Lab Testing and Modeling of Archimedes Screw Turbines

by
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A Thesis
Presented to
The Faculty of Graduate Studies
of
The University of Guelph

In partial fulfilment of requirements
for the degree of
Master of Applied Science
in
Engineering

Guelph, Ontario, Canada
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ABSTRACT

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Archimedes Screw Turbines (ASTs) are being utilized across Europe and are beginning to be utilized in North America for micro-hydropower (less than 500 kW) production. There is currently no mathematical model in the literature which describes the power production of an AST of arbitrary geometry and at partial-fill conditions, and only limited literature on laboratory and full-scale testing of AST performance.

A custom lab test apparatus for scale-model ASTs was developed, and five different scale model ASTs were tested to quantify performance in terms of power production. In addition, testing was performed on a small prototype AST installed on a waterway in Ontario. A quasi-static numerical model is presented which describes the performance of an arbitrary AST at normal operating conditions, including varying rotation rates, flows, and fill levels. The model uses the laboratory and prototype test data for validation. Under normal operating conditions the model accurately predicts AST power production. Some limitations exist in the model regarding accurately quantifying leakage through the turbine, as well as operation at very low fill levels.
Acknowledgements

I would like to express my gratitude to my graduate advisor, Dr. William David Lubitz for his wisdom, guidance, and advice throughout the project, as well as his patience while I completed documenting it.

My appreciation also goes to Dr. Doug Joy for his support for this project.

This research would not exist without the leadership and hard work of Brian Weber and Tony Bouk of Greenbug Energy. Without their tireless effort and drive this research would not have happened.

I must acknowledge my friends and research colleagues in the School of Engineering, specifically Scott Simmons for his help with research and testing, and Brian Ho-Yan, Rajendra Sapkota, Qiyue Song, Tim Lambert, and Dr. John Runciman, for their advice and support.

I also thank my good friends Alex Leveille, Graham Poulin, Matt DiCicco, Dan Hoang, Jordan Knapman, Kevin Miller, and Jane Moore, for their constant encouragement and unwavering support throughout this project.

I am grateful for the technical and administrative support from the University of Guelph staff, namely Barry Verspagen, Nate Groendyck, Martha Davies, Laurie Gallinger, Lucy Cremasco, Phil Watson, Ken Graham, and Mary Payne.

This research was supported by the Natural Sciences and Engineering Research Council (NSERC) through the Engage and Collaborative Research and Development (CRD) programs.
# Table of Contents

Acknowledgements ........................................................................................................ iii
Nomenclature .................................................................................................................. viii
Glossary of Terms .......................................................................................................... ix

Chapter 1: Introduction ................................................................................................. 1
  1.1 Archimedes Screw Turbines .................................................................................... 2
  1.2 Principles of Hydroelectric Power Generation ..................................................... 5
  1.3 Motivation ............................................................................................................... 6

Chapter 2: Literature Review ......................................................................................... 7
  2.1 Microhydro Power Generation .............................................................................. 7
  2.2 History of Modeling Archimedes Screws ............................................................ 7
  2.3 Archimedes Screw Generators ............................................................................ 9
  2.4 Fish-Friendliness of ASGs .................................................................................... 10
  2.5 Other Literature ................................................................................................... 11

Chapter 3: Research Objectives .................................................................................... 13
  3.1 Problem statement and Significance ..................................................................... 13
  3.2 Partners .................................................................................................................. 13

Chapter 4: Prototype Screw ......................................................................................... 15
  4.1 Flow ...................................................................................................................... 18
  4.2 High water levels ................................................................................................. 20
  4.3 Summary ............................................................................................................... 20

Chapter 5: Lab Testing ................................................................................................ 22
  5.1 Lab Test Setup ...................................................................................................... 22
  5.2 Measurement Uncertainty .................................................................................... 24
  5.3 Test Procedure ...................................................................................................... 26
  5.4 Pitch ...................................................................................................................... 30
  5.5 Lower Basin Fill Height ..................................................................................... 33
  5.6 Volume Flow Rate ............................................................................................... 35
  5.7 Inclination Angle .................................................................................................. 40
  5.8 Lab Test Summary ............................................................................................... 42

Chapter 6: Power Model of an Archimedes Screw Turbine ........................................ 45
  6.1 Previous Models in Literature .............................................................................. 45
    6.1.1 Muller and Senior ......................................................................................... 45
  6.2 Model Overview .................................................................................................... 45
  6.3 Model Variables ................................................................................................... 49
    6.3.1 Archimedes Screw Model .......................................................................... 50
    6.3.2 Water Plane .................................................................................................. 52
    6.3.3 Bucket Fill Height ...................................................................................... 57
  6.4 ‘Bucket Volume’ .................................................................................................. 58
    6.4.1 Bucket Volume Prediction Accuracy ............................................................ 60
  6.5 Rotation Rate ........................................................................................................ 63
  6.6 Torque from a bucket ........................................................................................... 64
    6.6.1 Defining Mesh ............................................................................................... 65
    6.6.2 Component Force ....................................................................................... 66
    6.6.3 Component Torque Generated by Component Force .................................. 69
  6.7 Losses .................................................................................................................... 72
    6.7.1 Bearing Losses ............................................................................................. 72
    6.7.2 Leakage losses .............................................................................................. 73
List of Tables
Table 1 – Basic Geometry of an AST ................................................................. 3
Table 2 - Prototype AST S1 parameters ........................................................... 17
Table 3 - Model Lab Screw Parameters .......................................................... 22
Table 4 - Geometric Parameters of an AST ....................................................... 50
Table 5 - Additional Model Variables .............................................................. 50
Table 6 - Predictions of volume with Solidworks test and model, Pr = 1, β = 15°, N = 3, D = 0.146 m ................................................................. 62
Table 7 - RMSRE errors between select lab tests and model output .................... 94

List of Figures
Figure 1 – Basic Geometry of an AST ............................................................... 3
Figure 2 - Archimedes Screw Pump (Rorres 2000) ............................................ 4
Figure 3 - Turbine Application Range Chart, Williamson, et al. (2011) ................. 5
Figure 4 - Prototype AST installed at Delhi, Ontario ........................................ 16
Figure 5 - Example of lower end flooding on Prototype AST ......................... 16
Figure 6 – S1 viewed from outflow ................................................................. 17
Figure 7 - S1 in Weatherproof Enclosure ....................................................... 18
Figure 8 – Power and Efficiency vs Flow for Prototype AST, June 27 2012 .......... 19
Figure 9 – Flow Rate vs Efficiency for Prototype AST, June 27 2012 ................. 19
Figure 10 - Normalized Power at Wire vs Normalized Head, Prototype AST. Nominal flow 71 L/s, Rotation Rate 69.84 RPM, on 2012-01-29 ........................................... 20
Figure 11 - Experimental Setup ................................................................... 23
Figure 12 - Load Cell and Hall Effect Magnet Configuration ............................. 24
Figure 13 - Lower Basin Fill Categories ....................................................... 28
Figure 14 – Rotational velocity vs torque, test screw A, β = 22.7°, screw lower end unsubmerged, Q = 0.755 L/s ............................................................. 29
Figure 15 – Example rotation rate vs power, test screw A1, β = 22.7°, end unsubmerged, Q = 0.755 L/s ............................................................. 30
Figure 16 - Rotation rate vs power for various pitches, lab screws ...................... 31
Figure 17 - Rotation rate vs efficiency for various pitches, lab screws ................. 32
Figure 18 - Rotation rate versus torque for different lower basin levels .............. 33
Figure 19 - Rotation rate vs power for different lower basin levels .................... 34
Figure 20 – Rotation rate vs efficiency for different degrees of submergence of screw outlet lower basin levels .................................................. 35
Figure 21 - Rotation rate versus power, lab screw B (Pr = 1.4), β = 20.5° ............ 36
Figure 22 - Rotation rate versus efficiency, lab screw B (Pr = 1.4), β = 20.5° ....... 37
Figure 23 - Rotation rate vs power, lab screw B (Pr = 1.4), β = 24.6° ............... 38
Figure 24 - Comparison of efficiency vs flow for various turbines ...................... 39
Figure 25 - Flow rate vs power for screw B, rotation rate = 5.57 ±0.24Rad/s .......... 40
Figure 26 - Power versus rotation rate, screw A (Pr = 1.0), flow rate 1.07± 0.06 L/s . . . 41
Figure 27 - Efficiency versus rotation rate, screw A (Pr = 1.0), flow rate 1.07± 0.06 L/s .... 42
Figure 28 – Bucket volume with force vectors, top view ................................... 47
Figure 29 - Basic geometry of Archimedes screw ........................................... 49
Figure 30 - Coordinate system of AST, d and D of two flights are shown, point P is on the D of
the first flight................................................. 51
Figure 31 – Model coordinate system and water plane............................................. 53
Figure 32 - Defining \( r_{eff} \).................................................................................. 54
Figure 33 - Convergence of points for finding the lower limit of a bucket; black lines are the
lower flight’s \( D \) and \( d \) bounds, yellow lines are the upper flight’s bounds........... 56
Figure 34 - Example of fill height increment size on model output............................... 58
Figure 35 – Volume element .................................................................................. 60
Figure 36 - Solidworks model of bucket volume .......................................................... 62
Figure 37 - Sensitivity analysis of number of elements for volume calculation .............. 63
Figure 38 - Torque meshes..................................................................................... 65
Figure 39 - Element mesh for calculating torque .......................................................... 66
Figure 40 - Finding \( P_2 \) (\( x \)-axis out of page) ............................................................ 69
Figure 41 – Defining the pressure vector at \( P_1 \) (\( z \) out of page) ..................................... 70
Figure 42 - Sensitivity analysis of number of elements for torque calculation .............. 71
Figure 43 - Peak power output with rotation rate between 0.5 and 25 rad/s, versus outer diameter
.......................................................................................................................... 80
Figure 44 - Peak efficiency versus outer diameter ....................................................... 81
Figure 45 - Peak torque and rotation rate at peak torque versus outer diameter ............. 82
Figure 46 - Peak efficiency versus pitch ratio, \( Pr \) ......................................................... 83
Figure 47 - Peak torque versus pitch ratio.................................................................. 84
Figure 48 - Peak efficiency versus ratio of inner to outer diameter ............................. 85
Figure 49 - Ratio of inner to outer diameter versus peak torque ............................... 86
Figure 50 - Peak power versus inclination angle, constant head............................... 87
Figure 51 - Peak power versus inclination angle, constant length ............................... 88
Figure 52 - Peak Efficiency versus Inclination Angle, \( \beta \), constant length ................. 89
Figure 53 - Screw B, \( \beta = 24.6^\circ \), \( Q_f = 0.74 \pm 0.02 \) L/s, Lab Test 20130527_0930 ......................................... 92
Figure 54 – Leakage models applied to prototype AST S1....................................... 96
Figure 55 – Power versus rotation rate, screw B, \( \beta = 20.5^\circ \), \( Q_f = 0.43 \pm 0.02 \) L/s, Lab Test
20130418........................................................................................................ 98
Figure 56 – Power versus rotation rate, screw B, \( \beta = 20.5^\circ \), \( Q_f = 1.17 \pm 0.02 \) L/s, Lab Test
20130509........................................................................................................ 99
Figure 57 - Power versus rotation rate, screw B, \( \beta = 20.1^\circ \), \( Q_f = 0.54 \pm 0.02 \) L/s, Lab Test
20130424........................................................................................................ 100
Figure 58 - Power versus rotation rate, screw B, \( \beta = 24.6^\circ \), \( Q_f = 0.74 \pm 0.02 \) L/s, Lab Test
20130527_0930........................................................................................................ 101
Figure 59 - Power versus rotation rate, screw C, \( \beta = 22.7^\circ \), \( Q_f = 0.71 \pm 0.02 \) L/s, Lab Test
20120503_2........................................................................................................ 102
Figure 60 - Power versus rotation rate, screw A, \( \beta = 34.4^\circ \), \( Q_f = 1.10 \pm 0.02 \) L/s, Lab Test
20130614_1300 (note the differing scale due to the larger power produced).............. 103
Figure 61 – Power versus rotation rate, screw B, \( \beta = 20.7^\circ \), \( Q_f = 0.38 \pm 0.02 \) L/s, Lab Test
20130502........................................................................................................ 104
Figure 62 - Lab stall test compared to model predictions, screw B, \( \beta = 20.1^\circ \), \( Q_f = 0.54 \pm 0.02 \) L/s,
Lab Test 20130424........................................................................................................ 106
Figure 63 - Model predictions match calculated lab bucket fill .................................. 109
## Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g$</td>
<td>Gravitational acceleration at surface of Earth</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Density of water, taken as 1000 kg/m$^3$ throughout</td>
</tr>
<tr>
<td>$h$</td>
<td>Head drop across turbine [m]</td>
</tr>
<tr>
<td>$Q$</td>
<td>Flow rate [m$^3$/s]</td>
</tr>
<tr>
<td>$D$</td>
<td>Outer diameter of turbine flighting, [m]</td>
</tr>
<tr>
<td>$d$</td>
<td>Inner diameter of turbine flighting (diameter of inner tube), [m]</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Ratio of inner diameter of turbine flighting to outer diameter of turbine flighting [-] = $d/D$</td>
</tr>
<tr>
<td>$P$</td>
<td>Pitch of turbine flighting [m]</td>
</tr>
<tr>
<td>$Pr$</td>
<td>Ratio of pitch of turbine flighting to outer diameter [-] = $P/D$</td>
</tr>
<tr>
<td>$L$</td>
<td>Length of turbine flighting along centerline of turbine, [m]</td>
</tr>
<tr>
<td>$Lr$</td>
<td>Ratio of length of turbine flighting to outer diameter of turbine flighting [-] = $L/D$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Angle of installation of turbine between global horizontal and centerline of turbine using global coordinate system, [rad]</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of flights [-]</td>
</tr>
<tr>
<td>$n_b$</td>
<td>Number of buckets</td>
</tr>
<tr>
<td>$k$</td>
<td>Dimensionless angle of installation of turbine between global horizontal and centerline of turbine [-]</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Angle between centerline of turbine and water surface in a ‘bucket’ using bucket coordinate system, [rad]</td>
</tr>
<tr>
<td>$r_{off}$</td>
<td>Offset between water surface in a ‘bucket’ and centerline of turbine, at turbine datum [m]</td>
</tr>
<tr>
<td>$Ni$</td>
<td>Current flight number [-]</td>
</tr>
<tr>
<td>$f$</td>
<td>Fill factor of screw bucket [-]</td>
</tr>
<tr>
<td>$V_p$</td>
<td>Volume contained in one pitch of the screw [m$^3$]</td>
</tr>
<tr>
<td>$V_b$</td>
<td>Volume of one ‘bucket’ [m$^3$]</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Rotation rate of the AST [rad/s]</td>
</tr>
<tr>
<td>$T$</td>
<td>Torque [N-m]</td>
</tr>
<tr>
<td>$P_{avail}$</td>
<td>Power available at a site [W]</td>
</tr>
<tr>
<td>$P_p$</td>
<td>Power Produced by turbine [W]</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Efficiency of system [-]</td>
</tr>
<tr>
<td>$A_{bearings}$</td>
<td>Coefficients of a polynomial fit representing the relationship between bearing resistive torque and rotation rate</td>
</tr>
<tr>
<td>$B_{bearings}$</td>
<td></td>
</tr>
<tr>
<td>$p$</td>
<td>Hydrostatic pressure</td>
</tr>
<tr>
<td>$G_w$</td>
<td>Gap width between the outer edge of the flights and the trough [m]</td>
</tr>
</tbody>
</table>
## Glossary of Terms

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>AST</td>
<td>Archimedes Screw Turbine. The screw turbine itself, without gearbox, generator, controllers, etc.</td>
</tr>
<tr>
<td>ASP</td>
<td>Archimedes Screw Pump. An Archimedes screw used to pump water.</td>
</tr>
<tr>
<td>Flight</td>
<td>Helix wrapped around the central shaft of an AST</td>
</tr>
<tr>
<td>Chute</td>
<td>Space between two flights</td>
</tr>
<tr>
<td>Bucket</td>
<td>Continuous volume of water between two flights.</td>
</tr>
</tbody>
</table>
Chapter 1: Introduction

With rising energy demands and dwindling non-renewable energy sources, the need for renewable energy is increasing. One primary renewable energy sources is hydroelectric power.

Hydroelectric power makes up a large percentage of the electricity produced in Canada. This is due to the relatively low cost of hydroelectric installations, the reliability and maturity of hydropower technology, and the dispatchable nature of large-scale hydro installations (Natural Resources Canada, 2008).

Most large-scale approaches to harnessing hydropower involve construction of large dams, reservoirs, and power houses containing large Francis or Kaplan turbines. In contrast, hydro-generators with capacities less than 100 kW, often called microhydro generators (Natural Resources Canada, 2009), are typically run-of-the river. While microhydro systems are often undispachetable, and exhibit lower efficiencies (typically 65 – 80%) than large hydro installations (Natural Resources Canada, 2009), they typically have minimal effects on river conditions up- or downstream of the installation, reducing their ecological impact compared to large installations.

Most available microhydroelectric sites have low available head and low or moderate flows. There are many feasible microhydro sites in Canada, however, relatively few potential microhydroelectric sites, particularly low head sites, have been developed in Ontario (Natural Resources Canada, 2008).

There is a wide range of technologies and approaches utilized in microhydroelectric power production. Examples include Turgo turbines, cross flow turbines, and pump-as-turbine setups.
Several of these are reviewed by Williamson, et al. (2011). The choice of technology depends on the available head, flow, intermittency, and other characteristics of a particular site. A relatively new development in microhydroelectric power production is the Archimedes screw turbine (AST). These turbines are typically used at sites with very low heads and relatively large flow rates (Williams et al, 2005).

The first reported AST installation was in Europe in 1993 (Nuernbergk and Rorres, 2012). Since then hundreds have been installed and are now operating in Germany, Austria, the United Kingdom, and other parts of Europe (Hawle et al., 2012b). Archimedes screw generators (AST) have been widely adopted in Europe (Kantert, 2008) due to their ability to effectively harness power at low-head sites, efficiency, ease of installation, and minimal ecological impacts.

1.1 Archimedes Screw Turbines
An Archimedes Screw Turbine (AST) consists of a central cylindrical tube about which are wound a number \( N \) of helical flights (Anderson, 2011), creating a screw (Fig. 1). The distance across the outer edges of the helical flights is the outer diameter \( D \), and the central shaft’s diameter is the inner diameter \( d \). Each of the helical flights have the same pitch \( P \). The total length \( L \) of the flighted section is measured along the centerline of the screw, from the point where the flights start to the point where the flights end. The flights are evenly offset from each other in an axial direction, the spacing of which is dependent on the number of flights. Many of these variables can also be expressed in a non-dimensional form by normalizing to the outer diameter (Table 1).

The screw itself sits in a semi-circular to circular trough. Some designs fix this trough to the screw and cause the entire assembly to rotate. More commonly, the trough is fixed and the screw
rotates within it. In this case, the edge of the screw’s flights are separated from the trough by a small gap. The size of this gap depends on a number of factors, discussed later.

A space between two consecutive flights is termed a chute. When the screw is inclined at some angle ($\beta$) to the horizontal, each chute may be divided into separate ‘buckets’, contiguous volumes of water separated by air gaps within the chute.

![Figure 1 – Basic Geometry of an AST](image)

**Table 1 – Basic Geometry of an AST**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Meaning</th>
<th>Nondimensionalized Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D$</td>
<td>Outer Diameter of Flights</td>
<td>n/a</td>
</tr>
<tr>
<td>$d$</td>
<td>Inner Diameter of Flights</td>
<td>$\delta = d/D$</td>
</tr>
<tr>
<td>$P$</td>
<td>Pitch of Flights</td>
<td>$Pr = P/D$</td>
</tr>
<tr>
<td>$L$</td>
<td>Length of Flights</td>
<td>$Lr = L/D$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Inclination of Screw from Horizontal</td>
<td>n/a</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of flights</td>
<td>n/a</td>
</tr>
</tbody>
</table>

Design methods for ASTs are generally based on experience with Archimedes screw pumps (ASPs), devices which have existed since antiquity (Fig. 2) and still see use today (Koetsier, 2004). Archimedes screw pumps’ main advantages are their ability to move liquids which contain
large amounts of debris; one major application is in wastewater treatment plants. ASPs operate at relatively low rotational velocities (on the order of 10-100 RPM), allowing them to also be employed in fish farms without fear of harming the fish (Kibel, 2008).

![Figure 2 - Archimedes Screw Pump (Rorres 2000)](image)

Generating electricity using an Archimedes screw involves attaching an AST to a generator, typically through a constant-ratio gearbox. An assembly of screw turbine, trough, gearbox, and generator is termed an Archimedes screw generator. Archimedes screw turbines are a recent addition to the available range of microhydro generation technologies.

ASTs are best suited to low-head microhydro sites (<5 m head, <100 kW, see Fig. 3) (Williamson, et al. 2011). They can maintain high efficiencies across a large range of flows (Hawle et al., 2012b).
1.2 Principles of Hydroelectric Power Generation

The energy associated with the flow of water over a height difference may be extracted mechanically from the flow using an energy converter. Typically, this converter is termed a turbine, and may be of several different designs. The mechanical energy captured by the turbine may then be converted to electrical energy using a generator. If the kinetic energy in the flow is negligible, the theoretical power, \( P_{\text{avail}} \), available in a water course is:

\[
P_{\text{avail}} = \rho g h Q
\]  

(1)

where \( h \) is the height difference (head) between free surfaces above and below the system, \( Q \) is the volumetric water flow rate, \( \rho \) is water density and \( g \) is the gravitational constant. The actual amount of energy which can be obtained from the flow will always be less than this, due to inefficiencies in the conversion system.

The efficiency of an AST (or any hydroelectric system) is the ratio between the power produced by the system \( (P_p) \) and the total available power in the flow:
\[ \eta_{\text{turbine}} = \frac{P_p}{P_{\text{avail}}} \]  

(2)

For an Archimedes screw generator, the efficiency of the electrical generator and the gearbox connecting the AST to the generator must also be taken into account, resulting in an efficiency of:

\[ \eta_{\text{system}} = \frac{P_p}{P_{\text{avail}}} \eta_{\text{gearbox}} \eta_{\text{generator}} \]  

(3)

1.3 **Motivation**

Most available literature on Archimedes screws focuses on the performance or applications of ASPs rather than the performance of ASTs. Notable exceptions are Muller and Senior (2009) and Nuernbergk and Rorres (2012) which will be discussed below. Even in the literature concerning ASTs, implicit assumptions are often made that ASTs will act with similar characteristics and efficiencies to equivalent ASPs. It should not always be assumed that an AST will act the same as the equivalent ASP. A fuller understanding of the physics underlying AST operation is needed to confidently predict the behaviour of an AST system.

Except for the simplified model proposed by Muller and Senior (2009), there currently exists no physically-based model which describes the behaviour of an AST with arbitrary geometry, under arbitrary operating conditions. Such models are needed to allow the design of optimal AST systems in industry.
Chapter 2: Literature Review

2.1 Microhydro Power Generation

Microhydro generation technologies are defined as systems that convert hydraulic energy to electrical energy, with a capacity less than or equal to 100 kW (Natural Resources Canada, 2009). For most hydroelectric systems, including the AST, performance is viewed in terms of the efficiency of the system – how much of the total available power the AST is capable of capturing. Micro-hydro systems typically run at efficiencies between 65-80%, with larger systems in general being more efficient (Natural Resources Canada, 2009).

Williamson, et al. (2011) discuss the relative merits of various microhydro technologies, including a design approach for selecting appropriate technology for a specific site. Compared to other generation technologies, ASTs have the greatest potential at low head sites (below about 5 m). Unlike conventional reaction or impulse turbines, ASTs have the potential for maintaining high efficiencies even as the head approaches zero (Williamson et al., 2011).

2.2 History of Modeling Archimedes Screws

ASPs have existed since antiquity; and several attempts have been made to model their behaviour. Koetsier (2004) provides a review of various analyses of the ASP throughout history, starting in antiquity, and ending with the work of Rorres (2000). This review examines the state of the art of analysis of an ASP for various time periods, and includes analyses done by Da Vinci, Cardano, Galilei and Bernoulli, among others. While the usefulness of most of the methods discussed were limited by limited availability of analytical tools, it is useful to examine prior attempts at modeling this system, and potentially incorporate previously developed concepts using modern techniques.
The Archimedes screw pump handbook by Nagel (1968, originally published in German) is likely the most influential book on ASPs. Nagel gives a comprehensive method for designing an Archimedes Screw Pump (ASP), based on field experience and some graphical analysis. In addition to methods of optimizing pumping efficiency, the author also discusses practical design concerns such as ease of manufacture and operation. There are several specific heuristics provided by Nagel for the design and operation of an ASP which may also apply to ASTs. For example, the recommended maximum gap width between the outer edge of the screw flights and the trough given for ASPs may also be adopted for ASTs.

Rorres (2000) lays out an analytical method to optimally design an Archimedes screw geometry for pumping applications. This problem is framed as maximizing the amount of water which can be lifted with each turn of the ASP. Though the method given is focused on use for ASPs, specific parts of the method may be applied to AST geometry as well, as seen in the work of Nuernbergk and Rorres (2012).

Muller and Senior (2009) created a simplified model for ASTs which idealizes the turbine’s blades as moving weirs. Based on this idealization, a comparison is made to the hydrostatic pressure wheel which is described in Senior et al. (2010). Muller and Senior (2009) concluded that an AST’s efficiency is a function of both turbine geometry and mechanical losses, and that efficiency increases with an increased number of flights (N) as well as with decreased installation angle (β). While the data presented shows good agreement with that given by Brada (1999) on AST performance, the idealizations made in Muller and Senior’s model prevent it from being able to accurately predict the efficiency of a full range of operation conditions for an AST, because the effects of rotation speed and torque variations, and geometric details, are not included.
Nuernbergk and Rorres (2012) did not directly examine the performance of an AST, but developed a model for determining the inflow head to an AST, allowing a specified bucket fill level to be achieved given a flow through the turbine. It is notable in this work that it is assumed that an optimum filling point for an AST is the same as the optimal filling point for the equivalent ASP. However, the model developed by Nuernbergk and Rorres is suitably robust to calculate the optimal inflow head for an arbitrary filling point, and therefore may be applied regardless of the accuracy of this assumption.

2.3 *Archimedes Screw Generators*

Commercial ASTs providing power to an energy grid have been installed in Europe for over a decade. In spite of this, there is almost no literature on the performance of such installations. Only three studies are known to the author at this time, the works of Bard (2007), Brada (1999), and of Hawle et al. (2012b).

Bard (2007) provided a report on the performance of an AST installed on the River Dart in Devon, UK. While the report itself only reaches preliminary conclusions as to the system performance with a constant-speed drive versus performance with a variable-speed drive, detailed performance data was reported for various flow conditions, making this work a useful source of validation data for future studies.

Brada (1999, originally published in German) discusses an overview of Archimedes screw pumps versus ASTs, as well as ASTs as a technology versus other hydropower systems of the same size.

Hawle et al. (2012b) performed a survey of existing commercial AST sites in Europe. This survey covered both quantitative aspects of the ASTs installed (such as power output, plant cost, plant
efficiency) as well as qualitative aspects (e.g. noise while operating). In addition to the survey itself, some comparison was done between empirical (survey) data and the models developed by Muysken (1932, in German), and Nagel and Radlik (1988), which were found to be in good agreement in terms of outer diameter versus design flow and fairly good agreement with rotation speed versus design flow. In addition, this survey states that the most common AST design used in Europe is a screw with $N = 3$, $\beta = 22^\circ$, and $\delta = 0.5$.

Hawle et al.’s (2012b) analysis of different plant efficiencies, sorted by speed control method is also of interest, as it shows that for flows between 75 and 115% of design flow, fixed speed ASTs are at least as efficient as variable speed ASTs. The introduction of a variable-speed drive to an AST design would significantly increase system costs. This data suggests that for sites with low variance in flow, such speed-control is not needed to maintain high efficiencies. It also highlights the need for detailed data on flow variance throughout a typical year when designing a site-specific AST installation.

Hawle et al. (2012b) also found that most of the surveyed operators experienced icing problems with the screw, with interruptions to plant operation occurring for many plants at temperatures below -10°C. The typical solution to the icing problem was to build an insulating enclosure around the screw.

2.4 Fish-Friendliness of ASGs
An additional advantage of the AST over conventional technologies is their ability to safely pass fish, as well as moderately-sized debris. McNabb (2003) provides a study of fish passage through ASPs, and found no significant difference between fish moved through an ASP and those in a control group. Kibel (2007, 2008) conducted a more comprehensive study of fish passage through
an ASG installed on the River Dart in Devon, UK, and concluded the ASG could safely pass fish across the full range of operation of the AST. The United Kingdom Environment Agency (2012) recommends coarse ‘trash screens’ to be placed at the inlet to AST systems, to prevent larger fish which could be harmed by the turbine from entering, as well as an upper limit on the tip speed of the AST and the installation of rubber bumpers.

Kibel (2009) found that at turbine blade tip speeds up to 4.5 m/s, AST systems are safe for fish under 1 kg without any protection on the blade’s leading edge. However, the report recommends that, unless screens are placed to prevent fish larger than 1kg from entering, screw turbines should either not operate at tip speeds above 3.5 m/s and/or install compressible rubber bumpers on the leading edge of the flights to prevent damage to large fish. Turbines with tip speeds below 3.5 m/s were found to only need hard rubber bumpers for large fish.

Kibel (2009) concluded intake screens are not needed for the inlet of an AST, and a typical AST system is not likely to cause any fish injury. However, a minor modification to the upper edge of the turbine blades is recommended, in the form of a rubber extrusion covering the edge. This is thought by the study’s author to reduce potential wear on the blade leading edge by debris, therefore preventing the edge from developing sharp edges. Such a modification should have no noticeable effect on the power generation capabilities of an AST.

2.5 Other Literature

Recently, more research has been conducted examining ASTs in detail, both from a lab-testing perspective such as Hawle et al. (2012a), and from a model-driven perspective such as Shimomura and Takano (2013), Schleicher et al. (2014) and Raza et al (2013).
There also exists non-English literature related to ASTs; while the contents of these publications is not directly used in this work, they bear mentioning as having influenced the development of AST systems in Europe. Notable examples are Hellmann (2003), Brada (1996), Aigner (2008), Schmalz (2010), Lashofer et al (2011), and Muysken (1932).
Chapter 3: Research Objectives

3.1 Problem statement and Significance
There currently exists no model, other than the one presented here, to predict the power produced by an arbitrary AST operating at specified speeds, or at partially-filled conditions. No literature to date discusses the operation of ASTs in partial-fill conditions. As many ASTs are installed as fixed-speed systems (Hawle, Lashofer and Pelikan, 2012b), and are typically installed at run-of-river sites, ASTs necessarily operate some of the time under partially-filled conditions. A model which accounts for such partial-fill conditions, and which describes power production from any arbitrary AST geometry is needed to optimally design an AST for a specific site. This model should use input parameters as outlined in Section 1.1 to define the AST, and output the power produced by the turbine.

This model is to be developed from first principles of mechanics and fluid statics/dynamics, both to allow the optimization of an AST’s geometry and to provide insights into the behaviour of the AST, notably its performance under partial-full conditions and any general trends between the input parameters and the performance of the AST. This understanding will allow for the development of heuristics to aid in later optimization.

Testing of small laboratory-scale ASTs, as well as data collection on a prototype AST will provide data as to the performance of ASTs under different conditions to supplement that found in the literature. This data will also be used for model validation.

3.2 Partners
This research project was conducted in collaboration with Greenbug Energy (Delhi, ON; greenbugenergy.com) which is developing AST technology. The author worked with Greenbug
Energy to develop the lab AST system used in this project. Greenbug Energy also provided logistical and technician support as well as access to a prototype AST for data collection. Funding for this research was provided by NSERC through the Engage and Collaborative Research and Development (CRD) programs.
Chapter 4: Prototype Screw

To gain an understanding of the dynamics of an AST system, a small AST (hereafter “S1”) was installed on a small watercourse in southern Ontario (Canada), and data from the site was recorded from 2011-11-18 to 2012-06-27. The goal of the data logging was to provide real-world data from a fixed-speed AST system, for validation of the mathematical model. Of particular interest were effects of flow variation, as well as the resulting variation in head. Qualitative effects of cold weather and varying flow conditions on the system were also sought by Greenbug Energy.

AST S1 was installed at a concrete weir on Big Creek, Dehli, Ontario with approximately 0.9 m of available head (Fig 4). The upper head level could be modified by placing or removing stop logs in the weir, and the downstream head was affected by downstream conditions (for example, Fig. 5 flooding, and Fig. 6, design head).

S1 was designed with a design flow rate \( (Q_{\text{design}}) \) of 70 L/s, and a design head \( (H_{\text{design}}) \) of 0.915 m. The AST was connected to a synchronous induction generator with a constant-speed gearbox with a gear ratio of 25.71:1; the AST was designed to run at 70 RPM. The target power output of the AST was 400 W. Geometric parameters for this AST can be found in Table 2.
Figure 4 - Prototype AST installed at Delhi, Ontario

Figure 5 - Example of lower end flooding on Prototype AST
Upper inlet channel and lower water basin levels were measured using pressure transducers-based depth meters (Instrumentation Northwest Inc. PS9805). Inflow water velocity was measured using a propeller type submerged flow meter (General Oceanics Model 2031H Electronic Flowmeter). The inlet channel had a width of 0.73 m. The volume flow rate into the AST was calculated using the inlet channel width and measured depth and the inflow water velocity. Temperature probes were installed inside the generator housing and outside the housing (see Fig. 7 for layout of generator and screw housing). In addition, the upper depth gauge was capable of measuring water temperature. The rotation rate of the generator was measured using the pulse output from an Electro-Sensors AP1000 Digital Tachometer, and the electrical power output of the AST was measured using a Continental Control Systems WattNode Pulse watt meter. All
measurements were recorded using a Campbell Scientific CR1000 datalogger at 5 second intervals.

![Figure 7 - S1 in Weatherproof Enclosure](image)

The downstream water level at the site was noted to rise significantly during high seasonal flows. This would result in the lower end of the turbine becoming flooded. Flow rates at the site varied greatly with season, however a mean flow rate of approximately 72 L/s was observed flowing through the prototype (when in operation) over the course of a year.

### 4.1 Flow
As $P_p = \tau \omega$ ASTs are typically installed as run-of-the-river systems (without active control of the flow rate through the $P_p = \tau \omega$), the effects of flow variation on the performance of the prototype AST was explored. Over the course of logging, different flow rates were observed. As well, the flow to the AST could be varied across a short time span by adjusting the sluice gate upstream of the AST. The sample data presented in Fig. 8 is representative of the general performance of the prototype.
Normalizing the data in Fig. 8 in terms of design flow rate and maximum power production, the data may be compared to other ASTs in literature. An example comparison to the survey data compiled by Hawle et al. (2012b) for fixed-speed ASTs is given below (Fig. 9).

The ASTs surveyed by Hawle et al. had an average outer diameter of 2.15 m (minimum 1 m). The prototype, with an outer diameter of 0.59 m, is much smaller than these, one possible
explanation for the lower efficiencies in Fig 9. In spite of this, the general trend shows that the prototype may be considered as representative of a typical (small) commercial AST.

4.2 High water levels
Another concern was that seasonal flooding of the lower reach would cause a large decrease in the effective head across the AST. Over the course of the data logging, the net head across the AST saw reductions of up to 50% from the design head, primarily due to high seasonal flow rates raising downstream water levels (the worst cases were in early spring, due to snowmelt). Power reductions of up to 80% were observed at the lowest net head, during high flow conditions.

To control for other variables, the flow was artificially blocked for a few hours during high flow conditions, causing the lower reach level to drop. The flow was then allowed to slowly return to normal, and the effects of changing lower reach level on power output were observed. The data is presented in Fig 10.

![Normalized Power at Wire vs Normalized Head, Prototype AST. Nominal flow 71 L/s, Rotation Rate 69.84 RPM, on 2012-01-29](image)

It is clear that power production decreases at a roughly linear rate as the net head decreases due to the lower reach’s level increasing. As ASTs are typically run-of-the-river systems, it is not practical to control the lower reach’s level in such high flow conditions. As one of the possible
parameters which can be optimized is the length of the flighted section, these results suggest that when designing for a particular site, the typical head variation at the site should be examined in addition to the site’s typical flow variation.

4.3 Summary

A maximum power output of 409 W at an efficiency of 74% was achieved from the prototype AST over the course of testing. This occurred at a flow rate of 60 L/s, lower than the design flow rate of 70 L/s. This is likely due to the fact that, at the design flow rate, the effective head across the AST was less than the design head of 0.915 m, due to these flows causing an increase in the lower reach water level.

Over the course of testing, the power output at the AST’s design flow was, on average, 360 W, with an average efficiency of 58%. The choice of design flow for S1 was based on the mean yearly flow at the test site, without consideration given to the effective head likely to occur at that flow rate. This suggests that in order to design a suitable AST for a particular site, both the flow and head characteristics of that site must be considered.

Operating during icing conditions showed that when the screw was open to the environment, significant ice buildup could occur, particularly near the bottom of the screw. Addition of a proper insulating structure effectively eliminated icing of the turbine, even at temperatures reaching -20°C. As long as water continued to flow through the turbine, the insulating structure maintained an ambient temperature above freezing.
Chapter 5: Lab Testing

5.1 Lab Test Setup
Laboratory experiments allow measurements of AST performance across a wide range of conditions and parameters, such as turbine rotation speed, slope, and fill point, which are not easily obtained from field measurements.

Lab tests with different conditions were carried out; Table 3 gives a summary of the different test screws’ geometry. The first three screws were identical save for their pitch; the last two were the same as screw A, but with modifications made to the end flights.

<table>
<thead>
<tr>
<th>Screw</th>
<th>D [m]</th>
<th>δ</th>
<th>P [cm]</th>
<th>N</th>
<th>Lr</th>
<th>Modification to end</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.15</td>
<td>0.52</td>
<td>0.15</td>
<td>3</td>
<td>4.00</td>
<td>n/a</td>
</tr>
<tr>
<td>B</td>
<td>0.15</td>
<td>0.52</td>
<td>0.21</td>
<td>3</td>
<td>4.00</td>
<td>n/a</td>
</tr>
<tr>
<td>C</td>
<td>0.15</td>
<td>0.52</td>
<td>1.2</td>
<td>3</td>
<td>4.00</td>
<td>n/a</td>
</tr>
<tr>
<td>D</td>
<td>0.15</td>
<td>0.52</td>
<td>1.5</td>
<td>3</td>
<td>4.00</td>
<td>Angled</td>
</tr>
<tr>
<td>E</td>
<td>0.15</td>
<td>0.52</td>
<td>1.5</td>
<td>3</td>
<td>3.50</td>
<td>Shortened</td>
</tr>
</tbody>
</table>

The laboratory tests all followed a similar procedure. The AST to be tested was installed in the test rig (Fig 11), and the pump and weirs were configured to produce a desired volume flow rate. Readings were taken both by hand and through a data acquisition device (DAQ) (National Instruments NI USB-6009), depending on the reading. Upper and lower basin water levels were measured by noting the depth of water in vertical sighting tubes installed in each basin. Rotation rate was measured via a Hall effect sensing circuit located in proximity to the path of rare earth magnets affixed to the upper end of the AST shaft (Fig 12). For every revolution, the passage of the magnets would cause the circuit to send 2 pulses to the DAQ. Torque at the shaft was
measured using a mechanical brake connected to a load cell (Omegadyne LCM703-25) at a known distance from the center of rotation. The load cell was connected to an amplifier, and then to the DAQ. The rotation rate of the screw could be varied by varying the mechanical brake. Power was calculated from the torque and rotation rate using \( P_p = \tau \omega \). Flow was measured using a flow tank with a V notch weir in early trials, or a flow tank with a fixed submerged outlet (see Appendix A for details) in later trials. The second flow tank was installed on 2013-02. In both cases, the flow through the tank was calibrated against a graduated scale measuring tank water depth prior to testing.

Figure 11 - Experimental Setup
5.2 Measurement Uncertainty
The upper and lower basin depth measurements were taken from sight tubes adjacent to a scale, graduated in millimetres. These measurements were done by eye, and the maximum expected error for the upper and lower head readings are estimated to be 0.5 mm.

There were multiple calibrations of the flow measurement apparatuses during the testing program. The uncertainty in the flow rate measure is a function of the uncertainty of the measure of depth of the flow measurement apparatus in question and the calibration curve of that apparatus. For tests from 2012-03-07 to 2012-11-03, the maximum uncertainty in the flow rate is 0.11 L/s. For tests between 2013-03-04 and 2013-04-15, it is 0.03 L/s. For tests from 2013-03-19 to 2013-07-26, it is 0.02 L/s.

The rotation rate of the screw shaft was measured using a Hall Effect sensor which sent a pulse every time a magnet affixed to the rotating shaft passed it. There were two magnets on the shaft of the AST, resulting in 2 pulses per revolution. Speed readings were taken for 60 seconds for all test points. The error in the NI-DAQ’s timing circuit may be considered negligible; therefore the
only source of error is in the counting of the pulses. It is possible for the readings to begin just after a magnet had passed the sensor, resulting in an error of half a revolution. Similarly, the reading could end just before one of the magnets reached the sensor, resulting in another half-revolution error. As the time that each pulse occurred was measured by the DAQ, any error in the time reading may be considered negligible. The maximum expected error in the rotation rate measurements is therefore 1 RPM.

The uncertainty in the torque arises from uncertainty in the measurement of the moment arm length, and uncertainty in the measured force. The moment arm length was measured in parts. The distance from the surface of the shaft on which the brake was mounted to the strain gauge was manually measured using a caliper with 0.5 mm graduations; the uncertainty in this measurement is 0.25 mm. The diameter of the shaft was also measured using a caliper with 0.5 mm graduations; the uncertainty in this measurement was 0.25 mm.

The force measured by the load cell was determined from the load cell amplifier output voltage using a calibration curve. Calibration was performed with the load cell used during testing. Known masses were hung from the load cell, and the resulting signal representing the force applied by the weight of the object from the DAQ was recorded. Each mass was tested three times for each calibration; seven different masses were used in the first calibration, and ten were used in the second. In both calibrations, a calibration curve between the known weights and the DAQ output was computed as a first-degree polynomial using a least-squares fit to the data.

To estimate the uncertainty in the load cell, the relative standard error of the calibration curve’s least squares fit between the DAQ output and the measured force resulting from hanging each known mass was determined. The relative standard errors for varied between 2.8% and 10.6%. The upper bound of 10.6% was chosen as a conservative estimate of uncertainty in the strain
gauge-DAQ system.

Observations during lab testing revealed that the mechanical brake setup was not concentric, resulting in a variation in both the effective length of the moment arm and the angle at which the force was applied to the strain gauge. The lack of concentricity in the setup is suspected to result from improper assembly of the test rig. To account for this variation, an additional uncertainty on the moment arm length of 5 mm was added.

Using Eqn 4 (Taylor, 1982), the total percent uncertainty, $\delta q/|q|$, for the torque was determined for each lab data point. Values were typically between 7.5% – 13%.

\[
\frac{\delta q}{|q|} = \sqrt{\left(\frac{\delta x}{x}\right)^2 + \cdots + \left(\frac{\delta z}{z}\right)^2 + \left(\frac{\delta u}{u}\right)^2 + \cdots + \left(\frac{\delta w}{w}\right)^2}
\]  

(4)

5.3 **Test Procedure**

In each case, a test consisted of:

1. Selection and installation of AST and setting of test conditions.
2. Spin-down Test
3. Free-wheel Test
4. Stall and Leakage Loss Test
5. Main Performance Test

When installing an AST in the test rig, the bearing pillow blocks were shimmed to ensure that no contract between the screw flights and the trough would occur as the AST rotated. Alignment of the bearings was verified at this time as well, to ensure that the AST could rotate freely, without the bearings binding. A piece of reflective tape was added to the inner tube to allow RPM
readings to be taken by a hand-held tachometer.

A spin-down test was conducted for each test condition to quantify the power loss from the bearings which support the AST. The spin-down test was conducted as follows: with the brake removed and no water flowing through system, the AST was spun up to at least 300 RPM by hand and the rotation rate was measured using a hand-held optical tachometer every 10 seconds for the first 2 minutes of free-wheeling. The time at which rotation stopped was also recorded. The change in rotation rate of the AST over the course of this test was then used to determine the effective resisting torque acting on the AST from the bearings (see Section 6.7.1). The rotation rate decreased linearly with time, suggesting that the resisting torque of the bearings is proportional to rotation rate.

The free-wheel test involved running the AST at full test flow with no brake installed, to determine the maximum no load rotation rate.

For the stall and leakage test, the brake was then installed and set to prevent the screw’s rotation. The volume flow rate through the screw, and the torque produced by the stalled AST were measured a minimum of three times with the screw at evenly spaced rotational angles. This was done to minimize the effects of inconsistencies in the flight-trough spacing on the measured leakage flow, as well as to limit the effects of any possible lacks of symmetry on the measured torque output at stall. It should be noted that in most cases, when the water level entering the screw was at its maximum depth, filling half of the circular trough when looking down the length of the screw, the water in each bucket was at a depth allowing it to spill over into the next bucket.

The main test consisted of taking at least 25 data points across the achievable range of rotational velocities and loadings. For each data point, the brake was adjusted to set rotation speed and
torque, and the system allowed to reach steady state, rotation speed and load cell force were recorded for 60 seconds through the DAQ. While these values were being recorded, the upper and lower basin levels, and flow tank’s level were measured from sight gauges and recorded.

When the effects of high water levels in the lower basin were not being examined, the lower basin fill level was lowered below the lower edge of the outlet of the AST. This eliminated any secondary effects from the lower basin level affecting tests of other variables. Due to this, the lower basin water level was measured differently depending on the level relative to the outlet of the AST. When the lower level was at or below the outlet (Fig 13, A), the effective lower basin level was taken at the lower edge of the outlet, determined by raising the lower basin level to just touch the lowest edge of the AST outlet prior to testing. At higher levels (Fig 13, B), the water level used for efficiency calculations was the measured basin water level.

![Figure 13 - Lower Basin Fill Categories](image)

While high efficiencies may be obtained with fixed-speed ASTs, the performance of each AST
under each test condition was analysed across the full range of possible rotational velocities, from almost free-wheeling to almost stalled. This allowed a characteristic torque versus rotation rate curve of the type pictured in Fig 14 to be constructed for each test condition. This has been found to be the typical relationship for all tested configurations, though the scaling and steepness of the curve varies greatly between different tests.

![Graph](image)

**Figure 14 – Rotational velocity vs torque, test screw A, β = 22.7°, screw lower end unsubmerged, Q = 0.755 L/s**

The maximum possible rotation rate at free-wheel and the maximum torque at stall may be easily seen on this graph, and general performance of multiple ASTs may be compared directly. This characteristic curve does not, however, directly suggest efficiencies of the AST in question, nor the power produced from the AST. Plotting power and efficiency as a function of rotation rate (Fig 15) clearly shows these trends. The decision to plot in terms of rotation rate and not in terms of torque was made because many ASTs are designed for a particular rotation rate (or range of rotation rates), and torque will depend on the flow through the system. It is therefore more useful to design for a rotation rate which gives maximum power.
In addition to gaining a general understanding of the dynamics of ASTs, the effects of screw pitch, lower basin fill height, flow, and screw inclination angle on power output and efficiency were investigated.

### 5.4 Pitch

The pitch of the AST may be easily selected during the design of the system. It has been suggested that pitch can have a large effect on the efficiency of Archimedes screws (Rorres 2000). However, the most detailed analyses currently available for optimizing the pitch of Archimedes screws, such as Rorres (2000) and Nagel (1968), are intended for ASPs, and may not be the optimal values for ASTs. Three lab screws with different pitches were produced to test the effects of pitch on the power produced by an AST (screws A, B, and C in Table 3). Each screw was tested at the same flow and slope. In all cases, the flow rate through the screw was set to 0.7±0.1 L/s, the slope was $\beta = 22.5^\circ$, and the head drop was 0.22 m. Figs 16 and 17 show the results for power and efficiency, respectfully.
According to these results, the efficiency (and power produced) from the screw with the larger pitch is higher. This is counter to the findings of Rorres (2000) for ASPs, reinforcing the idea that optimally designed ASPs will differ from optimally designed ASPs.
Figure 17 - Rotation rate vs efficiency for various pitches, lab screws

Changing the pitch of the screw changes both the volume in each bucket and the number of buckets. With a decrease in volume, there should be less force per bucket acting on the AST, however, there will be an increase in the number of buckets. The number of buckets in an AST may be calculated as

\[ n_b = \frac{L r N}{Pr} \]  \hspace{1cm} (5)

Under the assumption that the lower end of the AST is not submerged. As the pitch ratio increases, the number of buckets decreases. Fig. 17 shows that as the pitch increases, the total power at a given speed (and therefore the total torque) increases. Therefore, within the range of pitches tested and for a smaller screw, the torque per bucket must decrease at a greater rate than the number of buckets increases with decreasing pitch.
5.5 **Lower Basin Fill Height**

Observations of operational ASTs suggested that during extremely high flow conditions, it is possible for the lower end of the turbine to become submerged due to elevated lower reach water levels. Data from the Greenbug test site indicates that this flooding can cause a reduction in power produced from the AST.

In order to quantify power losses from outlet flooding, lab screw A was tested at several different lower basin fill levels. These levels ranged from leaving the outlet unsubmerged, to submerging two thirds of the entire turbine. Each test consisted of on average 39 (minimum 24, maximum 47) data points, at different rotation rates. The performance curves as a function of the outlet water level are shown in (Fig. 18).

![Figure 18 - Rotation rate versus torque for different lower basin levels](image-url)
Figs 18 and 19 show that torque and power production decreased with increasing lower basin fill level. Note that the maximum power production for every case occurred within a small range of rotation rates (4-5 rad/s, see line in Fig. 19). This becomes important in the selection of an operational rotation speed for a turbine, as the introduction of a variable-speed controller can add to the cost and complexity of an AST installation. The relative lack of variation between rotation rates at peak power for different inlet conditions implies that variable-speed capabilities may not be needed in most cases, and may be limited to a small range of rotation rates if required. The fact that many commercial ASTs use induction generators and operate at a fixed rotation rate further supports this idea.

![Figure 19 - Rotation rate vs power for different lower basin levels](image)

In terms of efficiency (Fig. 20), the small variation in rotation rate at peak power output is more
noticable. In addition, it is apparent that while the power output does change, the peak efficiency of the AST still remains fairly high — over 50% efficient with more than 50% of the screw submerged.

Figure 20 – Rotation rate vs efficiency for different degrees of submergence of screw outlet lower basin levels

5.6 **Volume Flow Rate**

Since ASTs are typically installed as run-of-the-river systems, an understanding of how an AST will perform under different flow conditions was needed. It should be noted that most prior literature only reports results for systems running with full buckets, and data for Archimedes screws operating in partial full states could not be found in the literature. Lab screw B was tested at a range of different flows. In each case, the lower basin level was low enough to prevent any flooding of the lower turbine end. The data is summarized in Figs 21, 22 and 23.
As would be expected, as the flow available to the turbine increases, the power production of the turbine also increases. It is interesting to note that the last two data sets (flow equal to 1.1 L/s and 1.2 L/s) overlap, suggesting that the maximum flow rate that the test screw can utilize is approximately 1.1 L/s. This is further supported by the fact that the efficiency curve for the 1.1 L/s test is slightly above the 1.2 L/s flow (see Fig. 22).
Examining Fig. 22, it is clear that there is a general trend of decreasing efficiency with decreasing flow. However, in the mid-flow ranges, the efficiency of the AST does not change with the flow rate. Additional data collected (Fig 23), shows that there is little change in the efficiency of the AST at mid-flow. It may then be concluded that the efficiency of an AST remains fairly steady over high and mid-high flow rates, only decreasing at low flow rates.
Due to the size of the lab screws, there was some doubt as to how analogous their results are to a full-sized AST. Hawle et al. (2012b) surveyed several ASTs across Europe and summarized their efficiencies; of these, 18 were fixed-speed systems. The efficiencies of these systems, as well as that of the prototype screw S1, may be compared to the efficiencies of the lab screw as shown in Fig. 24, by normalizing the observed flows by the full-bucket flow rate \(Q_0\) for each system.

This comparison suggests that the efficiencies of the lab tests, as well as the data from S1, follow the same trend with respect to normalized flow rate, and may be taken as representative of a full-size system. Note, however, that the reported efficiencies for S1 and the sites surveyed by Hawle et al. (2012b) are for electrical power, while the efficiencies for the lab screws are power at the shaft.
Since the efficiency of a turbine at the shaft is always higher than the electrical efficiency of the turbine-generator system, it would be expected that the lab screws’ efficiencies would be greater than those of S1 or the data gathered by Hawle et al. (2012b). It is expected that the small size of the lab screws (comparatively larger losses due to the relatively large trough gap compared to the size of the screw) results in the observed efficiencies at the shaft being lower than would be expected of an AST.

![Graph showing comparison of efficiency vs flow for various turbines](image)

**Figure 24 - Comparison of efficiency vs flow for various turbines**

Picking a rotation rate near the peak of each curve in the lab tests, a clear trend for maximum power versus flow rate may be observed (Fig. 25). Values were taken such that the same approximate rotation rate occurred for all points.
The roughly linear relation between power produced and flow rate for all but very low flows reinforces the idea that efficiency is largely independent of flow rate at flows above $5 \times 10^{-4}$ m$^3$/s.

A qualitative result of the lab testing is that, for any given test condition, the faster the AST is turning, the lower the water level in each bucket. This is because the net flow through the system remains the same for a particular flow condition, regardless of the rotation rate. At faster rotation rates, there is a faster linear velocity of the water through the AST, and a corresponding drop in bucket water level. The inflow water level would also change with bucket fill height.

### 5.7 Inclination Angle

When installing an AST, the inclination angle may be freely chosen, based on the overall length of the AST and the total available head at the site. Muller and Senior (2009) suggest that the efficiency of an AST will increase with decreasing screw installation angle, however for a given screw a decrease in the installation angle would correspond to a (sometimes large) decrease in the
overall head drop across that AST. Tests were conducted to determine the efficiency gains from decreasing the installation angle, and what effects such a change would have on the overall power produced from a particular AST geometry.

Lab screw A was tested at different installation angles ranging from $17.2^\circ$ to $34.4^\circ$, the minimum and maximum installation angles achievable with the test apparatus. The data is summarized in Figs 26 and 27.

![Figure 26 - Power versus rotation rate, screw A (Pr = 1.0), flow rate 1.07 ± 0.06 L/s](image)

Examining Fig 26 would suggest that the performance of the AST increases greatly with increasing installation angle, however it must be noted that the total available power due to the head drop across the AST will also increase with larger installation angles, for a given length of turbine. Examining the data in terms of efficiency (Fig 27) shows that, while in general the
efficiency does increase slightly with increasing installation angle, the peak efficiency remains almost constant except at the lowest installation angle.

Figure 27 - Efficiency versus rotation rate, screw A (Pr = 1.0), flow rate 1.07 ± 0.06 L/s

5.8 Lab Test Summary

The lab testing provided a range of insights in AST operation. The insights are summarized below.

The efficiency of an AST will increase with increased flow through the AST, to the limit of the maximum flow which the AST can handle without overflowing.

The efficiency of an AST reaches a maximum for a particular configuration at a particular
rotation rate, typically close to the stall rotation rate of the system. It is expected that this is because this condition maximizes the amount of filling in the screw’s buckets, resulting in the maximum torque capable of being produced by that particular geometry and setup.

At low flow rates, the efficiency of the system drops. It is expected that this is caused because at lower flow rates the available power approaches the magnitude of the system’s losses; the losses in the system will likely not decrease as the flow decreases, therefore the losses will consume a larger portion of the available power at lower flow rates.

The power output from an AST is adversely affected by submersion of the lower end. The efficiency is also adversely affected, though not as drastically as would be assumed, as the total available power is also reduced with the higher water levels on the downstream end which cause submersion. Peak efficiency is lowered by 18%, but the rotation rate at which peak efficiency occurs shows negligible change. It is expected that the efficiency drop is due to the action of the submerged section of the screw on the water in the lower basin, causing resistance to the motion of the screw.

It is interesting to note that the peak efficiency of the unsubmerged case (Fig. 20) is less than the next two cases (22% and 35% submerged). This suggests that the very bottom of the AST does not contribute to power production. The last bucket at the outlet of the AST is constantly being unformed as water exits the turbine, and therefore over a long span of time would not contribute as much to the power output of the turbine as the other buckets further from the outlet. This implies that taking the head across the AST from the lowest edge of the screw when it is unsubmerged does not give an accurate measure of effective head for the purposes of efficiency calculations; the effective head across the screw will vary depending on the fill level of the screw, but will always be above the lowest edge of the AST.
Efficiency increases with increasing inclination angle, contrary to the model proposed in Muller and Senior (2009). Theoretically, as the pitch decreases the amount of leakage between buckets should also decrease, as the effective length of the edge of the flight between the buckets decreases. This would lead to a higher efficiency due to less leakage. However, it may also be argued that the larger pitch of screw B allows for more of the static pressure a screw flight from the water in a bucket to be in the direction of rotation, therefore contributing more to the torque used to turn the screw. The three pitches examined in the lab were not enough to fully determine the relationship between efficiency and pitch, but qualitatively suggest that a pitch ratio greater than 1 allows for higher efficiencies than pitch ratios ≥ 1.

Screws D and E were tested to compare to screw A, but no conclusions could be drawn from these tests; it is expected that the effects of the differing end geometry is smaller than the level of noise in the test apparatus, resulting in no clear difference in the performance of screws D and E, compared to screw A.
Chapter 6: Power Model of an Archimedes Screw Turbine

6.1 Previous Models in Literature

6.1.1 Muller and Senior

Prior to this work, the most comprehensive model of power production from an AST in the literature to the author’s knowledge is the work of Muller and Senior (2009), which constructs a simplified model of an AST. Muller and Senior’s approach models the flights of the screw as vertical weirs which propagate along the centerline of the screw. Hydrostatic pressure against the weir (flight) is the prime motivator to transfer power to the screw.

The model assumes a representative wetted area of unit width, and basic hydrostatics are used to equate hydrostatic pressure of a bucket upstream and downstream of a flight to a net force on the flight in the axial direction. The work done on the screw is then expressed as a function of the wetted area and the longitudinal velocity of the screw flights along the axis of the screw.

Muller and Senior conclude that the efficiency of an AST is independent of the rotation rate, and is a function of the screw geometry and inflow depth. Lab data given in Section 5 shows that a given AST’s efficiency is not independent from its rotation rate, for a given flow. The data Muller tested against was from an AST run at a single rotation rate (Brada, 1999), which appears to correspond to the maximum fill condition only, implying no data comparing bucket fill amounts was available at the time Muller developed his method.

6.2 Model Overview

Similar to the model proposed by Muller and Senior, the model developed in this thesis assumes that the power produced by the AST arises due to the hydrostatic pressure of the bucket volume acting on the flight surface. The model developed here takes into account the full geometry of the
system in order to evaluate the performance of an AST based on an arbitrary set of input parameters.

The model assumes a quasi-static state for the water within the buckets, and does not attempt to model effects arising due to fluid motion within the bucket. It is expected that at low rotation rates losses due to such motion may be considered negligible.

The hydrostatic pressure acting on each flight is converted into a force vector, which in turn is converted into a torque acting on the AST. The torque from each bucket is summed, and used with the resistance of the bearings upon which the AST turns to derive a total net output torque for the AST.

Using the calculated rotation rate and torque, as well as a calculation for the total available power in the flow, the performance of the AST is determined for a particular combination of input parameters.

The model was developed assuming that each bucket within the AST is identical to all the other buckets within it. Each bucket will have a ‘front’ flight surface and a ‘back’ flight surface (Fig. 28). The component of the hydrostatic force acting in the direction of rotation of the screw will contribute to the total net force rotating the screw. On the lower flight, this component force adds to the total net force rotating the screw; on the upper flight, this component force is in the opposite direction and therefore subtracts from the total net force. The total net force acts about the centerline of the screw, resulting in a net torque on the screw’s shaft.
The total force on a flight (Fig. 28, lower flight) is the hydrostatic pressure acting on the surface of that flight from the bucket above (Fig. 28, A) and below (Fig. 28, B) that flight multiplied by the areas over which the hydrostatic pressure acts (White 2003). The net torque placed on a flight from the buckets is caused by the components of these forces which is along the direction of rotation of the screw (Fig. 28, a₁ and b₁).

The net torque caused by each bucket is the vector sum of these torques placed on the front and back flights. The torque caused by each bucket is then a function of the bucket volume and the geometry of the screw; for a given screw geometry, a given bucket fill will result in a particular torque net torque from the water, regardless of other conditions such as rotation rate.

The maximum torque occurs when a bucket is full; for a given screw with a given flow, this will occur at a specific rotation rate. Turning the screw any slower will result in water pouring out of
the top of each bucket down into the next, effectively creating additional overflow leakage in the system.

In practice, the maximum rotation rate of the screw occurs when there is no load on the system save rolling resistance of the bearings, and therefore occurs when the torque produced by the bucket fill is equal to the power needed to overcome this rolling resistance. This was observed in the lab to correspond to almost-empty buckets. It should be noted that this maximum rotation rate assumes a set flow rate. If a larger flow was supplied to the screw, the maximum rotation rate would increase.

The mechanical power output from the AST is \( P_p = \tau \omega \), where \( \tau \) is the total net torque (N m) on the screw, and \( \omega \) is the rotation rate (rad s\(^{-1}\)) of the screw.

The volume in a bucket is modeled using a numerical finite-element mesh. The mesh is bounded by the inner and outer diameters of the screw (\( d \) and \( D \), see Fig. 29), the front and back flights for that bucket, and the water plane. For any given AST to be modeled, the inner and outer diameters must be given. Section 6.3.1 describes the mathematical representation of the front and back flights of the AST. Section 6.3.2 describes the representation of the water plane. Section 6.4 describes how inner and outer diameters, the flights, and the water plane are used to determine the volume in a single bucket for an AST with the chosen parameters.

Once the volume in a bucket and the flow rate through the screw are known, the rotation rate of the AST may be determined (Section 6.5). To determine the power produced by the AST, the torque on the center shaft must also be known. Using the geometry determined previously, this torque may be found as described in Section 6.6.
6.3 Model Variables
A set of input parameters, shown in Fig 29 and summarized in Table 4, may be used to fully define the geometry of the AST itself. For any given installation angle and operating condition, there will be additional parameters which define the AST’s angle from the horizontal and the depth of the water in each bucket. These parameters are listed separately in Table 4.

The model implementation also uses several derived intermediate variables. These are summarized in Table 5.

The model uses cylindrical coordinates \((\theta, r, z)\), such that the \(z\) axis is along the centerline of the screw, and inclined at an angle \(\beta\) to the global horizontal.

![Figure 29 - Basic geometry of Archimedes screw.](image)
### Table 4 - Geometric Parameters of an AST

<table>
<thead>
<tr>
<th>Variable</th>
<th>Meaning</th>
<th>Variable Ratio when normalized by $D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D$</td>
<td>Outer Diameter of Flights</td>
<td>n/a</td>
</tr>
<tr>
<td>$d$</td>
<td>Inner Diameter of Flights</td>
<td>$\delta = d/D$</td>
</tr>
<tr>
<td>$P$</td>
<td>Pitch of Flights</td>
<td>$Pr = P/D$</td>
</tr>
<tr>
<td>$L$</td>
<td>Length of Flights</td>
<td>$Lr = L/D$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Inclination of Screw from Horizontal</td>
<td>n/a</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of flights</td>
<td>n/a</td>
</tr>
<tr>
<td>$r_{off}$</td>
<td>Water plane offset at $z = 0$</td>
<td>n/a</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Water plane angle</td>
<td>n/a</td>
</tr>
<tr>
<td>$f$</td>
<td>Relative fill level; 0 implies no fill, 1 implies maximum fill without overflow.</td>
<td>n/a</td>
</tr>
</tbody>
</table>

### Table 5 - Additional Model Variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>A point on a flight for the current iteration of the torque module.</td>
</tr>
<tr>
<td>P2</td>
<td>A point on the water plane vertically above P1 in the global coordinate system</td>
</tr>
<tr>
<td>$r_{offmax}$</td>
<td>Value of $r_{off}$ corresponding to maximum fill</td>
</tr>
<tr>
<td>$r_{offmin}$</td>
<td>Value of $r_{off}$ corresponding to no fill</td>
</tr>
<tr>
<td>$x$</td>
<td>Offset from P1 to origin along $\beta = 0$</td>
</tr>
<tr>
<td>$k$</td>
<td>Integer representing which bucket is being examined, from the start of the screw. Set to 1 (first bucket) throughout model.</td>
</tr>
</tbody>
</table>

#### 6.3.1 Archimedes Screw Model

The geometry of an Archimedes screw can be modeled as a set of helices wound around a central shaft. Helices of this form may be conveniently modeled using cylindrical coordinates. Defining the $z$-axis as along the centerline of the helix (as in Section 6.3) a point on a helix of arbitrary radius $r$ and unit pitch is given by the triple $P(\theta, r, z)$, such that (see Fig. 30),

\[
\begin{align*}
    r(z) &= r \\
    \theta(z) &= z
\end{align*}
\]
However, these equations do not represent the full geometrical complexity of the Archimedes screw. Introducing the screw geometry parameters discussed in Section 1.1 which define an arbitrary Archimedes screw, a new set of equations may be constructed to represent any point on a given flight of the screw. The parameter $N_i \in \{0, 1\ldots N-1\}$ is introduced to select which flight on the AST the equation refers to.

\[
\begin{align*}
    r(z) &= r \\
    \theta(z) &= 2\pi \left( z + \frac{N_i}{N} \right)
\end{align*}
\]  

(15)

Figure 30 - Coordinate system of AST, $d$ and $D$ of two flights are shown, point P is on the D of the first flight.

For the specific case of a point on the outer edge of a flight, $r_{outer}(z) = D/2$; similarly for an inner edge, $r_{inner}(z) = d/2$. Thus, $r \in [d/2, D/2]$. For a screw of length $L$, $z \in [0, L]$. To allow for comparisons between differently sized screws, the parameters introduced in Section 1.1 may be used. Thus, Eqn. 15 becomes

\[
r(z) = r
\]

(16)
\[ \theta(z) = 2\pi \left( \frac{z}{PrD} + \frac{N_l}{N} \right) \]

The derivation of Eqn 15 is given in Appendix B. Note that equation 16 may be re-arranged for either \( \theta = f(z) \) or \( z = f(\theta) \). Both forms are used in the model.

6.3.2 Water Plane

The free surface of water in the bucket is one of the bounding surfaces of the bucket volume. In order to determine the bucket volume (needed to determine the rotation rate, see Sections 6.4 and 6.5), and the torque acting on the center shaft of the AST (Section 6.6), this free surface must first be defined.

The water surface in a bucket may be defined as a plane inclined at an angle \( \alpha \) from the screw’s y-axis, (Fig. 32), such that

\[ \alpha = \frac{\pi}{2} - \beta \] (17)

As the water plane is parallel to the global horizontal, this angle encapsulates the inclination of the screw. In addition to an angle of inclination, the plane defining the water surface will have an offset from the origin along the line \( \theta = \pi/2, z = 0 \), called \( r_{off} \) (Fig 31).
Given any two of \((r, \theta, z)\), the third variable will define a point on the water plane according to the relationship given by Eqn. 18, the derivation of which is found in Appendix C.

\[
z = \tan \alpha \left( r_{off} - r \cos \left( \theta - \frac{\pi}{2} \right) \right)
\]  

(18)

The value of \(r_{off}\) is dependent on a ratio \(f\) which represents the depth of water in the bucket. At \(f = 0\), there is no water in the bucket, and the water plane is tangent to the outer edge of the front flight at \(\theta = \pi/2\). At \(f = 1\), the bucket is as full as possible without overflowing, and the water plane is tangent to the inner edge of the front flight, at \(\theta = 3\pi/2\) (Fig. 32).
The value for any $r_{off}$ is calculated based on a linear interpolation between these two limits as outlined below:

The required $r_{off}$ for the no-fill condition is computed. This is defined as the $r_{off}$ at which the water plane is tangent to the outer edge of the front flight. When viewed from the side, a flight of the AST may be described as a sine wave along $z$, such that

$$y = \frac{D}{2} \sin \left( \frac{2\pi}{PrD} z + \frac{N_i}{N} \right)$$  \hspace{1cm} (19)$$

The water plane may also be determined in this manner as a line with a slope of $-\alpha$. Solving for the first point above $z = 0$ where the derivative of Eqn 19 (given in Eqn 20) is equal to this slope yields a point $P(0, y, z)$. The $z$ value is then used in Eqn 16 to find a corresponding $\theta$, and the $\theta$ and $z$ values along with $r = D/2$ are then used in Eqn 18 to solve for $r_{offmin}$.

$$y = \frac{\pi}{Pr} \cos \left( \frac{2\pi}{PrD} z + \frac{N_i}{N} \right)$$  \hspace{1cm} (20)$$

The value $r_{offmax}$ is computed in a similar manner, except that when using Eqn 18, $r = \delta D/2$, and
the z-value used is the second point above $z = 0$ where Eqn 20 equals the slope $-\alpha$ after the point used to find $r_{offmin}$.

The value $r_{off}$ is calculated using Eqn. 21.

$$r_{off} = r_{offmin} - h_{fill}|r_{offmax} - r_{offmin}|$$  \hspace{1cm} (21)

An upper and lower limit on each of the bounding flights is also computed at this time. The bounding flights are defined based on the flight number used to generate them. The bucket of interest is defined as the first bucket above $z = 0$ such that any point ($\theta, r, z$) in the bucket will lie between the corresponding $z$-values of flight $N_i = 0$ (the ‘back’ flight) and flight $N_i = N-1$ (the ‘front’ flight).

The limits are used to calculate the volume and the torque. To find them, a small (no more than one ten-thousandth of the length of the AST) step size is first defined. For each of the bounding flights, points ($\theta, r, z$) on the outer helix ($r = D/2$) of that flight are found iteratively from $z = 0$ to $z = Lr D$ (the upper end of the screw), moving in the $z$ direction by the step size. For each point, the $z$-value on the water plane given by that point’s ($\theta, r$) is also computed.

The lower bound on each flight is conceptually the first point where the $z$-value for the helix and the $z$-value for the water plane are equal (Fig 33, A); this is where the water plane first crosses the bucket.
Figure 33 - Convergence of points for finding the lower limit of a bucket; black lines are the lower flight’s $D$ and $d$ bounds, yellow lines are the upper flight’s bounds.

The model gives a close approximation to this point by finding the first point where the $z$-value on the helix is greater than the $z$-value on the water plane. Testing for equality was found not to work in MatLab, due to rounding errors, and a non-infinitely-small step size. Given a small enough step size, this point approximates the true value to allow for accurate calculations.

The upper bound on each flight is conceptually the second point of intersection between the outer edge of the helix and the water plane (Fig 33, B). This point is calculated based on the first point. Between the $z$-value of the first point, and the upper limit of the screw, the first point where the $z$-value on the helix is less than the $z$-value on the water plane is found and used as the upper bound. If a valid value for the lower bound has been determined, a valid value for the upper bound is guaranteed, given a sufficiently small step size.

If there is any water in the bucket, upper and lower bounds on $z$ for the front flight are guaranteed. However, this is not always the case with the back flight. In the case that the back
flight is not wetted, the upper and lower bounds on the back flight will be found above the upper bound on the front flight (conceptually in the next bucket). In this case, the calculations for resisting torque on the back flight are omitted (see Section 6.6).

6.3.3 Bucket Fill Height
The fill height of the bucket is also determined numerically. The algorithm generates volumes at several different fill heights from \( f = 0 \) to \( f = 1 \) full and compares these to the calculated volume for the desired flow and rotation rate (Section 6.5, Eqn. 23). As fill height is calculated numerically, the step size between used to calculate volume affect the obtained solution. A typical solution to this type of problem is to try multiple step sizes until convergence to a single output value is noted; step sizes coarser than the one at which this occurs are expected to give a poor answer, and step sizes finer than this will increase computation time for little or no gain. Fig. 34 demonstrates the convergence test used for the fill height step size (for an AST with \( Pr = 1, \beta = 22^\circ, D = 0.5 \text{ m}, \delta = 0.3 \text{ to } 0.8, Lr = 4, N = 3, Q_j = 0.4 \text{ L/s}, \omega = 1 \text{ to } 9 \text{ rad/s} \)). Multiple points were tested to ensure that the output value was properly converging. Based on this analysis, a \( \Delta f = 0.01 \) step size causes the solution to converge. All other cases tested for convergence give similar results.
Figure 34 - Example of fill height increment size on model output

6.4 ‘Bucket Volume’

A ‘bucket’ is the region bounded by two consecutive flights, the inner and outer diameters of the helices, and the water plane (Fig. 32). The volume of a bucket must be determined in order to calculate the rotation rate of the AST (see Section 6.5).

Due to the variations in the general shape of a bucket as the geometric parameters change, it is not easy to define a closed-form analytical solution to calculate the volume of water held within a ‘bucket’ for any arbitrary AST geometry (Rorres, 2000). Therefore, a numerical model of the volume is used. The equations developed above for the water plane and the screw flights are periodic (repeating for each bucket), requiring a specific bucket to be chosen prior to solving, and a phase-shift made in the equations to properly apply these equations to any bucket other than the
first bucket, which starts at \( z = 0 \). As such, the algorithm used in this model makes the following assumptions:

- The bucket being modeled is the lowest complete bucket along the z-axis of the screw.
- The lower bounding flight (in terms of the screw’s z-axis) is flight \( Ni = 0 \)
- The upper bounding flight is \( Ni = N-1 \) (remembering that \( Ni \in \{0, 1, \ldots, N-1\} \))
- The water plane has been defined prior to this algorithm’s execution.
- The minimum and maximum bounds in \( z \) for the volume in question have already been defined.

In addition, the model assumes that any differences in filling between individual buckets are negligible.

The upper limit on \( z \) for the bucket volume occurs at the same point as the upper limit on the front flight (the second point of intersection, Fig. 33, C). The lower limit on \( z \) is the first point of intersection on the back flight (the first point of intersection, Fig. 33, A). If the volume element does not extend to touch the back flight, \( z = 0 \) is assumed to be the lower limit.

For any point \( P(\theta, r, z) \) within the bounds \( r = [\delta D/2, D/2], \theta = [0, 2\pi], z = [z_{\text{min}}, z_{\text{max}}] \), that point will lie within the bucket’s water volume if \( z \) is greater than the \( z \)-value calculated by Eqn. 18, and \( \theta \) is less than the \( \theta \)-value given by Eqn. 16 with \( Ni = 0 \) (back flight), but greater than \( \theta \)-value given by Eqn. 16 with \( Ni = N-1 \) (front flight).

Eqn 22, based on a difference of circular sectors with height \( z \), may be used to find the volume element pictured in Fig. 35. Integrating Eqn. 22 within the bounds and limits outlined above (inner and outer diameters, water plane, flight edges) would give an exact value for the volume within a
bucket. As of this writing, such an analytical closed-form solution has yet to be found (Rorres, 2000).

Instead, a mesh is defined within those bounds and limits; the spacing between each mesh point in \((\theta, r, z)\) being defined as \((d\theta, dr, dz)\). With a sufficiently small mesh spacing, an acceptable approximation of the volume of the bucket may be found by applying Eqn 22 at each point within the mesh and summing the resulting volume elements.

\[
dvol = \left( \frac{(r + dr)^2 d\theta}{2} - \frac{r^2 d\theta}{2} \right) dz
\]

\[(22)\]

\[\text{Figure 35 – Volume element}\]

6.4.1 Bucket Volume Prediction Accuracy

To validate the bucket volume model outlined, the model was implemented in MatLab, and a representative volume element was created using Solidworks 2011 (Fig. 36). Solidworks was
chosen to generate this representative volume as it is an industry standard solid body modeling software. Similar to the method described above, Solidworks uses finite element approximation to determine the volume of a solid body. Give a properly formed solid body, the volume calculated by Solidworks may be taken as an accurate representation of reality. The MatLab model was used to calculate the bucket volume for several different AST geometries. The Solidworks model was also used to determine the bucket volume for these geometries.

Due to the implementation of the Solidworks bucket volume, the values for $r_{off}$ and $\alpha$ were also needed as inputs. These were also output from the MatLab model. The nature of the Solidworks model was such that, for many of the parameter combinations tried, Solidworks solver errors had to be manually resolved (these errors were related to the order of steps used to model the volume in Solidworks, and once resolved under human supervision, did not have any effect on the accuracy of the volume prediction). This limited the number of parameter combinations which could be run, as the Solidworks model required an operator to be present to review the output of each combination.

A sample comparison between bucket volume predicted by Solidworks and predicted by the model is given in Table 6.

The model is in good agreement with the volume predictions given by Solidworks, with the relative error between the model predictions and the predictions given by Solidworks less than 0.09 in all cases examined.
Table 6 - Predictions of volume with Solidworks test and model, Pr = 1, β = 15°, N = 3, D = 0.146 m

<table>
<thead>
<tr>
<th>Solidworks Bucket Volume Prediction (m³)</th>
<th>Model Bucket Volume Prediction (m³)</th>
<th>Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.36E-04</td>
<td>3.56E-04</td>
<td>5.95</td>
</tr>
<tr>
<td>3.02E-04</td>
<td>3.21E-04</td>
<td>6.29</td>
</tr>
<tr>
<td>2.54E-04</td>
<td>2.70E-04</td>
<td>6.30</td>
</tr>
<tr>
<td>2.17E-04</td>
<td>2.31E-04</td>
<td>6.45</td>
</tr>
<tr>
<td>1.87E-04</td>
<td>2.00E-04</td>
<td>6.95</td>
</tr>
<tr>
<td>1.68E-04</td>
<td>1.79E-04</td>
<td>6.55</td>
</tr>
<tr>
<td>1.51E-04</td>
<td>1.61E-04</td>
<td>6.62</td>
</tr>
<tr>
<td>1.38E-04</td>
<td>1.46E-04</td>
<td>5.80</td>
</tr>
<tr>
<td>1.23E-04</td>
<td>1.31E-04</td>
<td>6.50</td>
</tr>
<tr>
<td>1.14E-04</td>
<td>1.22E-04</td>
<td>7.02</td>
</tr>
<tr>
<td>9.88E-05</td>
<td>1.05E-04</td>
<td>6.28</td>
</tr>
<tr>
<td>8.01E-05</td>
<td>8.70E-05</td>
<td>8.61</td>
</tr>
</tbody>
</table>

As the performance model uses a numerical process to find the volume, a sensitivity analysis was run on the number of volume elements used to generate the overall volume. The results are summarized in Fig. 37. Based on this analysis, using on the order of 500,000 volume elements will cause the volume prediction to converge; an increased number of volume elements does not change the volume prediction appreciably. This element size also gives a result which has a percent error for the volume estimation between the model and Solidworks predictions which is
less than 3%.

Figure 37 - Sensitivity analysis of number of elements for volume calculation

6.5 Rotation Rate

Based on the geometry of the AST, the flow through it, and the volume held within a bucket, the rotation rate of the AST may be determined analytically. The problem may be stated as: Given a bucket volume $V_b$ and a flow rate $Q_w$, at what rotation rate would the AST need to turn in order to maintain both $V_b$ and $Q_w$? The process is given below.

Isolate a single pitch-length of the screw, and examine the flow into and out of the isolated region. Let the volume within a single pitch $P$ of the screw equal $V_p$.

In one rotation of the screw, the entire volume $V_p$ will have moved through one pitch of the screw. As the flow rate is assumed constant, this implies that $V_p$ will flow through a single length
of the pitch and in a single revolution. There are N buckets per pitch, thus \( V_p = N V_b \).

This implies that in one rotation, \( N V_b \) will have moved past a point. Given a rotation rate of \( \omega \) rev/s, this implies the flow rate will be \( \omega V_b \) m\(^3\)/s

We have already defined the flow rate as \( Q_w \) m\(^3\)/s. Therefore, \( \omega N V_b = Q_w \). Solving for \( \omega \),

\[
\omega = \frac{Q_w}{NV_b} \text{ rev/s} \quad \text{or} \quad \omega = \frac{2\pi Q_w}{NV_b} \text{ rad/s}
\]  \hspace{1cm} (23)

### 6.6 Torque from a bucket

The method for determining the rotation rate for an AST based on the flow rate and bucket volume was described in Sections 6.3-6.5. To calculate the power produced from an AST, the torque transmitted to the center shaft must also be known. The method used to calculate this torque is discussed here.

Due to the complexity of the bucket’s shape, the torque generated from the water in a single bucket cannot easily be modeled analytically. A numerical solution is employed, similar to that used in Section 6.4 for determining the volume of the bucket. It is described in brief here, and in more detail below: First, meshes representing the wetted surfaces of the front and back flights are first defined (Fig. 38).
Next, an element area surrounding each mesh point is defined. The hydrostatic pressure at each mesh point, and the resulting force applied to the point’s element area is determined, followed by the distance from the point to the center of rotation of the screw. The component force which is translated into torque to turn the screw is then calculated, and added to the total net torque applied by the water volume acting on the flight. Note that, in the case of the back bucket, the torque acts against the direction of rotation, and therefore the torque values calculated are negative.

**6.6.1 Defining Mesh**

The method used to find the upper and lower limits of the wetted areas of the flights bounding the water volume is identical to that used in finding the upper limit of the bucket volume. The lower limit for the ‘front’ flight is the first point of intersection found in that method, while the upper limit is the second point of intersection (Fig. 33, C). The upper and lower limits on the ‘back’ flight are found using an identical method on with that helix (Fig. 33 B and A, respectively).
At each point on the mesh, denoted as point P₁ for that iteration, an element area surrounding that point and equal to the mesh spacing at that point is calculated (Fig. 39).

![Element mesh for calculating torque](image)

**Figure 39 - Element mesh for calculating torque**

With sufficiently small element areas, a vector normal to the flight at P₁ may represent the direction along which the hydrostatic force acts at P₁. Only a portion of this force will contribute to turning the turbine, as only a portion of it is in the direction of rotation of the turbine.

Note that, for any point P on the mesh, the z-coordinate of that point P will be greater than the z-coordinate of the water plane at P’s r and θ coordinates. This is guaranteed by the method used to find the upper and lower limits (in z) of the mesh, as well as by the definition of the bucket.

**6.6.2 Component Force**

The hydrostatic pressure exerted on the flight surface generates a torque on the turbine, causing it to turn. The hydrostatic pressure at a point P₁ on one of the bounding flights acts on the elemental
area of the flight around \( P_1 \), creating an elemental force on the flight. A component of this force generates an elemental torque, which when summed across the entire mesh, gives the total net torque produced on that flight by the bucket in question. The hydrostatic pressure, \( p \), may be found using (White 2003)

\[
p = \rho gh
\] 

(24)

Where \( \rho \) is the fluid density (assumed as 1000 kg/m\(^3\) for water throughout the model), \( g \) is the gravitational acceleration constant (assumed as 9.81 m/s\(^2\)), and \( h \) is the vertical depth below the water plane. This distance is equal to the length of the line from the water plane to the mesh point being analyzed (distance between \( P_1 \) and \( P_2 \)), and normal to the water plane. For the algorithm to find this distance, the assumption that \( P_1 \) lies above the water plane in \( z \) must be fulfilled. This is guaranteed based on the definition of the mesh.

The definition of the slope between a line normal to the water plane and the \( y \)-axis, \( A \), may be given by the following (see Fig 40):

\[
A = \tan \left( \frac{\pi}{2} - \alpha \right)
\] 

(25)

The equation for this line in Cartesian coordinates is then given by:

\[
z = Ay + B_1
\] 

(26)

Given the cylindrical coordinates for each point in the mesh \( i \), the Cartesian representation of these points may be easily found using:

\[
x_i = r_i \cos \theta_i \\
y_i = r_i \sin \theta_i \\
z_i = z_i
\] 

(27)
Then, the equivalent B values for the point on the line given by Eqn. (26) corresponding to a point on the mesh may be found using:

\[ B_{1,t} = z_t - Ay_t \]  \hspace{1cm} (28)

Similarly, the line which lies along the plane \( x_i \) and along the water plane may be defined by

\[ z = -\tan(\alpha) y + B_2 \]  \hspace{1cm} (29)

Where \( B_2 \) for this family of lines may be found using the known point, \( y = r_{off}, \; z = 0 \):

\[ B_2 = 0 + \tan(\alpha) r_{off} \]  \hspace{1cm} (30)

The intersection of these two lines for a particular point on the mesh is the point on the water plane such that the point is along a line normal to the water plane, passing through the mesh point. The distance between these two points is the depth which the point on the mesh is below the water plane. The intersection of the two lines may be found by setting the equations for those lines equal to each other and solving for \( y \):

\[ z = Ay + B_1 = -\tan(\alpha)y + B_2 \]  \hspace{1cm} (31)

\[ y = \frac{B_2 - B_1}{A_1 - A_2} \]  \hspace{1cm} (32)

The \( z \)-value of this point may then be found using either Eqn. 26 or Eqn. 29.
Figure 40 - Finding P2 (x-axis out of page)

This point, which is on the water plane, is termed P₂. Creating a Cartesian vector \( \mathbf{P}_1\mathbf{P}_2 \) (Eqn. 34) and calculating the length of this vector gives the distance between the two points:

\[
|\mathbf{X}| = \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}
\]

and is equivalent to the Pythagorean theorem for \( n \) dimensions.

\[
\mathbf{P}_1\mathbf{P}_2 = \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} r_1 \cos \theta_1 - r_2 \cos \theta_2 \\ r_1 \sin \theta_1 - r_2 \sin \theta_2 \\ z_1 - z_2 \end{bmatrix}
\]

6.6.3 Component Torque Generated by Component Force

To calculate the component force which contributes to torque, the vector which represents the force must first be found. This is calculated as outlined below:

First, a point \( \mathbf{P}_c \) on the centerline of the screw \((r, \theta = 0, \text{ equivalently})\) is defined at the same \( z \)-height as the point \( \mathbf{P}_1 \) on the flight. The line between \( \mathbf{P}_1 \) and \( \mathbf{P}_c \) is given by:
\[
\overrightarrow{P_1P_c} = \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} r_1 \cos \theta_1 - r_c \cos \theta_c \\ r_1 \sin \theta_1 - r_c \sin \theta_c \\ z_1 - z_c \end{bmatrix}
\]  

(35)

Next, a point \(P_4\) is defined as

\[
P_4 = \begin{bmatrix} r \\ \theta \\ z \end{bmatrix} = \begin{bmatrix} \sqrt{r_1^2 + g k^2} \\ \theta_1 - \tan^{-1}\left(\frac{k}{r_1}\right) \\ z_1 + k \tan\left(\frac{\pi}{2} - \tan^{-1}\left(\frac{Pr D}{2 \pi r_1}\right)\right) \end{bmatrix}
\]

(36)

where \(k\) is an arbitrary distance (Fig. 41). Note that a point \(P_3\) (at the same \(z\)-height as \(P_1\)) is used as an intermediary in calculating the location of point \(P_4\).

The force exerted by the hydrostatic pressure at the point \(P_1\) acts along the vector \(P_4P_1\).

![Figure 41 – Defining the pressure vector at \(P_1\) (\(z\) out of page)](image)

The component of this force in the direction of rotation of the turbine is what acts to turn the turbine. Remaining force from the hydrostatic pressure contributes to other factors such as thrust on the lower bearing, and flex of the central shaft. The geometry of the turbine is such that, when in motion, the turbine will turn in the direction of increasing \(\theta\); at point \(P_1\) this direction is normal to both the vector \(P_1P_c\) and the centerline of the screw. Treating the centerline of the screw as a
unit vector along z,
\[
\vec{T} = \overrightarrow{P_1P_c} \times \hat{z}
\]  \hspace{1cm} (37)

is the vector in the direction of rotation of the turbine at P₁.

The distance from P₁ to the center of rotation, normal to the center of rotation, is the vector \( \overrightarrow{P_1P_c} \), and may be determined using the length of this vector.

While no baseline model was available for the torque; comparisons were made between different mesh sizes for the torque output. The results are summarized below.

![Figure 42 - Sensitivity analysis of number of elements for torque calculation](image)

Beyond \( 1.5 \times 10^4 \) elements in the mesh, the overall torque becomes relatively constant. In order to
preserve accuracy while saving computation time, the number of torque mesh elements should be set to this value.

6.7 Losses
The efficiency of the AST system is partially determined by the AST’s geometry and partially by losses in the system. Those losses which have the largest impacts on the efficiency are taken into account by the model.

6.7.1 Bearing Losses
The friction in the bearings used to suspend the AST will consume some power. This power loss may be modeled as a torque resisting the rotation of the AST. As bearings typically may be modeled as viscously-damped devices (Wensing, 1998), the effective resisting torque will vary with the rotation rate of the AST. Assuming that a function for the cumulative resistive torque of the bearings is known, \( \tau_{\text{bearings}} = f(\omega) \), an equation for the total net torque on the AST may be formed.

\[
\tau_{\text{net}} = \tau_{\text{bucket}} \frac{LrN}{Pr} - \tau_{\text{bearings}} - \tau_{\text{consumed}} \tag{38}
\]

where \( LrN/Pr \) gives the total number of buckets in the system, and \( \tau_{\text{consumed}} \) is the power transferred to the gearbox/brake. At steady-state, \( \tau_{\text{net}} = 0 \), and so

\[
\tau_{\text{consumed}} = \tau_{\text{bucket}} \frac{LrN}{Pr} - \tau_{\text{bearings}} \tag{39}
\]

\( \tau_{\text{bucket}} \) may be found based on the screw geometry and placement of the water plane, as described above.

If \( \tau_{\text{consumed}} = 0 \), ie the system is free-wheeling without a connected brake or gearbox, steady-state will be reached when \( \tau_{\text{bucket}} \cdot LrN/Pr = \tau_{\text{bearings}} \). The AST will be unable to rotate any faster under the force exerted by the buckets alone. However, the model’s calculation of rotational velocity
does not take this condition into account. As such, it is possible for Eqn 39 to give a negative value for \( \tau_{\text{consumed}} \) in the model, and to give an unrealistic value for the rotation rate of the AST. Therefore, if in the model \( \tau_{\text{bucket}} Lr \frac{N}{Pr} \leq \tau_{\text{bearings}} \), the calculated rotation rate is greater than the free-wheel rotation rate for the given conditions. As these conditions are impossible, \( \tau_{\text{consumed}} \) and \( \omega \) are set to zero.

In the lab testing, no function \( \tau_{\text{bearings}} = f(\omega) \) was supplied with the bearings. Based on the spin-down data taken before each test (see Section 5), a linear relationship was assumed, and modeled as

\[
\tau_{\text{bearings}} = f(\omega) = A_{\text{bearings}} \omega + B_{\text{bearings}}
\]  

(40)

See Appendix D for details and derivation of the coefficients \( A_{\text{bearings}} \) and \( B_{\text{bearings}} \).

The calculated values of \( \tau_{\text{bearings}} \) obtained using these coefficients was typically within 0.5-4\% of the total net torque created by the water in the turbine’s buckets, for high fill rates. As the AST rotated faster, the value of \( \tau_{\text{bearings}} \) typically increased by a small amount, though due to the decreased bucket fills at higher rotation rates (with constant flow) the reactive torque caused by the bearings could reach upwards of 30\%.

6.7.2 Leakage losses
While it is possible to construct an AST which has the outer trough attached to the flights, preventing all leakage loss, this design is less popular than fixed-trough ASPs, because of their complexity and cost of implementation. With a fixed-trough setup, a gap must exist between the outer edge of the flights and the trough to prevent the flights from striking or rubbing against the trough. The width of this gap is typically kept small (Nagel 1968), however some leakage will still occur. This lost flow is termed \( Q_l \); the flow which acts to push the AST is termed \( Q_w \); thus

\[
Q_w = Q_f - Q_l
\]  

(41)
Nagel (1968) suggests that the leakage losses are dependent on geometry and trough gap width alone, and gives an empirical relation such that:

\[ Q_l = 2.5G_w DD^{0.5} \]  \hspace{1cm} (42)

where \( G_w \) is the gap width between the trough and the outer edge of the screw flight in meters, and \( Q_l \) is in cubic meters per second. Nagel goes on to suggest a maximum allowable gap width of

\[ G_w = 0.0045 D^{0.5} \]  \hspace{1cm} (43)

to allow Eqn. 42 to apply. It should be noted that Eqn. 42 assumes a full bucket, and may need adjusting for non-maximally-filled conditions.

Muller and Senior (2009) found good agreement with published data after adjusting his model to account for leakage losses using Eqn. 42.

The model uses a modified version of Eqn. 42, which testing has shown to be more accurate for the cases studied than that proposed by Nagel (1968) (see Section 7.2). The modified equation is given below:

\[ Q_l = 5G_w DD^{0.5} \]  \hspace{1cm} (44)

The coefficient in Eqn. 44 was chosen by selecting a range of coefficients between 1 and 10 and comparing the model output using each to the lab data; a coefficient of 5 best fit the lab data.

It must be noted that while this modified leakage equation gives better results, it is expected based on comparisons to lab data (Section 7.2) that Eqn. 44 is not sufficient to fully describe the leakage through the AST under all conditions. Under normal operating conditions (AST bucket fill \( f > 0.5 \), AST rotation rate less than half the free-wheel rotation rate), Eqn. 44 serves as a good
approximation to the leakage through the AST.

6.8 Performance Calculation Algorithm
The relations in Sections 6.3 to 6.7 can collectively be used to determine the torque and rotation rate of a screw with a given geometry and operation conditions. The algorithm used by the model is described step-by-step here.

1. The algorithm begins by taking a set of input parameters which define the geometry and rotation rate of the AST, and the desired volume flow rate.

2. The gap width is calculated using Eqn. 43. If existing gap width values are known, they are used instead.

3. The leakage losses are estimated using Eqn. 44. If existing leakage data is available, those values may be used instead. Based on the leakage losses, $Q_l$ is computed using Eqn. 41.

4. If bearing coefficients are known for Eqn. 40, these are used. If instead spin-down data is available, for example when comparing to the lab screws, this data is used to compute the bearing coefficients as discussed in Appendix D. Otherwise, the bearing losses must be omitted from the calculations. The bearing losses may be calculated at any time prior to determining the total power output of the AST; they are calculated at the beginning of the algorithm run for convenience.

5. The maximum possible $r_{off}$ and maximum possible volume for this geometry and angle is
computed by setting $f$ to 1. The required volume for the desired rotation rate is computed using Eqn. 23.

6. If the required volume is greater than the maximum possible volume, it may be deduced that the torque required to spin the screw under this condition is greater than the maximum torque the system configuration can produce. This in turn implies the current configuration would result in the system stalling. Therefore, the rotation rate is set to zero.

7. If the required volume is less than the maximum possible volume, values of $f$ ranging from 0\% to 100\% are used to generate different possible values for $r_{off}$, which are used to generate different possible volumes. Each of these volumes is tested against the required volume. The closest volume which is less than or equal to the required volume is used at the volume for this condition. The corresponding $f$ and $r_{off}$ are used for the remaining calculations. With a sufficiently fine graduation between the $f$ values used to generate the possible volumes, the values determined with this method will be close to the true values. To minimize the effects of floating point errors, the rotation rate required for the calculated volume is determined using Eqn. 36, and used as the output rotation rate. This value may not exactly match the input rotation rate, but will be close and due to the re-calculation, will be accurate to the rest of the outputs.

8. Using the selected $r_{off}$, the upper and lower limits on the bounding flights are determined as described in Section 6.2.2. The torque applied to the front flight is calculated as described in Section 6.4. If applicable, the resisting torque is also calculated. Otherwise, it is set to 0 for the remaining calculations. The net torque applied by the bucket volume is then calculated.
9. The total net torque based on the number of buckets, given by Eqn. 38, is determined. If
the data is available, the resisting torque from the bearings is subtracted from the total net
torque.

10. The power output from the AST is then determined by multiplying the total net torque by
the rotation rate. The efficiency of the AST may then be determined using Eqn. 2, based
on the head drop across the AST, which may be determined by the length of the AST and
the inclination angle.

11. The model outputs the power, efficiency, rotation rate, flow rate and leakage losses,
bucket volume and net torque, and the geometry originally input into an output file.
Chapter 7: Analysis and Discussion of Model

The model results will be presented as follows. First, general trends resulting from changing each of the geometry parameters will be presented in section 7.1. Next, the effects of leakage losses on the model predictions will be discussed, followed by the effects of high rotation rates. Comparisons between model predictions and lab data will then be examined, followed by a discussion of model limitations.

7.1 General Trends

To determine if the model gives results as would be expected for an AST, it was run with different input geometries/conditions. For each of the input parameters \((D, Pr, \delta, \beta, N, Lr, \omega)\), the model was run several times, varying that parameter and holding the others constant. In addition, the model was run varying multiple of the input parameters to keeping another geometry value constant (such as varying both \(\beta\) and \(L\) to maintain a constant head with different values of \(\beta\)).

The AST geometry used in the lab tests served as a baseline for each series. The baseline AST used was screw A, with the baseline parameters \(D = 0.146\) m, \(Pr = 1.0\), \(\delta = 0.55\), \(\beta = 20^\circ\), \(N = 3\), \(Lr = 4\), \(Q_f = 0.8\) L/s. This is the “typical” case for the lab screws discussed in Section 5.

An AST will run at its free-wheel rotation rate for the given flow under a no-load condition. A load must be present in order to utilize the power produced by the AST (the load absorbs the power, putting it to some use). Depending on the magnitude of the load, the AST would run at a different rotation rate. It is assumed that the rotation rate which gives the peak power output would be preferred when employing an AST. A range of rotation rates (from 0.5 rad/s to 25 rad/s) were tested for each parameter combination to determine at which rotation rate the peak power would be produced for that configuration, as lab testing had shown that this point will vary slightly depending on the conditions. These peak power points are compared below as each
parameter is changed. The peak efficiency is similarly compared where useful. Unless otherwise noted, each test listed below assumed a constant flow.

### 7.1.1 Increasing $D$
Increasing the outer diameter, $D$, while holding the pitch, $P$ (0.146 m for the baseline), the inner diameter, $d$ (0.080 m), the length of the screw, $L$ (0.584 m), the number of flights (3), the inclination angle (20°), and the flow (0.8 L/s) constant resulted in a decrease in power to a minimum value. Note that this implies $\delta$, $Pr$, and $Lr$ decrease as $D$ increases. In the case of the baseline used here, this minimum value was reached at an outer diameter of 0.22 m. Fig. 44 indicates that the efficiency also decreases with increasing $D$. It was initially assumed that a larger $D$ would result in higher power production, however examining the effects of increasing $D$ on the torque and rotation rate (Fig 45) it becomes apparent that, while the peak torque increases, the rotation rate at which peak torque is achieved decreases towards a steady value faster than the increase in torque, resulting in an overall decrease in power production until the increase in torque “catches up” with the decrease in rotation rate. The rotation rate decreases as a result of requiring the same flow rate through the turbine.
Figure 43 - Peak power output with rotation rate between 0.5 and 25 rad/s, versus outer diameter
Figure 44 - Peak efficiency versus outer diameter
The above results imply that, for a given amount of available power, an oversized AST will not be as efficient as an AST sized appropriately for that head and flow. For a given flow rate and rotation rate (implying a given bucket volume), a larger diameter turbine will have a lower bucket fill, and therefore the wetted area on the lower flight (which contributes to torque) will be smaller. In addition, the gap width, predicted using Eqn. 43, will be larger for larger outer diameters, resulting in increased leakage losses.

### 7.1.2 Altering $Pr$

Efficiency was found to increase with larger values of $Pr$, approaching a maximum at a relatively large value of $Pr$. Fig. 46 shows the peak efficiency of an AST for rotation rates between 0-25 rad/s, with $D = 0.146$ m, $\beta = 20^\circ$, $\delta = 0.55$, $N = 3$, and $Lr = 4$. 

Figure 45 - Peak torque and rotation rate at peak torque versus outer diameter
This finding is opposite what Muller and Senior (2009) obtained using their model. It is thought that the model proposed by Muller and Senior is too simplistic to accurately represent an AST system. As pitch ratio increases the volume of water in a full bucket increases. Due to the helical shape of the flights, the wetted area on which torque acts also increases. Up to a point, this increase in torque occurs at a greater rate than the loss of buckets along the length of the screw (longer buckets across the same length of screw requires less buckets). Beyond this point, \( Pr = 1.8 \) for the AST examined here (Fig. 47), the torque decreases.

![Figure 46 - Peak efficiency versus pitch ratio, \( Pr \)](image)
7.1.3 Increasing \( \delta \)

The ratio between the inner and outer diameter, \( \delta \), increased while the outer diameter \( D \) (as well as all other parameters) was held constant; this is equivalent to increasing the inner diameter. As \( \delta \) increased, the power and efficiency of the system increased slightly towards a maximum, then decreased rapidly after \( \delta \) reached a certain size (Fig. 48). As the inner diameter increases, the maximum possible volume within a bucket decreases, resulting in a higher rotation rate for a given bucket fill level. The wetted area on the lower flight, and therefore the torque generated by the water volume, also decreases, at a faster rate as the inner diameter increases.

The peak efficiency occurs at \( \delta = 0.67 \). Rorres 2000 has shown, for comparable ASPs, the optimal value of \( \delta \) is 0.54. The power produced from an AST will in general increase as bucket volume
decreases (leading to increased rotation rate), assuming a constant torque. The torque decreases at a higher rate as $\delta$ increases; the rate of increase in the rotation rate due to the decreased bucket volume is not fast enough to counteract the decrease in torque beyond a certain point, $\delta = 0.67$ in this case.

![Figure 48 - Peak efficiency versus ratio of inner to outer diameter](image-url)
7.1.4 Altering $\beta$ and $L_r$

The inclination angle, $\beta$, and the length ratio, $L_r$, of the baseline AST were varied so as to keep a constant head drop across the AST. As $\beta$ increased, peak power increased towards a constant value (Fig. 50), though the change was small. Since the head did not change, the efficiency likewise increased towards a constant value.

This suggests that, above a particular $\beta$ where the efficiency approaches a constant value, the choice of inclination angle should focus on optimizing the volume flow rate versus diameter (and therefore cost) of the AST. As the inclination angle increases, the bucket volume and torque at maximum efficiency both decrease.
Figure 50 - Peak power versus inclination angle, constant head
When the length of the AST is kept constant, and only $\beta$ is varied, a roughly linear increase in power output was noted (Fig. 51), as would be expected. The efficiency of the AST showed very little change with $\beta$ alone. For an AST with $Pr = 1$, $D = 0.146$ m, $\delta = 0.54$, $N = 3$, $Lt = 4$, the peak efficiency of 72% was found to lie at $\beta = 28^\circ$ (Fig. 52). This efficiency was within 1% of the efficiency at $\beta = 20^\circ$, the lowest installation angle tested. The increase in power output is due to the increase in the effective head across the AST – the steeper the angle for a set length of AST, the more power in the flow across that head drop.
While power output increases linearly with increasing turbine length, $L$ (or $Lr$), it should be noted that, if the overall installed height of the AST exceeds the total available head, the model will no longer predict meaningful results. Turbine efficiency is not noticeably affected with changes in $L$ alone.

7.1.5 Altering $N$

A marginal reduction in efficiency was found as the number of flights, $N$, increased above $N = 3$. A larger reduction in performance was found at $N = 2$. Rorres found that, for ASPs, the efficiency of the system should monotonically increase with increasing number of flights. However, the goal of optimizing an ASP is to increase the volume moved with each rotation of the screw, which is fundamentally different from extracting energy from a flow, so Rorres’ observations on ASPs...
cannot be compared to the results for ASTs, except to note that an efficient ASP does not suggest an AST of the same geometry will also be efficient for the same reasons.

7.2 Effects of Leakage Losses
The lab test screws have relatively large leakage losses, up to 10% of the design flow rate of 1.1 L/s. The model was run with the same parameters as the lab tests, with the leakage in the model being set equal to the measured leakage losses from the stall test for the lab test in question. The model was also run for the lab test using zero leakage, a leakage model as suggested by Nagel (1968) (Eqn. 42), and with a modified version of Nagel’s model, Eqn. 45.

\[ Q_l = 5G_w DD^{0.5} \] (45)

Using the measured leakage losses from the stall tests in the model resulted in power predictions lower than those observed for all lab tests examined. Using no leakage resulted in power predictions greater than observed. Using Eqn. 42 to determine the leakage through the AST also tended to result in over-prediction of the power output. Using Eqn. 45 showed the best agreement with the observed results. Below half of the free-wheel speed of the AST (as measured during a given test), the model using the modified leakage equation (Eqn. 45), gave good agreement with the lab data in most cases. However, in all cases examined, the model was not able to accurately predict the power output of the lab AST at rotation rates above half the free-wheel rotation rate. It is expected that this is due to secondary effects and/or a limitations on the leakage model (further discussed in Section 7.5). Any secondary effects would be negligible in a slowly rotating AST, but could become relatively large as the rotation rate increases towards the free-wheel speed. In addition, it must be noted that in all cases with the lab tests, the slower the AST rotated, the fuller the AST’s buckets were, as a result of a constant flow rate being present in a given test. It is
expected that Eqn. 42 (and also Eqn. 45 which is derived from it) only applies for relatively full buckets. As Nagel (1968) was intended for ASP design, and an ASP designer would desire to have the largest volume possible in the buckets when using the screw as a pump, Nagel (1968) likely did not examine any partial-full cases when constructing Eqn 42. At partial fill levels, there would be less area for leakage to occur between one bucket and the next, resulting in lower leakage than would occur in a full bucket case. In addition, the direction of flow (up in the case of an ASP, down in an AST) would also cause a change in the leakage between buckets. Fig. 53 gives a summary plot of a typical lab test run along with the outputs of the power model using the different leakage models.

As the peak power production of the AST occurs at low rotation rates in all cases known to the author, the divergence of the model from the observed data at high rotation rates does not prevent the model from being used to quantify the expected power output of an AST under normal operating conditions.
While it seems counter-intuitive that using the measured leakage values does not give as good of predictions as using a mathematical model of the leakage, it must be noted that the measured values for leakage were only able to be taken with the lab AST stalled. In all cases when the lab AST was stalled, the AST’s buckets were full at steady state (readings were taken at steady state). As discussed above for the equation purposed by Nagel (1968), leakage when the buckets are full would most likely not be the same as the leakage under partial-fill conditions. Additionally, the rotation of the AST under operating conditions may have some additional effect on the leakage, beyond that which can be described solely by the geometry of the system under the quasi-static assumption.

To better quantify the accuracy of the model’s output, an equation relating the predicted power
output to the rotation rate was fitted for each set of model output data. First- and second-order polynomial equations were also fitted to the model output; testing showed that a second-order polynomial best represented the model output, regardless of the leakage model used. The $R^2$ between the polynomial used to represent the model output and the model output itself was typically $>0.99$.

The model was run using a set of uniformly spaced rotation rates in all cases. Since the lab data was not taken at evenly spaced rotation rates, the polynomial representing the model output was used to allow a prediction between the model output and the lab data to be made at exactly the rotation rate measured in the lab. For each model output, a Root Mean Square Relative Error (RMSRE) between the lab and the model output was calculated using Eqn. 46.

$$RMSRE = \sqrt{\frac{\sum_{i=1}^{n} \left( \frac{P_{mi} - P_{li}}{P_{li}} \right)^2}{n}}$$

(46)

Where $P_n$ is the power recorded from the $i^{th}$ reading taken during the lab test, and $P_{mi}$ is the corresponding model prediction of power at the rotation rate for $P_n$. For the majority of lab tests compared, the no-leakage model typically resulted in the largest RMSRE error between the lab data and the polynomial fit to the model output. For a typical test case, the lowest RMSRE error between model runs and the lab data was for the model run using Eqn. 45.

However, in lab tests with very low flow rates such as test 20130502, or very shallow inclination angles, the model using a leakage predicted by Eqn. 45 did not always result in the smallest RMSRE error. A summary of the RMSRE errors between the model using different leakage equations and lab data is given in Table 7. The RMSRE error between the model output and the
prototype AST is also given, though no leakage data was available for the prototype AST, making a model run with leakage data impossible for the prototype AST.

<table>
<thead>
<tr>
<th>Test #</th>
<th>RMSRE Between Lab Data and Lab-measured Leakage Model</th>
<th>RMSRE Between Lab Data and No Leakage Model</th>
<th>RMSRE Between Lab Data and Nagel (1968) Leakage Model</th>
<th>RMSRE Between Lab Data and Modified Leakage Model</th>
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<td>1.41</td>
<td>0.79</td>
<td>0.20</td>
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<td>0.54</td>
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The power value measured for the prototype AST S1 with the datalogger was not power at the shaft, as with the lab test screws, but electrical power as output by the connected generator; losses from the gearbox and generator therefore had to be taken into account to allow the model output to be compared to the prototype data. The model output was scaled based on the efficiencies of the gearbox and generator used with the prototype AST. The gearbox efficiency used was that given from the gearbox datasheet. The generator used with the AST was originally developed as an electric motor; no generation efficiency was available. The efficiency of the device when used as a generator was assumed to be equal to that of the device as used as a motor. These values were 96% (gearbox) and 84% (motor).
Rather than the rotation rate varying as with the lab screws, the prototype AST S1 had a fixed rotation rate and varied flow rates. Similar to the lab test comparisons, a second order polynomial was fit to the model outputs for the prototype AST (after adjusting the model output from power at the shaft of the turbine to electrical power output) and used to compare the model output to the prototype AST data.

The least-squares error between the polynomial output representing the prototype AST power output and the polynomial output representing the model using Eqn. 45 to predict leakage was lower than those for no leakage and leakage predicted using Eqn. 42 (see Fig. 54).

Based on an analysis of the RMSRE values as well as visual inspection of the plotted data (such as shown in Fig. 53), the modified leakage equation (Eqn. 42) was selected as the best available representation of the actual leakage through the AST. This leakage model was used for the rest of the analysis.
The leakage losses only occur in Eqn. 41, as a modifier to the variable $Q_w$, which itself only appears in Eqn. 23 for relating the effective flow rate and the volume to the rotation rate of the AST. As effective flow rate and bucket volume are the only variables used to compute the rotation rate of the AST, the power output predictions of the AST are sensitive to the effective flow rate $Q_w$. When using the proposed equation for leakage losses, Eqn. 45, leakage losses are typically over 10%; therefore $Q_w$ and by extension the predictions of the model, are sensitive to the leakage losses in the system.

The method of determining the leakage losses used in this model is limited, as it assumes a constant leakage flow across all bucket fill heights. This is not realistic, and results in less accurate predictions than a variable leakage flow would. During the development of the model,
attempts were made to include the effects of bucket fill on leakage flow. However, as the leakage flow must be calculated prior to determining the bucket fill. The solution attempted used the static leakage flow calculation described in Section 6.2.2 to determine bucket fill, and then retroactively applied a linear interpolation on the leakage flow based on the bucket fill. The rotation rate was then re-calculated using the new leakage flow.

The linear interpolation of the leakage flow as described above produced output which did not fit the lab data as well as the model run without any leakage flow interpolation. Therefore, a static leakage flow was assumed for the purposes of this model. It must be reiterated that, while the linear interpolation described here did not work as well as the static leakage flow, the overall model would be improved with a correct non-static leakage flow calculation.

### 7.3 High Rotation Rates
At low rotation rates, the power output from the AST is well predicted using the assumption of quasi-steady state. At higher rotation rates, this assumption may no longer apply. This is seen in the model consistently over-predicting AST performance at high rotation rates – typically any rotation rate over approximately half of the free-wheel rotation rate, in the case of the lab screws.

Qualitative observations from lab testing show that at higher rotation rates the bucket volume does not exhibit significant churning along the length of the screw; therefore the larger losses observed in the system at high rotation rates is likely not caused by increased turbulence in the flow. This is discussed further in Section 7.5.

### 7.4 Model Predictions of Lab Data
In order to validate the model, it was run with AST parameters and flow rates the same as those
used in the lab tests. The model was run with the leakage losses predicted using Eqn. 45. Figs. 55 - 61 show comparisons between various lab tests and the model output for those lab tests at below half the free-wheel speed for each lab test. In general, the model predicts the power output from the lab ASTs within the bounds of the expected error of the lab tests. In addition, the comparison between the data logged from the prototype AST and the model output for the prototype (see Fig. 54) indicates that for larger ASTs running in their typical operating range the model can predict power output for that AST.

Figure 55 – Power versus rotation rate, screw B, $\beta = 20.5^\circ$, $Q_f = 0.43 \pm 0.02$ L/s, Lab Test 20130418
Figure 56 – Power versus rotation rate, screw B, $\beta = 20.5^\circ$, $Q_f = 1.17 \pm 0.02$ L/s, Lab Test 20130509
Figure 57 - Power versus rotation rate, screw B, $\beta = 20.1^\circ$, $Q_f = 0.54 \pm 0.02$ L/s, Lab Test 20130424
Figure 58 - Power versus rotation rate, screw B, $\beta = 24.6^\circ$, $Q_f = 0.74 \pm 0.02$ L/s, Lab Test 20130527_0930
Figure 59 - Power versus rotation rate, screw C, $\beta = 22.7^\circ$, $Q_f = 0.71 \pm 0.02$ L/s, Lab Test 20120503_2
At low flow rates (example, Fig. 61), the model does not accurately predict the performance of the lab ASTs. It is expected this is because the leakage losses make up a large percent of the total flow through the system, resulting in a case where the equation relating the rotation rate of the AST to the volume in a bucket (Eqn. 45) is no longer valid.
The model predicts that, given a constant flighted length (as is the case in the lab tests), an increase in inclination angle, $\beta$, will result in a very small increase in peak efficiency until $\beta = 30^\circ$, after which point the efficiency would begin to decrease (Section 7.1.4, Fig. 50). The lab tests (Section 5.8, Fig. 27) showed a negligible peak efficiency between $\beta = 17.2^\circ$ and $\beta = 30^\circ$, with a large increase in peak efficiency at $\beta = 34.4^\circ$. Below $\beta = 30^\circ$ the model predictions agree well with the lab results. Above this value, the predictions of the model cannot be said to agree with the observed results.

The large jump in efficiency observed in the lab suggests that the efficiency of an AST with respect to inclination angle is not well behaved above a certain value, for the lab screws between $\beta = 30^\circ$ and $\beta = 34.4^\circ$. While this possibility cannot be ruled out without more testing, the test at $\beta = 34.4^\circ$ was only performed once for the data presented, and the general well-behaved nature of
all the other correlations examined, as well as the well-behaved nature of the inclination angle data up to that point, suggest that the large jump in efficiency observed was due to experimental error for this particular test.

The model predicted that the efficiency of an AST would increase with increasing pitch ratio, up to a maximum ($Pr = 2.1$ for the lab screw, see Section 7.1.2, Fig. 46). Lab testing (Section 5.4, Fig. 17) showed that the larger pitch ratio (Screw B, $Pr = 1.4$) had a greater efficiency than the other two lab screws (screw A, $Pr = 1$ and screw C, 0.8). However, the lab tests did not show a clear difference in peak efficiencies between screws A and C. At high rotation rates screw A showed greater efficiencies than screw C, but as the model has trouble predicting AST performance at those high rotation rates, the model cannot be used to directly test against this observation.

The stall tests performed at the beginning of each lab test run may be used to determine the maximum torque produced when the lab screws are filled to the centerline of the screw at the screw’s inlet (the maximum fill height permitted by the geometry of the test setup). By definition, no power is produced at stall. Fig 62 shows a test run in terms of rotation rate and torque. The maximum torque (at $\omega = 0$) gives the torque at stall. Because the flights on the lab screws were not perfectly balanced, the stall test was performed three times for each lab test, with the AST rotated by approximately 120 degrees between each test. This allowed an average value to be determined for the maximum torque produced by each test setup. This value can then be compared to the maximum torque value calculated by the model.

For the majority of the lab tests, the standard deviation between the torques observed at the three stall tests was small (typically less 0.03 Nm), and their average may be taken as a good representation of the maximum torque capable of being produced by the screw at its maximum
fill level. As shown in Fig. 62, for the majority of cases the model under-predicted the torque which would be produced by the lab screws. As discussed in Section 5.3, when the stall tests were performed the water level in each bucket was high enough to leak over into the next; the buckets were over 100% full as defined by the model. Therefore, it is expected that the model would under-predict the stall torque when compared to the lab data. It cannot, however, be stated if the model accurately predicts torque at a 100% full bucket based on comparisons to the lab data.

![Figure 62 - Lab stall test compared to model predictions, screw B, $\beta = 20.1^\circ$, $Q_f = 0.54 \pm 0.02$ L/s, Lab Test 20130424](image.png)

The free-wheel tests were performed to determine the maximum rotation rate possible with the lab screws. They were performed by completely removing the torque brake from the system. Therefore, similar to the stall tests, by definition no power was produced at the screw shaft during the free-wheel tests.
As the model is unable to accurately predict the power produced by ASTs at rotation rates above half the free wheel speed, no meaningful comparison can be made between the model and the free-wheel tests. The free-wheel tests did give guidance during the testing procedure as to the range of rotation rates which should be examined for a particular experimental setup, and were also useful in confirming model’s inaccuracy at high rotation rates.

7.5 Limitations of Model
The model developed is based on a numerical approach, limiting its accuracy to the element-size used for the volume and torque meshes. As the element size is reduced, the number of elements used must increase, which results in an increase in computation time. While a single run of the model typically took only a few seconds to run, the model usefulness of the model as an analysis tool arises by allowing many simulations of different variable combinations to be easily performed. This necessitates a trade-off between the number of model simulations which are run in a given time-span and the accuracy of those simulations. While acceptable mesh sizes to balance this trade-off were determined in Sections 6.4 and 6.6, when looking for smaller changes in the model output, these mesh sizes may not be sufficient. Fig. 52 gives a good example where the mesh size chosen was close to being too large to give acceptable results.

Another limitation of the model is the sensitivity to leakage through the gap between the trough and the flights, as discussed in Section 7.2. While the leakage model proposed in this thesis does give acceptable results in the majority of cases where the rotation rate is relatively low and flows are not small, a more complete leakage model would be needed to allow accurate predictions under all conditions.
A third limitation is the need to take into account bearing losses for the lab screws, requiring spin-down data or similar to be readily available to the user. Comparisons between the data from the prototype AST and the model suggest that for larger systems these losses may be small or negligible, however this should not be assumed without further testing.

An additional limitation of the model is its inability to predict accurate results at very low bucket fill heights. This is most clearly seen in comparisons between the model and the lab results at high rotation rates, for example Fig. 53. For a typical lab test, the flow was held constant. As a result, when the AST was turning at high rotation rates, the buckets were relatively empty. With relatively empty buckets, a greater portion of the flow which passes through the AST will pass through the trough gap; the percent of the flow through the AST which is lost to leakage through the trough gap is higher at low bucket fills. Eqn. 23 relates the rotation rate and flow rate of the AST to bucket volume. The flow value used in this equation is calculated assuming a leakage as modeled at high bucket fills, based on the expected leakage given by Nagel (1968) for ASPs. It is suspected that this equation does not accurately model leakage through the AST at low bucket fill levels. The flow value calculated for use in Eqn. 23 will be greater than the actual flow acting to turn the AST. The result is a larger volume (higher bucket fill) being calculated at high rotation rates than is present in the lab tests. This could account for the divergence from the observed data.

The volumes for each point in the lab data are easily calculated using Eqn. 23, using the assumption that the leakage flow for each lab test is equal to the average leakage flow observed in the stall tests. Taking the maximum volume calculated for each non-stall test run in a given test as being a full bucket, the percent fill of each bucket may be determined for each test run. Because of the geometry of the lab test rig, overfilling in non-stall conditions would not have happened.

Comparing the power output at these fill levels to the model output shows that, for about half of
the lab tests, the model predictions of power as a function of bucket fill show good agreement with the lab data (e.g.: Fig. 63).

![Figure 63 - Model predictions match calculated lab bucket fill](image)

The same results were obtained assuming no leakage in the volume calculations of the lab data. As the leakage used was a static value, and the fill is calculated as a percent of the maximum calculated fill, this is expected. When a similar comparison is performed between bucket fill and rotation rate, the same level of agreement between the lab data and the model was found.
Chapter 8: Conclusions

An experimental lab test setup was developed for use with five small ASTs. The lab setup worked, allowing useful data to be gathered to help in developing an understanding of the AST system. The observed trends in the data gathered were consistent. Due to the small size of the ASTs, minor losses (e.g.: bearing losses) were large relative to the observed power output, contributing to variations in the data gathered. Though the data obtained from the lab setup was noisy, it was consistent, and able to be used to guide the development of, and to provide validation for, the mathematical modelling of the AST system. It was found that an AST with a larger pitch than 1.0 was more efficient, that an AST can still operate with a large portion of the lower end submerged, that increasing inclination angle does not have a large effect on efficiency, and that the AST system is fairly tolerant of changing flow rates. It could not be determined what affects, if any, the differing end geometry of test screw D had on power production and efficiency. Improvements to the system could be made, in particular the flow measurement method and the consistency of the brake used in the torque measurement system. Larger test screws would also allow for higher accuracy as the signal-to-noise ratio would be higher on a larger system, and some of the minor losses mentioned before would be further reduced.

Data logging of a prototype AST installed on a small reach in Southern Ontario was conducted over a number of months, both to gather data on AST performance with changing flow conditions, as well as to collect data on the use of an insulated enclosure to prevent freezing of the AST.

The model developed is able to predict the performance of an AST under full and partial-fill conditions, allowing for analysis of performance under low-flow conditions. This improves upon other models in the literature, which focus only on conditions where the AST’s buckets are full.
The model is useful to help understand the impact of how altering the parameters which define the geometry and operating conditions of an AST effects the power output of the AST.

Under typical operating conditions (rotation rates below half the free-wheel rotation rate) the model closely approximates the results obtained from the prototype testing. The model also closely approximates the results obtained from laboratory testing under typical rotation rates and moderate and high flow rates.

Under non-typical operating conditions, particularly rotation rates over 50% of the free-wheel speed of the system, the assumption of a quasi-static system made in the model no longer apply; as a result the model does not accurately predict under these non-typical operating conditions.

In addition, the model was found to be sensitive to the effective flow rate used, requiring accurate calculation of the available flow as well as leakage losses. A new equation, based on the work of Nagel (1968), but applying to ASTs specifically, has been proposed. Use of this new equation give more accurate results from the model than the other methods examined. However, this new equation still does not allow the model to accurately predict the performance of the AST at very low flow rates or at high rotation rates, suggesting that the leakage losses are not constant for a given geometry, but vary with the operating conditions.

Using the model, inferences as to the performance of an AST with geometry which differs from those available for study were made. The model predicts:

- as $D$ increases, efficiency decreases for a constant flow rate
- as $Pr$ approaches 1.4, efficiency increases; between 1.4 and 2.2 there is a very small increase in efficiency, with efficiency decreasing at pitch ratios larger than 2.2.
- As $\delta$ increases, there is little change in efficiency up to a point, with a rapid decrease in
efficiency beyond that point; the point at which this occurs is expected to vary for different geometries of AST.

- Efficiency does not vary largely with changes in $\beta$, within the range tested ($15^\circ \leq \beta \leq 45^\circ$).
Chapter 9: Recommendations

The effects of leakage losses through the gap between the flights and trough has a large effect on the accuracy of the developed model. The only leakage model found in literature does not accurately predict the leakage through an AST. The leakage model proposed here gives better agreement with experimental data; however, more research is needed to properly quantify these losses.

The numerical model element size underwent some by-hand tuning over the course of testing. This tuning should be continued more rigorously to optimize the calculation of bucket volume and torque, both for accuracy and for decreased processing time.

The level of noise in some of the lab tests’ power readings (eg. tests of different pitch, Section 5.4) was relatively high, making deduction of more than a general trend difficult. The test rig should be re-designed to allow larger ASTs to be tested at higher flow rates, in order to increase the signal-to-noise ratio of these tests.


Muysken, J. (1932). Berekening van het nuttig effect van de vijzel. *De Ingenieur,*(21), 77–91


Hydro Power Plant in Simulink MATLAB, 2(7), 2471–2477.


Appendix A: Flow Tank Calculations

The first lab setup used a custom V-notch weir to measure flow through the system. The V-notch weir was approximately 45 degrees, with an opening approximately 8 cm high. The weir was built into the side of a catchment tank. A scale graduated in millimeters was placed beside the weir opening to allow the depth of the water flow at the weir to be measured. The catchment tank was translucent, allowing the depth of water in it to be measured by the graduated scale without the use of a site tube. An empirical calibration curve was developed in situ as follows.

The system was set to allow a particular flowrate (as-of-yet unknown) by varying the rotation rate of the turbine and allowing excess flow from the pump to be re-directed over another weir. The flow through the turbine also flowed into the catchment tank and through the V-notch weir. A large (relative to the flowrate) basin was placed to intercept the flow out of the V-notch weir, and the time taken to fill the basin to over three quarters full was recorded using a stopwatch. The depth of water flowing through the V-notch weir was also recorded as measured with the adjacent graduated scale. The volume in the basin was then measured (accurate to the nearest millilitre) using a graduated cylinder. By making the basin volume large, random errors in the starting and stopping of the stopwatch as well as in the placement of the basin under/away from the flow through the V-notch weir were reduced low enough levels to allow acceptable accuracy in the measurements taken. This process was repeated several times with different flow rates through the system.

Flow rates were calculated by dividing the volume recovered in the basin during each test point by the time it took for that volume to fill the basin.
The second lab setup replaced the catchment tank-and-weir with a flow tank with a fixed submerged outlet. The flow tank was designed such that as more water flowed into the flow tank, the depth of the water increased until the pressure caused by the head in the tank forced an equal flow through the submerged outlet, reaching an equilibrium, with the depth of water in the flow tank corresponding to the flow rate through the system. A site tube connected to the flow tank allowed observation of the water depth; a graduated scale next to the site tube facilitated measurements. The graduated scale was accurate to 1/4". The flow tank was calibrated in situ using the same method as described above for the V-notch weir.

The V-notch weir was calibrated once over the course of testing; the flow tank was calibrated twice. In all cases, the empirical relationship between the depth of water measured and the flow through the structure was modeled as a first-degree polynomial.

The calibration of the V-notch weir used seven data points and had a maximum uncertainty in the flow rate of 0.11 L/s. The first calibration of the flow tank used 8 data points and had a maximum uncertainty in the flow rate of 0.03 L/s; the second calibration used 15 data points and had a maximum uncertainty of 0.02 L/s.
Appendix B: Derivation of Flight Equations

Defining a helix of diameter D, pitch P, and length L, the projection of this helix onto a plane which passes through the centerline of the helix takes the form of a sine wave. Using the form

\[ f(t) = A \sin(\theta t + \phi) \]  

B1

to represent the helix, the amplitude, A, is equal to the radius of the helix, \( t \) represents the point along the length of the helix from 0 to L, and \( \omega \) represents the pitch of the helix, thus

\[ A = \frac{D}{2} \]

\[ \theta = \frac{2\pi}{P} \]  

B2

\[ 0 \leq t \leq L \]

The variable \( \phi \) gives an indication of the starting point of the helix relative to the chosen plane. If the helix starts on the plane, \( \phi = 0 \); if the helix starts on a plane which is normal to the chosen plane and intersects the chosen plane through the helix centerline, \( \phi = \pi/2 \). As the goal is to represent an arbitrary flight in an AST, let

\[ \phi = N_i \frac{2\pi}{N} \]  

B3

where \( N \) is the total number of flights in the AST, and \( i \) is a representation of which flight is being modeled, such that \( 0 \leq i \leq N-1 \).

An Euclidian coordinate system is now defined such that the \( z \)-axis is along the centerline of the helix, and the \( y \)-axis passes through the starting point of the first helix in the AST. Using the above definitions, a triplet \([x,y,z]\) may be defined as

\[ x = \frac{D}{2} \sin \left( \frac{2\pi}{P} z + N_i \frac{2\pi}{N} \right) \]
\[
y = \frac{D}{2} \cos \left( \frac{2\pi}{P} z + N_i \frac{2\pi}{N} \right)
\]

The above triplet represents a helix located on the outer edge of a flight in an AST. The inner edge of the flight may be given by substituting \( d \) for \( D \), where the outer diameter of the AST is \( D \) and the inner diameter of the AST is \( d \). Similarly any point on the surface of the flight may be given with proper selection of the \( A \) term in Eqn. B1.

A helix is more easily represented in cylindrical coordinates than in Cartesian coordinates. Using the transform from Cartesian to cylindrical coordinates as given in Eqn. B5, using the stipulation that \( \theta \in [0, 2\pi] \), the helix described in Eqn. B4 may be represented as the triple \((r, \theta, z)\), as shown Eqn. B6:

\[
r = \sqrt{x^2 + y^2}
\]

\[
\theta = \begin{cases} 
-\tan^{-1} \left( \frac{y}{x} \right) & x \geq 0 \\
-\tan^{-1} \left( \frac{y}{x} \right) + \pi & x < 0
\end{cases} \quad \text{B5}
\]

\[
z = z
\]

\[
r = r
\]

\[
\theta(z) = 2\pi \left( \frac{z}{P} + \frac{N_i}{N} \right) \quad \text{B6}
\]

\[
z = z
\]

Note that Eqn. B6 will give values for \( \theta \) that exceed \( 2\pi \); due to the way the algorithms used in the overall AST model handle the triple \((r, \theta, z)\) this is in fact desirable.
Appendix C: Derivation of Water Plane Equation

Define the plane as begin parallel to the line defined by $\theta = 0$, and at some angle $\alpha \in (0, \pi/2)$ to the $y$-axis.

A radial line is a line through the $z$-axis, and may be defined as $\theta = \phi$, $r = r$, $z = c$. A line which is normal to a radial line $\theta = \phi$, crossing $\theta = \phi$ at point $(r_0, \phi)$ has the equation

$$ r(\theta) = r_0 \sec(\theta - \phi), \quad z = c. \quad \text{C1} $$

The equation of the plane described above would require $(r_0, \phi) = f(z, \alpha)$, as the plane is defined as parallel to $\theta = 0$, which is equivalent to being normal to the radial line $\phi = \pi/2$. Given a point $P$ which should lie on the plane, an $r_2$ may be calculated using Eqn C2 (see Fig. C2)

$$ r_2 = r \cos(\theta - \pi/2) \quad \text{C2} $$

Figure C1 - Defining water plane
The $z$-value of this point may be found as shown in the diagram below:

$$z = r_1 \tan \alpha$$
\[ r_1 = r_0 - r_2 \] \hspace{1cm} \text{C4}

\[ z = (r_0 - r_2) \tan \alpha \] \hspace{1cm} \text{C5}

\[ z = \left( r_0 - r \cos \left( \theta - \frac{\pi}{2} \right) \right) \tan \alpha \] \hspace{1cm} \text{C6}

Eqn C6 is the base equation for a point on a plane which is parallel to the line \( \theta = 0 \), defined by the angle \( \alpha \) and an offset from the \( \theta = 0 \) axis of \( r_0 \).
Appendix D: Calculating Moment of Inertia of Lab Screw and bearing torque equation of lab screw tests

Given a system suspended between two bearings, and rotating along the radial axis of the two bearings, if allowed to free-wheel with no input power, the system will slow down and stop due to the energy dissipated by the bearings. This energy dissipation may be thought of as a torque resisting the motion of the system. The time for the system to slow to a stop will depend on this torque, the moment of inertia (MoI) of the system, and the initial rotation rate of the system. The laboratory screw spin down test is an example of this type of system. The system’s energy dissipation due to the bearings may be modeled from the data collected in the manner described below. For the purpose of this work, the relationship between the rotation rate of the system and the ‘resisting torque’ was assumed to be linear.

The system was initially spun up with no brake attached to approximately 300 RPM, and the rotation rate of the system was recorded at known time intervals until the rotation rate fell to 0. The time at which 0 RPM was reached was also recorded. The RPM values were then converted to rad/s.

The MoI of the AST was also needed. It was calculated by calculating the MoI of each piece of the AST and then finding the sum of the MoIs of all the parts. For each part, a density of \( \rho = 7850 \text{kg/m}^3 \) (density of steel) was assumed, and the part masses were calculated using this value and a calculation of the volume. The summation of these masses were compared to the total measured weight of each screw, and found to be in good agreement.

The outer tube was modeled as a hollow cylinder, and Eqn. D2 was used to calculate its MoI. The volume of this shape is given by Eqn D1.
\[ V = \pi r_0^2 - \pi r_1^2 L_t \]  
\[ I = \frac{1}{2} \rho V (R_i^2 + R_o^2) \]

The center shaft was modeled as a solid cylinder. Its volume and MoI calculations are given in Eqns D3 and D4.

\[ V = \pi \left( \frac{1}{2} D_s \right)^2 L_s \]  
\[ I = \frac{1}{2} \rho V \left( \frac{1}{2} D_s \right)^2 \]

The two end caps were modeled as cylinders (Eqns. D5 and D6)

\[ V = \pi \left( \frac{1}{2} \delta D \right)^2 L_c \]  
\[ I = \frac{1}{2} \rho V \left( \frac{1}{2} \delta D \right)^2 * 2 \]

The two shaft collars which hold the center shaft in pace were modeled as hollow cylinders (Eqns. D1 and D2).

The flights’ MoIs were modeled as thin rectangular plates rotating about an axis offset from their edge. The flights themselves are essentially such a rectangle, wrapped about the inner tube. The volume calculation and MoI calculation for the flights are given in Eqns. D8 and D9. The lengths of the rectangles which model the flights were found using equation D7.

\[ L_h = \left( \frac{D}{2} \right)^2 + \left( \frac{PrD}{2\pi} \right)^2 \]  
\[ V = W_f (D - \delta D)L_h \]  
\[ I = \frac{1}{3} \delta VD^2 \]

Each point on the spin-down data, \((t, \omega)\), corresponded to a single reading of rotation rate, \(\omega\), and the corresponding time from the start of the test, \(t\). For each point, the difference in \(t\) and \(\omega\)
between it and the one preceding it was calculated, giving a set of points \((dt, d\omega)\). By dividing these, a set of values \(d\omega/dt\) were found which approximate the slope of the spin-down test data between the two points which generated the \((dt, d\omega)\). As the definition of torque is

\[
\tau = M\omega \frac{d\omega}{dt}
\]

The torque resisting the rotation of the system could be found for each point of the spin-down data. A function

\[
\tau_{\text{bearings}} = f(\omega) = A_{\text{bearings}} \omega + B_{\text{bearings}}
\]

was then found by using a least-squares approximation to fit a first order polynomial between the rotation rate at each point and the corresponding torque at that point. The values \(A_{\text{bearings}}\) and \(B_{\text{bearings}}\) are the coefficients of the polynomial fit.