Modelling Ontario Agricultural Commodity Price Volatility
with Mixtures of GARCH Processes

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Abstract

Modelling Ontario Agricultural Commodity Price Volatility with Mixtures of GARCH Processes

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Agricultural commodity price volatility is a critical global issue that may affect market participants quite differently. Previous work has studied factors that may affect price changes, such as supply and demand factors, climate change, role of speculation in futures and option trading markets, etc. Modeling price volatility helps to understand causes, patterns and impacts of price changes, making it possible to mitigate the aforementioned risks and negative effects. To demonstrate the price volatility for future pricing forecasts used in the business risk management program, this thesis estimates price models based on mixture distributions. The price is assumed to follow a normal mixture distribution Generalized Autoregressive Conditional Heteroscedasticity (GARCH) (1,1) with separate GARCH (1,1) processes in each mixture.
Dedication

This thesis is proudly dedicated to my beloved parents, Mr. Wang Shiyong and Mrs. Zheng Xiangying, for their endless love and support.
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Chapter 1  Introduction

High and volatile agricultural commodity prices are essential global issues that require a great deal of attention. World agricultural commodity prices from 2006 to 2012 are presented in Figure 1.1 for illustrative purposes. These demonstrate recent trends in prices as well as show a series of dramatic swings. In 2008, the prices for almost all agricultural commodities, and especially the price of rice, reached significant peaks, following a slow build-up of prices that began in 2006. Prices for corn and soybean spiked again in 2012. Overall, commodity prices have been on a rollercoaster ride since the mid-1990s (Cashin and McDermott, 2002; Jacks, O’Rourke, and Williamson, 2011; Roache, 2010; Bellemare, 2011). Price volatility, also referred to as price uncertainty, has been recognized by economists for quite a long time (Engle, 1982; Buguk, Hudson, and Hanson, 2003); the speed and magnitude of the current run-up in agricultural commodity prices have created a level of increasing risks not only for agricultural producers, but also for consumers, traders, and policy makers.

Previous studies have discussed factors that may explain evolutions in price changes (Abbott, Hurt, and Tyner, 2009; Gilbert, 2010; Gilbert and Morgan, 2010). The first factor that affects price changes is variations in supply and demand factors. On the demand side, the rising global demand driven by economic growth is often cited as a contributory factor. On the supply side, underinvestment in agricultural technology and low commodity prices are often emphasized as responsible for declines in agricultural production growth. In addition, some other factors, such as climate change, changes in trade polices in exporting and importing countries, and feedback between price expecta-
Figure 1.1: World Commodity Prices

Figure 1.1: World Commodity Prices

tions and market responses, have also been proposed to influence agricultural commodity price volatility. Also, crop yields, and therefore prices, are strongly affected by adverse weather and climate conditions such as excessive rain, drought, wind, and hail. As such, climate change could alter the prevailing volatility in crop prices. Furthermore, the role of speculation in future and option trading markets may influence prices (Gilbert, 2010; De Schutter, 2010).

Some empirical studies have suggested influential factors that can change agricultural commodity prices. Changes in exchange rates, mainly through the mechanism of international purchasing power, and their effects on margins for producers with non-U.S. dollar costs, can affect commodity prices (Meyers et al., 1986; Gilbert, 1989; Taylor and
Spriggs, 1989). However, some researchers have argued that monetary impacts are not key factors for food prices (Isaac and Rapach, 1997). In 2000, Kliesen and Poole argued that monetary policy can only affect prices indirectly under circumstances, such as low inflation, stable inflation expectations, and low interest rates. Frankel (2006) suggested that when market participants expect interest rate shocks to persist, interest rates also affect food prices.

Price volatility affects both producers and consumers. For producers, price volatility increases production and investment risks, especially when the production cycle is long. For consumers, volatile prices make it difficult to make consumption decisions, especially when their income or budget is limited. Volatile prices can also reduce the accuracy of producers and consumers price forecasts, and thereby causing welfare losses (Binswanger and Rosenzweig, 1986; Sahn and Delgado, 1989). Price volatilities may have negative impacts on economic growth and poverty, which are most damaging to poor country economies (Rodrik, 1999). Furthermore, higher and more unpredictable volatility may lead to an economic crisis (Acemoglu et al., 2003).

Modeling price volatility helps to understand causes, patterns, and impacts, making it possible to mitigate the aforementioned risks and negative effects. Quantitative methods can be used to forecast the absolute magnitude, quantiles, and distribution of price changes, and thus are widely used in derivative pricing, portfolio selection, and risk management. Fitting a price discrete time volatility model can help producers and traders make proper production and investments decisions under conditions of price uncertainty. Producers need to know whether specific product prices are likely to decline in the future, while option traders want to know the expected price volatility along the life of a contract. In addition, futures traders use buy or sell stops to maintain their
financial losses at comfortable levels, and to make profit based on future price trend predictions. Governments as well as policy makers need better fitting models to manage the economy in volatile times, especially when the economy is affected by financial crisis or major depression. Such models also help them understand how markets behave during crashes.

An ample body of research has studied price volatility using a generalized autoregressive conditional heteroscedasticity (GARCH) framework, which was pioneered by Engle (1982) and Bollerslev (1986). A common assumption of volatility models is that error terms are conditionally normally distributed. However, some researchers have suggested that for daily or higher-frequency data, observed non-normalities in both conditional and unconditional returns are higher than can be predicted by normal GARCH model (Bollerslev, 1987; Baillie and Bollerslev, 2002; Nelson, 1996). With this in mind, many non-normal conditional densities have been considered within the GARCH framework. For example, Bollerslev (1987) proposed a GARCH model with a Student’s t-distribution, Engle and Gonzalez-Rivera (1991) have developed the semi-parametric ARCH model, and Fernández and Steel (1998) have extended the Student’s t-distribution to a skewed t-distribution.

In recent years, some studies have used a mixture of two normal distributions GARCH to model volatility processes. Haas, Mittnik, and Paolella (2004) first proposed the general symmetric Normal Mixture GARCH model (NM-GARCH). Alexander and Lazar (2006) further investigated the NM-GARCH (1,1) model. In their paper, they emphasized that the NM-GARCH (1,1) model with two components performs better than both the symmetric and skewed Students’ t-GARCH models in modeling exchange rates. In addition, the two-state NM-GARCH models are more powerful when modeling volatility.
prices in financial market (Alexander and Lazar, 2009). The two states in such NM-GARCH models can represent two different components. The component with a lower variance represents a “stable” state, which occurs most of time, while the component with a higher variance represents a crash or stressful state that rarely occurs. This paper uses this NM-GARCH model to consider the agricultural commodity markets.

The purpose of this thesis is modeling agricultural commodity prices in Ontario, in order to study the determinants of price volatility for future price forecast used in the business risk management program. The rest of this thesis proceeds as follows. Chapter two presents the literature review, which specifically introduces different technological methods for price modeling, and describes the development of these methods. Chapter three investigates the empirical methods and explains the selected data of this study. Chapter four analyzes and compares the results of different models. Finally, the main conclusion and findings of this study are summarized in Chapter five.
Chapter 2  Literature Review

2.1  Introduction

The purpose of this thesis is to extend the literature on modeling agricultural commodity price volatilities. To that end, this chapter reviews the literature on: (i) the different reasons for price volatility; (ii) the influences of price volatility on different market participants; and (iii) the development of modeling price volatility methodologies.

2.2  Price Volatility in Literature

Several previous studies on agricultural markets detected leptokurtic returns in agricultural futures prices (Hudson, Leuthold, and Sarassoro, 1987; Hall, Brorsen, and Irwin, 1989). Price volatility is important when studying in agricultural commodities, such as corn, soybean, wheat, beef, etc. (Kinnucan, 1986; Hudson and Coble, 1999; Goodwin and Schnepf, 2000; Buguk, Hudson, and Hanson, 2003). Prices often exhibit sudden, unexpected and discontinuous changes (Koekebakker and Lien, 2004). During the commodity bull-and-bear cycle of 2006 through 2009, the volatility of agricultural commodity prices was exceptionally high (Carter, Rausser, and Smith, 2011; Karali and Power, 2013). The sustained periods of high price volatility increased business difficulties. Challenges associated with high agricultural price volatility, suggest the significance of identifying determinants of price volatility for agricultural producers, consumers, market regulators, and governments (Karali and Power, 2013).
In developed countries, agricultural food commodities cover a relatively small part of a households budget. However, households allocate a large part of their income to food expenses, which indicates that the price increases could lead to a reduction in real income and higher volatility in agricultural markets. As a result, price volatility could affect consumers, especially those who come from low-income countries, and price volatility may lead to future price-range variations (Apergis and Rezitis, 2003). This can reduce the accuracy of producers forecasts of future agricultural commodity prices, consequently leading to welfare losses (Binswanger and Rosenzweig, 1986; Sahn and Delgado, 1989).

In order to hedge contracts and set profits and limits at a comfortable level, traders must understand how price volatility could respond differently in the market (Alexander and Lazar, 2009). For policy makers, it is crucial to pay more attention to the degree of price volatility before making appropriate policies (Apergis and Rezitis, 2003). Several previous studies have explored price volatility degrees in farm output and the retail food market. Natcher and Weaver (1999) researched the price volatility spillovers in vertically linked beef markets through a multivariate time-series approach, while Kesavan, Aradhyula, and Johnson (1992) explored price volatility in farm retail livestock price relationships. Modeling price volatility is attractive to researchers and of interest to consumers, producers, traders and governments (Alexander and Lazar, 2009).

The growing literature on agriculture finance are focusing on finding appropriate ways for market returns, option pricing, and risk analysis. One of the most attractive methodologies is the mixture of normal densities, which is introduced by Ball and Torous (1983). The normal mixture (NM) densities are the weighted sums of the normal density,
which is:

$$\eta(x) = \sum_{i=1}^{K} p_i \varphi_i(x) \quad (2.1)$$

where \([p_1, p_2, ..., p_k]\) is the positive mixing law, with \(\sum_{i=1}^{K} p_i = 1 \) (\(p_i \in (0, 1)\)), and \(\varphi_i(x) = \varphi(x; \mu_i, \sigma_i^2)\) as normal density functions. The innovation is characterised by a density function, which is \(X \sim NM(p_1, ..., p_k; \mu_1, ..., \mu_k; \sigma_1^2, ..., \sigma_k^2)\), for a random variable.

The normal mixture distribution may perform better than the Student’s t model, as the intuitive interpretations can be placed in the normal mixture framework (Ball and Torous, 1983; Kon, 1984; Alexander and Lazar, 2004a). In the normal mixture model, the individuals in the mixture density could represent different market states, and the mixing law shows the probabilities of these states. For example, a mixture of two normal densities could differentiate between ‘normal’ and ‘unusual’ market conditions.

The purpose of this thesis focuses on an econometric approach to modeling commodity price volatility. The predominant model among a large body of literature is the Generalized Auto-Regressive Conditional Heteroscedasticity (GARCH) model, which was proposed by Engle (1982), and extended by Bollerslev (1986). Specifically, the single-symmetric normal GARCH (1,1) model is represented by the conditionally normal error processes, which is gathering from the return \(y_t\):

$$y_t = f(t) + \epsilon_t, \quad \epsilon_t | I_{t-1} \sim N(0, \sigma_t^2) \quad (2.2)$$

where \(I_{t-1}\) is the information set at time \(t - 1\), and the conditional variance in the error
processes is presented as:

\[ \sigma^2_t = \omega + \alpha \epsilon_{t-1}^2 + \beta \sigma^2_{t-1} \]  

(2.3)

where \( \omega, \alpha \) and \( \beta \) are greater than zero, and \( \alpha + \beta < 1 \).

One of the most important usual assumptions of the proposed volatility GARCH model, as previously mentioned, is that error terms are conditionally normally distributed. In a normal GARCH (1,1) model, the conditional excess kurtosis is zero, but some degree of leptokurtosis in the unconditional returns can be captured (Alexander and Lazar, 2004b). In addition, Bollerslev (1987), Baillie and Bollerslev (2002), and Johnston and Scott (2000) mentioned that in both conditional and unconditional returns, the observed leptokurtosis is usually higher than what is predicted by the normal single GARCH model. The standard GARCH model is not appropriate to capture heavy tails, large kurtosis and the occurrence of extreme events.

As an extension, some researchers have further developed the classic GARCH framework. Those extensions are included in the literature either by assuming different distributions of the error structure (Bollerslev, 1987; Baillie and Bollerslev, 2002; Engle and Gonzalez-Rivera, 1991; Nelson, 1996; Fernández and Steel, 1998; Wang et al., 2001) or by adding asymmetric terms, such as leverage effects, in the variance process (Alexander and Lazar, 2006).

Some researchers considered using heavy-tailed conditional densities in the GARCH framework. For example, Bollerslev (1987) introduced the Student’s t-GARCH model, which is characterized by constant conditional skewness and kurtosis, while Fernández and Steel (1998) extended the Student’s t-GARCH further, considering the skewed t-distribution. However, these extensions did not consider both the time-varying condi-
tional skewness and kurtosis. Wang et al. (2001) developed the class of the GARCH models. They captured GARCH models with a flexible parametric error distribution based on the exponential generalized beta, allowing for the conditional skewness and kurtosis.

Contrary to the heavy-tailed conditional densities distributions, Engle and Lee (1993) developed a component model with a trend for the long-term volatility, which increases the amount of unconditional leptokurtosis captured by the normal GARCH model. However, this GARCH model cannot account for the time-varying conditional higher moments. Furthermore, Engle and Gonzalez-Rivera (1991) considered non-distributional models--the semi-parametric ARCH model -- as another extension for GARCH processes.

Usually, squared financial returns are highly auto-correlated (Taylor (1986)). The normal GARCH model can only partly capture the auto-correlation (Ding, Granger, and Engle (1993)). Therefore, some researchers (Ding and Granger (1996)) used the Long Memory ARCH model to overcome this issue. To be more specific, that model could mimic the empirical auto-correlations, but still could not perform well when accounting for time-varying higher moments.

Several researchers (Vlaar and Palm, 1993; Bauwens, Bos, and Van Dijk, 1999; Roberts, 2001; Haas, Mittnik, and Paolella, 2004; Alexander and Lazar, 2004b) have considered the normal mixture GARCH (NM-GARCH) model. In that model, the error processes have a normal mixture conditional distribution with GARCH variance components. The NM-GARCH model can account for the time-varying conditional skewness and kurtosis density.

Among them, Vlaar and Palm (1993) introduced a restricted NM-GARCH model. According to their study, in a mixture of two normal distributions they assumed that
the difference between the conditional variances of components is constant. Meanwhile, Roberts (2001) introduced another simple model for the normal mixture form. In his research, the error processes are a mixture of two normal densities with one constant variance component. This restricted model improves the fit compared to the single GARCH model, and also accounts for most of the skewness and kurtosis in data. Some studies (Bauwens, Bos, and Van Dijk, 1999; Bai, Russell, and Tiao, 2003) introduced another restricted NM-GARCH model, where the ratio of the two conditional variances is constant.

More recently, the general symmetrical NM-GARCH model was developed by Haas, Mittnik, and Paolella (2004), and further investigated by Alexander and Lazar (2006). According to their study, they investigated the generalized two-component NM-GARCH (1,1) model to estimate the exchange rates and they provided evidence that NM-GARCH (1,1) performs better when compared with the symmetric and skewed Student’s t-GARCH. One of the most appealing advantages of the NM-GARCH is that it allows for multiple states, which could contribute to economic interpretation. After a while, Alexander and Lazar (2009) explored the fact that when using NM-GARCH, the two different states can capture different situations. A component with relative low variance could represent a ‘stable’ state, which generally occurs, while a component with high variance could represent a ‘crash’ or ‘stressful’ state, which rarely occurs.

There are also other important extensions of GARCH processes with non-normal error distributions. Hamilton and Susmel (1994) introduced the Markov Switching (MS) ARCH model, and Gray (1996) further investigated the MS-GARCH model. In the MS-GARCH model, the key assumptions are: the error terms are non-normal distributions; there are more than one volatility regime; and it has $K$ individual conditional variance
The majority of the previous literature on agricultural price volatility focused on the dependence of price volatility across related markets and price volatility determinants (Shively, 1996; Buguk, Hudson, and Hanson, 2003; Balcombe, 2009; Rezitis and Stavropoulos, 2010; Stigler, Prakash et al., 2011; Serra, 2013; Serra, Zilberman, and Gil, 2011; Serra and Gil, 2012; Karali and Power, 2013). Buguk, Hudson, and Hanson (2003) analyzed the price volatility spillovers in the United States catfish supply chain using an exponential-GARCH (E-GARCH) model. Their results indicate that there were volatility spillovers in all agricultural markets. Additionally, policies and events would increase volatility in basic agricultural commodity markets, such as corn and soybean markets, and would also effect the vertically-related markets, such as catfish producers and processors.

Serra and Gil (2012) studied the corn price fluctuations in the United States during the last two decades through the multivariate-GARCH models. They suggested that in some specific periods of low stocks it is necessary to search for a powerful tool to mitigate food price for the public stock management. Also, their findings provide evidence that the United States corn price volatility could be explained by volatility clustering, the influence of ethanol markets, corn stocks and global economic conditions.

Nomikos and Pouliasis (2011) forecasted the petroleum futures market’s volatility using both Mix-GARCH and MRS-GARCH models. Their results indicate that those two models capture the persistence in volatility more accurate than the single GARCH model. Furthermore, market participants may benefit from identifying different volatility components for normal and highly volatile periods.

Karali and Power (2013) built on the spline-GARCH framework of Engle and Rangel
to explain price volatility in the United States agricultural, energy, and metal futures markets. In their study, the low frequency volatility is affected by changes in inflation, industrial production, inventories, and both long- and short-term interest rates spreads for most commodities. In addition, the increase of volatility will provide both positive and negative changes in inflation and the industrial production index.

However, most studies investigating NM-GARCH for estimating exchange rates, seldom studied the use of the NM-GARCH model to estimate agricultural price. Therefore, it is interesting to use NM-GARCH (1,1) models with different components to capture the relevant properties of agricultural commodity prices.
Chapter 3  Methodologies and Data

3.1  Introduction

Modeling price volatility is central to understanding price fluctuations. It is also necessary to develop powerful methods of forecasting future price trends. This chapter introduces the methodologies, and their respective advantages, of modeling price volatility in agricultural commodities. In addition, the data used in this thesis are introduced, including an explanation of data sources and data collection methods. Furthermore, descriptive statistics will be developed and presented to measure the features of the data.

3.2  Price Model Methodologies

Price volatility tends to vary, and show clustering behavior (Myers, 1992; Serra and Gil, 2012). In order to clearly understand this pattern, this chapter introduces the methods, comprising combinations of GARCH processes, that are used to measure commodity price volatility. Specifically, the price model follows the GARCH processes in variance equations which studied by Haas, Mittnik, and Paolella (2004) and Alexander and Lazar (2006). According to the literature, there is no obvious advantage in using more than one lag in each of the individual conditional variance equations. Hence, the form of NM-GARCH model in this thesis is termed the NM(K)-GARCH (1,1) model.

The mixture of normal densities, introduced to the financial community by Ball and
Torous (1983) and Kon (1984), consists of the weighted sums of normal densities, which can be represented as:

\[ \eta(x) = \sum_{i=1}^{K} p_i \varphi_i(x) \]  

(3.1)

where \([p_1, ..., p_k]\) represents the positive mixing law, in which \(p_i \in (0, 1), i = 1, ..., k,\)

\[ \sum_{i=1}^{K} p_i = 1, \]  

and the \(\varphi_i(x) = \varphi(x; \mu_i, \sigma_i^2)\) are normal density functions.

The innovation is denoted by the error processes, \(\epsilon_t\), which is assumed to follow a mixture of \(K\) Gaussian distributions with component mean, \(\mu_i\), and component variance, \(\sigma_i^2\), as shown in equation,

\[ \epsilon_t | I_{t-1} \sim NM(p_1, ..., p_k, \mu_1, ..., \mu_k, \sigma_1^2, ..., \sigma_K^2), \sum_{i=1}^{K} p_i = 1, \sum_{i=1}^{K} p_i \mu_i = 0 \]  

(3.2)

where \(I_t\) is the information set available at time \(t\) with different means, \(\mu_i\), and different time-varying variances, \(\sigma_i^2\), for \(i = 1, ..., K\) and \(K (\geq 2)\).

The normal mixture density distributions usually have non-zero excess kurtosis and skewness. If we use zero mean densities in normal mixture GARCH processes, it would be termed a symmetric NM(K)-GARCH (1,1) models. Such models have the zero conditional skewness. General NM(K)-GARCH (1,1) models with non-zero means can be used to analyze markets with skewed and heavy-tailed return densities, such as foreign exchange markets (Alexander and Lazar, 2004a).

There are \(K\) conditional variance components, which following by the GARCH (1,1) process. However, this thesis only considers scenarios with one, two or three possibilities.

(i) NM(K)-GARCH (1,1):

\[ \sigma_{it}^2 = \omega_i + \alpha_i \epsilon_{t-1}^2 + \beta_i \sigma_{it-1}^2 \quad \text{for } i = 1, ..., K \]  

(3.3)
where $\alpha_i$ represents the volatility reaction parameter, which refers to the effect of marketing shocks on volatility; and $\beta_i$ represents the volatility persistence parameter, which indicates the extent of inertia in volatility.

According to equations (3.2), the mixture’s mixing parameter and the mean of the density can be expressed as a function of other parameters. Consequently, the NM($K$)-GARCH (1,1) model consists of $5K - 2$ parameters that need to be studied, represented in equation (3.4) as:

$$\theta = (p_1, \ldots, p_{K-1}, \mu_1, \ldots, \mu_{K-1}, \omega_1, \alpha_1, \beta_1, \omega_2, \alpha_2, \beta_2, \ldots, \omega_K, \alpha_K, \beta_K)' \quad (3.4)$$

To subtract the means of each series, and estimate parameters in NM($K$)-GARCH (1,1) models, the expectation maximization algorithm (EM algorithm) (Dempster, Laird, and Rubin, 1977) has been developed. The EM algorithm is an iterative method for finding the maximum likelihood (ML) parameters in the mixture GARCH models. The EM algorithm seeks to get the ML parameters by iteratively applying two steps to estimate parameters - the expectation step (E-step) and the maximization step (M-step) - until they converge.

The EM algorithm begins with assumptions of starting values for the parameters in the distributions, such as $\lambda_i$, $\mu_i$ and $\sigma_i^2$, and then follows with the E-step and the M-step. In the E-step, the weights are calculated to indicate to which component each data point belongs. In the M-step, the weights are used to maximize the likelihood function and then gathering new estimation parameters. The final parameter results are given when the estimation parameters converge.
3.3 Price Data Collection

In order to analyze agricultural commodity price volatility in Ontario, the major crops which are most commonly grown in Ontario were selected for study. According to Agricorp, there are several major crops in Ontario: black beans, canola, corn (grain corn and farm feed), cranberry beans, hard red winter wheat, Japanese and other beans, Kidney beans, soft red winter wheat, soft white winter wheat, soybeans (incl. organic, tofu and natto varieties), spring grain (oats, barley and mixed grain), spring wheat and white beans.

This study analyzed the county-level daily cash prices of three major crops in Ontario: corn, soybeans, and soft white winter wheat (which will be called winter wheat in the follow chapters). The historical price data, covering the period from 2006 to 2012, I gathered those data from the Ridgetown College (University of Guelph Ridgetown Campus) data resource center. In the futures market, there are two prices that will occur with certainty everyday: one is the highest price of the day and the other is the lowest price of the day. In this research, the daily county average cash prices, in Canadian dollars\(^1\) per bushel, represent the average prices each day. Next Chapter will further explain how the daily county average data are used for modeling price volatility.

Since the main objective of this paper is to model and analyze price volatility for future Business Risk Management program\(^2\) studies, only the harvest period crop prices

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1 All monetary units mentioned in this paper are in Canadian dollars.
2 BRM programs include: (i) AgriInvest, which helps cover small margin declines; (ii) AgriStability, which aims to assist in the case of large margin declines caused by circumstance, such as low prices, and rising input costs; (iii) AgriInsurance, which is the program mainly addressed in this paper and offers protection for production losses related to specific crops or commodities caused by natural disaster; (iv) AgriRecovery, which helps producers return their farm business to operations following disaster situations; and (v) the Advance Payments Program, which is a complementary federal-only program to help crop and livestock producers with cash flow and provides flexibility for marketing of commodities. (The source comes from Agriculture and Agri-Food Canada)
were selected. The harvest time periods vary for different commodities; specifically, the location from which daily cash prices in the model are derived is the municipality of Chatham-Kent in southwestern Ontario, where three agricultural commodities with different harvest periods are considered:

i. Corn: Ontario daily cash price series in September and October for the 2006 - 2012 period (297 observations)

ii. Soybean: Ontario daily cash price series in September and October for the 2006 - 2012 period (297 observations)

iii. Winter wheat: Ontario daily cash price series in July and August for the 2006 - 2012 period (308 Observations)

3.4 Data Characteristics

The historical daily cash commodity prices contain important information about the price pattern; therefore, it is worthwhile to learn the price trends over the past several years, as well as the statistical characteristics of the price data for each commodity, and then study the price volatility issues.

The commodity price data in harvest period show different degrees of volatility due to a variety of factors, such as supply and demand factors, monetary policy and climate change, etc. The following three descriptive plots of the volatility of commodity prices against time are derived from the raw price data in Ontario:
Figure 3.1: Corn price volatility historical plot, Ontario
Figure 3.2: Soybean price volatility historical plot, Ontario
Figure 3.3: Wheat price volatility historical plot, Ontario
The descriptive graphs from Figure 3.1 through 3.3 indicate significant price volatility during the past several years. The discontinuous spots of price represent that these two prices belongs to different harvest years. The general trends of the prices of corn, soybean and winter wheat have continued to increase over the past decade; however, each commodity has experienced a different level of local price volatility during different harvest periods.

From these graphs, it is easy to observe the characteristics of the daily cash price data for each crop. Over the past several years, corn prices have undergone the smallest change, the change in the soybean price has been larger than the change in the price of corn, and the price of soybeans has varied the most.

The appropriated functional form of commodity prices with respect to time must be determined first. Price trend modeling, based on historical data, is the key to measuring the volatility of prices. More details will be discussed in Chapter 4.

In addition to the price trends, there are other characteristics of the price data that can be explored further through descriptive statistics:

| Table 3.1: Descriptive statistics over historical prices ($/Bushel) |
|-------------------------|--------|--------|--------|----------|--------|-------|
| Crop               | N      | Minimum | Mean  | Maximum | Std. Dev. | CV    |
| Corn               | 297    | 2.338   | 4.515 | 7.375    | 1.251   | 0.277 |
| Soybean            | 297    | 5.272   | 9.839 | 16.652   | 2.403   | 0.244 |
| Winter Wheat       | 308    | 3.830   | 7.178 | 10.070   | 1.701   | 0.237 |

As indicated in Table 3.1, corn has the lowest minimum and maximum prices among the three commodities. The standard deviation of the corn price is the smallest. However, it has the largest coefficient of variation \( CV \). This indicates that corn prices are easily influenced when facing factor change uncertainty. The winter wheat price has a
large standard deviation, but a smallest CV, which suggests that it is the most stable price among these three commodities. Soybean price has the highest mean, and maximum value, but the CV is not the highest. Hence, the price of soybean is not as high volatile as corn.

Data on historical daily cash commodity prices are used to estimate the model parameters. For each agricultural commodity, the continuously compounded percentage changes of prices, \( \text{Return}_t = 100 \times (\log P_t - \log P_{t-1}) \), with an autoregressive moving average (ARMA \((u,v)\)) model are being used, as shown in equations (3.5),

\[
\text{Return}_t = c + \epsilon_t + \sum_{i=1}^{u} a_i \text{Price}_{t-i} + \sum_{j=1}^{v} b_j \epsilon_{t-j}
\]

and the Akaike Information Criterion (AIC) is used to select appropriate values of \(u\) and \(v\). Further details are presented in next chapter.
Chapter 4  Price Model Estimation Results

4.1 Introduction

This chapter presents the price model estimations results, including the details of the stationary test, differencing the data, and identifying the ARMA model. Also, the estimation results of NM($K$) restricted and unrestricted models for different values of $K$ are presented along with the model selection criteria used to compare different GARCH models.

4.2 Unit Root Test

As mentioned in the previous chapter, the ARMA model is used to fit the compounded percentage changes of prices. Both the ARMA and GARCH methods are based on the assumption of stationary time series data. Therefore, a unit root test is necessary in order to gather the error terms and modeling price data. Based on the test, the non-stationary price data need to be converted to stationary.

The Dickey-Fuller (DF) and the augmented Dickey-Fuller (ADF) tests identify the unit root using time series data. In the DF and ADF tests, the statistics are negative. The more negative these numbers are, the stronger the reasoning for rejecting the hypothesis of unit root is (Greene, 2003).
The procedure for the ADF test is:

\[
\Delta y_t = \alpha + \beta t + \gamma y_{t-1} + \delta_1 \Delta y_{t-1} + \cdots + \delta_{p-1} \Delta y_{t-p+1} + \varepsilon_t,
\]

(4.1)

where \(\alpha\) is a constant, \(\beta\) is the coefficient of the time trend, and \(p\) is the lag order of the autoregressive process. If the test statistic is less than the critical value after the ADF test is completed, the data is regarded as stationary and null hypothesis is rejected. Otherwise, the data is non-stationary.

The ADF results for different commodities are shown in Table 4.1.

<table>
<thead>
<tr>
<th>Crop</th>
<th>Test Statistic</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corn</td>
<td>-2.9115</td>
<td>0.1921</td>
</tr>
<tr>
<td>Soybean</td>
<td>-3.6424</td>
<td>0.0294</td>
</tr>
<tr>
<td>Wheat</td>
<td>-1.9121</td>
<td>0.6134</td>
</tr>
</tbody>
</table>

### 4.3 Differencing Data Set

Non-stationary time series price data seldom used in economic analysis. Thus, the differencing method is used to convert non-stationary series data to stationary data. Non-harvest period data are not taken into consideration, hence the price data are not continuous from different years. Figures 4.1 through 4.6 show the differences among the corn, soybean and wheat. For example, in the Figure 4.1, the first graph represents the original time series of corn prices from 2006 to 2012; and the second one represents the differences in corn prices from 2006 to 2012\(^3\). In the Figure 6, the first graph shows the

\(^3\)Price data comes from different years could not be differences, so that is why the difference price data not continue.
log price of corn from 2006 to 2012; and the second one shows the differences in the log corn prices from 2006 to 2012. The log corn price data is smoother and more linear than the raw price data.

In financial time series data, the log transformation are usually used to linearized the data (as shown in Figures 4.1 through 4.6). Thus, this thesis uses the differencing log price to analyse price volatility. ADF test is used to test the stationary data of the differenced log price data, which are stationary this time. (Results are shown in Table 4.2.)

<table>
<thead>
<tr>
<th>Crop</th>
<th>Test Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corn</td>
<td>-6.7614***</td>
</tr>
<tr>
<td>Soybean</td>
<td>-6.2371***</td>
</tr>
<tr>
<td>Wheat</td>
<td>-6.7077***</td>
</tr>
</tbody>
</table>

*** indicate values significant at 10% significance levels respectively.
Figure 4.1: Corn Price & Differencing Corn Price
Figure 4.2: Log Corn Price & Differencing Log Corn Price
Figure 4.3: Soybean Price & Differencing Soybean Price
Figure 4.4: Log Soybean Price & Differencing Log Soybean Price
Figure 4.5: Wheat price & Differencing Wheat Price
Figure 4.6: Log Wheat price & Differencing Log Wheat Price
4.4 ARMA Model Identification

The ARMA model is used to fit the changes of price data, and the model is given by:

\[ X_t = c + \epsilon_t + \sum_{i=1}^{u} \alpha_i X_{t-i} + \sum_{i=1}^{v} \beta_i \epsilon_{t-i} \]  

(4.2)

where \( X_t \) is the changes of price data, which are equal to 100 \( \times (log P_t - log P_{t-1}) \).

The Box-Jenkins method provides a way to identify the ARMA \((u, v)\) model based on the auto-correlation (ACF) and partial auto-correlation (PACF) graphs\(^5\). Figure 4.7 shows the ACF and PACF graphs for different commodities.

Alternatively, the AIC provides another quantitative method that can be used to help identify the ARMA model. The AIC calculation is:

\[ AIC = 2K - 2 \ln(L) \]  

(4.3)

where \( K \) is the number of parameters in the model, and \( L \) is the maximised value of the likelihood of the selected ARMA model. The details of AIC results for ARMA identification are presented in Table 4.3 for illustrative purposes.

Based on the AIC results combined with the ACF and PACF graphs, the selected ARMA models are: ARMA\((1,0)\) for corn, ARMA\((0,1)\) for soybean, and ARMA\((0,1)\) for wheat.

---

\(^4\)\( u \) represents the autogressive parameter; and \( v \) represents the moving average parameters

\(^5\)If ACF cuts off after lag \( n \) and the PACF dies down, the MA\((v)\) model can be used; If the ACF dies down and the PACF cuts off after lag \( n \), then the AR\((u)\) is identified (Greene (2003))
### Table 4.3: AIC Results for ARMA Identification

<table>
<thead>
<tr>
<th>Model</th>
<th>Corn</th>
<th>Soybean</th>
<th>Wheat</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1,0)</td>
<td>-946.99</td>
<td>-1018.84</td>
<td>-1108.3</td>
</tr>
<tr>
<td>(0,1)</td>
<td>-946.88</td>
<td>-1018.92</td>
<td>-1108.36</td>
</tr>
<tr>
<td>(1,1)</td>
<td>-945.88</td>
<td>-1017.85</td>
<td>-1106.36</td>
</tr>
<tr>
<td>(2,0)</td>
<td>-945.4</td>
<td>-1019.15</td>
<td>-1106.39</td>
</tr>
<tr>
<td>(2,1)</td>
<td>-943.94</td>
<td>-1017.21</td>
<td>-1104.39</td>
</tr>
</tbody>
</table>
4.5 Estimation Results & Model Selection

For NM($K$) models with $K = 1, 2$ and $3$ respectively, there are estimations for three different commodities that use the historical daily cash commodity price data. Notably, the NM(1) is equivalent to the GARCH (1,1). In addition to the general normal mixture GARCH model, a restricted model is also estimated. The restriction is based on the zero-mean normal model in the mixture GARCH processes. Estimation results are presented in Table 4.4.

In the NM($K$)-GARCH(1,1) framework, 

$$
\sigma^2_{it} = \omega_i + \alpha_i \epsilon^2_{i-1} + \beta_i \sigma^2_{it-1} \quad \text{for } i = 1, ..., K \tag{4.4}
$$

$\alpha_i$ is defined as the volatility reaction parameter, which represents the short term price volatility, and $\beta_i$ is the volatility persistence parameter, which indicates the long term volatility.

It is easy to fit the NM(2) model with the mixture distribution components, as the lower long-term volatility component has the highest mixing parameter value. The model captures two distinct market regimes in terms of agricultural commodity price volatility. A lower volatility component occurs with higher probability, which is represented as a “usual market circumstance” (the usual regime) that occurs most of the time. There is also a higher volatility component with lower probability that represents an “extreme market circumstance” (the unusual regime). This component rarely occurs, but its volatility is still higher than the “usual” one. The estimated weights, $p_i$, in the mixing law explained the frequencies at which these two circumstances have occurred over the last several years.
<table>
<thead>
<tr>
<th></th>
<th>Corn</th>
<th>Soybean</th>
<th>Wheat</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Normal</td>
<td>NM(2)</td>
<td>NM(2)μ = 0</td>
</tr>
<tr>
<td>( p_1 )</td>
<td>0.8933</td>
<td>0.9024</td>
<td>0.5646</td>
</tr>
<tr>
<td>( \mu_1 )</td>
<td>-0.0944</td>
<td>0</td>
<td>-0.2045</td>
</tr>
<tr>
<td>( \omega_1 )</td>
<td>5.6366</td>
<td>0.8007</td>
<td>0.7780</td>
</tr>
<tr>
<td>( \alpha_1 )</td>
<td>1.23E-08</td>
<td>1.49E-08</td>
<td>1.49E-08</td>
</tr>
<tr>
<td>( \beta_1 )</td>
<td>0.7617</td>
<td>0.8877</td>
<td>0.8274</td>
</tr>
<tr>
<td>( p_2 )</td>
<td>0.1067</td>
<td>0.0976</td>
<td>0.3891</td>
</tr>
<tr>
<td>( \mu_2 )</td>
<td>0.7901</td>
<td>0</td>
<td>0.3431</td>
</tr>
<tr>
<td>( \omega_2 )</td>
<td>18.9143</td>
<td>20.3945</td>
<td>1.3047</td>
</tr>
<tr>
<td>( \alpha_2 )</td>
<td>1.49E-08</td>
<td>1.49E-08</td>
<td>1.49E-08</td>
</tr>
<tr>
<td>( \beta_2 )</td>
<td>0.9005</td>
<td>0.9020</td>
<td>0.8920</td>
</tr>
<tr>
<td>( p_3 )</td>
<td>0.0463</td>
<td>0</td>
<td>( p_3 )</td>
</tr>
<tr>
<td>( \mu_3 )</td>
<td>-0.3987</td>
<td>0</td>
<td>( \mu_3 )</td>
</tr>
<tr>
<td>( \omega_3 )</td>
<td>38.5187</td>
<td>0</td>
<td>( \omega_3 )</td>
</tr>
<tr>
<td>( \alpha_3 )</td>
<td>1.49E-08</td>
<td>0</td>
<td>( \alpha_3 )</td>
</tr>
<tr>
<td>( \beta_3 )</td>
<td>0.9037</td>
<td>0</td>
<td>( \beta_3 )</td>
</tr>
<tr>
<td>( E\sigma_1 )</td>
<td>2.0614</td>
<td>2.1232</td>
<td>1.7919</td>
</tr>
<tr>
<td>( E\sigma_2 )</td>
<td>13.7849</td>
<td>14.4241</td>
<td>3.4757</td>
</tr>
<tr>
<td>( E\sigma_3 )</td>
<td>20.0017</td>
<td>0</td>
<td>( E\sigma_3 )</td>
</tr>
</tbody>
</table>
According to the NM(2) model results, corn prices have the lowest and negative mean components in the usual regime (-0.09% per day), and the unconditional volatility is also low at 2.06%. In the usual regime, corn prices exhibit the least reactive and most persistent volatility. In the unusual regime, soybean prices show the highest unconditional volatility (over 18% per day) and the highest persistent parameter, but small reactive parameter. The NM(2) models with zero-mean restriction provides the same results as the NM(2) model without restriction for each commodity.

When fitting the NM(3) model with three different mixture distribution components, in all cases, the component with the second highest mixing parameter, \( p_i \), has an average long-term volatility. The other two components have lower and higher long-term volatilities, one with a smaller mixing parameter, and the other with a larger mixing parameter. The NM(3) model captured two “exceptional circumstances” (Alexander and Lazar, 2004a): one represented the unusually tranquil markets and the other represented unusually volatile markets. In addition, adding another component to the two normal mixture distributions decreased the weights for both the high- and low-volatility components. The NM(3) model analyzed the medium- and low- price volatilities.

According to the NM(3) estimation results, the corn series have the lowest unconditional volatility (1.79% per day) in the usual regime, as well as the largest persistent parameter and smallest reactive parameter. Although corn has the highest and negative mean component (-0.2% per day), it still presents the least reactive and most persistent features. In contrast, wheat prices have the highest unconditional (3.76% per day) volatility, as well as the lowest mean component (-0.10% per day), which indicates that wheat has the highest volatile price.

AIC is used to determine which model has the best feature. The results are presented
in Table 4.5. The highest likelihood statistics are obtained using the NM(2) model with restrictions. However, there are no significant differences between the AIC for NM(2) with restrictions and NM(3). The normal GARCH(1,1) model shows the worst performance under the criteria for all three commodities. Figure 4.8 through 4.10 shows the density plots of corn, soybean and wheat for different methods.

<table>
<thead>
<tr>
<th></th>
<th>Log-likelihood</th>
<th>AIC</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Corn</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Normal</td>
<td>614.82</td>
<td>-6.8427</td>
</tr>
<tr>
<td>NM(2)</td>
<td>715.4191</td>
<td>-7.14</td>
</tr>
<tr>
<td>NM(2) $\mu = 0$</td>
<td>751.5593</td>
<td>-7.2443</td>
</tr>
<tr>
<td>NM(3)</td>
<td>745.4096</td>
<td>-7.2279</td>
</tr>
<tr>
<td><strong>Soybean</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Normal</td>
<td>612.551</td>
<td>-6.8353</td>
</tr>
<tr>
<td>NM(2)</td>
<td>726.5295</td>
<td>-7.1766</td>
</tr>
<tr>
<td>NM(2) $\mu = 0$</td>
<td>727.2389</td>
<td>-7.1785</td>
</tr>
<tr>
<td>NM(3)</td>
<td>724.46</td>
<td>-7.1709</td>
</tr>
<tr>
<td><strong>Wheat</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Normal</td>
<td>575.2544</td>
<td>-6.7096</td>
</tr>
<tr>
<td>NM(2)</td>
<td>603.7834</td>
<td>-6.8064</td>
</tr>
<tr>
<td>NM(2) $\mu = 0$</td>
<td>606.4315</td>
<td>-6.8151</td>
</tr>
<tr>
<td>NM(3)</td>
<td>606.1168</td>
<td>-6.8141</td>
</tr>
</tbody>
</table>
Figure 4.8: Corn Density
Figure 4.9: Soybean Density
Figure 4.10: Wheat Density
Chapter 5 Conclusions

Previous studies, such as Balcombe (2009), Stigler, Prakash et al. (2011) and Karali and Power (2013) have investigated agricultural commodity price volatility as well as its effects. Moreover, a large body of literature has applied modeling the commodity price volatility model without comparing different methods and most of those studies have just considered the single state GARCH (1,1). This thesis considers three main agricultural commodities in Ontario and analyzes their volatility during the harvest periods. In addition, the performance of Normal Mixture GARCH (1,1) (NM-GARCH (1,1)) models with different components is examined.

In the NM-GARCH (1,1) models, the error terms follow normal mixture distributions with different numbers of GARCH (1,1) variance components. Previous studies have failed to detect the state-dependent volatility dynamics in cases in which there are multiple states. Most of those studies have assumed that the commodity price volatility model follows a single-state GARCH(1,1) process with constant conditional skewness and kurtosis. In contrast, the NM(K)-GARCH(1,1) models capture the volatility behavior and the time-varying conditional skewness and kurtosis of different states (Haas, Mittnik, and Paolella, 2004; Alexander and Lazar, 2009). This thesis models agricultural commodities price volatility using NM(K)-GARCH (1,1) with one, two and three different market states.

The estimation results for the three different agricultural commodity daily cash prices illustrate the relationship between the usual price change and the unexpected volatility dynamics across (which are reflected by $\alpha$, and $\beta$). For each commodity there is an
expected negative price change, usually with a smaller volatility persistence ($\beta$) and a larger reaction volatility ($\alpha$), which indicates that the volatility tends to be more reactive when the price drops. This is similar, in part, to the findings of Haas, Mittnik, and Paolella (2004) regarding the equity markets; the volatility is more responsive in negative price change circumstances and more persistent in positive price change circumstances.

Model selection criteria is used to select the best fitting model for analyzing commodity prices. The NM(3) models perform better than NM(2) models without restriction, while NM(2) models with restriction perform better than NM(3). However, the NM(3) does not have an obverse advantage when compared with the NM(2) with restriction according to the AIC results. The normal GARCH (1,1) performs the worst among all the methods. The results indicate that the NM($K$)-GARCH (1,1) models are better at capturing the persistence in volatility than the single state GARCH (1,1) model.

Although NM(3) models perform well based on the AIC results, using these models to analyzing price volatility is still not highly recommended. The reason is that the mixing parameter on the third component is always too small, which might lead to biased parameters and conditional and unconditional kurtosis estimates (Alexander and Lazar, 2004a). As an alternative, the results of the NM(2) processes for each of the commodities are appealing. Among the different commodity, the estimation parameters of each conditional variance are clearly distinguished. Based on the empirical results, the higher mixing weight always has lower variance than the lower mixing weight. This interpretation illustrates the point that if agricultural economic intuition provided with attractive methods, two different volatility ‘states’ could exist in the agricultural commodity market. Thus, the results presented in this thesis may lead to a recommendation for using the general unrestricted NM(2) models and/or the restricted NM(2) models.
for agricultural commodity price volatility modeling.
Appendix A

R Code for GARCH (1,1) process:

```r
# generate garch (1,1)
# initial value
n <- 500
a <- c(0.5, 0.4, 0.2)
u <- rnorm(n, mean=0, sd=1)
h <- rep(0, n)
e <- rep(0, n)
e[1] <- rnorm(1, mean=0, sd=sqrt(a[1]/(1.0-a[2]-a[3])))

for (i in 2:n)
{
  h[i] <- a[1] + a[2]*e[i-1]^2 + a[3]*h[i-1]
e[i] <- u[i]*sqrt(h[i])
}
e.hat <- e

sigma <- array(var(e.hat), dim=c(n, 1))
llh <- array(0, dim=c(n, 1))
params <- c(8.270296e-01, 5.000000e-02, 5.000000e-02)
garch.LLH = function(garch.opt)
{

```
for (j in 2:n)
{
  sigma[j] <- max(0.000001, (garch.opt[1])^2 + (garch.opt[2])^2 * (e.hat[j - 1])^2 + (garch.opt[3])^2 * (sigma[j - 1]))
  llh[j] <- -.5 * log(sigma[j]) - .5 * ((e.hat[j])^2 / sigma[j])
}

-sum(llh)

start1.garch <- array(1, dim = c(3, 1))

result.1.garch <- optim(start1.garch, garch.LLH)

para.est <- result.1.garch$par^2
Appendix B

Code for NM(K)-GARCH (1,1) process

```r
nn <- 500
t <- runif(nn)
lambda <- 0.25

a <- c(0.5, 0.4, 0.1)
u1 <- 1
e1.random <- rnorm(nn) + u1
e1 <- e1.random
h1 <- array(1, dim = c(nn, 1))

for (i in 2:nn) {
e1[i] <- u1 + rnorm(1) * h1[i]^0.5
}

b <- c(12, 0.25, 0.3)
u2 <- 1
h2 <- array(9, dim = c(nn, 1))
e2.random <- rnorm(nn) + u2
e2 <- e2.random

for (i in 2:nn) {
}
```
h2[i]<-b[1] + b[2]*(e2[i-1]-u2)^2 + b[3]*h2[i-1]
e2[i]<-u2+rnorm(1)*h2[i]^0.5
}
e<-e1

for(j in 1:nn)
{
  if(t[j]<lambda) e[j]<-e2[j]
}
k<-2

NM_Garch = function(data,k=2){
  Dist = function(z, h) { dnorm(x = z/h)/h }
  n <- length(data)

  # M Step for EM of N-Mixture Model
  M< function(phi){
    sumphi<- colSums(phi)
    mu0<- colSums(phi*data)/sumphi
    temp<-matrix(0,n,k)
    for (j in 1:k){
      z<-data-mu0[j]
      temp[,]<-phi[,]*z^2
    }
sigma2 <- abs(colSums(temp))/sumphi
  }
}
pr <- colSums(phi)/n
z <- rep(0, n)

for (i in 1:n) {
  j <- sample(k, size = 1, prob = phi[i,])
  z[i] = j
}

v1 <- rep(0, n)
v2 <- rep(0, n)
Mean = c(sum(data[z==1])/length(z[z==1]), sum(data[z==2])/length(z[z==2]))

v1[z==1] = data[z==1] - Mean[1]
v2[z==2] = data[z==2] - Mean[2]

Var = c(sum(v1^2)/length(z[z==1]), sum(v2^2)/length(z[z==2]))

mu <- rep(0, k)
a <- rep(0, k)
b <- rep(0, k)
c <- rep(0, k)

for (j in 1:(k-1)) {
  Loglike.1 <- function(theta) {
    mn[j] <- (theta[1])
a[j] <- (theta[2])
b[j] <- (theta[3])
c[j] <- (theta[4])

    llh <- matrix(0, n, 1)

    # garch(1, 1)
  }

  # garch(1, 1)
temp <- mean((data)^2)

h <- rep((temp*b[j]+a[j])/(1-c[j]), n)

for (i in 2:n) {
  h[i] = a[j] + b[j]*((data[i-1])^2 + (c[j]*h[i-1]))
}

hh = sqrt(abs(h))

llh = log(dnorm(data, mu[j], hh))*phi[j]

ll <- -sum((llh))

return(ll)

}

S = (sqrt(.Machine$double.eps))

theta = c(mu0[j], 0.1*sigma2[j], (.1), (.8))

lowerBounds = c(mu = -10*abs(mu0[j]), a = S^2, b = S, c = S)

upperBounds = c(mu = 10*abs(mu0[j]), a = 100*sigma2, b = 1-S, c = 1-S)

result <- nlminb(start=theta, objective=Loglike.1, lower = lowerBounds, upper = upperBounds)

mu[j] <- (result$par[1])
a[j] <- (result$par[2])
b[j] <- (result$par[3])
c[j] <- (result$par[4])

}

mu[k] <- sum(pr[1:(k-1)]/pr[k]*mu[1:(k-1)])

Loglike.2 <- function(theta){
  a[k] <- (theta[1])
b[k]<-(theta[2])
c[k]<-(theta[3])

llh<-matrix(0,n,1)

temp<-mean((data)^2)
l<-rep((temp*b[k]+a[k])/(1-c[k]),n)

for (i in 2:n ) {
    h[i] = a[k] + b[k]*(data[i-1])^2+ (c[k]*h[i-1])
}

hh = sqrt(abs(h))
llh = log(dnorm(data,mu[k], hh)*phi[k])

ll <- -sum((llh))
return(ll)
}

S = (sqrt(.Machine$double.eps))
theta =c(0.1*sigma2[k], (.1), (.8))

lowerBounds = c( S^2, S, S)
upperBounds = c(100*sigma2[k], 1-S, 1-S)
result<-nlminb(start=theta, objective=Loglike.2, lower = lowerBounds, upper = upperBounds)

a[k] <-(result$par[1])
b[k]<-(result$par[2])
c[k]<-(result$par[3])
list(pr=pr,mu=mu,a = a, b = b, c=c)
E <- function (pr, mu, a, b, c) {
  phi <- matrix(0, n, k)
  h <- phi
  temp <- solve(diag(k) - diag(c) - t(t(b)) * sum(pr * mu^2))
  h[1,] <- temp
  for (i in 2:n) {
    h[i,] = a + b*(data[(i-1)])^2 + (c*h[(i-1),])
  }
  for (j in 1:k) {
    hh = sqrt(abs(h[,j]))
    phi[,j] = Dist((data- mu[j]), hh)*pr[j]
  }
  phi <- phi/rowSums(phi)
  return(phi)
}

Loglike <- function(theta)
{
  pr <- (theta[1:k])
  mu <- rep(NA, k)
  mu[1:(k-1)] <- (theta[(k+1):(2*k-1)])
  mu[k] <- sum(pr[1:(k-1)]/pr[k]*mu[1:(k-1)])
  a <- (theta[(2*k):(3*k-1)])
}
b<-(theta[(3*k):(4*k-1)])
c<-(theta[(4*k):(5*k-1)])
llh<-matrix(0,n,k)
inner = rep(0,n)
b<-llh
temp<-solve(diag(k)-diag(c-t(b)$t(pr))$t(b))*sum(pr*mu^2)
h[1,]<-temp
for (i in 2:n ) {
  h[i,] = a + b*(data[i-1]$^2 + (c*h[i-1]))
  for (j in 1:k){
    hh = sqrt(abs(h[,j]))
    llh [,j] = Dist((data-mu[j]), hh)*pr[j]
    inner =inner+llh [,j]
  }
  ll <- sum(log(inner))
}
n <- length(data)
mcl<-Mclust(data, 2)
cl<-summary(mcl)
Mean<-cl$mean
clcluster<-cl$classification
size.1<-length(clcluster[clcluster==1])
size.2<-length(clcluster[clcluster==2])
v1 <- rep(0, n)
v2 <- rep(0, n)
Var <- c(sum(v1^2) / size.1, sum(v2^2) / size.2)

phi <- E(pr = c(length(cluster[cluster==1])/length(cluster), length(cluster[cluster==2])/length(cluster)), mu = Mean, a = c(.1 * Var), b = c(.1, .1), c = c(.8, .8))

pr <- (M(phi)$pr)
mu <- M(phi)$mu
a <- M(phi)$a
b <- M(phi)$b
c <- M(phi)$c

temp <- order(pr, decreasing = TRUE)
pr <- pr[temp]
mu <- mu[temp]
a <- a[temp]
b <- b[temp]
c <- c[temp]
theta = c(pr = pr, mu = mu[1:(k-1)], a = a, b = b, c = c)
nll <- Loglike(theta)
theta
l1 <- Inf
tol <- .0001
vars<-rep(0,k*5)

while ((ll+tol< nll) ){
  ll <-nll
  phi<- E (pr, mu, a, b, c)
  pr<-M( phi)$pr
  mu<-M( phi)$mu
  a<-M( phi)$a
  b<-M( phi)$b
  c<-M( phi)$c

  temp<-order (pr, decreasing=TRUE)
  pr<-pr[temp]
  mu<-mu[temp]
  a<-a[temp]
  b<-b[temp]
  c<-c[temp]
  theta = c (pr=pr, mu=mu[1:(k-1)], a=a, b=b, c=c)
  nll<- (Loglike (theta))
}
sigma<-sqrt (solve (diag(length(mu)))- diag (c)-t (t(b))%*%(t(pr)))%*%(t(a)+t(t(b))*sum (pr*mu^2))

list (pr=pr, mu=mu, a=a, b=b, c=c, sigma=sigma)
Bibliography


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