter. Therefore, to make the uncertainty calculations easier, these calculations were all scripted in MATLAB. Refer to Appendix A to see the associated MATLAB scripts.

In order to perform the uncertainty analysis, baseline model parameters were chosen for all input variables, representing typical operating conditions. In this thesis, there are two basic experimental laboratory AST setups of different scales. The first experimental setup utilizes ASTs with screw outer diameters of approximately 15.2 cm and lengths of approximately 50 cm. A second, larger experiential AST setup was also used. In this second setup, screws with outer diameters of 32.5
cm and length of 1.229 m were utilized. Therefore, this analysis was performed twice using baseline values consistent with both experimental setups. Table 3-3 shows the baseline parameters applied in the Lubitz et al. (2014) model for the uncertainty analysis.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Chapter 4 Model Parameters</th>
<th>Chapter 5 Model Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>B (rad)</td>
<td>0.4328</td>
<td>0.4189</td>
</tr>
<tr>
<td>S (cm)</td>
<td>0.14461</td>
<td>0.33</td>
</tr>
<tr>
<td>N</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>D₁ (m)</td>
<td>0.07626</td>
<td>0.168</td>
</tr>
<tr>
<td>D₀ (m)</td>
<td>0.14461</td>
<td>0.325</td>
</tr>
<tr>
<td>L (m)</td>
<td>0.584</td>
<td>0.1229</td>
</tr>
<tr>
<td>F</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>ω (rad/s)</td>
<td>4.1888</td>
<td>4.1888</td>
</tr>
<tr>
<td>Gₘ (m)</td>
<td>0.00111</td>
<td>0.00199</td>
</tr>
<tr>
<td>C</td>
<td>0.90</td>
<td>0.90</td>
</tr>
<tr>
<td>Θsteps</td>
<td>1000</td>
<td>1000</td>
</tr>
<tr>
<td>Rsteps</td>
<td>1000</td>
<td>1000</td>
</tr>
</tbody>
</table>

The uncertainty values on all experimental measurements presented in Chapters 4 and 5 were performed using these uncertainties analysis models. The uncertainties were of particular importance for the gap flow measurements (Chapter 4) since the gap flow measurements themselves are very small relative to the experimental input parameter uncertainties, both due to the nature of the gap flow measurements, but also the small scale size of the experimental apparatus used.
3.5.2.2 Model convergence test

The Lubitz et al. (2014) MATLAB model solves for all output variables using numerical integration. The numerical accuracy of this model is reliant on the numerical step size, or the number of divisions between successive radial and angular distances ($r_{steps}$ and $\theta_{steps}$). Therefore, a convergence test was performed to determine a satisfactory number of radial and angular divisions that yields numerically accurate results for the AST geometry. The number of radial and angular steps were increased starting from zero until the differences between successive output results were negligible. Figure 3-5 shows the results of the convergence test. As seen in Figure 3-5, all model output parameters converge to within 0.1% difference within approximately 1000 radial and angular steps. Therefore, all model runs were run with $r_{steps}$ and $\theta_{steps}$ values of 1000.

![Figure 3-5. Percent differences in Lubitz et al. (2014) model outputs between successive radial and angular numerical steps](image-url)
3.5.2.3 Sensitivity analysis

As part of the uncertainty analysis, the sensitivity of the Lubitz et al. (2014) MATLAB model output parameters to its input parameters was analyzed. The basic output polynomials for each output were mapped to a single input parameter, while the remaining input parameters were held constant. The resulting percentage difference from the baseline case are plotted in Figure 3-6 and Figure 3-7. These figures show the percentage change in the model output parameters power ($P$), torque ($T_{tot}$), total flow ($Q$), bucket flow ($Q_b$), bucket flow ($Q_l$) and bucket volume ($V$) resulting from a percentage change-from-baseline in the various single input parameters. Both experimental AST configurations show similar trends. The modeled power, torque, and predicted flow rates are most sensitive to the outer diameter ($D_o$), fill height ($f$), and gap width ($G_w$). Therefore, measured uncertainties in these parameters will contribute the largest portions in the Lubitz et al. (2014) model output uncertainties.
Figure 3-6. Lubitz et al. (2014) MATLAB model sensitivity analysis performed using Chapter 4’s experimental AST input parameters. $B = \text{blue}, S = \text{red}, N = \text{violet}, Di = \text{orange}, Do = \text{cyan}, L = \text{salmon}, f = \text{dark green}, \omega = \text{yellow}, Gw = \text{light green}, C = \text{teal}$
Figure 3-7. Lubitz et al. (2014) MATLAB model sensitivity analysis performed using Chapter 5’s experimental AST input parameters. B = blue, S = red, N = violet, Di = orange, Do = cyan, L = salmon, f = dark green, ω = yellow, Gw = light green, C = teal
3.6 Problem Formulation – Concluding Remarks

In Chapter 3, the methods of quantifying the geometry of an AST, consistent with the Lubitz et al. (2014) AST power model, are presented. Additionally, the Lubitz et al. (2014) AST flow and power models have been outlined. In this thesis, a power loss model will be developed based on the key mathematical concepts and framework examined here in Chapter 3. The Lubitz et al. (2014) power model only offers an ideal power prediction model for an AST operating under quasi-static conditions. Using this foundation, amendments to this model will be developed using theoretical arguments and insights gained from experimental data. The result, which will be developed in the subsequent chapters, is a power model with loss prediction capabilities.
Chapter 4: Validation of Gap Leakage Flow Models

4.1 Experimental Objective

The two types of gap leakage flow models presented in Chapter 3.4.2 (i.e., the empirical Nagel model and the fluid mechanics-based Muysken/Neurenbergk/Lubitz model) represent an energy loss that, depending on the size of the gap width and overall screw flow, can represent significant power losses to an AST.

For ASTs operating with fill heights of 100% and lower (i.e., \( f \leq 1.0 \)), there exists two flow categories: primary, or bucket, flow that occurs between the helical plane flights and a secondary leakage flow that occurs between the screw edges and the containing trough. The theoretical efficiency of an otherwise perfect AST impacted by gap flow leakage is:

\[
\eta = 1 - \frac{Q_{gl}}{Q} \tag{4-1}
\]

This makes an accurate gap leakage model an important component to any AST performance model. While both the Nagel (1968) and Lubitz et al. (2014) gap leakage models have been previously used, neither has been validated against actual experimental data. This is primarily because measuring gap leakage flow in an operating AST is highly problematic; separating the primary bucket and secondary leakage flows is difficult to do while an AST is turning. Typically, only the total flow can be easily measured, however, on commercial ASTs, even this can at times be difficult. The purpose of the experiment in this chapter is to measure the gap leakage flow \( (Q_{gl}) \)
through a scale-laboratory AST model and compare it the predictions made by the Nagel (1968) and Lubitz et al. (2014) leakage models.

4.2 Experimental Apparatus

Gap leakage measurements were made in a laboratory-scale AST. A typical AST has three flow pathways: primary bucket flow, secondary gap leakage flow, and a secondary overflow (that occurs when buckets are filled past the 100% fill level). Separately measuring these three flow regimes would be extremely difficult. Therefore, rather than attempting to measure the different flow types, indirect measurements were taken to allow the various flow types to be calculated. Since the overflow measurements were not the primary interest, all of the following experiments were conducted such that buckets were never filled passed the 100% fill level to ensure no overflow leakage ever occurred.

The experimental setup consisted of an AST situated between two basins, one located slightly above the other. An additional collection basin held the available water for each of the experiments. The upper basin acts as the AST water source, providing water to the inlet of the AST. Water was pumped from the collection basin to the upper basin. The water in the upper basin was then routed to the AST inlet and allowed to flow through the screw. Flow leaving the AST is discharged to the lower basin. The flow is then routed from the lower basin through a flow meter down to the collection basin. The AST was built by Greenbug Energy Inc. out of painted steel, supported at each end by low-resistance bearings, and positioned within a rigid transparent acrylic tube that served as the trough while allowing optical access. The dimensions of the AST
can be seen in Table 4-1. Figure 4-1 shows the schematic diagram, with flow directions for the experiment. Figure 4-2 shows a photograph of the experimental AST and setup for the experiment.

Table 4-1. Dimensions of the scale-laboratory AST used to measure gap leakage

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inner Diameter (mm)</td>
<td>76.26 ± 0.97</td>
</tr>
<tr>
<td>Outer Diameter (mm)</td>
<td>144.36 ± 0.14</td>
</tr>
<tr>
<td>Screw Length (mm)</td>
<td>584 ± 3</td>
</tr>
<tr>
<td>Angle (Degree)</td>
<td>24.8 ± 0.3</td>
</tr>
<tr>
<td>Pitch (mm)</td>
<td>144.61 ± 3.67</td>
</tr>
<tr>
<td>Gap Width (mm)</td>
<td>1.11 ± 0.15</td>
</tr>
<tr>
<td>Casing Thickness (mm)</td>
<td>3.43 ± 0.28</td>
</tr>
<tr>
<td>Head (m)</td>
<td>0.24</td>
</tr>
</tbody>
</table>

Figure 4-1. Gap leakage measurement experimental setup
All available head was provided using a simple constant flow rate submersible electric sump pump, capable of providing a flow rate of approximately 5400 L/h at about 1 m of head. Flow rates through the AST were regulated using a weir situated at the back of the upper basin, opposite to the screw. This back weir functioned as a secondary bypass, allowing excess flow to be diverted away from the AST and back to the collection basin, while maintaining a consistent water level in the upper basin.

Control of the fill height level was regulated by altering the rotational speed of the screw. The screw's rotational speed was controlled through a simple friction Prony brake. For a constant flow rate, the volume of water, and therefore bucket fill height, is determined by the screw's rotational speed. Increasing the screw rotation speed causes the bucket fill heights to decrease because the screw is physically capable of moving more water for a set time period. Similarly, slowing the
screw caused the fill heights to increase as a slower moving screw can transport less water, therefore larger bucket volumes are required to transport the same amount of water.

**4.3 Experimental Procedure**

**4.3.1 Basic procedure**

The gap leakage flow occurs between the surface of the trough surrounding the AST, and the outer edges of the screw surfaces. Gap measurements were approximated by taking the average difference between the inside diameter of the screw casing and outer diameter of the screw. The casing measurements were made at 20 evenly spaced locations around its circumference using calipers with a precision of 0.01 mm. The screw outer diameter was measured using a Mitutoyo Digimatic Height caliper by rotating the screw on a level table and measuring the displacement tip-to-tip displacements at 24 equally spaced locations. This gap was determined to be approximately 1.11 mm. As the screw rotates, the leakage gaps translates through space relative to the water contained within the buckets and the AST trough. Directly measuring the leakage through these small, moving gaps is not feasible. Therefore, this experiment utilizes a novel approach to achieve the gap flow measurements.

Instead of measuring the gap flow directly, the gap flow is calculated using geometric measurements of the water volumes physically in the buckets and the rotational speed of the screw. In this experiment, the bucket fill heights ($f$) were measured allowing the bucket volumes to be determined from knowledge of the screw geometry. Because the helical geometry of the screw is accurately known, the bucket volumes can be calculated directly from the fill height measurements as demonstrated in Chapter 3.2. Once the bucket volumes are known, an idealized bucket flow
rate \( (Q_b) \) can be calculated using Equation (3-16). This idealized bucket flow rate assumes there is no gap present between the helical planes and the outer casing. Finally, the gap leakage \( (Q_{gl}) \) can be deduced by measuring the total flow \( (Q) \) that passes through the screw; the gap leakage is simply the difference between the total measured flow and the calculated bucket flow rates.

In this experiment, three experimental runs were conducted. Each run consisted of a distinct total flow rate and varying AST rotation speeds that ranged approximately from 0 to 25 rad/s. For each run, approximately 60 individual measurements were taken. Each measurement consisted of a set of fill height measurements, obtained using high-speed photography, in addition to the rotational speed of the AST, and the total volume flow rate of water through the AST. Measurements of flow rate and rotation speed were averaged over a 60 second sampling period.

The calculated leakage flows were very small and within the experimental measurement uncertainty of the total volume flow rate. Therefore, an uncertainty analysis was completed on all measurements in order to ascertain the uncertainty of the overall leakage values. All physical dimensions of the laboratory AST and sensors were quantified. The gap width, in which the gap leakage occurs, was determined by taking the difference between the inner diameter of the outer screw casing and the outer screw diameter. Since the bucket volumes were calculated using the Lubitz et al. (2014) MATLAB model, the experimental measurements were combined with the uncertainty analysis results applied to the Lubitz et al. (2014) model, completed in Section Chapter 3.5.2.

### 4.3.2 Fill height measurements
Accurate measurement of the water bucket fill height levels in the screw was essential to the experiment. The outer casing on the experimental AST unit was a clear, acrylic cylindrical tube. This clear tube allowed for high-speed photographic images to be taken of the screw while it rotated. Photographs were taken using a tripod-mounted Nikon D3200 camera fitted with a remote shutter. For each experimental run, four separate images were taken at 15 second intervals. Since the screw rotational speeds were often high enough to create motion blur in the photographs, fast shutter speeds between 1/800 s and 1/1400 s were utilized, depending on the rotational speed of the specific run. An external halogen light source was used allow higher shutter speeds. The camera was positioned 1.5 m away from the AST while the maximum optical zoom was applied to reduce lens curvature distortion effects. Additionally, tests were conducted to assess the impact of optical distortions due to the curved acrylic tube. Photographs of rulers placed both inside and outside the casing tube were analyzed to see if the tube introduced optical distortions. It was determined that the distortion effects from the acrylic casing were negligible.

The physical height measurements were ascertained using leveled straight-edged rulers fixed to the frame of the AST in locations where they would be included in the photographs. Water distortion effects made observing the helical plane bottoms, and therefore the minimum bucket fill height levels, difficult. Instead, measurements were made from the bottom of the outer casing. The actual fill heights were then adjusted by both the measured outer casing thickness and gap width. Similarly, the maximum possible water level was impossible to see because the curvature of the helical planes obstructed the point at which the center shaft contacted the planes. Therefore, the 100% fill height level was determined from the screw geometry and matched to the photographic images. The fill heights for all runs were obtained from the measurement
photographs using the open-source Java program Plot Digitizer 2.6.3. Figure 4-3 shows a typical fill height image, including the minimum and maximum depths for a bucket as well as the water level in the bucket.

![Fill Height Measurement](image.png)

*Figure 4-3. Example fill height measurement for the indicated water bucket*

There was great variation in the fill height measurements leading to significant uncertainties. The fill height measurement had the highest uncertainty of all the measurements taken. The uncertainties of the individual measurements themselves were relatively low since the fill heights could be ascertained precisely using plot digitization software. However, the fill height levels often varied between different buckets within the screw at any instant of time. There are likely two causes for these variations. Slight disparities in the screw flight lengths could have resulted in slight variations in the gaps, causing variations in the bucket fill heights. Additionally, the rotation of the screw causes waves to propagate within the upper basin that supplies water to the AST. These waves lead to slight variations in the supplied flow at the inlet of the screw, which could further create variations in the volume of the buckets and corresponding fill heights. Therefore, it was deemed that the individual measurement uncertainties would provide an insufficient indicator of the actual measurement uncertainty since the other measurements were aggregated and averaged.
over a 60 second time period. Similarly, the maximum deviation from the mean was found to be too conservative and would be an overestimate of the fill height uncertainties. Therefore, it was assumed that each set of fill height measurements were normally-distributed and the standard deviation was taken as the fill height measurement uncertainty.

### 4.3.3 Rotational speed measurements

The turbine rotational speeds were measured using a recessed magnetic switch, positioned above the inner turbine shaft at the inlet. A small rare earth magnet fastened securely to the central shaft of the turbine caused the magnetic switch to close once per revolution. These electrical pulses were recorded through a National Instruments USB-6009 Data Acquisition (DAQ) unit using acquisition software written in LabVIEW specifically for this experiment. The rotational speed was determined by measuring the time between pulses and averaged over the entire duration of the experimental run. The pulse train from each 60 second rotational speed measurement was clipped prior to the first recorded pulse and after the last recorded pulse to ensure only complete rotations were taken into account, thereby reducing the uncertainty of the measurements. Speed measurements were validated using a manually operated optical tachometer.

The magnetic switch data was recorded at 1000 Hz. At this sample rate, the rate at which measurements were made far exceeded the rotational frequency of the screw. This ensured that no revolutions were missed. The digital nature of the signal also means that the measurements were not subject to noise. Therefore, uncertainty in the angular velocity measurements were deemed negligible.
4.3.4 Flow measurements and flow sensor calibration

The flow sensor was a submerged orifice situated at the outlet of the AST. Water discharged from the AST was routed through a single vertical pipe before exiting through the orifice sensor located at the bottom of the pipe. The vertical pipe ensured that the orifice remained completely submerged and pressurized within the water column. This allowed the height of the water column to act as an indicator for the flow measurements. A calibration equation was derived for the relationship between the water level in the vertical pipe and the outflow discharge. The complete flow meter calibration procedure, results, and uncertainty analysis can be found in Appendix B.

4.4 Experimental Results and Discussion

Figure 4-4a through 4-4c show both the measured total flow ($Q$) for all runs and the non-gap bucket flow ($Q_b$) for all three experimental runs. It should be noted that for each of the runs, the flow rate through the screw is reduced as the angular velocity approaches zero. This occurs because, in this experimental setup, the flow has two pathways: through the AST or through the upper basin overflow back weir. As the rotational speed of the screw is slowed, the amount of water that can enter the AST diminishes. This causes the upper basin water level to rise. With less water flowing through the AST and a higher water level in the upper basin, and therefore a higher head above the weir crest, a greater amount of water flows through the back weir. This causes a reduced flow rate through the screw and can be seen in Figures 4-4a through 4-4c as the angular velocity approaches zero.

When the angular velocity is 0 rad/s, the screw is stopped and the flow is entirely gap leakage flow. At this point, the total flow represents the gap leakage and the bucket flow can be seen to
be reduced to 0 cm$^3$/s. The flow rate was varied for each experimental run. The maximum flow rates, corresponding to the maximum, free-wheeling rotational speed (i.e. no applied braking), for runs 1, 2, and 3 were found to be 735 cm$^3$/s, 906 cm$^3$/s, and 1097 cm$^3$/s respectively. As the angular velocity approached free-wheeling speeds, the total flow and bucket flow can be seen to converge as the gap-leakage is reduced to zero.

Figures 4-4d through 4-4f show the fill heights as a function of angular velocity. As expected, for all three experimental runs, the fill heights decreased with increasing angular velocity. This occurs because as the angular velocity increases, the amount of water the screw is capable of transporting within a set time period increases. Therefore, at higher speeds, a smaller bucket volume and fill height are required in the buckets to move the same amount of water that a slower moving screw would require. Since the Lubitz et al. (2014) model is essentially a pressure driven model, decreasing the fill heights should result in lower pressures at the bucket gaps. This suggests, for a given flow rate, as the rotational speeds increase, the gap-flow should be reduced.

The measured gap leakage was estimated by fitting polynomials to both the measured total flow ($Q$) and the calculated bucket flow ($Q_b$) for each of the experimental runs. These polynomials can be seen as the fitted curves in Figures 4-4a through 4-4c and the associated polynomial equations can be found in Table 4-2. The gap leakage was approximated by taking the difference between the total flow rate and the bucket flow rate polynomials. Figures 4-4g through 4-4i show the actual measured gap leakages for each of the runs as a function of rotational speed. Also included in Figures 4-4g through 4-4i are the modeled Nagel (1968) and Lubitz et al. (2014) gap leakage predictions.
Interestingly, the observed gap leakage trends were not expected. The Nagel (1968) model made no prediction of any variations in gap leakage under varying rotational speeds and flow rates. As seen in Figures 4-4g through 4-4i, the Nagel (1968) model predicts a constant leakage value for all runs, at all rotational speeds. This is because the Nagel (1968) model was intended for pumps, not ASTs, operating at the most efficient bucket volume and speeds. (Rorres (2000) showed that an Archimedes screw pump would be most efficient when operating with full buckets.) This model predicts the leakage based entirely on the screw geometry. However, despite this, Nagel's leakage model provides a reasonable order of magnitude estimate for the measured gap leakage.

The Lubitz et al. (2014) model provides a physical description of the potential gap leakage based on the actual fluid mechanics of the water in the buckets. This model was expected to better predict the dynamics of the gap leakage trends. However, this was not the case. Figures 4-4g through 4-4i all show the gap leakages increase with increasing rotational speeds, peaking at approximately 16-18 rad/s. After this peak, the gap leakage flows are all seen to decrease until there is no gap leakage at all at free-wheeling (maximum0 speeds. Interestingly, the gap leakage flows for all three runs appear to peak at approximately the same magnitudes despite the different total flow rates. The peak gap leakage flow rate for all experimental runs was seen to be approximately 260 cm$^3$/s. It is not clear if this is a property of the screw itself, or if the differences in the total flow rates were not significant enough to yield differences between the experimental runs within the experimental uncertainties.
This result was not expected. The experimental procedure was carefully reviewed and the author does not believe the results are due to experimental error. According the Lubitz et al. (2014) gap leakage model, as the rotational speeds increase, and the fill heights decrease, the pressure at each of the gaps should decrease as well. Therefore, the Lubitz et al. (2014) model predicts a decrease in gap leakage with increasing rotational speeds. The opposite was observed. As the rotational speeds increased, the gap leakage could be seen to increase as well, before peaking and being reduced to zero.

Figure 4-4. Measured total flow and bucket flow for: a) Run 1 b) Run 2 c) Run 3. Fill heights for: d) Run 1 e) Run 2 f) Run 3. Comparison of measured leakage to Lubitz (Equation 3-18) and Nagels (Equation 3-17) for: g) Run 1 h) Run 2 i) Run 3
Table 4-2. 6th order polynomial fits for flow rates vs angular velocity, for total and bucket flows

<table>
<thead>
<tr>
<th>Run</th>
<th>Polynomial</th>
</tr>
</thead>
<tbody>
<tr>
<td>Run 1 - Total Flow</td>
<td>$Q = (1.709 \times 10^{-5}) \omega^6 - 0.001237 \omega^5 + 0.02641 \omega^4 + 0.07763 \omega^3 - 10.21 \omega^2 + 135.2 \omega + 114.8$</td>
</tr>
<tr>
<td>Run 1 - Non-gap Flow</td>
<td>$Q = (-6.973 \times 10^{-5}) \omega^6 + 0.004086 \omega^5 - 0.08554 \omega^4 + 1.069 \omega^3 - 14.41 \omega^2 + 139.2 \omega + 0.03932$</td>
</tr>
<tr>
<td>Run 2 - Total Flow</td>
<td>$Q = (4.056 \times 10^{-5}) \omega^6 - 0.003571 \omega^5 + 0.1138 \omega^4 - 1.436 \omega^3 + 1.048 \omega^2 + 124.8 \omega + 90.04$</td>
</tr>
<tr>
<td>Run 2 - Non-gap Flow</td>
<td>$Q = (8.020 \times 10^{-6}) \omega^6 - 0.001671 \omega^5 + 0.08217 \omega^4 - 1.324 \omega^3 + 1.560 \omega^2 + 115.7 \omega + 0.1914$</td>
</tr>
<tr>
<td>Run 3 - Total Flow</td>
<td>$Q = (7.123 \times 10^{-5}) \omega^6 - 0.006357 \omega^5 + 0.2115 \omega^4 - 3.070 \omega^3 + 13.35 \omega^2 + 107.2 \omega + 114.9$</td>
</tr>
<tr>
<td>Run 3 - Non-gap Flow</td>
<td>$Q = (7.385 \times 10^{-5}) \omega^6 - 0.006863 \omega^5 + 0.2399 \omega^4 - 3.657 \omega^3 + 18.06 \omega^2 + 87.81 \omega + 0.6349$</td>
</tr>
</tbody>
</table>

One possible explanation for the differences between the Lubitz et al. (2014) model and the measured gap leakage is that at lower rotational speeds static pressure dominates the leakage flow within the screw. This conclusion is reasonable as the measured gap leakage and the Lubitz et al. static pressure driven model seem to converge at slower speeds. However, as the rotational speeds increase, the dynamic effects created by the rotation of the screw begin to play a larger role. At higher speeds, the shear stress created by the turbine blades and the external casing can generate complex internal flow patterns within the buckets themselves. At higher speeds, these unaccounted for dynamic effects may begin to drive the gap leakage, leading to the unexpected increase and peak in leakage.

Additionally, at speeds above 10 rad/s, the turbine shear on the water in the buckets can actually cause water to be elevated out of the buckets and “pumped” into the air. It is possible that some of the calculated gap leakage flow rate is not gap leakage at all but rather water that leaves contact with the buckets due to the high rotational speeds. Furthermore, at free-wheeling speeds, the Lubitz et al. (2014) model still predicts a gap leakage because there exists a fill height level in the screw. It is likely that at extreme speeds, the gap leakage is actually reduced to zero because the turbine blades are traveling at the same speeds as the water buckets themselves. This was seen in the measured leakages but not in the Lubitz et al. (2014) predicted leakages. From these...
experimental results, it is clear that current gap leakage models can provide an order of magnitude estimate of potential gap leakages, but future gap leakage models will need to account for the dynamic flow effects that occur in the screw to better quantify gap leakage.

4.4.1 Experimental uncertainty

Due to the small scale of the gap leakage flow rates (under 300 cm³/s), and all the input measurements required to calculate this leakage flow rate, the final leakage results were subject to relatively large uncertainties. The gap leakage flow rate uncertainties were propagated from all the input measurements through all the gap leakage calculations. The gap leakage flow rate was most sensitive to the fill height measurements, therefore the uncertainties in the fill height measurements dominated the gap leakage flow uncertainties. The average fill height measurement uncertainty was approximately 7.9% of the actual measurements. While this amount seems small, a small deviation in fill height can correspond to a much larger deviation in bucket volume and therefore flow rate.

The uncertainty in the fill height measurement was primarily due to the flow patterns within the screw rather than uncertainty in the measuring sensors, which were just standard straight-edge rulers. During the experiment, noticeable differences between the fill heights of adjacent water buckets were visually observed and measured at a single instant. For an ideal screw operating at steady-state, with constant flow rate and rotational speed, the fill heights in all the buckets should be the same. This is particularly true for the most upstream buckets, which sometimes exhibited widely varying fill heights. Visually from the photographic images, it seems that the internal
currents in the water buckets caused by water entering the screw take 1/3 to 2/3 pitch lengths down the axial length of the screw to stabilize, before optical fill height measurements can be taken.

An average fill height for a given run was used to predict the bucket volumes and flow rate. Since this average was comprised of significant variations in the bucket, this lead to high uncertainties. When propagated through the bucket flow uncertainties, these uncertainties were typically quite large; the mean and maximum bucket flow uncertainties, relative to the actual measurement, were approximately 17.2% and 69.3% respectively. Furthermore, when propagated to the gap leakage flow rate uncertainties, the uncertainties relative to the measurements were even larger. For all experimental runs, the mean uncertainty relative to the leakage measurements was approximately 95%. Figure 4-5 shows the gap leakage measurement uncertainties relative to the measurements for all three experimental runs. This high uncertainty was expected since the gap leakage is quite small relative to the total flow and occurs through a gap that is 1.11 mm or approximately 0.77% of the outer AST diameter. These high measurement uncertainties mean the observed resulting leakage flow rate trends should be treated with caution. Nonetheless, it seems that the leakage trends were repeatable and consistent across all experimental runs with varying flow conditions.
4.5 Experimental Conclusions

The gap flow measurements performed in Chapter 4 provide a measure of the possible scale and flow trends for the gap leakage within a laboratory-scale AST. These measured gap leakage trends were compared to two accepted gap leakage models for Archimedes screws (Nagels; Equation 3-17 and Lubitz et al.; Equation (3-18). While both models provide reasonable estimates for the magnitude of the leakage flow, both models also failed to predict the observed leakage flow rate patterns particularly at high rotational speeds. The leakage flow rate values themselves have very high measurement uncertainties relative to the actual measurements and therefore, the observed flow trends should be regarded with caution. Several other factors, such as the shear force between the water and the helical planes causing water to be “pumped” out of the buckets could be contributing to the observed leakage values and therefore causing overestimates of the leakage flow rate values.
Chapter 5: AST Power Loss Model

In this chapter, a complete loss model for an AST is developed. Currently, the literature only contains attempts at quantifying AST leakage losses and inlet losses ((Rorres, 2000), (Lubitz et al., 2014)) that occur as water transitions from an upstream channel into an AST (Nuernbergk & Rorres, 2012). There has yet to be a complete loss model that takes into account the full spectrum of losses that an AST experiences. These losses include:

- Inlet entrance head loss
- Outlet exit head loss
- Hydraulic frictional head loss
- Bearing friction head loss
- Outlet drag torque head loss

5.1 Relevant head loss principles

The power losses occurring inside an AST are to be obtained by first experimentally measuring AST power for a range of operating conditions. These measured power values can then be compared to the idealized power predictions made by the Lubitz et. al (2014) AST power model. The difference between the measured and predicted powers is considered the energy losses within the system. The implicit assumption made in predicting energy losses as the difference between the experimentally measured power and the Lubitz et al. (2014) model is that Lubitz et al. (2014) model represents the ideal or maximum power output of the screw. This assumption is reasonable because the Lubitz et al. (2014) model represents the maximum theoretical torque that could be
applied to a screw under static conditions. Furthermore, all experiments showed measured power less than the Lubitz et al. (2014) predictions. Each of the energy losses are to be analyzed separately, through a combination of known hydraulic and mechanical theory, as well as dimensional analysis.

5.1.1 Inlet Head and Outlet Exit Power Losses

5.1.1.1 Inlet Delivery Channel, Outlet Channel, and Average Screw Water Depths

The inlet channel entrance and outlet channel exit losses are dependent on the water depths and areas of the water entering as well as leaving the screw. Additionally, these loss calculations depend on the water levels inside the screw. Therefore, in order to approximate these losses, the water levels upstream and downstream of the AST must be defined. Additionally, an average water depth, and associated area (termed characteristic average flow area) are defined.

Figure 5-1 shows the positions of the inlet delivery channel depth \( (h_d) \) and cross-sectional area \( (A_d) \), the outlet channel depth \( (h_o) \) and outlet channel cross-sectional area \( (A_o) \), and the average water depth \( (h_{avg}) \) in the screw and associated characteristic area \( (A_c) \).

![Figure 5-1. The upstream delivery channel, downstream outlet channel and internal average elevations and cross-sectional areas.](image)
At the inlet of the AST, the helical planes cause discrete bucket formations that continually adjust the water levels at the AST entrance. Additionally, they can cause upstream waves or oscillations particularly at slow speeds. These oscillations are usually minor and dampen out up stream for long inlet channels. Due to these conditions, it is difficult to analytically determine the upstream water level at any given time.

However, for a screw operating at steady-state, the inlet water depth just prior to entering the screw is approximately constant and can be averaged over time. This average water depth can be determined by analyzing the screw as if the helical planes do not exist and the AST is just comprised of the outer trough and the central cylindrical shaft, so that the water buckets do not form. Under this idealized condition, the water depth would reach an average depth that remains constant throughout the screw. This average depth \( (h_{avg}) \) would be the water height at the entrance of the AST. If it is assumed that this average AST water depth meets the delivery channel’s water surface at the transition between the inlet channel and the AST, the delivery channel’s water depth \( (h_d) \) can be calculated as the vertical projection of \( h_{avg} \).

The average AST depth \( (h_{avg}) \) is calculated first by determining an average characteristic flow area \( (A_c) \) that corresponds to this average depth within the screw. This is done by first determining the total volume of water inside the screw at any given instant. If volume of a single bucket \( (V_b) \) is first determined through Equation 3.8, then the total volume \( (V_{tot}) \) of water in the screw is volume of a single bucket multiplied by the number of buckets in the screw:
\[ V_{tot} = V_b \frac{LN}{S} \]  \hspace{1cm} (5-1) 

where \( L \) is the length of the screw, \( N \) is the number of planed surfaces and \( S \) is the screw pitch.

The average characteristic flow area is then calculated as:

\[ A_c = \frac{V_{tot}}{L} = V_b \frac{N}{S} \]  \hspace{1cm} (5-2) 

The average AST water depth is determined from the average characteristic flow area and the screw geometry. The relationship between the average AST water depth \( (h_{avg}) \) and the characteristic flow area \( (A_c) \) is seen in Figure 5-2.

![Figure 5-2: Idealized characteristic flow area in relationship to the average AST water depth and inner and outer radii](image)

The characteristic area can be determined from the screw's inner and outer radii \( (r_i \) and \( r_o) \) and idealized average water height \( (h_{avg}) \) as:
The average AST water depth ($h_{avg}$) was found numerically, by iterating through values of $h_{avg}$ until the desired $A_c$ was calculated. With the average AST water depth known, the water depth in the delivery channel immediately upstream of the AST can be found by taking the vertical projection of $h_{avg}$ onto the delivery channel:

$$
h_d = \frac{h_{avg}}{\cos \beta} \tag{5-5}
$$

where $\beta$ is the AST's angle of inclination.

### 5.1.1.2 Inlet Entrance Head Loss

Currently, there has been one attempt to quantify the energy losses that occur as the water makes the transition from the inlet channel to the screw trough by Nuernbergk and Rorres (2012) using the well-known Borda-Carnott entrance loss relationships. Nuernbergk and Rorres suggested the head loss through the transition could be modelled using the one-dimensional Bernoulli energy equation with a Borda-Carnott head loss. Using the channel water levels defined in the previous Chapter 5.1.1.1, the energy equation across the inlet transition can be expressed as:
\[ h_d + \frac{v_d^2}{2g} = h_{avg} + \frac{v_t^2}{2g} \left( 1 + \zeta_i \right) \]  

(5-6)

where \( h_d \) and \( v_d \) are the upstream water level and flow velocity in the inlet delivery channel and \( h_{avg} \) is the average height in the AST and \( v_t \) is the transport velocity. The transport velocity \( (v_t) \) is the rate at which the water buckets are translated through space down the axial length of the screw. It is defined as:

\[ v_t = \frac{S \omega}{2\pi} \]  

(5-7)

where \( \omega \) is the angular velocity of the turbine and \( S \) is the pitch. The head loss through the inlet transition is quantified through the Borda-Carnot head loss coefficient \((\zeta_i)\):

\[ \zeta_i = \left( \frac{A_c}{A_d} - 1 \right)^2 \]  

(5-8)

where \( A_d \) is the cross-sectional area, immediately upstream, in the delivery channel area and \( A_c \) is cross-sectional area of the flow just after it crosses the inlet transition.

The inlet energy loss expressed in Equation (5-6) has no impact on the power loss experienced at the turbine for a given flow rate. Rather, the head loss is experienced upstream in the delivery channel, requiring additional head in the supplying reservoir to overcome this loss in order to deliver the desired volume flow rate to the AST. In-depth detailed calculations of this procedure are outlined in Chapter 7.
5.1.1.3 Outlet Exit Energy Loss

Similar to the inlet energy loss, there is a head loss at the outlet of the screw as the water transitions from the AST trough to the outlet channel. For most ASTs, this loss is generally an expansion loss as the cross-sectional area of the AST is typically smaller than the cross-sectional area of the outlet discharge channel. Often, ASTs discharge directly to a stream, which is also an expansion loss. Since there is a geometry change from the trough to the outlet channel, again the Borda-Carnot head loss equation can be used to estimate the head loss. Research has shown that Borda-Carnot head loss calculations are suitable for culvert exit losses (Tullis and Robinson, 2008) which are similar in structure and operation to AST troughs. Therefore, similar to the entrance loss, the Borda-Carnot exit loss coefficient is determined as:

\[ \zeta_o = \left(1 - \frac{A_c}{A_o}\right)^2 \]  

The overall head loss from the exit transition is then:

\[ h_{f,o} = \zeta_o \frac{v_t^2}{2g} \]  

The overall exit power \( P_{loss, exit} \) loss from the screw can be determined as:

\[ P_{loss, exit} = \rho g Q h_{f,o} \]  

5.1.2 Hydraulic Friction Power Loss

5.1.2.1 Transport Friction Power Losses
The hydraulic frictional forces that occur within the screw can be divided into two categories: drag friction in the transport direction and drag friction in the rotational direction. The friction losses in the transport direction can further be separated into two components: the friction between the water and the enclosing trough, and friction between the water and the central cylindrical shaft. This occurs because as the water is transported down the axial length of the screw, it drags along both the trough and center shaft creating a frictional force that does work on the fluid, causing resistance to the motion. Both the friction along the trough and central cylindrical shaft can be theoretically modeled using the Darcy-Weisbach head loss equation for flow in an open-channel:

\[ h_f = f_{DW} \frac{L}{4R_h} \frac{v^2}{2g} \]  \hspace{1cm} (5-12)

where \( h_f \) is the head loss factor, \( f_{DW} \) is the Darcy-Weisbach friction factor, \( L \) is the trough length, \( R_h \) is the hydraulic radius, \( v \) is the fluid velocity and \( g \) is the gravitational acceleration constant (9.8 m/s\(^2\)). The hydraulic radius is defined as:

\[ R_h = \frac{A}{P_w} \]  \hspace{1cm} (5-13)

where \( A \) is the flow cross-sectional area and \( P_w \) is the wetted perimeter. The Darcy-Weisbach frictional factor is a function of material roughness, defined as the relative roughness of the material to the diameter of the pipe (\( \varepsilon \)), and the level of turbulence in the flow (as characterized by the Reynolds Number, \( R_e \)):

\[ f_{DW} = f(\varepsilon, N_r) \]  \hspace{1cm} (5-14)
Typically, the friction factor is determined through the well-known Moody diagram, but there are several empirical approximations for various flow regimes such as open-channel flow, pipe flow, fully turbulent, laminar, and laminar-turbulent transition flows. The friction factor can be determined experimentally for a given AST but ideally this factor should be predicted more generally for a given trough and turbine material to allow for scalability to any AST. Nuernbergk (2012) suggested that AST flow is similar to open-channel flow in which the well-known Manning's equation friction factor \( (n) \) can be used to approximate the shear friction along the rough walls and center shaft. Under this assumption, the Darcy-Weisbach friction factor for the walls and center shaft \( (f_{DW,1}) \) can be determined from the Manning's \( n \) value through:

\[
f_{DW,1} = \frac{8gn^2}{R_{h}^{1/3}}
\]

(5-15)

The Darcy-Weisbach friction factor can be converted to a shear stress \( (\tau_s) \) at the surfaces of the trough and central shaft through:

\[
\tau_s = f_{DW,1} \frac{\rho V_t^2}{8}
\]

(5-16)

where \( \rho \) is the density of water and \( V_t \). If the wetted, or submerged areas of the trough and center shaft are known, the frictional force on the trough surface \( (F_{At}) \) and the frictional force at the central cylinder surface \( (F_{A_c}) \) on these surfaces can be found as:

\[
F_{At} = \tau_s n_b A_t \quad \text{and} \quad F_{Ac} = \tau_s n_b A_c
\]

(5-17)
where \( n_b \) is the number of buckets, and \( A_t \) and \( A_c \) are the wetted areas of the trough and center shaft respectively, for a single bucket. \( A_t \) and \( A_c \) can be found through integrating the wetted area portions of the trough and center shaft. For the trough:

\[
A_t = \int_{\theta=0}^{2\pi} dA_t
\]

where

\[
dA_t = \begin{cases} 
0 & z_2 > z_{wl}, z_1 > z_{wl} \\
\frac{z_{wl} - z_1}{z_2 - z_1} \frac{S}{N} r_o d\theta & z_2 \geq z_{wl}, z_1 \\
\frac{S}{N} r_o d\theta & z_2 < z_{wl}, z_1 > z_{wl}
\end{cases}
\]

(5-19)

For the center cylinder:

\[
A_c = \int_{\theta=0}^{2\pi} dA_c
\]

where

\[
dA_c = \begin{cases} 
0 & z_2 > z_{wl}, z_1 > z_{wl} \\
\frac{z_{wl} - z_1}{z_2 - z_1} \frac{S}{N} r_i d\theta & z_2 \geq z_{wl}, z_1 \\
\frac{S}{N} r_i d\theta & z_2 < z_{wl}, z_1 > z_{wl}
\end{cases}
\]

(5-21)

The work done on the trough and center cylinder surfaces by the fluid over the entire length of the AST (\( W_{wall friction} \)) is then:

\[
W_{wall friction} = (F_{A_t} + F_{A_c}) \cdot L
\]

(5-22)
According to Nuernbergk (2012), the work down by the shear stress on the walls can also be formulated as

\[ W_{\text{wall friction}} = \rho \cdot V_{\text{water}} \cdot g \cdot h_f = \rho \cdot n_b V_b \cdot h_f \]  

(5-23)

where \( V_{\text{water}} \) is the total volume of water inside the screw and \( h_f \) is the free water surface height due to wall friction. The overall power loss \( (P_{\text{loss, wall friction}}) \) can be determined from the wall friction, free water surface height and the AST flow rate:

\[ P_{\text{loss, wall friction}} = \rho g Q h_f \]  

(5-24)

5.1.2.2 Rotational Friction Power Losses

Similar to the transport hydraulic friction power losses, there are drag friction losses that arise from the rotational motion of the screw itself. These rotational friction power losses can also be separated into two components: friction between the water and the center shaft, and friction between the water and the helical planes. The latter of these two components tends to dominate the friction power losses, particularly at high rotational speeds because the relative velocity between the planes and the water in the buckets is much greater than the transport velocity. The rotational shear stress can be modelled similar to the transport shear stress losses on the trough and center shaft, however, the numerical determination of the friction factor \( (f) \) is less clear and more problematic, because the relative motion and velocities between the planes and the water buckets is more complicated. The Darcy-Weishbach friction factor is derived for steady-state, fully developed pipe and channel flow, with the movement of the water in the direction of the axial
length of the pipe or channel. In contrast, the rotational shear stress losses have varying orientations and directional motion depending on the position of the planes being considered. Furthermore, the circular type rotation of the helical planes creates an uneven shear on the water buckets. Additionally, this introduced shear is no longer in the direction of bucket translation but rather opposite to the relative motion between the water buckets and screw at every given point along the helical planes. The blade shear would potential create secondary currents in the water buckets at high velocities that are not in the transport direction. It is assumed that these currents are negligible (though they certainly are not, particularly at high rotational speeds).

The relative velocity \( v_{r,c} \) between the center shaft and the water in the rotational direction of motion is simply:

\[
v_{r,c} = r_i \omega \quad (5-25)
\]

The shear stress can be found similarly as:

\[
\tau_s = f_{DW,2} \frac{\rho v_{r,c}^2}{8} \quad (5-26)
\]

Where \( f_{DW,2} \) is the Darcy-Weisbach friction factor for the rotational motion of the center shaft. Since the distance between the contact point between the water and the center shaft, relative to the center axis of the screw remains constant and the rotational speed of the screw is constant, the power loss due to the rotational shear along the center shaft can be determined as:
The rotational shear stress that arises at the helical planes is more complicated. Since the frictional force opposes motion, the relative shear stress at any given point on an elemental area on the helical plane would be in the opposite direction to the rotational spin, applied at a distance $r$ from the center shaft as seen in Figure 5-3.

![Figure 5-3. Simplification of a helical plane surface, showing the shear stress from the water acting on the plane.](image)

Unlike for the center shaft, the relative velocity between the planes and the water is not constant at the contact surfaces. The relative motion can be separated into two components, one in the radial direction, orthogonal to the rotational axis, and the other in the transport direction (down the axial length of the screw). Only the radial direction component opposes the rotational motion of the shear, therefore only this component needs to be considered.
Since the relative velocity between the water and the plane changes with position, and both the shear stress and power are dependent on the radial position, the power loss must be found by integrating across the entire helical plane surfaces for the entirety of the wetted plane surfaces:

\[
P_{\text{loss, plane friction}} = n_b \int_{r=r_i}^{r=r_o} \int_{\theta=0}^{2\pi} \frac{f_{DW,3} \rho}{8} r^3 \omega^3 dA_1 + n_b \int_{r=r_i}^{r=r_o} \int_{\theta=0}^{2\pi} \frac{f_{DW,3} \rho}{8} r^3 \omega^3 dA_2 \quad (5-28)
\]

where \( f_{DW,3} \) is the Darcy-Weisbach friction factor for the rotational motion of the planes. Both the upstream and downstream areas of the planes must be determined separately because they have different wetted areas. If only the wetted portion of the planes are considered, the downstream \((dA_1)\) and upstream \((dA_2)\) infinitesimal areas are:

\[
dA_1 = \begin{cases} 
0 & z_1 > z_{wl} \\
\frac{\sqrt{4\pi^2 r^2 + s^2}}{2\pi r} r dr d\theta & z_1 \leq z_{wl}
\end{cases} \tag{5-29}
\]

\[
dA_2 = \begin{cases} 
0 & z_2 > z_{wl} \\
\frac{\sqrt{4\pi^2 r^2 + s^2}}{2\pi r} r dr d\theta & z_2 \leq z_{wl}
\end{cases} \tag{5-30}
\]

The rotational shear stress loss can be simplified if the frictional factor \( f_{DW,3} \) is considered constant. Total wetted areas on the downstream \((A_1)\) and upstream \((A_2)\) planes for a single bucket can be calculated as:
\[ A_1 = \int_{r=r_i}^{r_o} \int_{\theta=0}^{2\pi} dA_1 \] (5-31)

\[ A_2 = \int_{r=r_i}^{r_o} \int_{\theta=0}^{2\pi} dA_2 \] (5-32)

Since the rotational shear stress power losses on the plane surfaces are increase with the cube of the radial position, weighted average wetted radii for the downstream \( \bar{r}_1 \) and upstream \( \bar{r}_2 \) can be determined as:

\[ \bar{r}_1 = \left( \frac{\int_{r=r_i}^{r_o} \int_{\theta=0}^{2\pi} r^3 dA_1}{A_1} \right)^{1/3} \] (5-33)

\[ \bar{r}_2 = \left( \frac{\int_{r=r_i}^{r_o} \int_{\theta=0}^{2\pi} r^3 dA_2}{A_2} \right)^{1/3} \] (5-34)

The rotational friction losses on the planes can then be simplified using the wetted areas and average weighted radii as:

\[ P_{loss, \text{plane friction}} = n_b \int_{DW} \rho \omega^3 \left( \bar{r}_1^3 A_1 + \bar{r}_2^3 A_2 \right) \] (5-35)

5.1.3 Bearing Friction Power Loss
The mechanical friction loss that occurs in the bearings is typically dependent on the rotational speed. Assuming that the mechanical torque ($\tau_m$) losses are linearly proportional to the rotational speed ($\omega$):

$$\tau_m = k\omega$$  \hspace{1cm} (5-36)

where $k$ is a loss coefficient dependent on the bearing material properties. Then the mechanical power loss would be proportional to the angular velocity squared. Additionally, the weight of the water inside the screw would create additional loading on the bearings, potentially increasing the bearing losses. Therefore, it is reasonable to assume the bearing loss coefficient $k$ would be dependent on the volume of water in the screw ($V$):

$$P_{bl} = k(V)\omega^2$$  \hspace{1cm} (5-37)

### 5.1.4 Outlet Torque Power Loss

The outlet head loss is a function of the depth of water above the bottom of the screw outlet. The relative height of the water above the bottom of the screw to the outer diameter ($\Psi$) is defined as:

$$\Psi = \frac{h_o}{D_o\cos(\beta)}$$  \hspace{1cm} (5-38)

where $h_o$ is the outlet channel water level above the bottom of the screw, $D_o$ is the outer screw diameter, and $\beta$ is the screw angle of inclination.
Power losses tend to increase with increasing $\Psi$ as the higher water level creates an increased resistive drag and back pressure at the screw outlet. This resistive back pressure functions as an increased drag force that reduces torque, and therefore power for an operating screw. Early experimental data presented in Chapter 5.3 suggests that this outlet torque loss ($P_{\text{loss, outlet torque}}$) is a function of the outlet water level, rotational speed, volume flow rate and $\Psi$:

$$P_{\text{loss, outlet drag}} = f(\omega, Q, \Psi)$$  \hspace{1cm} (5-39)

The outlet drag power loss can be non-dimensionalized through a torque loss coefficient

$$\eta_T = \frac{T_{\text{loss, outlet drag}}}{T_{\text{modelled}}} = \frac{P_{\text{loss, outlet drag}}}{P_{\text{modelled}}}$$  \hspace{1cm} (5-40)

A non-dimensional flow rate ($Q_{nd}$) is derived that relates the rotational speed of the screw ($\omega$), the total flow rate ($Q$), and the dimensional scale of the screw. The dimensional scale of the screw is represented by the open portion of the screw’s cross-sectional area that is potentially exposed to flow. This yields a non-dimensional flow rate ($Q_{nd}$) of:

$$Q_{nd} = \frac{v_t(D_o^2 - D_i^2)}{Q} = \frac{\omega S}{2\pi Q}(D_o^2 - D_i^2)$$  \hspace{1cm} (5-41)

$Q_{nd}$ is a function of the transport velocity, with itself is directly proportional to the AST’s angular velocity. Since $\eta_T$ and $Q_{nd}$ are non-dimensional, the relationships between the torque loss coefficient and non-dimensional flow should yield discernable patterns for unique values of $\Psi$. 

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The experimental results are used to ascertain these relationships and derive a non-dimensional empirical loss model that describes the outlet torque drag power losses for a generic AST.

5.2 Experimental Methodology

A laboratory experiment was devised and performed to determine AST energy losses over a range of operating conditions. Power measurements were taken for a range of flow rates, water levels, and speeds. The following is an outline of this experiment.

5.2.1 Experimental Setup

The experimental setup consisted of an AST situated between two basins, an upper and lower basin. The upper basin lied approximately 60 cm above the lower basin. Water was continuously pumped from the lower basin to the upper basin through a return pipe containing a flow meter. Water from the upper basin was introduced to the inlet of the screw and allowed to pass through the screw by gravity driven flow. Upon exiting the screw outlet, the water flowed back into the lower basin allowing the cycle to continuously repeat. The pump was a variable head pump capable of supplying a maximum flow rate of approximately 14 L/s.

Water levels in both the upper and lower basins were controlled. The upstream water levels were regulated by the rotation speed of the AST. Regulating the screw speed also regulates the fill height; slowing the screw down increases the fill height for a given flow rate because the AST transports water at a slower speed down the axial length of the screw, meaning it has to transport more water per revolution to move the same amount of water. Similarly, speeding up the AST decreases the fill height as the same volume flow rate of water is divided between more buckets.
The lower basin water level was controlled simply by adding or removing water from the basin as needed. Additionally, both the flow through the screw and upper water level could be further attenuated through the use of an overflow weir located at the back of the upper basin. Figure 5-4 shows the experimental setup.

![Experimental setup, with the location of the upper and lower basin](image)

The lab screw used was 122.1 cm long ($L$) with inner ($D_i$) and outer ($D_o$) diameters of 16.8 cm and 32.6 cm respectively. The screw pitch ($S$), or distance between successive helical planes, was 31.8 cm. The screw was inclined at an angle ($\beta$) of 24°. Table 5-1 outlines the AST dimensions.

The rotational speed of the screw controlled by a gear motor driven by a variable frequency drive (VFD) attached to the central shaft of the AST. The gear motor maintained accurate user-set rotation speeds ranging from 20 to 250 RPM. The gear motor and VFD effectively functioned as a combined speed control and brake, dissipating the excess torque that would normally have provided power to the electrical generator.
Table 5-1. Dimensions of the scale-laboratory AST used to power losses

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inner Diameter (mm)</td>
<td>168. ± 0.02</td>
</tr>
<tr>
<td>Outer Diameter (mm)</td>
<td>325.0 ± 0.3</td>
</tr>
<tr>
<td>Screw Length (cm)</td>
<td>122.9 ± 0.9</td>
</tr>
<tr>
<td>Angle (Degree)</td>
<td>24.4 ± 0.3</td>
</tr>
<tr>
<td>Pitch (mm)</td>
<td>33.0 ± 0.5</td>
</tr>
<tr>
<td>Gap Width (mm)</td>
<td>1.99 ± 0.05</td>
</tr>
</tbody>
</table>

5.2.2 Experimental Measurements

The flows supplied by the pump and diverted over the back weir were both measured with two Omega FTB740 flow meters. The upper and lower basin water levels were measured using two Keller Valueline depth gauges, one per basin. The rotational speed of the screw was measured using a magnetic switch that was triggered once every screw rotation by the passing of a rare earth magnet fastened to the AST central shaft.

Power measurements were made using an Omegadyne LC703-25 load cell, fastened to the screw at a distance of 26.1 cm from the center of the screw shaft via an arm attached to the gear motor. The load cell prevented the gear motor from rotating with the screw shaft, converting the rotation of the screw into a torque (Figure 5-5).
The product of the measured torque measurement and the angular velocity measurement yielded mechanical power at the shaft:

\[ P = \omega \tau \]  \hspace{1cm} (5-42)

where \( \omega \) is the angular velocity (rad/s) and \( \tau \) is the torque (Nm). All data acquisition was achieved through custom data acquisition software written in LabVIEW. All input sensors (torque, rotational speed, depth gauges) were recorded for a duration of 1 minute, at a sampling frequency of 1000 Hz. (The high sampling rate was used to accurately resolve the triggering time of the magnetic speed sensor.) All measurements were recorded using a National Instruments NI DAQ 6009 data acquisition unit interfaced to a laptop computer running National Instruments LabView software. Processing of recorded data was performed in MATLAB. Approximately 2 minutes was given between changes to experimental speed and flow conditions and recording all measurements.
to allow the experimental AST to reach steady-state. All measurements were made with a water temperature of approximately 20°C.

5.2.2.1 Torque cell calibration

The torque sensor was a simple tension link load cell that measures the force of an applied load across the cell. An OMEGADYNE LC703 load cell was used in these experiments. The force was converted to a torque by attaching the load cell to a rigid arm attached to the VFD in such a way that the load cell is only point of contact preventing the VFD from rotating with the screw shaft. Since the load cell measures the force acting at a known distance along the moment arm, it effectively measures the torque produced by the AST. A calibration equation was derived for the load cell, describing the relationship between the load cell’s output signal voltage and the input applied load. The complete calibration results and equations can be found in Appendix D.

5.2.3 Experimental Runs

The AST was operated at four different volume flow rates: 4 L/s, 7.5 L/s, 10 L/s, and 12.5 L/s. For each of these volume flow rates, measurements were taken across a wide range of rotational speeds. Measurements were taken starting at rotational speeds of 20 RPM, increasing incrementally until the AST no longer produced power. Each experimental run consisted of approximately 10 to 25 measurements. For each measurement, the AST was brought to steady state, with a constant flow rate, rotational speed and power. All sensors were then recorded for a duration of 1 minute. The first set of measurements were taken with the outlet water level below the screw such that no part of the screw outlet was immersed in water, meaning the non-dimensional outlet water depth \( \psi \) was 0. For each subsequent run, the lower water level was
increased by 5 cm until the end of the screw was more than completely submerged. For each flow rate, \( \psi \) values ranging from 0 to 1.31 were tested. The power and basin water levels were measured for each of these conditions. In total, 520 measurements were made across the four flow rates (105 measurements at 4 L/s, 113 measurements at 7.5 L/s, 156 measurements at 10 L/s and 146 at 12.5 L/s). For each flow rate, a set of 8 different water levels (\( \psi = 0, \psi = 0.25, \psi = 0.43, \psi = 0.60, \psi = 0.77, \psi = 0.94, \psi = 1.10, \) and \( \psi = 1.31 \)).

Figure 5-6 shows a typical power curve for a single experimental run. The measured data (square, red) is typical of power measurements made on the scale-laboratory AST models. For any given point on the power curve, the power is comprised of both the rotational speed and torque generated by the screw (\( P = \omega \tau \)). A stalled screw (\( \omega = 0 \) rad/s) would experience the highest level of torque because it would create the highest fill heights and least hydraulic/bearing losses. While the power should increase with increasing rotational speed, this is actually not the case. As the rotational speeds increase, the torque is reduced because the fill heights decline and the torque losses due to hydraulic friction and bearing friction increase until the torque losses eliminate the induced torque on the screw completely. This creates a unique power “peak” for each measured power curve at a particular rotational speed where the combined induced torque and rotational speed produce the highest possible power.

For comparison, the ideal power output from the Lubitz et al. (2014) model is also shown (dot, blue) in Figure 5-6. The power losses are considered to be the difference between these two power curves and represent the total power loss experienced by the screw, excluding power losses associated with lost flow due to gap and overflow leakage. Note that the idealized power curve
produced by the Lubitz et al. (2014) model fails to produce the decline in power with increasing rotational speed.

![Figure 5-6. A typical AST experimentally measured and idealized Lubitz et al. (2014) power curves for increasing angular velocity.](image)

### 5.3 Experimental Results and Discussion

#### 5.3.1 General AST Loss Regions

Power losses in ASTs are highly speed dependent, and generally increase with increasing angular velocity. At low rotational speeds, and therefore low transport velocities, the shear frictional losses tend to be minor relative to the overall power losses. Muysken (1932) defined an empirical limit for the rotational speeds at which excessive shear frictional and centrifugal forces tend to dominate the power losses (Nagel, 1968; Neurnbergk and Rorres, 2012). Based on his observations, he suggested this rotational speed limit should be:

\[ RPM \leq \frac{50}{(2R_o)^{2/3}} \text{ (min}^{-1}\text{)} \]  

(5-43)
This RPM will subsequently be referred to as the “Muysken Limit.” For the experiment performed in this chapter, this limit is approximately 66.6 RPM (6.97 rad/s).

As will be shown in further detail below, analysis of the measured data lead to observations of two general regions with different dominant power loss mechanisms. For rotational speeds less than approximately 6.97 rad/s, it was observed that the losses are dominated by the outlet torque losses, that is, the torque drag created by the lower water level at the end of the screw. As rotational speed increases above 6.97 rad/s, the frictional shear losses begin to dominate the overall energy losses. This, of course, is a simplification, as the power losses at any given rotational speed are comprised of all the losses simultaneously. However, since the measurements taken in this experiment only measure the total AST mechanical power at the shaft, it is impossible to measure the losses independently. Therefore, by analyzing the loss regions where the torque outlet and shear frictional losses dominate, the individual component losses can be estimated from the experimental data.

Figure 5-7 shows all the experimentally measured AST data for varying lower water levels ($\Psi$). Also included in the plots are the Lubitz et al. (2014) power model results for the same experimental data and the Muysken Limit line at a rotational speed of 6.97 rad/s. It is assumed that below this rotational speed, the energy losses are dominated by the torque outlet losses from the lower water level. Above this this threshold, it is assumed that the losses are dominated by frictional shear forces between the water inside the screw and the trough, center shaft, and helical planes.
Figure 5-7. Experimental power measurements made on a 1.2 m long AST, $D_0 = 0.325 \text{ m}$, $D_i = 0.168 \text{ m}$ for various lower water levels ($\Psi$). The idealized Lubitz et al. (2014) power model is also included. The red line is the Muysken threshold. i) $Q = 4 \text{ L/s}$, ii) $Q = 7.5 \text{ L/s}$, iii) $Q = 10 \text{ L/s}$, iv) $Q = 12.5 \text{ L/s}$.

Taking the power losses to be the difference between the Lubitz et al. (2014) power model and the experimental measured mechanical power at the AST shaft, total measured power losses can be seen in Figure 5-8.
Figure 5-8. Experimental power loss measurements made on a 1.2 m long AST, $D_0 = 0.325 \text{ m}, D_i = 0.168 \text{ m}$ for various lower water levels ($\Psi$). The idealized Lubitz et al. (2014) power model is also included. The red line is the Muysken threshold. i) $Q = 4 \text{ lps}$, ii) $Q = 7.5 \text{ lps}$, iii) $Q = 10 \text{ lps}$, iv) $Q = 12.5 \text{ lps}$.

It should be noted that the lowest experimentally tested flow rate ($Q = 4 \text{ L/s}$) had deviating trends compared to the rest of the data. At this flow rate, the water buckets that formed were small, generally with very low $f$ values. Fill heights never exceeded 100% ($f > 1.0$) even at the slowest operating rotational speeds (i.e. no overfilling occurred). Therefore, for the analysis that follows in this chapter, these experimental runs were omitted. This highly suggests that energy losses described in this chapter pertain to flow rates in which nearly full or overfilled buckets form. The energy losses at low flow rates require separate examination.
Also note that for the low flow rate of \( Q = 4 \) L/s, the experimental run of \( \Psi = 0 \) is not present. This was due to technical difficulties during the laboratory test.

### 5.3.2 Inlet Head Loss Model

To date, the inlet losses as described by the Borda-Carnot relationship for AST inlets had yet to be verified. The data presented here attempt to validate the Bora-Carnot equations for use at AST entrances. It appears that there are weaknesses in using this model for the modeling the energy losses. The Borda-Carnot relationship was derived for pipe expansions; the application of this relationship to contractions requires sufficient downstream length after the contraction to allow the flow streamlines to normalize, something that does not occur with the rotating screw in an AST immediately downstream. Therefore, it is not clear whether these relationships are applicable from a theoretical perspective.

However, the energy losses using this model appear to be a small proportion of the total energy loss. *The inlet contraction losses are not power losses to the screw directly; they are a component loss to the upstream delivery channel and therefore have implications for the overall reservoir head (water level in the reservoir) but the power lost at the screw shaft itself.* Therefore, this loss cannot be analyzed directly from the measured AST power measurements like the other losses outlined in this chapter. One method that can be used to potentially validate the Borda-Carnot relationships for entrance losses, as suggested by Nuernbergk and Rorres (2012), is to use these relationships to attempt to predict the inflow head of an AST.
The inflow head is first predicted simply using mass continuity between the inlet channel and the AST. Using equation (5-1), the total volume of water in the screw at any instant can be calculated. This volume of water is converted to an equivalent water depth (assuming no helical planes exist in the screw to change the orientation of the water into “buckets”) through equations (5-2) through (5-4). The inflow head in the delivery channel is then calculated using equation (5-5).

Nuernbergk and Rorres (2012) predicted that as the water transitions from channel to AST, the contraction forces a slight draw down in water level. The magnitude of this draw down depth is equal to the predicted Borda-Carnot head loss. Figures 5-9i through 5-9iv shows the AST inlet heights for flow rates of 4 L/s, 7.5 L/s, 10 L/s and 12.5 L/s, including both the inlet heights as predicted using mass conservation only and the predictions amended using Borda-Carnot. These water levels are relative to the bottom of the upper basin. The bottom of the inlet opening relative to the bottom of the basin was approximately 0.250 m. Both mass conservation and Borda-Carnott predicted inlet heights tend to over-predict the inlet height at low speeds and provide reasonable predictions between 5-10 rad/s.
The difference between the mass conservation and the Borda-Carnot predictions are negligibly small at speeds below the Muysken Limit (6.67 rad/s). At higher speeds, mass conservation predictions begin to under-predict the inlet height, while the Borda-Carnot relationships over-predict it.

These experimental results suggest that the Borda-Carnot relationship may not be accurate since it predicts higher inlet energy than was possible. For this experiment, the drawdown effects were not visible within the experimental data. At the low speeds typical for running ASTs, mass...
conservation is enough to provide a reasonable inlet height prediction, and there is no difference apparent when applying the Borda-Carnot relationship.

At extremely high speeds, the high AST transport velocity suggests an energy loss that would create a far higher inlet head than was observed. This is because the contraction energy loss is dependent on the velocity of the water. At high rotational speeds, the Borda-Carnot relationships tend to suggest velocities that create energy losses that produce upstream heads (water levels) greater than what was actually observed.

Nuernbergk and Rorres (2012) reported general agreement between the Borda-Carnot inflow head predictions and their experimental values. This experiment also shows general agreement for the inflow head using Borda-Carnot at the same range of speeds tested by Nuernbergk and Rorres. However, Nuernbergk and Rorres did not report the actual inlet head losses and only provided inflow head data for low speeds, unlike this experiment where high rotational speeds were observed.

It should be noted that in these tests, the inlet height was measured both upstream of the inlet (based on the upper basin depth sensor) and at the inlet directly (by physical measurement of water depth at the point where it entered the front face of the top end of the screw) in an attempt to directly measure the drawdown. For these experiments, no drawdown (difference between the two measurements) was observed within the uncertainty of the experimental measurements. For the physical measurement of the inlet depth, a simple ruler was used with an uncertainty of approximately ± 0.5 mm. For the electronic depth sensor, the uncertainty was approximately ± 0.1 mm for still water (this value likely increases when the gauge is subjected to flowing water).
The location of the depth sensor was approximately 0.20 meters upstream from the AST inlet. The relative close proximity of the depth sensor to the AST inlet might have had an effect on why this drawdown effect was not observed.

5.3.3 Experimental Outlet Exit Power Loss

As water transitions from the end of the AST trough into the outlet channel, it experiences a change in geometry. For this experiment, the discharge flows into a rectangular discharge basin with a base width of 90 cm. The exit losses from this expansion were calculated for all experimental runs based on the average characteristic flow area \( A_c \) for each of the runs and the water level at the lower end of the screw using Equations (5-9) through (5-11). The downstream area \( A_o \) was calculated from the absolute bottom of the basin (not the screw) where \( A_o = 0.90 \text{ m} \times (0.354 \text{ m} + h_o) \), where the distance from the bottom of the screw outlet to the lower basin floor was 0.354 m.

Since the flow cross-sectional area varies with the transport velocity and the lower water level varies with \( \Psi \), the Borda-Carnot predicted exit power losses are plotted against both of these values in Figure 5-10. The calculated exit losses due to expansion represent a small portion of the total power losses. On average, the exit power losses represented approximately 4.9% of the total power losses across all the experimental runs, with a maximum power loss of 14.1% (at rotational speed of \( \omega = 25 \text{ rad/s} \) and the highest flow rate of \( Q = 12.5 \text{ L/s} \)). In the range that ASTs typically operate (rotational speeds between 20 and 50 RPM), these predicted exit power losses were on average 2.0% of the total power losses. It should be noted that these values could not be independently verified from the experiments conducted in this study because these power losses cannot be
adequately isolated from the other power losses, particularly the outlet loss due to the lower water level drag (discussed in section 5.3.6).

Figure 5-10. Exit power losses due the flow expansion from the AST trough to the discharge outlet channel for i) $Q = 4 \text{ L/s}$, ii) $Q = 7.5 \text{ L/s}$, iii) $Q = 10 \text{ L/s}$, and iv) $Q = 12.5 \text{ L/s}$

### 5.3.4 Experimental Bearing Power Loss

Determining the bearing losses under water-loaded conditions is difficult because the water also adds a power generating torque to the screw. It is only possible to measure the total power, so the bearing losses get encompassed into the total power measurement. However, dry tests were
conducted, to determine the bearing power losses without additional loading from the weight of the water. While determining the bearing power losses in this way will most likely cause inaccuracies in the predicted losses, it does provide the scale of the power losses for a range of speeds.

The bearings used in the experimental ASTs were SKF Deep Groove Ball Bearings (Model 6203-2RSH/C3), with 17 mm and 40 mm inner and outer diameters respectively. Prior to each set of tests, bearings were checked for free rotation and proper alignment to ensure the screw flights did not make contact with the trough.

The bearing losses were found to be proportional to the rotational speed directly, rather than the rotational speed squared as seen in Figure 5-11. This is likely unique to this experimental setup as real-world ASTs likely use a different bearing type (such as water-lubricated bearings). Figure 5-11 also shows a slight offset to the bearing loads which suggests a slight pre-load on the load cell used to measure torque.
Figure 5-11. The mechanical bearing losses of the AST running without water

Figure 5-12 shows the predicted dry bearing losses based on Fig. 5-10 and the measured screw rotation speed, for rotational speeds between 0 and 20 rad/s. Also shown are the observed total power losses for the four experimental flow rates tested. As the rotational speed of the screw increases, the relative proportion of the power loss that is consumed by bearing friction declines.

The scale and trend of the bearing loss may not be completely accurate for an AST operating under wet conditions. An AST containing water entrapped in the buckets would likely experience different loading conditions on the bearings compared to a stationary AST. Particularly, buoyant forces acting on the screw would likely alter the bearing loads on the screw in both the perpendicular and axial directions. Therefore, a more complete bearing loss model capable of including the additional loading/unloading effects on the bearings created by the inclusion of water in the AST is still needed. It is assumed for this study that the additional adjustments due to these
conditions are negligible relative to the overall power losses. Across all experimental runs, the bearing losses for this experimental setup were found to be, on average, 15.9% of the total energy losses.

![Graph showing dry bearing power loss relative to the total power loss observed for flows of Q = 4 L/s, Q = 7.5 L/s, Q = 10 L/s, and Q = 12.5 L/s](image)

Figure 5-12. Dry bearing power loss relative to the total power loss observed for flows of Q = 4 L/s, Q = 7.5 L/s, Q = 10 L/s, and Q = 12.5 L/s

### 5.3.5 Fluid Shear Friction Power Loss Model

#### 5.3.5.1 Shear Stress on Trough and Center Shaft

The predicted power loss associated with the shear stress on the AST containing trough and center shaft was calculated for all experimental runs, in accordance with section 5.1.2.1. The AST trough and center shaft were constructed of painted steel. Therefore, for power loss calculations for all the experimental runs, a Manning’s $n$ friction factor was chosen to be consistent with smooth steel ($n = 0.013$) (Ward, 2003).
Darcy-Weisbach friction factors \((f_{DW,i})\) for the trough and center shaft were calculated for every experimental run using Equation (5-15). The resulting Darcy-Wiesbach friction factors varied little across all of the tests. The mean friction factor was 0.03375 with a standard deviation of 0.001779 across 518 experimental runs. This suggests a fairly consistent friction factor within an AST bucket, regardless of the flow level and rotational speed. The friction factor was assumed to be constant in this study.

The calculated wall shear stress power losses (i.e. combined losses from shear between surfaces and water on the trough and center shaft) for each of the experimental runs are shown in Figure 5-13. Figure 5-13 shows that the wall shear stress power losses amount to very little of the total observed power loss. For all experimental runs, less than 1 W was lost due to shear stress on the trough and center shaft. On average, the wall shear stress power loss amounted to approximately 0.36% of the total observed energy loss.

![Figure 5-13. The predicted shear stress power loss from the trough and center shaft. The red, dashed line represents the Muysken Limit.](image)
5.3.5.2 Shear Stress on Helical Planes

Calculation of power losses due to the shear stresses acting on the helical planes as described in Chapter 5.1.2.2 requires experimentally obtaining a suitable friction factor \( f_{DW,2} \). This friction factor is likely highly dependent on factors including the material of the planes and flow conditions such as rotational velocity, bucket volume and other dynamic flow effects. It is likely that variations in the friction factor are small, and therefore assuming a constant friction factor could provide a reasonable approximation for the shear stresses on the helical planes.

The shear frictional power loss on the planes described in Equation (5-28) suggests the resulting power loss created by the shear frictional forces on the plane should increase with the average radial position (relative to the center screw axis) and velocities to the third power. The general power loss trends in Figure 5-8 suggest an approximately cubic relationship between the observed power losses and the rotational speed. This suggests that the frictional factor \( f_{DW,3} \) can be assumed to be approximately constant for a given flow regime.

In order to predict the helical plane friction factor \( f_{DW,3} \), the frictional shear losses on the planes must be isolated from the all other power losses (outlet torque losses, trough and center shaft shear stress losses, bearing friction losses, and outlet exit losses). To do this, a subset of the entire measurement data space was chosen in which the plane shear stress forces are likely to dominate. This subset is for rotational speeds above the Musyken limit (6.97 rad/s) and for the case in which the AST was completely unsubmerged at the lower end \( \Psi = 0 \). This measurement data is shown in Figure 5-14.
Figure 5-14. Experimentally measured losses, Lubitz et al. (2014) predicted power, and hydraulic frictional power losses for i) $Q = 4 \text{ L/s}$, ii) $Q = 7.5 \text{ L/s}$, iii) $Q = 10 \text{ L/s}$, and iv) $Q = 12.5 \text{ L/s}$

It should be noted that data that for this criteria to represent the shear dominated region is reasonable for long screws. In long screws (in this case pitch-to-diameter ratios of approximately 4), the inlet and outlet losses relative to the total energy losses should be minor. Water entering into the screw typically settles into the bucket formation well before traversing the length of the screw, therefore the associated energy losses should be small relative to the over power measurements. Similarly, outlet losses from the transition from the AST to the discharge basin should be minor as well in these conditions.
The Lubitz et al. (2014) model only considers the torque and power created by a single ideal bucket, between two adjacent planes. The implicit assumption in this model is all of the buckets in the screw have the exact same bucket formation at all times. This is approximately true for all buckets. However, this assumption breaks down slightly at the inlet and outlet of the AST. At the inlet, where buckets first form when water enters the screw, the formation of a bucket into its “static” shape and position often does not occur immediately; internal currents within the bucket created from the transition from the delivery channel to the screw often require time to disperse. Similarly, at the outlet end of the screw, formed buckets lose cohesion as the water in the bucket is discharged into the downstream channel. At this end, the assumption of a constant back-pressure from the water contained in the downstream bucket does not hold. In reality, if the receiving basin water depth is below the screw, this backpressure does not occur near the end of the screw, as water beings to exit the screw and fall away as the rotation of the screw end opens the last bucket to the surrounding air and the water rapidly exits. There would always be drawing down of water for the last bucket as the water discharge from the final bucket into the discharge basin. Effects associated with this drawdown are not considered, but for a long screw, these effects relative to the overall power generated from all the AST buckets, should be minor.

The measured power loss based on the data in Figure 5-13 was adjusted to remove the power loss from the bearings, trough and center shaft shear stress, outlet and outlet exit loss exposing the power loss just due to shear friction:

\[ P_{\text{loss, plane friction}} = P_{\text{loss, measured}} - P_{bl} - P_{\text{loss, exit}} - P_{\text{loss, wall friction}} \]  (5-44)
The outlet power loss due to the drag created by how the water level in the discharge basin interacts with the lower end of the screw. These losses are non-existent for cases where the screw is unsubmerged. These losses are further discussed in section 5.3.6.

Plotting $P_{loss, plane friction}$ versus $\omega T_p n_b / f_{DW,3}$ yields an approximately linear relationship where the slope is equal the shear stress friction factor for the helical planes. For the experimental data, $\omega T_p n_b / f_{DW,3}$ was solved by numerically integrating Equations (5-33) and (5-34) for both the upstream and downstream helical plane surface radii and Equations (5-31) and (5-32) for the wetted areas, then calculating $T_P$, without the friction factor.

Figure 5-15 shows the linearized helical plane friction power loss plotted against the predicted plane friction loss power divided by the friction factor ($f_{DW,2}$). In this figure, all flow rates are plotted except for the lowest flow ($Q = 4$ L/s). The lowest flow rate ($Q = 4$ L/s) was not considered because the power losses for this low flow did not follow the general trends found with flow rates that resulted in full buckets are normal operating rotational speeds suggesting low rates have additionally dynamic losses not taken into account.
The friction factor $f_{DW,2}$ can be approximated as the slope of the line of best fit from the linearized plane friction power loss data presented in Figure 5-15. This gives a value for $f_{DW,2}$ of 0.034. For the experimental setup conducted in this experiment, the material of the AST planes was painted steel. This friction factor derived from the shear-stress dominated data in Figure 5-15 is nearly identical to the average friction factor of 0.034 determined for the trough and center cylinder. The percent difference between the ascertained helical plane friction factor and the average friction factor for the trough and cylinder was approximately 1.9%. This suggests that the Manning’s $n$ value for the construction material of the planes may provide a decent approximation of the friction factor of the helical planes generally, and therefore, if no experimental data is available to determine the friction factor for a given AST, one can still be successfully approximated.

Figure 5-15. Power loss primarily due to the shear stress on the helical planes/friction factor ($f_{DW,2}$)
Therefore, a shear stress frictional factor \( (f_{DW}) \) of 0.34 is an approximation for painted or coated steel constructed ASTs operating under standard conditions.

Using a shear stress friction factor of 0.34, the helical plane shear stresses and resulting power losses for all the experimental data were predicted using the procedure outlined in Chapter 5.1.2.2. The resulting predicted power losses for all experimental runs are seen in Figure 5-16. As previously predicted, the shear frictional power losses on the planes makes up the majority of the power loss at high rotational speeds (ranging between 50% and 70% of the total power loss). Conversely, at rotational speeds below the Muysken limit (6.97 rad/s), the shear plane frictional power losses comprise of a small fraction of total losses. On average, for all measurements below 6.97 rad/s, the shear frictional power loss on the planes amounted to only 6.4% of the total measured power losses, further suggesting that in this range of speeds, the overall power loss is dominated by the lower water level torque loss.

![Figure 5-16. The predicted shear stress power loss from the helical plane surfaces](image)

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5.3.6 Outlet Head Loss Model

In this section, a model describing the power losses due to outlet torque drag is described using the principles outlined in Chapter 5.1.4. The measured total power losses are again taken as the difference between the Lubitz et al. (2014) power model and experimentally measured power. Figure 5-17 shows the measured mechanical power at the shaft for all experimental tested flow rates and increasing lower water levels (Ψ) in the region dominated by torque outlet losses.

![Figure 5-17](image)

*Figure 5-17. Experimentally measured power for various outlet water level heights for the region dominated by the outlet torque drag losses for i) \( Q = 4 \) L/s, ii) \( Q = 7.5 \) L/s, iii) \( Q = 10 \) L/s, iv) \( Q = 12.5 \) L/s. Included (blue) is the Lubitz et al. (2014) power prediction for each experimental run.*
The power loss due to the outlet torque drag is isolated from all over power losses by first computing the other individual power losses for each experimental run and subtracting those from the measured power. These calculated power losses include all power losses described in Chapters 5.3.2 through 5.3.5 (outlet exit loss, bearing friction, shear stress on the trough and center cylinder, and shear stress friction on the helical planes). This means the outlet torque drag power loss can be calculated as:

$$P_{\text{loss, outlet drag}} = P_{\text{loss, measured}} - P_{\text{bl}} - P_{\text{loss, exit}} - P_{\text{loss, wall friction}} - P_{\text{loss, plane friction}}$$ (5-45)

The resulting outlet torque drag power losses for all of the experimental flow rates and lower water levels can be seen in Figure 5-18. As predicted, the outlet drag losses are at a minimum (close to zero) when the lower water level ($\Psi$) is zero and increase with increasing $\Psi$. There is a flow dependence to the outlet drag losses, as the magnitude of the power losses increases with increasing flow rate. Similarly, there is also a speed dependence as increasing rotational speed results in an increased outlet drag power loss, as expected.
Experimentally determined power outlet drag losses for various outlet water level heights for i) $Q = 4$ L/s, ii) $Q = 7.5$ L/s. iii) $Q = 10$ L/s, iv) $Q = 12.5$ L/s.

The torque and flow rates are non-dimensionalized according Equations (5-40) and (5-41) respectively and plotted in Figure 5-19. In this figure, all the experimental data was grouped together by the lower water level $\Psi$, with each line containing points from all the flow rates. As previously predicted, plotting non-dimensional outlet torque losses ($\eta_T$) versus non-dimensional flow ($Q_{nd}$) yields distinct relationships for the predicted outlet drag losses for each level of $\Psi$ tested.

It should be noted, the lowest flow rate experimentally tested ($Q = 4$ L/s) is not included in the results shown in Figure 5-19. This is because at this flow rate, the experimental data was not
consistent with the other runs and did not follow the observed trends. This, again, highly suggests at low flow rates, there are additional losses and dynamics that are not fully understood.

The quality of the outlet torque drag power loss and flow non-dimensionalization can be seen to improve with increasing $\Psi$. For $\Psi$ values below 0.60, the scatter in $\eta_T$ values suggests some remaining dependence on volume flow rate that is not captured is this non-dimensionalization.

![Figure 5-19](image.png)

*Figure 5-19. Non-dimensional outlet torque drag loss versus non-dimensional AST flow for varying levels of $\Psi$ and all flow rates.*

Second order polynomial fits for the relationship between $\eta_T$ and $Q_{nd}$ were made for each value of $\Psi$ from the experimental data. Figure 5-20 shows the resulting polynomials and Table 5-2 displays the resulting polynomial equations for the varying levels of $\Psi$. Note, as previously mentioned, the quality of the parameter non-dimensionalization can be seen to generally improve with increasing $\Psi$, as demonstrated with increasing $R^2$ values for the polynomials to the experimental data. The experimentally derived polynomials provided in Table 5-2 are limited to the range of values tested.
in this experiment. Therefore, $\Psi$ should be limited between 0 and 1.31. Similarly, $Q_{nd}$ should be restricted to values between 0.74 and 5.0 in order for this outlet model to be applicable.

Outlet depths $\Psi$ above 1.31 would represent lower water levels representing outlet submergence well beyond the screw’s outer diameter. At these extreme water levels, the power losses can often be double or triple the power losses observed at the most efficient points. Most real-world AST units do not operate in this region, making this restriction acceptable. Similarly, most practical AST rotational speeds and flow conditions limit the $Q_{nd}$ values that occur in practice. High flow rates that create $Q_{nd}$ values less than 0.74 typically would be well beyond the design flow rate for a practical AST. Similarly, low flow rates that create $Q_{nd}$ values greater than 5 would be well below the normal design flow rates. High rotational speeds associated with $Q_{nd}$ values greater than 5 would have significant shear stress losses while low rotational speeds that create $Q_{nd}$ values below 0.74 would be operating well below the maximum efficiency for a given AST. Therefore, the restriction range for $Q_{nd}$ should encompass most real-world AST units.

Figure 5-20. The experimental derived polynomial fits for the non-dimensional torque ($\eta T$) as a function of non-dimensional flow ($Q_{nd}$) for varying levels of $\Psi$. 
Polynomial equations for the non-dimensional torque ($\eta_T$) as a function of non-dimensional flow ($Q_{nd}$) for varying levels of $\Psi$:

<table>
<thead>
<tr>
<th>$\Psi$ (-)</th>
<th>$\eta_T = f(Q_{nd})$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$\eta_T = 0.02076 \cdot Q_{nd}^2 - 0.1545 \cdot Q_{nd} + 0.2885$</td>
<td>0.7732</td>
</tr>
<tr>
<td>0.25</td>
<td>$\eta_T = 0.01765 \cdot Q_{nd}^2 - 0.1307 \cdot Q_{nd} + 0.1989$</td>
<td>0.7544</td>
</tr>
<tr>
<td>0.43</td>
<td>$\eta_T = 0.02233 \cdot Q_{nd}^2 - 0.1394 \cdot Q_{nd} + 0.3330$</td>
<td>0.6902</td>
</tr>
<tr>
<td>0.60</td>
<td>$\eta_T = 0.02849 \cdot Q_{nd}^2 - 0.1429 \cdot Q_{nd} + 0.3750$</td>
<td>0.8952</td>
</tr>
<tr>
<td>0.77</td>
<td>$\eta_T = 0.02977 \cdot Q_{nd}^2 - 0.1136 \cdot Q_{nd} + 0.3907$</td>
<td>0.9611</td>
</tr>
<tr>
<td>0.94</td>
<td>$\eta_T = 0.03892 \cdot Q_{nd}^2 - 0.1282 \cdot Q_{nd} + 0.4734$</td>
<td>0.9804</td>
</tr>
<tr>
<td>1.10</td>
<td>$\eta_T = 0.04534 \cdot Q_{nd}^2 - 0.1395 \cdot Q_{nd} + 0.5589$</td>
<td>0.9707</td>
</tr>
<tr>
<td>1.31</td>
<td>$\eta_T = 0.05080 \cdot Q_{nd}^2 - 0.1576 \cdot Q_{nd} + 0.6564$</td>
<td>0.9845</td>
</tr>
</tbody>
</table>

The resulting non-dimensional polynomials seen in Table 5-2 link the flow properties as well as the scale of the geometry to the power losses associated with the outlet drag forces in a manner that should allow for scalability. Therefore, the power outlet drag losses indicated in Table 5-2 should apply to an AST of any size, provided the scale of the input parameters ($Q_{nd}$ and $\Psi$) are within the boundary range tested in this experiment.

### 5.4 Power Loss Model Summary

The power loss model presented in Chapter 5 describes five component AST losses:

- Inlet entrance head loss
- Outlet exit head loss
- Screw interior hydraulic fluid frictional head loss
- Bearing friction head loss
- Outlet drag torque head loss
Models were presented that allow for each of these losses to be determined independently. Four of losses translate to direct mechanical power losses at the screw (outlet exit head loss, hydraulic frictional head loss, bearing friction head loss, and outlet torque head loss). The inlet entrance head loss is an energy loss that propagates upstream in the delivery channel to impact the required reservoir head (the amount of head needed in the supplying reservoir to provide the desired design flow rate to the AST).

The models presented for the inlet entrance losses, outlet exit losses, hydraulic frictional losses and bearing friction losses are physics based models, grounded in the operating mechanics of the problem. The hydraulic frictional losses are divide into two categories: shear stress losses on the AST containing trough/center shaft and shear stress losses created by the rotation of the helical planes. Both of these shear stress losses are calculated independently. The outlet drag torque loss is an empirically-derived, non-dimensional model created to determine the scale of the outlet torque drag losses for a given AST.

The outlet head loss, and shear stress losses on the trough and center shaft can be considered minor losses, comprising only a small fraction of the observed total power losses. The average outlet exit and trough/center shaft shear stress losses were found to be approximately 2.0% and 0.36% of the total observed power loss respectively. The bearing losses were found to scale with rotational speed. On average, the bearing losses represented approximately 15.9% of the total observed power loss. These bearing losses were specific to the experimental AST units used and may not be indicative of bearing losses in full-scale real-world ASTs.
The two dominant power losses were the outlet torque drag and helical plane shear stress power losses. At low speeds, below the Muysken limit, the outlet torque drag power losses were dominating, typically representing approximately 50% to 70% of the total power losses. At speeds about the Muysken limit, the shear stress friction on the helical planes dominated. At speeds approaching 200 to 250 RPM in the test AST, the frictional shear stress power losses on the planes represented approximately 70% of the total observed energy losses. On average, the shear stress from the helical plane surfaces represented 23.1% of the total observed power losses.
Chapter 6: Validation of AST Power Loss Model

In this chapter, the complete AST power loss model described in Chapter 5 is used to predict the power losses for a real-world, grid-connected AST unit. The purpose of this chapter is to determine whether the AST power loss model derived drawing from experience with laboratory-scale AST units is applicable to full-sized AST units operating in the actual world. To complete this validation, measurements were conducted of electrical power output, water volume flow rate, and inlet and outlet water levels on a grid-connected 7 kW AST installation. The AST power output was also modeled using the Lubitz et al. (2014) power model, with the power results amended using the Chapter 5 power loss model to account for the additional losses experienced by the AST. A comparison is made between the measured real-world AST power measurements and those predicted with the loss-adjusted Lubitz et al. (2014) power model.

6.1 Experimental ASG unit

The ASG installation examined in this experiment was located in Waterford, Ontario, Canada. It was commissioned in September, 2013 making it the first commercial grid-connected, power generating ASG in North America. It was installed at Fletcher's Horse World, a recreation arena for horse related events and training on Nanticoke Creek. The unit was designed and installed by Greenbug Energy Inc. (Delhi, Ontario, Canada). The Fletcher’s ASG has operated reliably since commissioning and continues in active use.

The ASG was installed on a pre-existing dam structure, retrofit into a small building that previously used to house a hydroelectric turbine more than half a century ago. The dam includes a height-adjustable weir through which water normally flows and continues downstream. Water from the
reservoir behind the dam is routed into a side channel through the building to reach the ASG. The outlet of the ASG is situated on the side of Nanticoke Creek just downstream of the dam. The discharged flow to return to the natural creek watercourse.

The ASG was installed in a net-metering configuration and is used to offset the power demands of the nearby equestrian center, which typically exceed the energy production of the ASG, allowing the property owners to the amount of electricity purchased from the power grid.

Figure 6-1 shows the building that now contains the ASG, before the start of construction. On the left side of this image, water can be seen flowing under a bridge over the creek; this is water that is discharged over the dam and is diverted away from the AST. Figure 6-2 shows the dam face and the ASG installation.
Figure 6-1. Dam and building before installation of the ASG.

Figure 6-2. The building after ASG installation with dam overflow on left. The angled enclosure contains the AST.
Figure 6-3 shows a schematic of the experiment ASG within the building. (A real image of the ASG was not supplied because the screw is contained in an enclosure that makes photography difficult.)

![Figure 6-3. Fletchers Horse World commercial AST unit used in validation experiment](image)

The specifications for the Fletcher’s ASG are given in Table 6-1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum Design Output</td>
<td>7.2 kW</td>
</tr>
<tr>
<td>Grid Connection</td>
<td>240 V single phase net metered</td>
</tr>
<tr>
<td>Maximum Design Flow Rate</td>
<td>536 L/s</td>
</tr>
<tr>
<td>Design Head</td>
<td>1.7 m</td>
</tr>
<tr>
<td>Rotational speed</td>
<td>44.5 RPM (4.66 rad/s)</td>
</tr>
<tr>
<td>Outer Diameter (m)</td>
<td>1.39</td>
</tr>
<tr>
<td>Inner Diameter (m)</td>
<td>0.761</td>
</tr>
<tr>
<td>Number of turns (-)</td>
<td>3</td>
</tr>
<tr>
<td>Parameter</td>
<td>Value</td>
</tr>
<tr>
<td>--------------------</td>
<td>--------</td>
</tr>
<tr>
<td>Screw Length (m)</td>
<td>4.53</td>
</tr>
<tr>
<td>Angle (º)</td>
<td>22</td>
</tr>
<tr>
<td>Pitch (m)</td>
<td>1.39</td>
</tr>
<tr>
<td>Gap Width (mm)</td>
<td>2</td>
</tr>
</tbody>
</table>

### 6.2 Experimental Methodology

A measurement campaign was conducted on November 13, 2014, to measure the electrical power output, water volume flow rate, and inlet and outlet depths during operation of the Fletcher’s ASG. The water levels in the reservoir and the downstream receiving basin were intentionally manipulated to provide measurements across a range of inlet and outlet depths and volume flow rates.

#### 6.2.1 Experimental Setup

Unlike the laboratory experimental setups described in Chapters 4 and 5, the Fletcher’s ASG had no basins with strict water level controls. Instead of an upper basin supplying water to the screw, the impounded reservoir behind the dam acted as the water source. Water was diverted from the impounded reservoir to the AST through an entrance weir and an inlet channel connecting the reservoir and the screw. Similarly, at the screw outlet, the ASG discharged directly to a naturally regulated water course instead of a height-controlled receiving basin. The downstream water level was determined by natural hydraulics of the stream. This made it much more difficult to control as the only control mechanism was the overflow weir in the dam itself. The water level downstream of the AST could be influenced by the amount of water flowing over the dam. However, changes to this water level required adjusting the dam crest height, which also affects
the amount of water supplied to the AST delivery channel, making this an imprecise control method.

Flow measurements were taken at the inlet of the AST channel using a Cipoletti weir, while power measurements were made from the grid-connected electrical meter. These power measurements were not made at the shaft but rather at the meter, meaning they include all the AST power losses described in Chapter 5, as well as additional losses from the gearbox, generator and electrical controls. The gearbox and electrical losses are not included in the power models described in Chapter 5 and are beyond the scope of this thesis.

Flow into the AST was controlled by regulating the amount of water allowed into the inlet channel, which was controlled by changing the water level in the reservoir. The higher the water in the reservoir, the greater the water entering into the delivery channel. Since the reservoir takes time to fill or empty, the experiment started with a very high water level in the reservoir. Over the course of several hours, the reservoir was drawn down by increasing the discharge over the dam (by lowering the dam crest height). As the water level declined in the reservoir, flow through the AST declined as well.

The experimental AST dimensions and properties can be found in Table 6-1. Since the Fletchers Horse World ASG is a grid-connected and utilizes an induction generator, the rotational speed is essentially fixed by the grid frequency. Therefore, the Fletchers Horse World AST rotated at a constant speed of 44.5 RPM (4.66 rad/s) when generating power.
6.2.2 Experimental Measurements

6.2.2.1 Flow Measurements

Several attempts to measure the flow inside the AST delivery channel were made, however, due to the channel design and shape, all attempts were unsuccessful. The delivery channel was both very deep (over twice the depth of the screw outer diameter) and relatively wide, with a few minor obstructions that resulted in an inlet of very low velocity with both dead current zones and secondary flows that made using propeller or ultrasonic flow velocity sensors impractical. Instead, a Cipoletti weir was installed at the entrance of the AST inlet channel to measure volume flow rate of water entering the inlet channel (and AST).

A Cipoletti weir is a fully contracted trapezoidal weir with non-vertical notch sides. The sides are angled to compensate for contraction losses through the weir, simplifying the governing equations. The volume flow rate over a Cipoletti weir is proportional to the water head above the weir crest. Cipoletti weirs are well-studied and understood in literature, and the relationships between the upstream head and weir flow are well established.

In this experiment, the reservoir water level, or weir crest height, was measured relative to the weir crest. The flow through the weir was then calculated according to Addison (1949):

$$Q = 0.63 \left(\frac{2}{3}\right) \sqrt{2gL_wh^{3/2}} \quad (6-1)$$
where \( Q \) (m\(^3\)/s) is the total discharge over the weir (and total flow through the AST), \( g \) is the gravitational constant (9.81 m/s\(^2\)), \( L_w \) (m) is the width of the bottom of the weir opening, and \( h \) is the water level above the weir crest. The side slopes of the Cipoletti weir used were 4:1 (vertical:horizontal). Figure 6-4 shows the layout and dimensions of the Cipoletti weir used. The bottom weir length \( (L_w) \) was 1.4 m.

![Figure 6-4. Cipoletti weir used to determine experimental AST water volume flow rates.](image)

### 6.2.2.2 Power Measurements

Power measurements were taken directly off the power meter, which connected the ASG to the grid. These measurements were manually recorded at intervals of approximately 5 to 10 minutes apart. Since the power measurements were taken at the meter, these power measurements include all power losses, including the losses outlined in this chapter, in addition to electrical power losses from the electrical equipment itself.
6.2.2.3 Upper and Lower Water Level Measurements

The water levels, both upstream and downstream of the AST were recorded using Keller America submersible Level Transmitter (Levelguage model) pressure-based depth sensors, connected to a Campbell Scientific CR1000 datalogger. Additionally, water depth measurements were taken manually using a metering stick to verify metered depths. The water levels, both just upstream of the AST as well as in the reservoir were recorded. The reservoir levels were tracked to ascertain the height of water above the Cipoletti weir crest, which was used with Equation (6-1) to calculate the volume flow rate entering the ASG.

6.3 Experimental Results and Discussion

A total of 44 observations were made over a period of 6 hours. For each observation, the Lubitz et al. (2014) model was applied to predict the ideal shaft power of the AST. Additionally, each of the AST power losses were computed independently. Chapter 5 outlines the power loss calculations for: outlet exit power loss, hydraulic frictional power loss, bearing friction power loss, and outlet drag torque power loss. The complete experimental data set can be found in Appendix E. These predicted values will then be compared to the measured electrical power output.

6.3.1 Outlet Exit Power Loss

A set of assumptions had to be made regarding the discharge outlet channel in order to calculate outlet exit power losses. The AST effectively discharges into a small bay adjoining a small stream. The bottom dimensions of the stream were not known but the vertical distance from the stream bed to the center of the screw was 79.5 cm at the end of the screw. Therefore, the outlet ‘channel’
was assumed to be rectangular, with a base width twice the size of the AST outer diameter \(2D_o = 2.78\) m). The Borda-Carnot loss coefficient and associated power losses for all points were calculated using Equations (5-9) and (5-11). Figure 6-5 shows the calculated outlet exit power losses \((P_{loss,\,exit})\) for all experimental measurements plotted against their corresponding flow rates.

![Figure 6-5. The outlet exit power losses for all experimental runs (Fletchers Horse World AST)](image)

6.3.2 Internal Hydraulic Friction Losses

6.3.2.1 Shear Stress on Trough and Center Shaft

Both the containing trough and the center shaft of the AST were constructed from smooth steel. A Manning’s \(n\) friction factor value of 0.012 was chosen to be consistent with smooth steel (Ward, 2003). The power losses due to the shear stress frictional forces on the trough and center shaft were calculated as outlined in Chapter 5.1.2.1. The wetted areas (portion of the trough and center shaft in contact with the water) were calculated through numerical integration of Equations (5-18)
and (5-20). The resulting predicted power losses due to shear frictional forces acting on the trough and center shaft are seen in Figure 6-6 for each measured flow rate.

![Figure 6-6. The trough and center shaft shear stress frictional power losses for all experimental runs (Fletchers Horse World AST)](image)

**6.3.2.2 Shear Stress on Helical Planes**

All power losses due to fluid friction shear stress on the helical planes were calculated in accordance with Chapter 5.1.2.2. The average weighted radii ($\overline{r_1}$) and ($\overline{r_2}$) were calculated through numerical integration of Equations (5-33) and (5-34) respectively. Similarly, the weighted areas on the upstream plane ($A_1$) and downstream plane ($A_2$) were solved by numerically integrating Equations (5-31) and (5-32). Darcy-Weisbach shear stress frictional factors ($f_{DW,2}$ and $f_{DW,3}$) of 0.034 was chosen for all runs. This number was obtained in Chapter 5.3.5.2 from the experimental measurements made on the scale-model AST unit. This value was consistent with Darcy-Weisbach friction factor values for the trough and center shaft. Furthermore, the Fletchers Horse
World AST unit was similarly constructed from smooth steel (similar to the scale-model AST). The shear stress losses for all experimental runs are plotted against the measured flow rates in Figure 6-7.

![Graph showing power loss vs flow rate]

*Figure 6-7. The helical plane shear stress frictional power losses for all experimental runs (Fletchers Horse World AST)*

### 6.3.3 Bearing Friction

The frictional forces, and associated power losses, on the bearings were specified by the bearing manufacturer were determined for each bearing separately. The unsubmerged upper bearing power losses were calculated according the manufactures specifications for the bearing type used in this commercial ASG unit. Operating at a constant velocity of 40 RPM and for the radial and axial loading applied to the bearing due to the weight of the screw, the upper bearing loss was taken as a constant 0.06 kW. The submerged downstream bearing power loss was manufactured directly by Greenbug Energy Inc. for this ASG installation. The lower bearing power losses were
determined empirically and experimentally to be approximately 0.048 of the total power produced by the screw. Therefore, the total bearing losses for each experimental run was determined as:

\[ P_{bl} = 0.048 \cdot P_s + 0.06 \]  \hspace{1cm} (6-2)

where \( P_s \) (kW) is the power the screw produces and \( P_{bl} \) (kW) is the power loss due to both bearings. Figure 6-8 shows the predicted bearing power losses for the Fletchers Horse World AST for all experimental runs.

![Figure 6-8. The bearing power losses for all experimental runs (Fletchers Horse World AST)](image)

### 6.3.4 Outlet Torque Drag Power Losses

The outlet power losses associated with the submerged portion of the screw were determined using the empirical non-dimensional model detailed in Chapter 5.1.4. The torque ratio drag coefficients \( (\eta_T) \) were obtained using the equations derived in Chapter 5.3.6 based on the experimental laboratory scale-model relationships in Figure 5-20 and the polynomial equations in Table 5-2.
For each data point, $\Psi$ values were calculated using Equation (5-38) from lower water measurements ($h_o$) relative the bottom the screw. Additionally, non-dimensional flow ($Q_{nd}$) values were obtained for each experimental measurement using Equation (5-41). Using the equations for $\eta_T$ as function of $Q_{nd}$, drag torque ratios were obtained for each measurement were obtained for their corresponding $\Psi$ values. When actual $\Psi$ values that lied in between the discrete $\Psi$ values corresponding to each equation in Table 5-2, $\eta_T$ were linearly interpolated between the curves above and below the required $\Psi$ value. The actual power losses due to the outlet torque drag force were obtained by multiplying the torque drag coefficient ($\eta_T$) by the modelled power ($P_{modelled}$) in accordance with Equation (5-40). The resulting outlet torque drag power losses for each experimental run is seen in Figure 6-9.

![Figure 6-9. The outlet torque drag power losses for all experimental runs (Fletchers Horse World AST)](image)
6.3.5 Generator Losses

This experiment did not measure the torque applied to the screw shaft to obtain power measurements, as was done in the experiments conducted in Chapter 5. Instead, measurements were made of the electrical power supplied to the grid. This final power measurement includes additional electrical power losses in the gearing and generator that both the Lubitz et al. (2014) and amended model outlined in Chapter 5 do not take into account. These additional losses are a function of the electrical equipment that accompany the AST. These losses were approximated through a proprietary electrical power loss model supplied by GreenBug Energy Inc. through personal correspondence with GreenBug Energy Modelling and Design Specialist Murray Lyons in December of 2015. The predicted electrical power gearing and transformer power losses are seen in Figure 6-10. The generator is known to be less efficient at lower output powers. This can be seen in Figure 6-10, the generator is less efficient and incurs a greater power loss at lower flow rates, corresponding to less output power from the AST.

![Figure 6-10. The generator power losses for all experimental runs (Fletchers Horse World AST)](image)

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6.3.6 Power Loss Composition

Figures 6-11 and 6-12 show the relative proportions of the independent loss components in comparison to the total predicted power losses for all experimental flow rates. Figure 6-11 shows the raw magnitudes of the power losses (in Watts) while Figure 6-12 shows the component losses as a percentage of the total losses. As was the case with the experimental laboratory scale-model AST tested in Chapter 5, the Fletchers Horse World AST’s power losses are dominated by the torque outlet drag losses and fluid friction shear losses on the helical planes. The Muysken limit (Equation (5-43)) for the Fletchers Horse World AST is approximately 40.1 RPM. The AST’s actual rotation speed was 40.91 RPM, which is practically at this limit, suggesting the AST is operating at a rotational speed where the losses begin to transition from outlet drag dominated to helical plane shear stress dominated. This is likely why the AST’s overall power composition is dominated by the lower water level outlet torque drag loss, but still has a significant component comprised of the loss from the shear stress on the helical planes.
Figure 6-11. The composition of the calculated Fletchers Horse World AST power losses (raw magnitude) relative to the total predicted power loss versus flow rate

Figure 6-12 shows the Fletchers Horse World AST predicted power losses as percentages of the total predicted power loss, for all measured flows. Again, this figure shows the overall power loss is dominated by the torque outlet drag loss and fluid friction shear power loss on the helical planes. The power losses associated with the shear friction on the helical planes is seen to increase with increasing volume flow rate. This is likely because the size of the wetted areas within the screw increase with increasing flow rate. According to Equation (5-27), the frictional shear stress on the planes increases as the wetted areas increase. The outlet torque drag loss is seen to decrease with increasing flow rate; this phenomenon is addressed below. On average, the torque outlet drag power losses, helical plane shear friction power losses, and bearing friction losses comprised approximately 41.2%, 14.3%, and 12.3% of the total predicted power losses respectively. This
represented over 67.6% of the power losses. There was a significant power loss associated with
the generator, which averaged approximately 26.5% of the total power loss for all experimental
runs, particularly at low flow rates.

![Graph showing power losses versus flow rate]

*Figure 6-12. The composition of the calculated Fletchers Horse World AST power losses (percentage) relative to the total predicted power loss versus flow rate*

Also seen Figure 6-12, the power losses from the shear stress wall friction (on the trough and center
shaft), bearing friction, and the exit loss at the outlet can be considered minor losses relative to the
overall power loss. On average, the wall shear stress frictional losses and exit losses were 1.38%,
and 4.47% respectively, of the total predicted power losses.
As previously mentioned Figure 6-11 and Figure 6-12 show that the overall power losses tend to decline with increasing volume flow rate. This is mainly due to the changing lower water level. Figure 6-13 shows the measured lower water level fraction ($\Psi$) plotted against the measured flow rates for all experimental runs. Throughout the experiment, the volume flow through the AST was gradually decreased. Reducing the volume flow rate supplied to the AST caused an increase in flow over the dam which, in turn, caused more flow to discharge into the downstream portion of the stream, raising its level. Furthermore, since the AST discharged into a natural stream, completely controlling the lower water level was difficult. As a result, the downstream lower water fraction ($\Psi$) increased as the flow rate through the AST increased, as shown in Figure 6-13.

![Graph showing lower water level fraction vs. flow rate](image)

*Figure 6-13. The lower water level fraction ($\Psi$) for all the experimental flow rates tested.*

Therefore, the declining power loss is attributed to the reduced levels of $\Psi$ that occurred at the high flow rates, and is not a result of the increasing flow directly. Figure 6-14 shows the same power
losses presented in Figure 6-11, but the power losses are plotted against the observed lower water level ($\Psi$). The predicted power losses can clearly be seen to increase with increasing lower water level which, by happenstance of the nature of the experiment, corresponded to reduced flow rates. Therefore, the apparent reduction in power loss with increased flow was actually caused by the variations in the water lower water level.

![Graph showing power loss vs lower water level](image)

*Figure 6-14. The composition of the calculated Fletchers Horse World AST power losses (raw magnitude) relative to the total predicted power loss versus lower water level fraction ($\Psi$)*

### 6.3.7 Power Loss Model Comparison

Figure 6-15 shows the measured electrical power output of the Fletchers Horse World ASG compared the Lubitz et al. (2014) AST power model and the Lubitz et al. (2014) model adjusted for the power losses described in Chapter 5. As seen in Figure 6-15, and as expected, the Lubitz et
al. (2014) model consistently over-predicts the measured power. The average percentage difference between the Lubitz et al. (2014) model and the experimental power data is approximately 89.7%. The greatest deviations between the Lubitz et al. (2014) predicted power and measured power occurred at low volume flow rates, where a maximum percentage difference of 206.8% occurred.

![Power vs Flow Graph](image)

*Figure 6-15. A comparison of the output power of a real-world AST (Fletchers Horse World) and the Lubitz et al. (2014) power model adjusted with the Chapter 5 power loss models.*

The amended Lubitz et al. (2014) model which includes the power losses detailed in Chapter 5 dramatically reduces these percentage errors. As seen Figure 6-15, the amended Lubitz et al. (2014) model converges closer to the experimentally measured powers. The average percentage difference between the amended Lubitz et al. (2014) model and the experimental data is approximately 14.0%, a reduction in relative error by 75.7%. However, the amended model also seems to have its greatest power over-predictions at low flows as the deviation between the amended model is greatest in this region. The maximum percentage difference between the
amended model and the experimental data was found to be 46.7\% and occurs at the lowest flow rate measured. This suggests that there are additional losses and flow dynamics at low flow rates that neither the Lubitz et al. (2014) model, nor the amended model that accounts for all the power losses in this flow region. This finding is consistent with all the experimental data collected in Chapter 5. All power trends collected in the laboratory-scale AST at the lowest flow rates in Chapter 5 also failed to align with the rest of the experimental data.

While the amended model creates much better approximations for the real-world AST unit, there is still a considerable gap between the predicted and measured powers. It should be noted this experiment measured only measured electrical power supplied to the grid, and not mechanical power at the shaft. Therefore, there are likely power losses associated with the electric generator and transformer that are not described in the power loss model outlined in Chapter 5; it is likely that these losses could be responsible for the observed difference between the present power loss model and the experimental data.

Another potential source of error could be the frictional factor ($f_{DW,2}$) used for the shear stress friction loss calculations. The value used was obtained from the laboratory scale-model experimental results presented in Chapter 5.3.5.2. It was assumed that the composition of the construction material of the scale-model and the Fletchers Horse World AST were the same, and that variations in this friction factor were negligible and therefore the friction factor remained constant. Both of these assumptions cannot be directly verified. It should be noted that adjusting the friction factor would make it possible to converge the predicted total power and the measured power to a much better degree.
6.4 Experimental Conclusions

In Chapter 6, real-world ASG power measurements were compared to the power measurements predicted by the Lubitz et al. (2014) model and the Lubitz et al. (2014) model amended to include the power losses described in Chapter 5. Using the power loss model from Chapter 5, the observed errors between the Lubitz et al. (2014) power model and the experimentally measured power data were seen to have better agreement. The average percent difference between the predicted and measured powers was reduced from 89.7% to 31.3%, for the Lubitz et al. (2014) and the power loss adjusted model respectively. Both the original and power loss adjusted models produced additional over-predictions of power at low flows that do not seem to be accounted for by the current model, suggesting additional, unaccounted for power losses at lower flows.

The disagreement between the power loss amended Lubitz et al. (2014) power model and the experimentally measured data most likely occurred because of either:

i. Inappropriate assigning of the plane shear stress frictional coefficient ($f_{DW,2}$)

ii. Additional, unaccounted for electrical losses in the generator and transformer

iii. Additional second-order dynamic effects magnified by relatively lower fill levels

iv. Additional dynamics in the bearings. (Both upper and lower bearings are of different types than those in the laboratory screw.)
Chapter 7: AST Delivery Channel Losses and Required Reservoir Head

For most commercial ASTs, flow is supplied to the screw through a diversion channel that routes water from an impounded reservoir to the entrance of the turbine. This upstream diversion, or delivery, channel has associated energy losses. This chapter will attempt to quantify the total required head above the AST screw bottom in order to provide the AST with a design flow rate. This required head is calculated in two parts. First, the water height at the inlet of the screw is approximated using idealized mass flow principles. Second, the delivery channel losses are calculated using standard open-channel flow hydraulic loss methods with boundary conditions imposed by an AST. This procedure results in an approximation for determining the head of water required in order to provide an AST with a desired flow rate. Finally, this chapter will present a theoretical test scenario for calculating the required head.

7.1 Delivery Channel Losses and Required Reservoir Head Calculation

Since ASTs are typically non-pressurized, open-channel flow devices, their delivery channels are usually open-channels as well. Therefore, the required head in the upstream reservoir needed to provide the AST with a desired flow rate can be calculated by ascertaining the energy losses that occur within the delivery channel. Most ASTs are coupled with a dam-reservoir system which acts as the water source. Therefore, AST inlet channel losses can be calculated using standard open-channel loss procedures, with downstream and upstream boundary conditions governed by the AST-controlled flow regime and the upstream reservoir depth. This section will outline the
required calculations for the delivery channel and overall head requirements at the reservoir to provide a desired flow rate.

7.1.1 AST Delivery Channel Losses

At the upstream end of the inlet channel, the reservoir is usually large relative to the AST approximately still water. At the downstream end of the inlet channel, the flow conditions and water level are governed directly by the AST and are a function of the flow and rotational speed of the AST. Chapter 5.1.1.1 outlines the calculation procedure for determining the delivery channel water depth just upstream of the AST. If the AST flow rate and rotational speed are constant, this water depth will remain constant. Similarly, reservoir depths are usually governed by overflow of a dam, and remain effectively constant for long periods of time. Therefore, the upstream and downstream boundary conditions can be considered to remain constant at both ends of the inlet channel, and the problem can be approached as quasi-steady state. Figure 7-1 shows a simple reservoir/delivery channel schematic. The procedure outlined in this section will provide a method of calculating the water level ($H_{res}$) in the reservoir needed to supply the AST with a desired flow rate for a simple rectangular channel.
Figure 7-1. Schematic of a simple AST reservoir and delivery channel. $H_{res}$ is the water level in the reservoir to supply the AST for a required flow rate.

The calculation of the energy losses along the AST inlet channel can be approached similar to a standard open-channel loss calculation problem known as the “two-lake” or “channel-delivery” problem, which is a method of calculating flow through a delivery channel situated between two reservoirs with constant depths (Jain, 2001). However, at the downstream end of the delivery channel, an AST acts as the downstream control mechanism.

At either end of the channel there are associated transition losses. At the outlet, the delivery channel exit head loss ($H_{d,\text{outlet}}$) can be predicted using the Borda-Carnot head loss model (Chapter 5.1.1). At the entrance of the delivery channel, a simple reservoir-to-channel contraction head loss ($H_{d,\text{inlet}}$) can be applied such that the head loss is proportional to the velocity head in the inlet channel:

$$H_{d,\text{inlet}} = K_e \frac{v_d^2}{2g} \quad (7-1)$$

where $v_d$ is the channel flow velocity just downstream of the entrance, $g$ is the gravitational constant (9.81 m/s$^2$) and $K_e$ is a loss coefficient dependent on channel geometry. For a sharp-edge
rectangular inlet channel, $K_e$ is approximately 0.5 (White, 2005); rounded edges can reduce this value, and associated head loss, dramatically.

To calculate the frictional losses and the water depth profile throughout the delivery channel, open-channel friction calculations are applicable. The flow regime in the inlet channel can be considered as one-dimensional gradually varying flow. For this type of flow regime, the two standard models for calculating channel friction losses and elevation profiles are the direct-step method and the standard-step method. Most ASTs utilize an inlet channel with a simple and constant geometry throughout, therefore, while both models apply, this chapter will outline the direct-step method.

In order to calculate the frictional losses along the AST inlet channel, the water profile type must be determined. First, the critical and normal depths in the channel must be determined, which are properties of the channel flow and geometry. The critical inlet channel depth ($y_{d,c}$) occurs where the Froude number is equal to 1. For a rectangular channel, it can be shown that the critical depth is determined by:

$$y_{d,c} = \left( \frac{Q^2}{gb^2} \right)^{1/3}$$  \hspace{1cm} (7-2)

where $Q$ is the volume flow rate of water (i.e., the flow rate in the channel and AST), and $b$ is the width of the channel. The inlet channel normal dept ($y_{d,n}$) is calculated using Manning’s equation. For a rectangular delivery channel, Manning’s equation becomes:
\[ Q = \frac{b(y_{d,n})}{n} \left( \frac{b(y_{d,n})}{b + 2(y_{d,n})} \right)^{2/3} (s_o)^{1/2} \]  

(7-3)

where \( n \) is the Manning’s \( n \) friction factor for the channel material and \( s_o \) is the channel bed slope.

To find the normal depth, Equation (7-3) can be iterated for values of \( y_{d,n} \) until the calculated flow rate \( Q \) matches the desired AST flow rate.

If the delivery channel normal depth \( (y_{d,n}) \) is greater than its critical depth \( (y_{d,c}) \), the water surface profile is considered a mild slope profile in which case open-channel M1, M2, and M3 type slope profiles apply. Alternatively, if the inlet channel normal depth is less than its critical depth, then the slope profile is considered steep and S1, S2, and S3 slope profiles apply (Jain, 2001). If the normal depths and critical depths are the same, then the slope profile is a critical slope (Jain, 2001).

This class of slope typically results in unstable flow regimes and should avoided in channel design.

The slope profile cases for the AST inlet channel can be seen in Table 7-1.

<table>
<thead>
<tr>
<th>Flow conditions</th>
<th>Slope Profile Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h_d &gt; y_{d,n} &gt; y_{d,c} )</td>
<td>M1</td>
</tr>
<tr>
<td>( y_{d,n} &gt; h_d &gt; y_{d,c} )</td>
<td>M2</td>
</tr>
<tr>
<td>( y_{d,n} &gt; y_{d,c} &gt; h_d )</td>
<td>M3</td>
</tr>
<tr>
<td>( h_d &gt; y_{d,c} &gt; y_{d,n} )</td>
<td>S1</td>
</tr>
<tr>
<td>( y_{d,c} &gt; h_d &gt; y_{d,n} )</td>
<td>S2</td>
</tr>
<tr>
<td>( y_{d,c} &gt; y_{d,n} &gt; h_d )</td>
<td>S3</td>
</tr>
</tbody>
</table>
The relative positions between the inlet channel depth, normal depth, and critical depths can be seen in Figure 7-2 for both mild and steep slopes. Both M3 and S3 type slope profiles typically result from an inlet restriction such as a sluice gate in which a hydraulic jump occurs. When designing an AST inlet channel, it is desirable to reduce all channel losses so that the flow carries the maximum possible energy when it reaches the screw. Hydraulic jumps dissipate a considerable amount of energy and should be avoided. AST inlet channels should be designed to avoid M3 and S3 slope profiles so, therefore, these will not be considered further.

For one-dimensional gradually varied flow, specific energy ($E$) changes along the channel, taking $x$ as distance along the channel, are equal to:
where $S_o$ is the delivery channel bed slope and $S_f$ is the energy friction slope, or the energy loss due to friction at any given point. Equation (7-4) is a non-linear, first-order differential equation that requires one boundary condition to solve. This boundary condition is the water level created by the AST operating at a set rotational speed and flow rate. The specific energy is defined as energy at the water surface for any given point in the channel:

$$E = y + \frac{v_d^2}{2g}$$

where $y$ and $v_d$ are the channel depth and flow velocity at any given point. For open-channel flow, the friction slope can be determined in several ways but the most common is through rearranging Manning's equation to yield:

$$S_f = \frac{n^2 v_d^2}{R_h^{1/3}}$$

where $n$ is the Manning's $n$ friction factor, $v_d$ is the channel flow velocity, and $R_h$ is the hydraulic radius. Using Equations (7-5) and (7-6), Equation (7-4) can be discretized to solve for the upstream position changes that correspond to an elevation change ($\Delta x$) such that:
\[ \Delta x = \frac{\Delta E}{S_o - S_f} = \frac{y_2 - y_1 + \frac{V_{d1}^2}{2g} - \frac{V_{d2}^2}{2g}}{S_o - S_f} \]  

(7-7)

where \( y_1 \) and \( y_2 \) are the water surface elevations at two adjacent points in the delivery channel, and \( v_{d1} \) and \( v_{d2} \) are the corresponding velocities at these points. \( S_f \) is the average friction slope between points 1 and 2. The backwater surface profile can then be calculated upstream of the AST. This is done by fixing the first elevation point to the boundary condition height \( (h_d) \) that is created by the AST. A step direction in the direction of elevation, or \( y \)-direction, is chosen to ensure calculations progress upstream of the AST. For M1 and S1 slope profiles, this direction should be chosen such that the water elevation profile decreases upstream. For M2 and S2 profiles, the step direction should be such that the elevation profile increases upstream. The water surface profile can be calculated all the way to the entrance of the delivery channel, so that the water elevation is known. Through this process, the frictional energy losses along the channel are calculated at each point.

### 7.1.2 Required Reservoir Head

The total head required in the impounded reservoir can be determined once the water elevation profile in the delivery channel has been calculated according to Chapter 7.1.1. The total required reservoir head \( (H_{res}) \) is simply equal to sum the of the elevation head, velocity head, and inlet channel entrance head loss:

\[ H_{res} = y_{d,\text{entrance}} + \frac{V_{d,\text{entrance}}^2}{2g} + K_e \frac{V_{d,\text{entrance}}^2}{2g} \]  

(7-8)
where $y_{d, \text{entrance}}$ and $v_{d, \text{entrance}}$ are the water elevation and velocity at the inlet channel entrance respectively. The total friction loss in the channel ($H_{df}$) can be determined by subtracting the energy head at the inlet channel's outlet from the reservoir head and the total elevation change upstream:

$$H_{df} = H_{\text{res}} + L_{\text{channel}}S_o - \left( y_{\text{exit}} + \frac{V^2_{\text{exit}}}{2g} \right)$$  \hspace{1cm} (7-9)

### 7.2 Sample Delivery Channel Calculations

In this chapter, a sample rectangular delivery channel with a base width of 3.13 m, maximum depth of 2.5 m, and total channel length of 25 m supplying flow to an AST is considered. The delivery channel is a concrete channel with a graded of 1% slope. A theoretical AST is used to determine the outlet boundary condition for the delivery channel. The delivery channel dimensions can be found in Figure 7-3 and the AST properties can be found in Table 7-2. The required reservoir head ($H_{\text{res}}$) is calculated to supply the AST outlined in Table 7-2 with flow rates covering the range from 1 m$^3$/s to 6 m$^3$/s. The results are compared with the open-channel flow software package HEC-RAS in order to validate the proposed delivery channel model.
Figure 7-3. Concrete AST delivery channel dimensions used for sample calculations

Table 7-2. AST dimensions used to determine the delivery channel flow calculations

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slope, $\beta$</td>
<td>24°</td>
</tr>
<tr>
<td>Pitch, $S$</td>
<td>2.799 m</td>
</tr>
<tr>
<td>Outer Diameter, $D_o$</td>
<td>2.799 m</td>
</tr>
<tr>
<td>Inner Diameter, $D_i$</td>
<td>1.3716 m</td>
</tr>
<tr>
<td>Length, $L$</td>
<td>13.52 m</td>
</tr>
<tr>
<td>Rotational Speed</td>
<td>28.28 RPM (2.96 rad/s)</td>
</tr>
<tr>
<td>Gap Width, $G_w$</td>
<td>0.0075 m</td>
</tr>
</tbody>
</table>

7.2.1 AST Delivery Channel Water Height Calculation

The volume of water in a single bucket ($V_b$) was calculated using the Lubitz et al. (2014) MATLAB model (see Section 3). Then, using Equations (5-1) and (5-2), the total volume of water in the AST ($V_{tot}$) and average flow characteristic area were determined. The average AST water height ($h_{avg}$) was determined using Equations (5-3) and (5-4) through numerical integration. Finally the delivery channel depth, just upstream of the AST was determined using Equation (5-5). The results are presented in Table 7-3.
Table 7-3. Delivery channel outlet boundary condition ($h_d$) calculations for flow rates ($Q$) of 1 m$^3$/s through 6 m$^3$/s.

<table>
<thead>
<tr>
<th>$Q$ (m$^3$/s)</th>
<th>$V_b$ (m$^3$)</th>
<th>$V_{tot}$ (m$^3$)</th>
<th>$A_c$ (m$^2$)</th>
<th>$h_{avg}$ (m)</th>
<th>$h_d$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>0.51</td>
<td>9.82</td>
<td>0.73</td>
<td>0.45</td>
<td>0.49</td>
</tr>
<tr>
<td>1.50</td>
<td>0.77</td>
<td>14.87</td>
<td>1.10</td>
<td>0.60</td>
<td>0.66</td>
</tr>
<tr>
<td>1.80</td>
<td>0.93</td>
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<td>0.75</td>
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<td>0.83</td>
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<td>0.97</td>
<td>1.06</td>
</tr>
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<td>3.20</td>
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<td>2.37</td>
<td>1.30</td>
<td>1.42</td>
</tr>
<tr>
<td>3.60</td>
<td>1.85</td>
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<td>2.65</td>
<td>1.48</td>
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<td>3.61</td>
<td>1.97</td>
<td>2.16</td>
</tr>
<tr>
<td>6.00</td>
<td>2.63</td>
<td>50.91</td>
<td>3.77</td>
<td>2.03</td>
<td>2.22</td>
</tr>
</tbody>
</table>

### 7.2.2 Delivery Channel Losses and Reservoir Head ($H_{res}$) Calculations

Using the direct-step model and energy calculations outlined in Chapters 7.1.1 and 7.1.2, the delivery channel's energy losses and required reservoir head were calculated using the delivery channel depth boundary conditions calculated in Section 7.1. A Manning's $n$ friction factor of 0.015 was used to approximate the friction of a concrete channel (Bedient et al, 2012). Figure 7-4 shows the calculated head losses within the theoretical delivery channel. Included in Figure 7-4 are the total, friction, transition losses as flow enters the AST, and the delivery channel entrance losses. Figure 7-4 suggests that the dominant energy loss in the delivery channel occurs as the flow is contracted as it transitions between the delivery channel and the AST. At low flow rates, the friction energy losses along the long of the channel are significant while the entrance losses
are quite small. As the flow increases, the entrance losses increase while along-channel friction is reduced to negligible levels. This is consistent with flow theory expectations. As the flow rates increase, the amount of flow experiencing forced contraction increases at the inlet causing increased head losses. Similarly, as the flow rate increases, the flow cross-sectional area in the delivery channel increases relative to the wetted perimeter, causing a decrease in frictional losses.

![Graph showing head loss vs. flow rate](Figure 7-4. Total, friction, AST inlet, and entrance losses for a concrete delivery channel with a base width of 3.13 m and length of 25 m)

Figure 7-5 shows the calculated reservoir head impounded within the dam needed to supply a given flow rate to the AST. As expected, increasing the flow rate requires a higher reservoir water level. Also included in Figure 7-4 are HEC-RAS results for a concrete rectangular channel with the same dimensions used in the delivery channel samples calculations (base width = 3.5 m, length = 25 m, and slope = 1%). The HEC-RAS model was subjected to the same downstream boundary conditions as the delivery model. As seen in Figure 7-5, there is strong agreement between the delivery channel and HEC-RAS models, suggesting that the proposed delivery channel and
reservoir head model would provide decent approximations to deliver the required AST flow rates simulated.

![Graph](image1)

*Figure 7-5. The reservoir head (Hres) to provide a flow rate (Q) to the AST for a rectangular channel with a case width of 3.13 m and length of 25*

### 7.3 Delivery Channel Losses and Required Upstream Reservoir Head Summary

This chapter detailed a method for calculating the required upstream reservoir head needed to supply a given AST with the needed design flow rate. Included in this chapter is a test scenario in which the reservoir head is calculated for a rectangular delivery channel. The reservoir head is determined by calculating the water level and all energy losses throughout the length of the delivery channel. The reservoir head is the sum total of elevation and energy loss heads. The frictional energy losses and elevation head in the delivery channel were calculated using the open-
channel Direct Step model, with a downstream boundary condition controlled by the AST. The reservoir head model was confirmed using the open-channel flow calculator HEC-RAS.
Chapter 8: Conclusions

This thesis examined the power losses inherent in Archimedes Screw Turbines (AST) and proposed methods to model these losses. The proposed loss models outlined were created using an already existing AST power model and are designed to be utilized as engineering design tools for sizing ASTs properly for a given site. In addition to the power loss model, a delivery channel loss model is also proposed capable of predicting the required upstream reservoir head needed to supply a design flow rate to an AST.

Current gap leakage models were examined and the first real-world experimental measurements of gap leakage flow were made. It was found that current gap leakage models fail to accurately represent gap leakage flow trends, but provide a reasonable order of magnitude estimate for gap leakage values. These measurements have identified key dynamic effects that influence gap leakage that were previously neglected. While there are three suggested flow pathways (bucket flow, gap leakage, and overflow leakage), the gap leakage validation experiment suggested a fourth, unaccounted for, pathway. It is possible that a type of pumped leakage, or water that is given flight at high rotational speeds due to the shear frictional forces of the blades, becomes a significant flow path that might need to be taken into account in order for the Lubitz et al. (2014) model to properly predict AST fill heights and, therefore, power at all operational speeds.

Additionally, a more complete AST power loss model was proposed that extends the Lubitz et al. (2014) AST power model to account for all the previously neglected power losses. These models were derived using a laboratory scale-model AST (Chapter 5) and validated using a real-world commercial ASG unit (Chapter 6). This amended loss model includes individual loss models for:
• Outlet exit power loss
• Hydraulic frictional power loss
  o Transport (center shaft and trough)
  o Rotational (helical planes and center shaft)
• Bearing friction power loss
• Outlet drag torque power loss

For the expansion losses at the outlet, the Borda-Carnot loss predictions appear to reasonably predict power losses for the expansion loss at the outlet of the screw. The power losses at the outlet, as predicted with the Borda-Carnot equations, were presented and seemed to be valid for AST outlet transitions where the flow is discharged to an outlet channel with a greater cross-sectional area than the average flow area inside the AST. Typically, the outlet expansion losses were minor losses, comprising, on average, only 4.47% percent of the total observed power losses.

The bearing losses are modelled with a simple dry bearing model. The bearing losses tended to increase with increasing rotational speed. On average, laboratory testing showed bearing losses of approximately 15.9% of the total power losses while the real world AST unit had bearing losses of 12.2%.

The hydraulic frictional forces can be separated into two categories: the shear stress on the trough walls/center shaft and the shear stress on the helical plane surfaces. The frictional losses on the trough and center shaft can be considered minor losses. In the laboratory AST scale-model, these
loses, on average, accounted for approximately 0.36% of the observed power losses; for the real-world AST the transport shear losses represented approximately 1.38%. This difference was because the laboratory tests included experimental runs with low fill heights, corresponding to smaller total wetted areas in the bucket. The shear stress on the helical planes is the dominant form of shear stress acting on the screw, and becomes particularly dominant at high rotation speeds that are in excess of the Muysken limit. For the laboratory model, the experimental results suggested that the helical plane shear stress power losses typically represented 50% to 70% of the total power losses; for the real-world AST unit rotational shear friction losses comprised approximately 10% to 25% of the total power losses. The smaller proportion on the real-world unit was due to the inclusion of generator losses on the real-world unit.

Finally, at low rotational speeds, the power losses tend to be dominated by the resistive torque drag induced at the screw outlet due to the screw’s submersion in water. This power loss tends to increase with increasing rotational velocity and outlet water level. An empirical, non-dimensional power loss model was developed that describes the outlet torque drag losses for a screw of any size. This empirical loss model was derived from the experimental results of a small-scale laboratory AST and validated using the real-world AST unit.

The new loss models improve the results of the Lubitz et al. (2014) performance model significantly. However, the amended model appears to still over-predict power at low flow rates, suggesting additional energy losses at low flow rates are still unaccounted for.
Finally, an AST delivery channel loss model is proposed that determines the required energy head in a supplying reservoir. This model was checked against a case study simulation using the open-channel flow simulation software HEC-RAS and showed very good agreement. The Lubitz et al. (2014) performance model, in conjunction with the power loss amendments from Chapter 5 and the Chapter 7 delivery channel loss model, provide a complete supply reservoir-to-receiving basin AST power output model that can be specifically used to design an AST for a given site.
Chapter 9: Recommendations

This thesis advances current AST research and provides a foundation for further research questions in this field. In particular, there still exists the potential to improve leakage models, both in terms of gap and overflow leakage. Existing gap leakage models were examined and previously unexplored key operating principles were identified. This could allow for an improved gap leakage model that includes many dynamic effects such as the inclusion of elevated flow lifted from within the buckets due to the shear friction along the plane walls. Similarly, while the overflow leakage model was not directly confirmed experimentally, the overflow leakage may have similar rotational velocity dependencies that were seen in the gap leakage model, suggesting the possibility of an improved model in the future.

Similarly, bearing friction losses have properties unique to Archimedes screws that were identified in this thesis but not completely addressed. Axial loading on the upper and lower bearings likely changes due to added weight from the water in the buckets as well as buoyant forces acting on the screw itself. These loading modifications could be included to create a more complete bearing loss model.

Additionally, this thesis identified the key components leading to losses at the outlet and presented an empirical non-dimensional outlet power loss model. This model has the potential for further improvement and could serve as the groundwork for a complete analytical outlet power loss model. Furthermore, the outlet model in this thesis has been validated using a dimensionally similar, but larger screw. Therefore, validation of the outlet power loss model for dimensionally dissimilar ASTs would be beneficial.
Chapter 10: References


Machines and Mechanisms, Proceedings HMM2004 (pp. 181-194).


*Bioscience, 50*(9), 766-775.


Appendix A: Sensitivity Analysis MATLAB Scripts

% This function generates the output of archimedesfh3 by varying a single parameter at a time, while all other
% parameters remain constant.
% There are no inputs, all needed input parameters are generated internally
% The outputs are:
% dataOutputs = 3 dimensional matrix that contains all physical model outputs for each run
% dataOutputPercentage = Same dataOutputs, but results in terms of percentages of the baseline output
% polyCoeffs = a matrix that contains the 6th order polynomial coefficients for the function of the each of
% the output variables and the varied input parameter. There is one function for each pair output and input
% variable
% polyCoeffsDerivatives = a matrix that contains the 6th order polynomial coefficients for the derivative
% function for all output variables and the varied input parameter

function [ dataOutput dataOutputPercentage polyCoeffs polyCoeffsDerivatives ] = uncertaintyAnalysis()

% This section will create a 3 dimensional matrix (dataOutput) and store the results of the sensitivity analysis.

% Baseline values representing the “typical” operating conditions for the Chapter 4 experiments
%betaDefault = 0.432841654; (rad) – screw angle
%PDefault = 144.61/1000; (m) - pitch
%NDefault = 3; (m) - # of turns
%diDefault = 76.26/1000; (m) – inner diameter
%doDefault = 144.61/1000; (m) – outer diameter
%LDefault = 584/1000; (m) – Screw length
%fDefault = 1; (-) – fill height
%omegaDefault = 4.1887902;
%GwDefault = 1.11/1000; (m) – Gap Width
%CDefault = 0.9; (-) – Leakage coefficient
%rsteps = 1000; radial steps
%thetasteps = 1000; angular steps

% Baseline values representing the “typical” operating conditions for the Chapter 5 experiments
betaDefault = 0.41887902; (rad) – screw angle
PDefault = 33.0/100; (m) - pitch
NDefault = 3; (m) - # of turns
diDefault = 168.0/1000; (m) – inner diameter
doDefault = 325.0/1000; (m) – outer diameter
LDefault = 122.9/100; (m) – Screw length
%fDefault = 1; (-) – fill height
%omegaDefault = 4.1887902; (rad/s) – rotational velocity
%GwDefault = 1.99/1000; (m) – Gap Width
%CDefault = 0.89; (-) – Leakage coefficient
%rsteps = 1000; radial steps
%thetasteps = 1000; angular steps

% Set the upper and lower limits of the input parameter variations
% from 25% to 175% of the default values
maxPercent = 2;
minPercent = 0.05;

% Analysis steps or divisions
aSteps = 100;

% Cycle through and vary all archimedesfh3’s 10 input variables and record the outputs
% variable 1 = beta
% variable 2 = P
% variable 3 = N
% variable 4 = di
% variable 5 = L
for variable = 1:10

% Restore all base line input parameters
beta = betaDefault;
P = PDefault;
N = NDefault;
di = diDefault;
do = doDefault;
L = LDefault;
f = fDefault;
omega = omegaDefault;
Gw = GwDefault;
C = CDefault;

% var = the varied variable, initially set to default value
if (variable == 1)
  var = betaDefault;
elseif(variable == 2)
  var = PDefault;
elseif(variable == 3)
  var = NDefault;
elseif(variable == 4)
  var = diDefault;
elseif(variable == 5)
  var = doDefault;
elseif(variable == 6)
  var = LDefault;
elseif(variable == 7)
  var = fDefault;
elseif(variable == 8)
  var = omegaDefault;
elseif(variable == 9)
  var = GwDefault;
elseif(variable == 10)
  var = CDefault;
end

% Find the upper and lower limits of the current variable
maxV = maxPercent*var;
minV = minPercent*var;

% j = place the dataOutput array to store current data output
j = 0;
for var = minV : (maxV - minV)/aSteps : maxV
  j = j + 1;

  % Set the current variable to the varied parameter
  if (variable == 1)
    fprintf('Variable = beta, j = %d, beta = %.4f
', j, var)
    beta = var;
  elseif(variable == 2)
    fprintf('Variable = P, j = %d, P = %.4f
', j, var)
    P = var;
  elseif(variable == 3)
    fprintf('Variable = N, j = %d, N = %.4f
', j, var)
    N = var;

elseif(variable == 4)
    fprintf('Variable = di, j = %d, di = %.4f \n', j, var)
    di = var;
elseif(variable == 5)
    fprintf('Variable = do, j = %d, do = %.4f \n', j, var)
    do = var;
elseif(variable == 6)
    fprintf('Variable = L, j = %d, L = %.4f \n', j, var)
    L = var;
elseif(variable == 7)
    fprintf('Variable = f, j = %d, f = %.4f \n', j, var)
    f = var;
elseif(variable == 8)
    fprintf('Variable = omega, j = %d, omega = %.4f \n', j, var)
    omega = var;
elseif(variable == 9)
    fprintf('Variable = Gw, j = %d, Gw = %.4f \n', j, var)
    Gw = var;
elseif(variable == 10)
    fprintf('Variable = C, j = %d, C = %.4f \n', j, var)
    C = var;
end

% Run archimedesfh3 model and store output
[V, Ttot, Power, Pavail, eff, Qb, h, nbuckets, T, Pavg, ltotal, lw, le, hue, Ql, Qo, Q, Fl, Fltot, Fb, Fbtot, A1, A2, Ac, At] = archimedesfh4( beta, P, N, di, do, L, f, omega, Gw, C, rsteps, thetasteps);

% Store output in 3 dimensional area, first parameter is always the varied input parameter
dataOutput(variable, j, 1)  = var;
dataOutput(variable, j, 2)  = V;
dataOutput(variable, j, 3)  = Ttot;
dataOutput(variable, j, 4)  = Power;
dataOutput(variable, j, 5)  = Qb;
dataOutput(variable, j, 6)  = T;
dataOutput(variable, j, 7)  = Ql;
dataOutput(variable, j, 8)  = Qo;
dataOutput(variable, j, 9)  = Q;
dataOutput(variable, j, 10) = Fl;
dataOutput(variable, j, 11) = Fltot;
dataOutput(variable, j, 12) = Fb;
dataOutput(variable, j, 13) = Fbtot;
dataOutput(variable, j, 14) = A1;
dataOutput(variable, j, 15) = A2;
dataOutput(variable, j, 16) = Ac;
dataOutput(variable, j, 17) = At;
end
end

dataOutputPercentage = dataOutput;

% Generate polynomial coefficients for the output variable and corresponding input variable
% Also take the derivative of the function, and store those coefficients (needed for generating uncertainties)
polyOrder = 6;
exponents = [6:-1:0];
polyCoeffs = zeros(10, 17, polyOrder + 1);
polyCoeffsDerivatives = zeros(10, 17, polyOrder);
for i = 1:10
    for j = 1:17
        tempCoeffs = polyfit(dataOutput(i, :, 1), dataOutput(i, :, j), polyOrder);
        for k = 1:polyOrder + 1
            polyCoeffs(i,j,k) = tempCoeffs(k);
            polyCoeffsDerivatives(i,j,k) = tempCoeffs(k)*exponents(k);
        end
    end
end

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% Create plots for each output parameter and varied input parameter pairs, for i = 1:10
for j = 1:17
    xPoly = min(dataOutput(i, :, 1)): 0.001 : max(dataOutput(i, :, 1));
    yPoly = polyCoeffs(i,j,1).*xPoly.^6 + polyCoeffs(i,j,2).*xPoly.^5 + polyCoeffs(i,j,3).*xPoly.^4 + polyCoeffs(i,j,4).*xPoly.^3 + polyCoeffs(i,j,5).*xPoly.^2 + polyCoeffs(i,j,6).*xPoly.^1 + polyCoeffs(i,j,7).*xPoly.^0;
    yPolyDeriv = polyCoeffsDerivatives(i,j,1).*xPoly.^5 + polyCoeffsDerivatives(i,j,2).*xPoly.^4 + polyCoeffsDerivatives(i,j,3).*xPoly.^3 + polyCoeffsDerivatives(i,j,4).*xPoly.^2 + polyCoeffsDerivatives(i,j,5).*xPoly.^1 + polyCoeffsDerivatives(i,j,6).*xPoly.^0;
    [x_label, y_label] = getAxisNames(i,j);
    fig = figure(1);
    set(fig, 'Visible', 'off');
    hold on
    plot(dataOutput(i, :, 1), dataOutput(i, :, j), 'o', 'Color', 'black');
    plot(xPoly, yPoly, '-', 'Color', 'black');
    xlabel(x_label, 'fontsize', 18, 'fontweight', 'bold');
    ylabel(y_label, 'fontsize', 18, 'fontweight', 'bold');
    hold off
    saveas(fig, strcat(num2str(i), '_', num2str(j), '.png'));
    close(fig);
    fig = figure(1);
    set(fig, 'Visible', 'off');
    hold on
    plot(xPoly, yPolyDeriv, '-', 'Color', 'blue');
    xlabel(x_label, 'fontsize', 18, 'fontweight', 'bold');
    ylabel(y_label, 'fontsize', 18, 'fontweight', 'bold');
    hold off
    saveas(fig, strcat(num2str(i), '_', num2str(j), '_D.png'));
    close(fig);
dataOutputPercentage(i,:,j) = dataOutputPercentage(i,:,j)/dataOutput(i, round(aSteps/2)+1,j);
end
end
end

% A function that generates the appropriate figure axis titles for plotting
function [x_label, y_label] = getAxisNames(i,j)
    if (i == 1)
        x_label = '\beta (rad)';
    elseif(i == 2)
        x_label = 'S (m)';
    elseif(i == 3)
        x_label = 'N (-)';
    elseif(i == 4)
        x_label = 'di (m)';
    elseif(i == 5)
        x_label = 'do (m)';
    elseif(i == 6)
        x_label = 'L (m)';
    elseif(i == 7)
        x_label = 'f (-)';
    elseif(i == 8)
        x_label = '\beta (rad)';
x_label = \(\omega (\text{rad/s})\);

elseif(i == 9)
    x_label = 'Gw (m)';
elseif(i == 10)
    x_label = 'C (-)';
end

if (j == 1)
    y_label = 'SAME';
elseif(j == 2)
    y_label = 'V (m^3)';
elseif(j == 3)
    y_label = 'Ttot (Nm)';
elseif(j == 4)
    y_label = 'Power (W)';
elseif(j == 5)
    y_label = 'Qb (m^3/s)';
elseif(j == 6)
    y_label = 'T (Nm)';
elseif(j == 7)
    y_label = 'Ql (m^3/s)';
elseif(j == 8)
    y_label = 'Qo (m^3/s)';
elseif(j == 9)
    y_label = 'Q (m^3/s)';
elseif(j == 10)
    y_label = 'Fl (N)';
elseif(j == 11)
    y_label = 'Fltot (N)';
elseif(j == 12)
    y_label = 'Fb (N)';
elseif(j == 13)
    y_label = 'Fbtot (N)';
elseif(j == 14)
    y_label = 'A1 (m^2)';
elseif(j == 15)
    y_label = 'A2 (m^2)';
elseif(j == 16)
    y_label = 'Ac (m^2)';
elseif(j == 17)
    y_label = 'At (m^2)';
end

% Generates the uncertainties for each of the output parameters based on the derivative polynomial
% functions created by the uncertaintyAnalysis() function and the input uncertainties of all input parameters
% [ dV, dTtot, dPower, dQb, dT, dQl, dQo, dQ, dFl, dFltot, dFb, dFbtot, dA1, dA2, dAc, dAt ] =
archimedesfh4_Calculate_Uncertainty( polyCoeffsDerivatives, dbeta, dP, dN, ddi, ddo, dL, df, domega, dGw, dC, V,
Ttot, Power, Qb, T, Ql, Qo, Q, Fl, Fltot, Fb, Fbtot, A1, A2, Ac, At)

% Cycle through all 17 output variables, note first parameter is
for j = 2:17
    dVar = 0; % define a total uncertainty of a generic variable and set it to zero

% Cycle through all 10 input variables and obtain its input uncertainty
for variable = 1:10
    if (variable == 1)
        var = dbeta;
    elseif(variable == 2)
        var = dP;
    elseif(variable == 3)
        var = 0;
    end
elseif(variable == 4)
    var = ddi;
elseif(variable == 5)
    var = ddo;
elseif(variable == 6)
    var = dL;
elseif(variable == 7)
    var = df;
elseif(variable == 8)
    var = domega;
elseif(variable == 9)
    var = dGw;
elseif(variable == 10)
    var = dC;
end

% Calculate the portion of the uncertainty contribution of each variable and add it to the total uncertainty
    dVar = dVar + (var*(polyCoeffsDerivatives(variable,j,1).*V.^5 + polyCoeffsDerivatives(variable,j,2).*V.^4 + polyCoeffsDerivatives(variable,j,3).*V.^3 + polyCoeffsDerivatives(variable,j,4).*V.^2 + polyCoeffsDerivatives(variable,j,5).*V.^1 + polyCoeffsDerivatives(variable,j,6).*V.^0))^2;

% Take the square root to obtain the actual uncertainty
    dVar = sqrt(dVar);

end

% Assign the uncertainty to the appropriate output variable
if(j == 2)
    dV = dVar;
elseif(j == 3)
    dTtot = dVar;
elseif(j == 4)
    dPower = dVar;
elseif(j == 5)
    dQb = dVar;
elseif(j == 6)
    dT = dVar;
elseif(j == 7)
    dQl = dVar;
elseif(j == 8)
    dQo = dVar;
elseif(j == 9)
    dQ = dVar;
elseif(j == 10)
    dFl = dVar;
elseif(j == 11)
    dFltot = dVar;
elseif(j == 12)
    dFb = dVar;
elseif(j == 13)
    dFbtot = dVar;
elseif(j == 14)
    dA1 = dVar;
elseif(j == 15)
    dA2 = dVar;
elseif(j == 16)
    dAc = dVar;
elseif(j == 17)
    dAt = dVar;
end
Appendix B: Gap Flow Validation Experiment Flow Sensor

Calibration

The following is the derivation of the calibration equation and uncertainty analysis for the flow measurement sensor used in the Gap Flow Validation experiment (Chapter 4). The experimental flow sensor was a water column with an orifice outlet and height gauge that could be used to measure height of the water in the water column. The orifice outlet restricted flow allowing for accumulation of water in the water column. This enabled the flow through the water column to remain pressurized. Figure B-1 shows the flow sensor used.

Figure B-1. Gap flow validation experiment flow sensor
The rate of water flow through the orifice is proportional to the square root of the water column height \((h)\) yielding the following calibration (White, 2005):

\[
Q = k_1 \sqrt{h} + k_2 \tag{B-1}
\]

where \(k_1\) is the discharge coefficient and \(k_2\) is an offset coefficient. Measurements of both the water column height and outflow were made simultaneously. Outflow was measured by collecting the discharged water over a set period of time. Water volumes of approximately 4 L were collected per measurement, with 5 separate measurements made per flow rate. The measurement time was recorded using Apple’s Logic Pro X audio recording software. The metronome feature was used to make measurements of a predetermined and precise timed period. This allowed the time uncertainty to be negligible relative to the uncertainty in the volume measurements; nearly all the flow measurement uncertainty was due to human error (the ability to start and stop water collection on time) and variations in the flow rate generated by the pump.

The volumes were measured by hanging the captured water from a load cell to yield the weight \((w)\). The density of water was determined by measuring the water temperature to be approximately 21.3°C. The density \((\rho)\) at this temperature was taken to be 0.9974 g/cm\(^3\). The flow rate is just the volume divided by time \((\Delta t)\):

\[
Q = \frac{w}{\rho g \Delta t} \tag{B-2}
\]

where \(g\) is 9.81 m/s\(^2\). Performing a weighted least squares linear regression on the flow rate \((Q)\) versus the square root of the water column height \((h)\) yielded the discharge coefficient \((k_1)\) and the associated intercept \((k_2)\) in Equation B-1. Since the uncertainty in the flow rates was considerably
higher than that of the water column height measurement, the calibration equation was determined
directly without the need of a transfer equation. Ideally, there would be no intercept but since it
is possible for flow to occur through the orifice without the orifice being completely submerged,
an intercept was required to allow the calibration equation to function correctly.

All the uncertainties in the calibration equation parameters (\( Q, h, k_1, \text{ and } k_2 \)) were required before
uncertainties could be propagated. Since there were not enough flow rate measurements to
estimate the uncertainty from the underlying distributions, the flow rate and water column height
uncertainties were conservatively approximated as the maximum magnitude measured from the
mean of 5 measurements. The uncertainties in \( k_1 \) and \( k_2 \) were determined from the weighted
regression statistics. The uncertainty in the calibration equation (\( \sigma_Q \)) was determined using the
appropriate error propagation techniques (Taylor, 1997):

\[
\sigma_Q = \sqrt{\left( \frac{\partial Q}{\partial h} \sigma_h \right)^2 + \left( \frac{\partial Q}{\partial k_1} \sigma_{k_1} \right)^2 + \left( \frac{\partial Q}{\partial k_2} \sigma_{k_2} \right)^2}
\]  

(B-3)

Solving Equation B-3 yields:

\[
\sigma_Q = \sqrt{\left( \sqrt{h} \sigma_{k_1} \right)^2 + \left( \frac{k_1}{2\sqrt{h}} \sigma_h \right)^2 + \left( \sigma_{k_2} \right)^2}
\]  

(B-4)

Figure B-1i shows the resulting flow sensor calibration plot and associated uncertainty range. All
of the flow measurements lie within the calculated uncertainty range of the flow sensor. The
discharge coefficient (\( k_1 \)) and the intercept (\( k_2 \)) were found to be (72.60 ± 0.77) cm\(^6\)/s and (-1.14 ±
8.92) cm\(^3\)/s respectively. The \( R^2 \) value of the calibration curve was approximately 0.9866,
suggesting a highly accurate calibration curve. Figure B-1ii shows the predicted flow rates from the calibration equations against the actually measured flow calibration data. This line is approximately 1:1 linear across all flow rates indicating a strong calibration was achieved.

![Flow sensor calibration curve](image)

*Figure B-2. Flow sensor calibration curve ii) Flow sensor calibration equation predictions compared to the experimental measurements*

The flow sensor has a lower limit of a water column height of approximately 80 mm (9 mm$^{1/2}$ in Figure B-1). Below this height, water bubbles could be seen forming in the outflow of the orifice suggesting that the orifice was not completely submerged and pressurized. This lower limit is acceptable because flow rates in this range fail to generate enough torque to overcome the AST’s internal frictional forces. Therefore, measurements in this range will not be taken during actual experimentation. No upper measurement limit was detected, so the effective upper limit would be the entire height of the water column itself. Again, the AST flow rates will never have approached this upper threshold, so this limit is inconsequential.

For the measured flow rates, insufficient measurement sample size prevented a statistical distribution assessment of the measurement uncertainties. Instead, the uncertainty of this parameter was conservatively defined as one standard deviation observed from the mean. The
uncertainty in all time (t) measurements used for the calculated flow rates were negligible due to
the high precision of the timing mechanism; instead all uncertainty was transferred to the volume
measurement. The average uncertainty of the flow sensor was determined to be approximately
±2.7% of the actual measurements.
Appendix C: Sample Gap Flow Measurement Photo

Figures C-1 through C-4 show

Figure C-1. Fill height measurement photograph for $Q = 800 \text{ mL/s, } \omega = 0 \text{ rad/s (Stalled)}$

Figure C-2. Fill height measurement photograph for $Q = 800 \text{ mL/s, } \omega = 4.32 \text{ rad/s}$
Figure C-3. Fill height measurement photograph for \( Q = 800 \text{ mL/s}, \ \omega = 12.31 \text{ rad/s} \)

Figure C-4. Fill height measurement graph for \( Q = 800 \text{ mL/s}, \ \omega = 19.8 \text{ rad/s} \)
Appendix D: Load Cell Calibration

The load cell, supplied with an input load of 5V has a rated capacity of 10 kgF. Load cell data acquisition is performed using a National Instruments’ NI USB-6525 DAQ and custom acquisition software written in LabVIEW. All load cell measurements were made for a period of 1 minute, sampled at a rate of 1000Hz. The load cell output voltage varies ($\Delta V$) linearly with applied load ($F_a$):

$$F_a = a_0 + a_1 \Delta V$$  \hspace{1cm} (D-1)

where $a_0$ and $a_1$ are regression coefficients. The measured torque ($\tau$) is simply the measured force multiplied by the moment arm length ($R_m$):

$$\tau = R_m(a_0 + a_1 \Delta V)$$  \hspace{1cm} (D-2)

For this experimental setup, the moment arm ($R_m$) is 0.261 m. The regression coefficients, $a_0$ and $a_1$, were determined by hanging 20 different masses from the load cell, allowing the acceleration due to gravity (g) to create a known force. The masses ranged from 0 g to 1000 g to completely cover the range of expected torque the AST could generate. Figure D-1 shows the resulting torque sensor calibration. As seen in Figures D-1i and D-1ii, the theoretically predicted torque values, approximated by converting the measured applied loads to a torque by multiplying by the moment arm length, all fall within the calibration equation’s uncertainty range. Figure D-1iii shows the calibration equation predictions for the torque sensor compared to the experimentally measured values. The actual measured torque value is based on the torque that the known masses used in
the calibration process would have generated had the force acted on the moment arm. As seen in Figure 4ii, all the points lie near the center line, suggesting all errors are minor.

Figure D-1. Torque sensor calibration curve ii) Torque sensor calibration equation predictions compared to the actual measurements

The regression coefficients $a_0$ and $a_1$ were found to be -0.1106 Nm and -45.04 Nm/mV respectively. The $R^2$ value of the calibration curve was approximately 0.9999932973, suggesting a calibration equation of high quality. The uncertainty in the slope ($a_1$) was approximately 4% of the actual slope which is deemed acceptable. The uncertainty in the intercept ($a_0$) was approximately 30% of the calculated intercept, however the intercept’s overall value is insignificant to the calibration equation output and this error falls within the uncertainty of the torque sensor itself. The moment arm length was measured to be 26.1 ± 0.1 cm.

Torque measurements with forces under 0.05 N may fall within the uncertainty of the load cell measurements themselves and not be accurate, representing a lower measurement limit. The upper measurement limit is approximately 90 N, as per the safety specifications of the load cell itself. This expected torque values from the model AST should never approach either the lower or upper
limits. Since the regression coefficients have small uncertainties, the overall uncertainty of the torque sensor is dominated by the accuracy of the moment arm length.
Appendix E: Lubitz et al. (2014) Model Results with Losses

\[ Q = 7.5 \text{ L/s} \]

\[ \psi = 0.00 \]

\[ \psi = 0.25 \]

\[ \psi = 0.43 \]

\[ \psi = 0.60 \]

\[ \psi = 0.77 \]

\[ \psi = 0.94 \]

\[ \psi = 1.10 \]

\[ \psi = 1.31 \]

- Measured
- Lubitz Model
- Lubitz Model (Amended)
$Q = 10 \text{ L/s}$

- $\psi = 0.00$
- $\psi = 0.25$
- $\psi = 0.43$
- $\psi = 0.60$
- $\psi = 0.77$
- $\psi = 0.94$
- $\psi = 1.10$
- $\psi = 1.31$

- Measured
- Lubitz Model
- Lubitz Model (Amended)
Q = 12.5 L/s

\[ \psi = 0.00 \]
\[ \psi = 0.25 \]
\[ \psi = 0.43 \]
\[ \psi = 0.60 \]
\[ \psi = 0.77 \]
\[ \psi = 0.94 \]
\[ \psi = 1.10 \]
\[ \psi = 1.31 \]