Remote Measurement and Analysis of Shallow Water Breaking Wave Characteristics

by

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REMOTE MEASUREMENT AND THE ANALYSIS OF SHALLOW WATER BREAKING WAVE CHARACTERISTICS

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This thesis is an investigation of one of the most important elements in coastal engineering design; shallow water breaking waves. Shallow water waves create large impact forces on coastal structures, drive sediment transport, regulate a number of biophysical and air-sea chemical processes, and ultimately control coastal morphology and storm inundation. The value of accurate measurements and prediction systems for breaking waves cannot be overstated - it is paramount to understanding the drivers of coastal processes, engineering design and hazard prediction.

A detailed review of key original empirical work is presented in this thesis to give a historical perspective of wave breaking research and identify avenues yet to be explored. The majority of the published relationships were semi-empirically extracted from scaled laboratory tests or limited field investigations. Traditional field investigation methods generally suffer from low spatial resolution data, high researcher safety risks and exorbitant costs. This study presents a precise, robust and low cost method to extract all relevant breaking wave properties from irregular waves, using simple consumer digital camcorder and global positioning system (total cost: ~ $350 USD).

The collective understanding of wave breaking characteristics is continually hampered by a lack of consistent definitions and measurement techniques for determining breaking, breaking depth and effective seafloor slope. In this detailed study, the dominant published definitions for the breaking
depth and effective seafloor slope are compared, their reliability tested and the best practices suggested. A novel method of calculating the breaking depth, corrected for optical wave trough water level variations, displayed the lowest variability and was determined to best define the effective breaking depth. A newly presented effective seafloor slope definition, based on individual wavelength to depth ratios, increased predictive ability over previous seafloor slope extraction methods. Finally, an optimized breaking wave height prediction method finds a root mean square relative error of just 1.7% within the ranges of measured dataset. Irregular waves investigated on an individual wave basis are shown to follow regular wave breaker height and depth prediction methods.

Investigations into plunging breaker “intensity” using wave vortex geometric parameters have been ongoing for 50 years with limited success. Previously published works, based on regular wave flume results or solitary wave theory, present contradictory results and conclusions. This thesis investigates the predictability of vortex parameters and validity of using vortex parameters as indicators of breaking intensity. Detailed analysis determined that vortex parameters cannot be accurately predicted and are not suggested as a possible indicator of breaking intensity.

Through this study, numerous innovative measurement methods and wave definitions were presented, defended and shown to increase our understanding of shallow water break events. The unique and highly detailed dataset of irregular breaking wave conditions allowed for unparalleled investigations into shallow water breaking waves.
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The following symbols are used in this paper:

\[ \begin{align*} 
A_c & = \text{wave vortex area (meters}^2) \\
A & = \text{co-efficient for breaker index formulae.} \\
a & = \text{wave amplitude (meters)} \\
a_c & = \text{wave crest amplitude (meters)} \\
a_t & = \text{wave trough amplitude (meters)} \\
c & = \text{individual wave celerity (meters/second)} \\
c_b & = \text{wave celerity at breaking (meters/second)} \\
c_g & = \text{wave group celerity (meters/second)} \\
E_r & = \text{primary individual wave} \\
ER & = \text{root mean square relative error} \\
F_p & = \text{spectral peak frequency (hertz)} \\
g & = \text{gravity (9.81 meters/second}^2) \\
H_b & = \text{breaking wave height (meters)} \\
H_o & = \text{offshore wave height (meters)} \\
H_{mo} & = \text{first moment wave height (meters)} \\
H_{sig} & = \text{significant wave height (meters)} \\
h_b & = \text{breaking wave water depth (meters)} \\
h_c & = \text{reef crest water depth (meters)} \\
h_t & = \text{wave trough depth (meters)} \\
h_c & = \text{water depth at trough corrected for optical effects (meters)} \\
h_{sd} & = h_c \text{ corrected for wave induced setdown (meters)} \\
L & = \text{wavelength (meters)} \\
L_o & = \text{offshore wavelength (meters)} \\
L_b & = \text{breaking wavelength (meters)} \\
L_{pb} & = \text{peak-frequency breaking wavelength. (meters)} \\
L_1 & = \text{horizontal wave jet length (meters)} \\
L_2 & = \text{vertical wave jet length (meters)} \\
L_3 & = \text{wave jet thickness (meters}^3) \\
l_c & = \text{wave vortex length (meters)} \\
l_w & = \text{wave vortex width (meters)} \\
m & = \text{seafloor slope} \\
m_{1/x} & = \text{slope calculated from } h_b \text{ to } h_{1/x} \\
m_1 & = \text{first wave moment} \\
R^2 & = \text{co-efficient of determination} \\
S & = \text{breaker type indicator proposed by Svedsen} 
\end{align*} \]
\( T \) = wave period (seconds)
\( T_p \) = spectral peak wave period (seconds)
\( X_p \) = wave plunge distance (meters)
\( X_s \) = wave splash distance (meters)
\( x_c \) = distance from camera to wave crest (meters)
\( x_t \) = distance from camera to wave trough (meters)
\( x_3 \) = optical wave trough correction distance (meters)
\( Y \) = wave vortex ratio
\( z_c \) = camera height (meters)
\( \beta \) = instantaneous wave height to depth ratio
\( \tau \) = wave period (seconds)
\( \gamma_b \) = breaker index
\( \gamma_1 \) = breaker index prior to bar
\( \gamma_2 \) = breaker index on reef crest
\( \eta_{(x,t)} \) = water level at position, \( x \), and time, \( t \) (meters)
\( \eta_c \) = water level at wave crest (meters)
\( \eta_t \) = water level at wave tough (meters)
\( \xi_o \) = offshore Surf Similarity Parameter
\( \xi_b \) = breaking Surf Similarity Parameter
\( \alpha \) = seafloor angle (degrees)
\( \phi \) = wave peel angle (degrees)
\( \theta \) = incoming wave angle - with respect to a flat coastline (degrees)
\( \theta_p \) = peak wave direction (degrees)
\( \theta_c \) = camera to wave crest angle (degrees)
\( \theta_t \) = camera to wave trough angle (degrees)
\( \delta_0 \) = offshore wave direction (degrees)
\( \delta_p \) = spectral peak wave direction (degrees)
\( \zeta \) = Surf Similarity Parameter
1 Introduction

1.1 Background and Rationale

Ocean waves have fascinated and entertained humans for thousands of years. While providing the simplest pleasures of playing in their nearshore forms, waves are also responsible for some of the most devastating natural disasters on the planet. Their generation, global propagation patterns and dissipation have intrigued and challenged scientists for centuries. Of all wave transformations, the moment when a shoaling wave reaches the point of instability and begins to break is both the most complex and difficult to predict.

The breaking of shallow water ocean waves is one of the most important elements in coastal engineering design. Shallow water waves create large impact forces on coastal structures, drive sediment transport, regulate a number of biophysical and air-sea chemical processes, and ultimately control coastal morphology and storm inundation. As a result, the value of accurate measurements and prediction systems for nearshore breaking waves cannot be overstated - it is paramount to any engineering planning or design concerning the surf zone. Some waves plunge spectacularly, shooting white water high into the air, while others crumble slowly and quietly.

Despite considerable effort, a complete and detailed understanding of shallow water wave breaking events still eludes scientists and engineers. Over a century ago, McCowan [1894] was the first documented to provide prediction methods for the most basic of wave breaking parameters; the wave height ($H_b$) and water depth ($d_b$). McCowan based his work on the ideal theoretical solitary wave over a horizontal seafloor and suggested that the breaker index ($\gamma_b$), or ratio of the breaking wave height to breaking water depth, maintains a constant value of 0.78. Unfortunately, this breaker index constant is still often erroneously quoted despite this approximation being almost 120 years old and disproven on numerous occasions.

Continued scientific effort has been directed to wave breaking studies since McCowan’s ground breaking work and has resulted in a far greater understanding of breaking events. Working on the visual evidence of differing forms of breaking waves, Iribarren and Nogales[1949] and Longuet-Higgins [1982] suggested analysing breaking wave intensities and breaking conditions via the location, size and presence of a wave vortex. The work of Iribarren and Nogales laid the foundation for the often used method of categorizing all shallow water breaking waves through the Surf Similarity Parameter. Additional classification methods using wave plunge distances [Smith and Kraus, 1991] and vortex ratios [Mead and Black, 2001] have resulted in new avenues and methods for categorization, as well as a greater understanding of wave breaking events.
Given the complex inter-connected nature of incoming wave conditions, complex seafloor slopes and the final breaking waves, the majority of research has been based on idealized laboratory investigations and numerical models rather than direct field collected data [Camenen and Larson, 2007].

Laboratory investigations are generally favoured since they allow researchers’ direct control over seafloor slopes, incoming wave conditions and host of other variables while also allowing for ease of data capture. Yet laboratory investigations are also inherently limited. First, most published experimental investigations generally dealt with regular monochromatic waves rather than the irregular waves that characterize most real life conditions. Traditional laboratory experiments are additionally limited by their inability to accurately scale physical characteristics of surface tension [Miller et al., 2010], bubble size, and wall friction [Johnson, 2009]. Many laboratory studies are simply physically unable to meet the minimum wave height and depth requirements of 100 mm [Battjes and Stive, 1985; Goda and Morinobu, 1998] in order to negate these adverse effects.

Numerical models hold great potential for wave prediction and have become a favoured method of prediction as computing power has exponentially increased. The vast majority of numerical models provide reliable predictions for deep and intermediate wave transformations yet break down when entering the shallow water zone. Many models simply impose a series of idealized assumptions to provide estimations of the complex events occurring at the point of breaking. Many of these idealized breaking wave relationships are based on empirically fitted equations using regular linear waves and parallel emergent beach slopes which, if applied to real-world irregular wave situations, could lead to considerable variations of final predicted breaking wave heights. As result, any planning or design resulting from these predicted breaking wave heights could be overly conservative and costly, or under designed and prone to failure.

Direct measurements of individual waves in nature obviously are the preferred method of building breaking wave relationships yet are complicated by their own set of complicating variables. Real world irregular waves feature a spectrum of incoming heights, periods, and directions. As a result, their breaking positions are spread over a wide spatial area and it is extremely difficult to mount in-situ single point measurement devices at the exact breaking position. Often cited large budget studies such as DUCK[Ebersole and Hughes, 1986], SUPERDUCK[Rosati et al., 1990] and DELILAH[Birkemeier et al., 1997] deployed large arrays of expensive in situ monitoring devices in an effort to increase data resolution and resulting accuracy of delineating the breaking point. While these studies considerably increased our understanding of breaking events, they were limited by the unsteady nature of the seafloor bathymetry at the study locales. The changing seafloor profiles and breaking depths greatly increased the uncertainty levels associated with their results.
The relatively low spatial resolution, non-permanent seafloor slopes at study locations, necessity of good breaking conditions, and researcher safety are often cited obstacles in field investigations. However, the cost-prohibitive nature of multiple array deployments may be the largest inhibitor for direct field measurement.

More positively, recent advances in video remote sensing have allowed for excellent spatial resolution of breaking position and breaking wave height at significantly lower costs and without risks to field researchers from hazardous conditions [de Vries et al., 2010; Gal et al., 2011; Almar et al., 2012; Shand et al., 2012]. Previous remote sensing studies have focused purely on measuring breaking wave heights. This study breaks new ground by integrating remotely sensed wave height and position locations, site specific seafloor bathymetric surfaces and directly measured shallow water incoming wave conditions, resulting in the highest possible resolution of breaking wave parameters.

In an effort to further quantify the physical differences within the plunging breaker category, Longuet-Higgins [1982] visually analyzed the laboratory breaking wave profiles from Miller[1957] and proposed fitting a cubic function to the enclosed vortex. Investigations into the breaking intensity of plunging breakers using of the geometric shape of the wave vortex have been ongoing for almost 50 years with limited success. However, this is still commonly cited as a possible avenue of research. This thesis thoroughly investigates the validity of using the vortex ratio and vortex angle as methods of predicting breaking intensity.

1.2 Experimental Objectives and Approach

The chief aims and objectives of the presented study are:

1. Review all literature pertaining to shallow water breaking wave characteristics and assess the current state of knowledge in this field. Through this process scientific knowledge gaps, the effect of assumed environmental conditions and the use of incongruent datasets will be explored in order to provide justification for a detailed irregular wave field collected dataset.

2. Develop a novel robust low-cost method to accurately extract wave breaking heights, depths, periods and vortex parameters at the instant of breaking. Traditional methods of extracting data from field breaking conditions are cost prohibitive and suffer from very coarse spatial resolution of breaking conditions. The development of a new cost effective technique will allow researchers to begin to better assess breaking conditions in many locations previously unsuitable for research and with higher spatial resolution.
3. Analyze and develop correlation procedures between incoming and breaking wave characteristics. Many previous studies have focused on predicting breaking conditions based on incoming wave conditions. However, this method is extremely sensitive to the local bathymetry and prone to large errors when applied to non-ideal locations. The ability to correlate breaking and incoming wave characteristics is essential for parameterising breaking wave characteristics.

4. Quantify and develop a consistent method to extract the seafloor slope relevant to the breaking process. The portion of the bathymetric slope which plays a role in the final breaking wave characteristics current is defined over a geographic distance, yet it is suggested that the effective slope would be better characterized using a wavelength dependent depth range. This suggestion will be thoroughly investigated throughout the study.

5. Evaluate the performance of published regular and irregular wave predictors and identify best performing methods. To the best of the author’s knowledge, the collected study data will be the most detailed set of field collected irregular breaking wave conditions ever assembled. This data will be used to evaluate the performance of all previously published breaking wave height predictors.

6. Develop an optimized breaking wave height predictor based on newly defined breaking depths, effective slopes, relevant periods and the performance of published predictors in Experimental Objective #5. Through a consistent and dependable method of extracting individual wave characteristics, the prediction terms involved with detailing effective seafloor slope and period parameters will be optimized for greater correlation with the measured dataset. A final optimized breaking wave height predictor will be developed for use when investigating irregular breaking waves in field conditions.

7. Identify dependencies between wave vortex profile parameters and breaking wave characteristics. The geometric wave profile parameters of vortex ratio and vortex angle have been cited as possible methods of characterizing breaking events. Due to noted incomplete datasets or interfering variables, no clear dependencies between these parameters and standard wave characteristics have been published using both laboratory and field data. This study will thoroughly investigate any possible trends between vortex parameters and breaking wave characteristics.

8. Investigate the feasibility of using vortex parameters as breaking intensity measures. The dominantly cited Surf Similarity Parameter has been shown to be unable to correctly classify breaking wave conditions. This has resulted in postulations that the vortex parameters may provide a better classification system yet no detailed investigations have been completed using a full suite of measured wave characteristics.
1.3 Novelty of Study

Characterisation and quantitative collection of all dominant parameters in the wave field has traditionally been very difficult, or not impossible, to achieve. However, recent technological advances have opened new avenues for collection. The methods provided in the thesis describe a wholly new and novel method to collect a full set of wave characteristics from the field. Additionally, the instant of breaking, the temporal location of interest in this study, is often cited as the most complex and difficult process to capture robust data from. Studying breaking waves in the real ocean, rather than in laboratories, is inherently more time consuming, dangerous and complex, however it is not subject to variables which inherently influence the ability to collect robust results in laboratory-based experiments.

Additionally, by investigating irregular breaking waves on an individual wave basis, rather than statistical spectral means, the effect of seafloor slope, period and height on individual waves can be quantified, explained and lead to a better understand of breaking events.

Finally, the postulation of the geometric breaker and vortex shape being reasonable measures of breaking “intensity” [Longuet - Higgins, 1982; Mead and Black, 2001] have long been discussed yet have never been thoroughly investigated through the quantitative measurement and analysis of wave profiles captured in field conditions. To the best of the author’s knowledge, this is the first study to collect all the relevant breaking wave characteristics of wave height, wave period, breaking depth, effective seafloor slope, vortex angle and vortex ratio in order to be able to provide a scientifically robust quantitative investigation of geometric-based postulations.

1.4 Thesis Outline

This initial chapter provides an introduction to the importance of understanding and being able to predict breaking waves and provides reasoning for this study. A quick review of the objectives and approach are presented.

Chapter 2 provides a broad introduction to near-shore wave processes, breaking waves and differing wave theories. This chapter provides the reader with enough background information to understand all terms, analysis decisions and assumptions throughout the rest of the document.

Chapter 3 focuses on giving a thorough and detailed review of all significant published quantitative research into shallow water wave breaking. Studies based on regular waves over plane slopes, regular waves over barred slopes and irregular waves are presented separately due to the fact that they are subject to different forcing mechanisms. Chapter 4 gives a brief insight to prior field studies devoted to the extraction of breaking wave information in the field.
Chapter 5 overviews the experimental procedure and methods used to collect the necessary data. This includes providing detailed reasoning for the specific choices of study locations and equipment deployments. Chapter 6 gives a full and detailed explanation of novel analysis and data processing methods developed as part of this study. Included is a full discussion of procedure uncertainties and inherent limitations of each extraction method.

Chapter 7 compares the extracted data against all previously published breaking wave height predictors and discusses their individual performance. Based on the performance of each prediction form and an optimized effective slope prediction, a new breaking wave height predictor is presented.

Chapter 8 investigates the feasibility of predicting breaking wave vortex parameters and their use as a breaking intensity measure. Possible alternate geometric breaking wave intensity measurements are suggested. Chapter 9 presents a summary of conclusions and recommendations from both the breaking wave height predictions and the vortex parameter investigations.

1.5 List of Literature Pertaining to Current Thesis

Detailed below is a list of accepted journal articles and peer reviewed conference papers resulting from the research conducted during the author’s PhD tenure. These papers constitute the bulk of the findings presented in this thesis. However, additional details are provided in the current traditional format thesis in order to better explain the rationale, logic flow, procedures and findings of this study.

Journal Articles:


Conference Papers:

- Robertson, B. & Hall, K. "Wave Vortex Parameters as an Indicator of Breaking Intensity". *International Conference on Coastal and Ocean Engineering*, Zurich, Switzerland, 2013.
2 Wave Theory Background and Review

A basic understanding of wave dynamics is necessary to fully understand the decisions and rationale used throughout the data extraction and analysis section of this report. This introduction of waves, their propagation and shallow water characteristics, is not designed to be exhaustive and give a complete understanding of basic wave theory, but rather to introduce many of the topics discussed and referenced later in this report. For a full description and breakdown of waves, see Holthuijsen [2008], Komar [1998] or the Coastal Engineering Manual[CERC-EW, 2008] amongst numerous others.

2.1 Wave Generation and Propagation

Elementally, oceanic waves are created and grow via surface wind forcing. Localized winds blow across the smooth ocean surface creating small ripples. Once small ripples are formed, the uneven sea surface causes pressure differentials, in phase with the waves, resulting in wave growth [Miles, 1957]. Winds blow heavily against the front face of the waves, creating high pressures zone, while eddies are created on the leeward side of waves, creating areas of lower pressure. As a result, wave height and period are dependent on the wind speed, the length of time the wind blows and the area over which the wind blows (fetch). Once waves have reached the maximum height and period based on these three major factors, and propagate beyond the extents of the wind forcing, they become referred to as swells. Ocean swells are numerically easier to explain than wind waves and are traditionally described via several different wave theories.

2.1.1 Airy Wave Theory

Airy linear wave theory is the simplest and most easily understood wave theory. Based on the Navier-Stokes equations of motion, Airy theory is reliable at describing the motion, orbital paths and energy transformations of fully developed deep water waves of uniform heights. Additionally it can be used to describe the linear summation of many sinusoidal waves that may cause a complex sea. Figure 2-1 illustrates basic wave characteristics and terms used to describe waves.

Airy linear wave theory makes several assumptions: 1) the wave height ($H$) is significantly smaller than the wavelength ($L$) and water depth ($h$), 2) that the water is incompressible, 3) the wave are regular, 4) that the water is subject to only one external force, gravity, 5) a horizontal seafloor and 6) that water particles may not leave the water surface or penetrate the seabed. Particle motion is assumed to be irrotational resulting in no net energy transfer.
Airy wave theory is also known as the base harmonic and the wave profile is calculated using:

\[ \eta(x, t) = \frac{H}{2} \cos(\omega t - kx) \]  

(1)

where \( k \) is the wave number \((2\pi/L)\) and \( \omega \) is the radian frequency. The dispersion relationship relates these two parameters and explains basic wave spreading:

\[ \omega^2 = gk \tanh(kh) \]  

(2)

or

\[ L = \left( \frac{g}{2\pi} \right) T^2 \tanh \left( \frac{2\pi h}{L} \right) \]  

(3)

Knowing the complex nature of \( \tanh() \) and \( L \) being featured on both sides of Eq. (3), the calculation of wavelength from this equation is not explicit. However when the relative water depth \((h/L)\) is greater than 0.5, \( \tanh \left( \frac{2\pi h}{L} \right) \) can be approximated by 1. In this deep water situation, the wavelength and wave celerity are thus approximated by:

\[ L_0 = \left( \frac{g}{2\pi} \right) T^2 \]  

(4)

\[ C_0 = \frac{g}{2\pi} T \]  

(5)

As a result, the offshore deep water wavelength and celerity are solely related to the wave period. These waves are generally undiminished by viscosity forces and the main restoring force for the surface
perturbations is gravity [Dean and Dalrymple, 1991], hence these waves are often referred to as gravity waves.

Once waves enter shallow water \((h/L < 1/20)\), wavelength and celerity can be calculated using:

\[
L_s = \frac{T}{\sqrt{gh}} \quad \text{(6)}
\]

\[
C_s = \sqrt{gh} \quad \text{(7)}
\]

In contrast to deep water waves, the celerity of shallow water waves is independent of period and is dependent on the water depth.

In intermediate water depths, both the water depth and wave period play roles according to the dispersion relation. Numerous authors have provided excellent approximations to the dispersion relation in this region [Eckart, 1952; Neilson, 1982; Fenton, 1990].

Unfortunately, Airy’s wave theory has limited applicability. For instance, linear theory has no ability to predict when the upper limit of the wave crest stability has been reached and the wave will break. For this and other more realistic physical conditions, non-linear wave theories are required.

2.1.2 Non-Linear Wave Theories

When waves do not become fully developed due to space, time or wind limitations, or enter shallow water, linear wave theory becomes increasingly inaccurate. Waves cease to conform to the linearized kinematic and dynamic free surface boundary conditions. Energy is transferred from high frequency waves to lower frequencies via non-linear wave-wave interactions [Holthuijsen, 2008], allowing growth of the low frequency wave spectrum as waves propagate further from the area of wind effects.

The degree of the inaccuracy (non-linearity) of waves can be quantified by the Ursell Number \((N_{\text{ursell}})\). \(N_{\text{ursell}}\) is a ratio of the wave steepness to the relative depth and is used to predict which of the subsequent non-linear wave theories provides the best representation of the wave field (see Eq. (8)).

\[
N_{\text{ursell}} = \left(\frac{H}{L}\right) \left(\frac{d}{L}\right)^3 = \frac{HL^2}{d^3} \quad \text{(8)}
\]

when \(N_{\text{ursell}} < 10\) Stokes theory is applicable, while when \(N_{\text{ursell}} < 26\) Cnoidal theory is the most appropriate. In the intermediate values, both theories are valid. Figure 2-2 illustrates the areas of applicability for each theory, depending on the non-dimensional wave height and depth.
Figure 2-2: Water wave theory applicability areas as defined by the Ursell number [Kraaijenest, 2012]. Note $\tau$ corresponds to $T$ in this thesis.

### 2.1.2.1 Stokes Higher Order Wave Theory

Sir George Stokes (1819-1903) was the first to provide methods to analyze wave motions and transformations without making assumptions about the wave height and depth. As shown in Figure 2-3, Stokes waves are not sinusoidal and feature narrow wave crests and broad flat wave troughs. Through the addition of extra harmonics to the base harmonic, Stokes was able to create a wave form which was a closer resemblance to reality.

Figure 2-3: Higher order Stokes Wave and first order Airy Wave profiles [Ditlevsen, 2002]
Higher order Stokes wave theories are the widely used and the second order wave profiles is calculated according to Eq. (9). The squared form of the correcting factor shows this equation to be a second-order correction:

\[ \eta = a \eta_1(x, t) + a^2 \eta_2(x, t) \]  

(9)

where \( a = \left( \frac{H}{2} \right) \) and \( \eta_1(x, t) = \cos(\omega t - kx) \)

Substitution shows the general solution in full:

\[ \eta = \left( \frac{H}{2} \right) \cos(\omega t - kx) + \frac{\pi H^2}{2L} \frac{\{ \cosh(kd) [2 + \cosh(2kd)] \}}{[\sinh(kd)]^3} \cosh[2(\omega t - kx)] \]  

(11)

While the deep water approximation is:

\[ \eta_0 = \left( \frac{H_0}{2} \right) \cos 2\pi \left( \frac{t}{T} - \frac{x}{L_0} \right) + \frac{\pi H_0^2}{4L_0} \cosh 4\pi \left( \frac{t}{T} - \frac{x}{L_0} \right) \]  

(12)

The second harmonic constructively raises the peaks of the wave profile and destructively flattens the troughs. These additive terms at the peak and trough of the wave form result in the still water line (SWL) maintaining a lower position in the wave form than the median predicted by Airy. This “raising” of the wave form is consistent with real world observations.

The phase speed of the extra/second harmonic is the same as the primary Airy wave phase speed and is known as a “bound” second harmonic. Higher order approximations result in increasing accuracy yet the mathematical complexity develops quickly.

While no difference is noted in the individual wave celerity at the second-order level, the third order solution shows the general and deep water wave celerities to be different:

\[ C = \frac{gT}{2\pi} \tanh \left( \frac{2\pi d}{L} \right) \left[ 1 + \frac{\pi H}{L} \right]^2 \frac{\left\{ 5 + 2 \cosh \left( \frac{4\pi d}{L} \right) + 2 \cosh^2 \left( \frac{4\pi d}{L} \right) \right\}}{8 \sinh^4 \left( \frac{2\pi d}{L} \right)} \]  

(13)

\[ C_0 = \frac{gT}{2\pi} \left[ 1 + \left( \frac{\pi H_0}{2L_0} \right)^2 \right] \]  

(14)

Stokes determined that if the particle motion at the crest of the wave was equal to the wave celerity, then breaking would occur. He deduced that this corresponded to a crest angle of approximately 120 ° and could be calculated by the following relationships in deep and shallow water respectively:
\[
\left(\frac{H_0}{L_0}\right)_{\text{max}} = 0.142 \approx \frac{1}{7} \tag{15}
\]

\[
\left(\frac{H}{L}\right)_{\text{max}} = 0.142 \tanh (kd) \tag{16}
\]

One fundamental difference between Stokes and Airy wave theories is that particle motion in Stokes theory is not a true orbital and the orbits do not ‘close’, resulting in a constant current or mass transport of water in the direction of the wave advance. This wave induced current is known as Stokes Drift and can be calculated using:

\[
U = \frac{1}{2} \left(\frac{\pi H}{L}\right)^2 C \frac{\cosh[2k(z + d)]}{[\sinh(kd)]^2} \tag{17}
\]

Stokes theory is able to adequately predict deep water wave profiles yet as waves enter shallower water, Cnoidal and Solitary wave theory begin to model events better than higher order Stokes theory.

### 2.1.2.2 Cnoidal Theory

First developed by Korteweg and de Vries [1895], Cnoidal wave theory spans the theoretical gap between sinusoidal linear waves, Stokes and Solitary wave theories. Cnoidal wave theory is widely regarded as one of the best descriptors for wave breaking but it is difficult to apply and therefore is generally used in conjunction with a tabulated series of generalized results.

The general wave profile is described by:

\[
\eta(x, t) = H \cn^2[2K(\kappa)\left(\frac{x}{L} - \frac{t}{T}\right), \kappa] \tag{18}
\]

where \(K(\kappa)\) is the complete elliptic integral of the first kind of modulus \(\kappa\) and \(\cn(r)\) is the Jacobian elliptic function. Jacobian Elliptic functions are well defined mathematical functions, much like sine and cosine, and are easily found in standard tables.

The celerity of a Cnoidal wave can be estimated using:

\[
C = \sqrt{\frac{gh_t}{1 + \frac{g}{h_t} \left(\frac{1}{\kappa^2} - 2\right)}} \tag{19}
\]

where \(h_t\) is the water depth at the wave trough, not the SWL. Cnoidal wave theory has applications when \(0 < \kappa < 1\), since when \(\kappa = 1\) the wavelength and period defined approach infinity and therefore act
according to solitary wave theory, or become a soliton. On the other extreme when \( \kappa = 0, \) \( cn(r, \kappa) = \cos(r) \) and the profile is the same as that of an Airy wave.

Cnoidal theory is generally computed to the third degree in order to correct for the effects of a finite depth. In much the same way as Stokes, Cnoidal approximations are multiplied by a correction factor, \( \beta \), at each degree increase (\( \beta \) is the ratio of amplitude to depth, \( H/h \)).

\[
\eta = \beta \eta_1 + \beta^2 \eta_2 + \beta^3 \eta_3 \ldots
\] (20)

Figure 2-4 below illustrates the excellent correlation of Cnoidal waves to experimental data:

![Figure 2-4: Correlation between higher order wave theories and laboratory extracted datasets [Komar, 1998]](image)

### 2.1.2.3 Solitary Wave Theory

As waves enter increasingly shallow water, their crests steepen and the troughs lengthen to the point where it may be assumed that each wave is discrete and independent of the rest in the wave train. This assumption greatly reduces the complexity of the relationships as the associated wavelength and wave period of the wave train are no longer applicable.

In Figure 2-5, the profile of a solitary wave is shown. Note that the entire wave sits above the SWL line, i.e. \( \eta(x, t) \) is never below zero. The profile follows the equation proposed by Boussinesq (1872):

\[
\eta(x, t) = Hsech^2 \left( \frac{3Hx}{4h^2} \right)
\] (21)
While the celerity of the wave can be calculated using:

\[ C = \sqrt{gh} \left( 1 + \frac{H}{2h} + \frac{3}{20} \left( \frac{H}{h} \right)^2 + \cdots \right) \]  

(22)

Solitary wave theory continues until the wave steepness reaches the 120° predicted by Airy, when the crest velocity of the wave reaches the phase celerity. However, due to the lack of a relevant wavelength and period, solitary wave theory is unable to account for wavelength dependent variations in refraction, diffraction and breaking.

![Solitary wave profile and defining parameters](image)

**Figure 2-5: Solitary wave profile and defining parameters**

### 2.1.2.4 Boussinesq Model

The three non-linear wave theories presented all deal with developed, non-evolving wave forms, i.e. their characteristics do not significantly change with time or horizontal movement (if no depth changes occur). Evolving non-linear waves replicate the motions of real water waves more accurately yet are extremely computationally intense. As a result, they are often only calculated over short distances e.g. a dozen wavelengths.

The Boussinesq model plays an integral role in modeling the motion of water waves in the transition depths between deep and shallow water wave equations. As waves enter shallow water, the particle motions in the surrounding fluid develop from their deep water circular patterns to completely horizontal motions, with no vertical component (see Figure 2-6). Once the vertical motions can be assumed to be negligible, the waves can be modeled by shallow water equations while, in the deep water instance, the relationships of Stokes and Airy give excellent correlations with the real world situation.
Once a wave has entered transitional water depths, a resident vertical velocity to the particle motion is still evident. The Boussinesq model gives the best approximation of the true wave profile and energy transformation changes by enforcing the vertical particle velocity to vary linearly with depth. Standard Boussinesq equations assume a constant horizontal particle velocity in the vertical plane and a horizontal seafloor.

Peregrine [1983] and Dingemans [1997] added to the standard Boussinesq equations to allow for non-horizontal seafloors resulting in:

\[
\begin{align*}
\frac{\partial \eta}{\partial t} + \frac{\partial}{\partial x} [(d + \eta) \bar{u}_x] &= 0 \\
\frac{\partial \bar{u}_x}{\partial t} + \bar{u}_x \frac{\partial \bar{u}_x}{\partial x} + g \frac{\partial \eta}{\partial x} &= \frac{1}{2} d \frac{\partial^3 (\partial \bar{u}_x)}{\partial t \partial x^2} - \frac{1}{6} d^2 \frac{\partial^3 \bar{u}_x}{\partial t \partial x^2}
\end{align*}
\]  

(23) 

(24)

where \( \bar{u}_x \) is the vertically averaged horizontal particle motion. The right hand side of the above equations deals with the vertical components of particle motion. These are expressed in terms of the associated horizontal motion due to the fact that Laplace equations, governing fluid motion, force the vertical motion to be expressed in terms of their horizontal velocities. As a result, the shallow water equations all have a zero on the right hand side to indicate no vertical motion.

### 2.2 Regular vs. Irregular Waves

The equations, predictions and relationships presented thus far have been developed using idealized regular waves. Regular waves feature consistent and time independent wave height and period intervals. In reality, the waves breaking on our beaches and coastlines are far from regular. Irregular waves are far more mathematically complex and are described by different parameters than regular waves. Figure 2-7 shows the surface track difference between a regular and irregular wave. The behaviour of regular and
irregular waves varies dramatically, particularly in the near shore zone, and interactions of individual waves play a significant role.

![Figure 2-7: Surface time series for irregular and regular waves](image)

Irregular wave conditions are generally quantified using $H_{\text{sig}}$ and $T_{\text{sig}}$ due to the lack of a consistent wave height or period. $H_{\text{sig}}$ and $T_{\text{sig}}$ represent the mean value of the top one third of recorded wave readings. Numerous other methods to describe irregular waves have been suggested, but these are beyond the scope of this thesis.

### 2.3 Shallow Water Wave Transformations

As described above, long period, fully developed waves travelling in deep water generally have a nearly sinusoidal shape to them. However once the wave begins to “feel” the ocean floor, at a depth of approximately half the wavelength, a systematic transformation takes place.

Figure 2-8 below shows the effect of shallower depths on the wave height, wave celerity and wavelength. As the water depth decreases, the wave height increases while the velocity and wavelength both decrease. It should be noted that the wave period remains constant through these transformations. In the intermediate zone, it is predicted that there is a small decrease in wave height and a small increase in wave velocity. This is due to the fact that the energy density remains constant and the slightly higher kinetic energy of the wave must result in a slight decrease in wave height.

The curves in the figure above are calculated using the following relationships:

\[
\frac{L}{L_\infty} = \frac{c}{c_\infty} = \sqrt{\tanh \left( \frac{2\pi h}{L_\infty} \right)}
\]

\[
\frac{H}{H_\infty} = \left( \frac{c_\infty}{2\pi c} \right)
\]
where $n$ is a non-dimensional number used to estimate the difference in celerity of individual waves as they progress from deep to shallow water. In deep water, $n = \frac{1}{2}$ while $n = 1$ in shallow water. For intermediate water depths:

$$n = 1/2 \left[ 1 + \frac{2kh}{\sinh(2kh)} \right]$$  \hspace{1cm} (27)

While the above prediction equations are easy to understand, they are based on idealized conditions and therefore do not always predict the true changes occurring in the dynamic surf zone. For instance, Goda [2010] showed that Eq. (26) underestimates the final breaking amplitude of waves in all but the ideal case. Numerous complex numerical methods exist to predict irregular waves breaking over non-uniform bathymetries yet are beyond the scope of this introductory chapter. Interested readers could refer to Goda [2009] for additional information.

![Figure 2-8: Idealized transformations from intermediate to shallow water environments [Komar, 1998]](image)

### 2.3.1 Wave Refraction

As waves propagate into intermediate and shallow water environments, the effects of seafloor bathymetry begin to alter the wave direction and amplitude. If the waves approach the beach slope at any angle other than directly perpendicularly, the non-constant depth along the wave crest causes the waves to bend towards a more seafloor contour parallel arrangement. This is easy explained by Eq. (25): as the water depth along the crest varies, the individual speed of the discrete sections of the wave crest will be moving at different speeds. The net effect is that waves begin to align themselves parallel to the depths contours of the area. This is illustrated in Figure 2-9.

17
Figure 2-9: Idealized wave refraction over a parallel seafloor sloping beach with regular incident waves

In areas of irregular bathymetry refraction can either cause significant wave height amplification from converging wave energy or a wave height reduction from wave spreading. Energy is generally always conserved in wave transformations and therefore the ratio of wave heights can be approximated using:

\[
\frac{H}{H_\infty} = \frac{1}{\sqrt{2n}} \frac{C_\infty}{C} \sqrt{\frac{d_{s,\infty}}{d_s}} \tag{28}
\]

where \(d_s\) indicates the distance between two wave rays or points in a swell. As shown on the right hand side of Figure 2-10, wave convergence results in the \(d_s\) distance being reduced, causing the extra energy to be converted into wave height amplification. The opposite is true of wave diverging refraction – see the left hand side of Figure 2-10 for a typical bay arrangement.

Figure 2-10: Idealized examples of wave divergence and convergence due to bathymetric slopes. Modified image from Coastal Engineering Manual [CERC-EW, 2008]
2.3.2 Wave Breaking

The most dramatic and most visually engaging aspect of the shallow water wave transformation process occurs when the wave propagates into shallow water and reaches a critical height, over-turns on itself and breaks. The breaking process is extremely non-linear and all but the simplest situations are currently beyond numerical explanation. As a result, most published predictions and theories surrounding wave breaking are generally based on empirical analysis.

Breaking wave heights and water depths vary considerably based on differing incoming wave conditions and the local bathymetry. Rather than predict the wave height or depth individually, most research has focused on the non-dimensional breaking index as the best descriptor for breaking conditions. The breaker index, $\gamma_b$, is the ratio of the breaker height ($H_b$) to breaker depth ($h_b$):

$$\gamma_b = \frac{H_b}{h_b}$$

While initially thought to maintain a constant value of 0.78 [McCowan, 1894], it has been shown that the breaker index is dependent on wave height, wave period and the effective seafloor slope. A detailed review of published breaker index formulations is presented in Chapter 3.

2.4 Surf Similarity Parameters

The Surf Similarity Parameter (SSP) was first proposed by Iribarren and Nogales [1949] in an effort to numerically differentiate between visually different breaking waves of similar breaker index values. Iribarren and Nogales proposed that all shallow water breaking events could be categorized into three broad categories depending on the output from Eq. (30) and the associated offshore wave steepness ($H_o/L_o$):

$$\xi_0 = \tan \alpha \frac{A_{0}}{L_0}$$

where $\alpha$ is the seafloor angle, $H_o$ is the offshore wave height and $L_o$ is the offshore wavelength. Table 2-1 details the SSP-based categories for both the breaking ($\xi_b$) and offshore SSP ($\xi_0$). Figure 2-11 illustrates the physical difference between spilling, plunging and surging/collapsing waves.

<table>
<thead>
<tr>
<th>Spilling</th>
<th>$0.4 &gt; \xi_b$</th>
<th>$0.5 &gt; \xi_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plunging</td>
<td>$0.4 &lt; \xi_b &lt; 2.0$</td>
<td>$0.5 &lt; \xi_0 &lt; 3.3$</td>
</tr>
<tr>
<td>Surging/Collapsing</td>
<td>$\xi_b &gt; 2.0$</td>
<td>$\xi_0 &gt; 3.3$</td>
</tr>
</tbody>
</table>
However, the SSP relies on the assumption of idealized conditions and has as a result been proven to be inaccurate in many situations. For example, Gourlay [1994] found that waves with identical deep water wave steepness and seafloor slope values broke in a plunging manner when the reef bar crest depth to deep water wave height ratio \( h_c/H_o \) was less than 1.8 and in a spilling manner when \( h_c/H_o \) was greater than 1.8. Additionally, sensitivity studies on the relationship between \( \gamma_b \) and \( \xi_0 \) showed that spilling waves \( (\xi_0 < 0.5) \) are more sensitive to offshore wave steepness values and are almost independent of slope. In contrast, plunging waves \( (\xi_0 > 0.5) \) are sensitive to seafloor slope and are almost independent of offshore wave steepness values.

### 2.5 Wave Breaking Vortex Ratio

In an effort to further differentiate between waves in the plunging category, Longuet-Higgins [1982] proposed fitting a cubic function to the breaking vortex profile of a plunging wave (see Figure 2-12). He postulated that the ratio of the vortex length to vortex width may describe the ‘intensity’ of a plunging wave breaking event.

Using a parametric description of an incompressible, irrotational flow, Longuet-Higgins used the following pair of equation to describe the flow pattern:

\[
\frac{x}{t^4} = -3\mu^2 + \frac{1}{3} \tag{31}
\]

\[
\frac{y}{t^4} = -\mu^3 + 2\mu \tag{32}
\]
By aligning the vortex length \((l_c)\) along the x-axis, the respective axis intersections occur at 
\((-\frac{1}{3},0), (\frac{17}{3},0), (0, \pm \frac{17}{27})\) (See the blue axis lines in Figure 2-12). Consequently, the vortex aspect ratio of Longuet-Higgins’ cubic function can be calculated as:

\[
Vortex\ Ratio: Y = \frac{vortex\ length}{vortex\ width} = \frac{l_c}{w_c} = 2.75
\]  

(33)

Longuet-Higgins based his findings on numerical wave theory and suggested further investigation using flume and field based wave profile images. Subsequently field and flume based work by Sayce [1997], Couriel et al. [1998], Scarfe et al. [2009], Blenkinsopp and Chaplin[2008] reported vortex ratio values varying between 1.73 and 4.43 depending on wave and seafloor characteristics.

Longuet-Higgins also suggested that the vortex angle, \(\theta\), may be another possible method to numerically categorize breaking wave events. However, the time dependence in Eq. (31) and Eq. (32) indicates a progression of the wave profile and an increase in the vortex angle during the breaking event.

A detailed review of published works dealing with the vortex ratio and vortex angle is given in Chapter 3.
2.6 Breaking Wave Parameter Definition Variance

The accurate and precise prediction of breaking wave parameters requires both a consistent definition for all wave characteristics and a consistent method of measurement. Unfortunately, the extraction and definition of wave characteristics in published papers are far from consistent and are even contradictory in some cases. As a result, many published relationships feature limited applicability bounds due to differing definitions for wave breaking, the water depth at breaking \( (h_b) \) and effective seafloor slope \( (m) \) [Scarfe et al., 2003; Johnson, 2009; Mead, 2010]. Measurement techniques follow a similarly inconsistent trend [Tsai et al., 2005; Rattanapitikon and Shibayama, 2006; Camenen and Larson, 2007].

The definition of wave breaking, the instant when a wave has reached its maximum stability and begins to break, is the most basic requirement for studying breaking waves. Rattanapitikon et al. [2003], Kamphuis [1991] and Fenton [1972] all defined breaking as the instant when the wave reaches its maximum height, while Grilli [1997], Blenkinsopp and Chaplin [2008], Iverson [1952], Smith and Kraus [1991] and Seyama and Kimura [1988] defined breaking as when the front wave profile becomes vertical. Johnson [2009] and Stokes [1847] used the instant the crest particle celerity is equal to the wave celerity to define breaking. Kraus and Larson [1988] and Haller and Catalan [2005] wait until the appearance of white water indicates the wave has begun to break. Unfortunately, the spatial and temporal location of the differing definitions varies considerably and restricts the ability to compare the accuracy of predicted relationships.

The variation in determining the breaking wave water depth is compounded by lack of consistency in its own measurement and associated variation in the definition of breaking. Black and Rosenberg [1992] note that “the analysis of \( \gamma_b \) experiments indicates a need of a consistent definition of water depth”. For example, Blenkinsopp and Chaplin [2008] used the mean water level (MWL) depth, averaged over the entire time series, when the wave face becomes vertical. In contrast, Johnson [2009] used the MWL depth but only when the crest particle celerity matched the wave celerity. The still water level (SWL) depth at the location of maximum wave height was used by Kamphuis [1991] while Smith and Kraus [1991] also used the SWL depth but only when a vertical wave front appears. Studies using Solitary wave theory [McCowan, 1894; Weggel, 1972; CERC-EW, 2008] are forced to use the SWL since a Solitary wave lies completely above the water surface and features no definitive wave trough.

Further complicating the breaking wave water depth definition is the effect of wave induced water level set-down. Longuet-Higgins and Stewart [1963] showed that individual waves will create temporary set-downs in the SWL in accordance with Eq. (34). This effect is not corrected for when using a SLW and may result in additional variation in dataset comparison.

\[
\eta = -\frac{1}{8} \frac{H^2(2\pi/L)}{\sinh(4\pi h/L)}
\] (34)
As with wave height and water depth, the extraction of the effective seafloor slope also lacks a standardized method. The effective slopes in published breaking wave studies vary between macro-scale features such as coastline approach slopes, to meso-scale features such as instantaneous bar slopes. For example, the Coastal Engineering Manual [CERC-EW, 2008] recommends the slope should be “the average bottom slope from the break point to a point one wavelength offshore” while Mead and Black[2001] calculated effective slopes by averaging seafloor slopes over 6 m to 8 m of water depth surrounding the break point. Additional uncertainty is introduced by small scale laboratory flume tests. Laboratory flume based experiments often present effective slopes based on the physical constraints of the wave flume, rather than those required by wave characteristics.

Further confusing the matter, Smith and Kraus[1991] discovered that waves breaking on barred beaches are influenced by different variables than those on plane sloped, emergent beaches. Figure 2-13 shows an illustration of plane emergent, non-emergent and barred beach configurations.

![Figure 2-13: Comparison of plane emergent, plane non-emergent and barred beach profiles](image)

When looking at wave trains, Kamphuis [1991] noted that irregular waves are subject to different forces than regular waves. Within the irregular wave train, individualized shoaling and breaking effects, wave-induced return current flows and changes in individual wave celerity all create additional variability in the collected irregular wave data. Consequently, direct comparison against regular wave results is not recommended [Thompson and Vincent, 1985].
While there is inherent variability within the breaking process of waves [Goda, 2010], additional uncertainty is introduced via the lack of consistent breaking wave parameter definitions and depth-correction techniques. As Gourlay [1994] notes, “[the analysis of dimensionless wave parameters] is diminished by the fact that the datasets from various sources are all subject to consistent bias arising from the use of different measurement methods, different criteria for determining breakpoint location, different procedures for allowing for wave reflections, etc.”.

### 2.7 Laboratory Scaling Effects

The majority of physical studies investigating breaking waves have been completed in laboratory wave tanks and flumes. Laboratory studies allow researchers to have direct control over the incoming wave conditions, seafloor slopes and wave field type, as well as providing the best possible platform for directly collecting data on the breaking conditions. Complicating field study based variables such as local wind conditions, multiple swell regimes and bathymetric inconsistencies are eliminated allowing for idealized wave breaking condition studies to occur. Additionally, wave flume and tank tests are cost-effective and the complete control over breaking conditions removes the need to wait for the ideal study conditions, as required in field tests. However, there are inherent drawbacks and limitations of wave flumes and tank based studies.

Firstly, wave reflection and wave-wave interaction is a major course of uncertainty in all studies completed using wave makers designed and built prior to the 1970’s. Prior to this date, wave makers were unable to actively absorb reflected waves, resulting in concurrent reflected and created wave regimes within the wave flume.

Wall friction has been identified as another source of error when attempting to extract wave vortex parameters and additional geometric wave characteristics [Johnson, 2009; Stagonas et al., 2011]. Depending on the amplitude of created wave and the overall width of the flume, breaking conditions across the width of the wave crest can vary due to the effects of wall friction. Additionally, bubble entrainment in laboratory studies have been shown not to follow Froude scaling laws and can create variation in the breaking wave structure and shape [Deane and Stokes, 2002; Stagonas et al., 2011]. Additionally, bubbles created by waves breaking in fresh water laboratory studies are subject to difference forcing mechanisms than those in salt water wave tanks [Slauenwhite and Johnson, 2012].

Surface tension can also play a restoring role when investigating scaled breaking waves. Miller [1972] investigated the effect of surface tension on solitary, periodic and standing waves and noted that surface tension plays a significant role, particularly in small scale laboratory tests. He additionally noted that salinity alters the surface tension and heavily-scaled experiments using only fresh water could include additional uncertainty when attempting to scale ocean waves. Few guidelines or studies exist to quantify
the alteration of laboratory breaking wave conditions due to surface tension. It is generally acknowledged that surface tension effects should be corrected for yet methods to quantify and correct for this effect are still unresolved.

In order to minimize scaling effects in wave flume and tank tests, Battjes and Stive [1985] recommend a minimum of 100mm wave height, while Goda and Morinbu [1998] recommend a minimum of 100mm water depth at the point of breaking. A basic analysis of the results from numerous published studies indicate that some of the published relationships were empirically extracted from wave flume experiments involving incoming and breaking conditions in contradiction of these recommendations, and may inherently be altered by the effects mentioned above. However, the laboratory results from large scale laboratories, such as those at Deltares, Netherlands and in Canada’s National Research Council Ottawa facility, are able to easily overcome the suggested minimum height and depth values and do produce realistic scaled breaking waves. Such facilities are able to create realistic breaking conditions and the data resulting from such studies is invaluable for coastal and naval engineering research.
3 Review of Breaking Wave Characteristic Predictors

Numerous methods and relationships designed to quantify the breaking water depth, wave height and geometry of waves have been presented and published. This chapter thoroughly reviews the most cited and influential works involving the prediction of the breaker index, the SSP, the vortex shape and the plunge distance of shallow water breaking waves.

The majority of Chapter 3 was fully accepted for publishing in the peer reviewed Coastal Engineering Journal on April 4th, 2013 [Robertson et al., 2013].

3.1 Breaking Depth Index

Breaker index formulae for regular waves can be classified into six differing functional forms. Sections are divided depending on whether deep or shallow water conditions were used, the mathematical identities involved and the number of independent wave parameters included. All forms are listed below and explained in more detail in subsequent sections:

\[ \gamma_b = \text{constant} \]
\[ \gamma_b = f(\text{slope}) = f(m) \]
\[ \gamma_b = f(\text{Surf Similiarity Parameter}) = f(\xi_o) \]
\[ \gamma_b = f(\text{breaking wavelength and height}) \]
\[ \gamma_b = f(\text{offshore wavelength and height, } m) \]
\[ \gamma_b = f(\text{offshore wavelength and height, exp}(m)) \]

Initial attempts to predict the breaker index were based on idealized conditions and generalized assumptions. As a result, many early relationships featured very limited areas of applicability. As wave flume and field measurement technology improved, the ability to extract increasingly accurate wave characteristic measurements resulted in continually improving correlation between measured and predicted breaking wave conditions across more diverse situations. The ultimate goal to find a universal relationship valid for all beach slopes and incoming wave conditions, rather than individual relations for the separate breaking conditions, is still elusive and inspires continual work in this field.

3.1.1 Regular Waves Breaking on Plane Beaches

While Thompson and Vincent[1985] note that “reference to monochromatic wave tests for irregular wave applications should be done with caution or avoided”, it is important to determine correlation with
regular wave-based relationships because the majority of published and industry used breaker height predictions are based on regular wave tests. Appendix A provides a quick review table detailing the presented relationships, validity regions and method of extraction detailed in the following sections.

### 3.1.1.1 Linear Relationships

In 1891, McCowan [1894] analyzed the case of a theoretical solitary wave over a horizontal seafloor and suggested that the breaker depth index was constant and independent of all incoming wave and seafloor slope parameters, yielding Eq. (35):

$$ y_b = 0.78 $$

(35)

In 1968, Yamada et al. [1968] revisited McCowans' solitary wave theory calculations and updated the breaking depth index to:

$$ y_b = 0.8261 $$

(36)

Tanaka et al.[1987] found agreement with McCowans' findings for the maximum wave height to depth ratio for stability yet notes that maximum unstable solitary waves can reach higher ratios. However, solitary wave theory is inherently limited by an inability to account for wave period dependencies and the lack of corresponding water depth datum to other theories. For solitary waves, there is no wave period parameter and the wave lies completely above the free water surface, with the wave crest height at $\eta_c = 1.784 h_b$ and the trough elevation ($\eta_t$) at $\eta_t = h_b$.

Interestingly, the US Army Corps of Engineers’ Coastal Engineering Manual (CEM) [2008] notes that $y_b = 0.78$ found better correlation with regular periodic waves than solitary waves despite McCowan using solitary wave theory to derive Eq. (35).

### 3.1.1.2 Slope Based Relationships

While researching breaker type classification, Galvin [1968] performed extensive breaking wave testing on three laboratory models with different sloped beaches ($m = 1/20, 1/10, 1/5$) and noted that the breaker index varied with the slope, in contradiction to McCowan and Tanaka et al. findings. Galvin, using data from Iverson [1952] and McCowan [1894], found an inflection point at $m = 0.07$ and proposed two empirically based relationships:

For $m < 0.07$:

$$ y_b = \frac{1}{(1.40 - 6.85m)} $$

(37)

For $m > 0.07$:

$$ y_b = 1.09 $$

(38)
Subsequent laboratory tests show that Eq. (37) and Eq. (38) feature large relative root-mean-square errors (RMSE) and are poor predictors for the breaker depth index [Rattanapitikoon et al., 2003].

In 1969, Collins and Weir [1969] and Camfield and Street [1968] concurrently used linear wave theory and proposed a single breaker depth index formulation for all slope values according to Eq. (39) [Collins and Weir, 1969] and Eq. (40) [Camfield and Street, 1968]:

\[
\gamma_b = 0.72 + 5.6m \\
\gamma_b = 0.75 + 25m - 112m^2 + 3870m^3
\]  

(39)  

(40)

Using new data, Madsen [1976] updated Eq. (39) and Eq. (40) and found better correlation using:

\[
\gamma_b = 0.72(1 + 6.4m)
\]  

(41)

Despite the lack of dependence on incoming wave characteristics, Black and Roseburg [1992] showed Eq. (41) to produce good predictions for individual wave heights in both laboratory and field experiments.

In 1984, the US Army Shore Protection Manual [1984] published experimental data detailing breaking index investigations using varying seafloor angles. Le Roux [2007] analyzed the published datasets and fitted Eq. (42) to the data:

\[
\gamma_b = 0.835 + 0.0843\alpha - 0.0036\alpha^2
\]  

(42)

Figure 3-1 illustrates the increasing nature of the breaker index with slope for all presented relationships. Notably, Camfield and Streets’ Eq. (40) was developed to predict maximum \(\gamma_b\) values and was based on shallow slopes \((m < 0.045)\) therefore creating unrealistic breaker index values when extrapolated to higher slopes.

Figure 3-1: Response comparison of slope dependent prediction relationships
Subsequent research has shown breaker index values to vary dramatically on constant slopes when incoming wave characteristics are varied. Hence relationships based solely on slope are limited in their applicability.

3.1.1.3 Surf Similarity Parameter Relationships

Published in 1949, the Surf Similarity Parameter (SSP, Eq. (30)) was proposed by Iribarren and Nogales [1949] as a non-dimensional characterization value to numerically categorize shallow water wave breaking events. The SSP requires seafloor slope and incoming wave characteristics, and assumes periodic, long crested waves travelling in an incompressible fluid towards a rigid, plane and impermeable slope that extends at a constant angle, \( \alpha \), from deep water to the water surface. Iribarren and Nogales [1949] suggested that the SSP may a valuable predictor for further research into breaking characteristics.

Working on this premise and using Galvin's[1968] data, Battjes [1974] proposed the following general relationship between the breaker depth index and the offshore SSP for the three discrete slopes investigated by Galvin:

\[
\gamma_b = 1.062 + 0.137 \log(\xi_0) \tag{43}
\]

Sunamura [1981] updated Eq. (43) to increase its validity over significantly larger slope range (0.02 < \( m \) < 0.3):

\[
\gamma_b = 1.1\xi_0^{1/6} \tag{44}
\]

Hoping to find greater correlation using wave characteristics at the break point, rather than the offshore conditions used in Eq. (30), Svendsen [1987] presented the breaking depth index in terms of breaking conditions and parameter \( S \), where:

\[
S = \frac{\tan \alpha}{(h_b/L_b)} \tag{45}
\]

Using laboratory data from Van Dorn, Battjes and Stive, and Okayasu et al. [Van Dorn, 1978; Battjes and Stive, 1985; Okayasu et al., 1986; Okayasu et al., 1988], Svendsen [1987] proposed:

\[
\gamma_b = 1.05S^{1/5} \tag{46}
\]

Larson and Kraus [1988] defined breaking as the instant white water appears on the wave crest and, using the results of Kajima et al.'s [1982] deep water flume results, proposed Eq. (47) based on the offshore SSP values. Larson and Kraus [1988] noted difficulties in precisely measuring conditions at breaking and used offshore wave characteristics.

\[
\gamma_b = 1.14\xi_0^{0.21} \tag{47}
\]
In 1993, Kaminsky and Kraus [1993] compiled 17 different datasets from various authors in order to investigate the breaker depth index. They found better correlation using Eq. (48) than breaker depth index relationships using the SSP. Additionally, they proposed that Eq. (48) was valid over a wider range of seafloor slopes and offshore wave steepness values \((0.009 < m < 0.2 \text{ and } 0.001 < H_o/L_o < 0.092)\) than previous relationships:

\[
\gamma_b = 1.20 \xi_0^{0.27}
\]  

Figure 3-2 illustrates all the SSP based breaker depth index formulations and shows the increase of the predicted breaker index as SSP values increase. Interestingly, as the seafloor slope and offshore wave condition validity ranges for each relationship has increased, so too has the predicted breaker depth index. The oft cited relationship of McCowan[1894] is included in Figure 3-2 as a reference line.

In their investigation of the breaker depth index, Seyama and Kimura [1988] note that “the clarifications with other factors, such as the surf similarity parameter, were not successful” and suggest the use of different parameters.

3.1.1.4 Relationships based on Breaking Steepness Values

Miche [1944] used theoretical orbital motions from linear wave theory and noted that the wave breaking height was dependent on the wavelength. Miche concluded wave breaking had begun when the particle velocity in the wave crest exceeded the wave celerity and proposed Eq. (49) to predict the wave height at breaking.

\[
H_b = 0.142L_b \tanh \left( \frac{2\pi h_b}{L_b} \right)
\]  

Figure 3-2: Response comparison of Surf Similarity Parameter relationships

In their investigation of the breaker depth index, Seyama and Kimura [1988] note that “the clarifications with other factors, such as the surf similarity parameter, were not successful” and suggest the use of different parameters.
The constant, 0.142, is based on the maximum theoretical deep water wave steepness limit [Michell, 1893] and overestimates breaking wave heights [Rattanapitikton and Shibayama, 2000]. Danel [1952] suggested adjusting the constant value to 0.12 when applying Miches’ formula to horizontal seafloors.

In 1979, Ostendorf and Madsen [1979] revisited Miches’ Eq. (49), based on newer works showing slope dependence, and suggested Eq. (50) and Eq. (51) to account for beach slope. They completed studies on both plane fixed bed and plane moveable bed beaches:

For \( m < 0.1 \)

\[
H_b = 0.14L_b \tanh\left( (0.8 + 5m)\frac{2\pi h_b}{L_b} \right)
\]

(50)

For \( m > 0.1 \)

\[
H_b = 0.14L_b \tanh\left( \frac{2.6\pi h_b}{L_b} \right)
\]

(51)

Despite being derived for plane sloped beaches, Eq. (50) and Eq. (51) have been shown to predict the breaking conditions for barred beach profiles better than many previous expressions [Camenen and Larson, 2007].

Figure 3-3 illustrates both the lack of slope dependence for Eq. (49) and the large discontinuity with Eq. (50) and Eq. (51).

![Figure 3-3: Response comparison of breaking steepness height predictors](image)

In 2011, Liu et al. [2011] proposed Eq. (52) for regular breaking waves. Liu et al. based Eq. (52) on 33 different plane and barred sloped datasets, yet found no dependence on seafloor slope on the final breaking conditions.
\[ 0.69 = \left( 1.21 - 3.30 \left( \frac{H_b}{L_b} \right) \right) \left( 1.48 - 0.54 \left( \frac{H_b}{R_b} \right) \right) \left( \frac{g H_b}{C_b} \right) \]  
(52)

where \( C_b = \sqrt{\frac{g z_b}{2\pi} \tanh \left( \frac{2\pi}{L_b} \left( h_b + \frac{H_b}{2} \right) \right)} \)  
(53)

However, Yao et al. [2012] noted that in practice, Eq. (52) is not convenient since \( C_b \) is not a quantity that can be given \textit{a priori} and is calculated iteratively. They also note satisfactory correlation with breaking waves of large steepness values, but disagreement when looking at low steepness breaking waves \( (H_b/L_b \leq 0.03) \).

### 3.1.1.5 Offshore Wave Steepness Relationships

The comparative ease of measuring offshore wave characteristics, as opposed to against those at breaking, resulted in the majority of research utilizing an offshore wave steepness \( (H_o/L_o) \) parameter as a determining factor for breaker height prediction.

Le Mehaute and Koh [1967] used energy flux equations empirically fitted to laboratory and field data in order to predict the breaking wave height. They determined that both the slope and offshore wave steepness played a role in the final breaker height.

\[ H_b = 0.76 H_o m^{1/7} \left( \frac{H_o}{L_o} \right)^{-0.25} \]  
(54)

In 1972, Komar and Gaughan [1972] revisited Eq. (54) and associated the energy flux equations to order define a simpler, more user friendly breaking height relationship. Calibrated against historical data from Iverson [1952], Galvin[1968], Komar and Gaughan [1972] and Munk[1949], a constant value of 0.56 was assigned by Komar and Gaughan to replace seafloor slope effects:

\[ H_b = 0.56 H_o \left( \frac{H_o}{L_o} \right)^{-1/5} \]  
(55)

In 1974, Sunamura and Horikawa [1974] investigated the correlation of both Eq. (54) and Eq. (55) against Goda’s 1970 laboratory dataset. Sunamura and Horikawa concluded that seafloor slope effects could not be represented by a constant and proposed that the breaking height could be more accurately predicted using:

\[ H_b = H_o m^{0.2} \left( \frac{H_o}{L_o} \right)^{-0.25} \]  
(56)
Unfortunately, Eq. (56) predicts that \( H_b = 0 \) when \( m = 0 \). This contradicts the findings of Riedel and Byrne [1986] who experimentally confirmed that waves do not diminish to zero on zero slopes. As a result, Eq. (56) has an inherent lower limit of applicability.

In 1978, Van Dorn [1978] found good correlation between experimental values and Eq. (57). In contrast to previous works, Van Dom found no slope dependence when \( H_o/L_o < 0.07 \):

\[
H_b = 0.4H_o \left( \frac{H_o}{L_o} \right)^{-1/3}
\]  

(57)

However, subsequent work by Tsai et al.[2005] showed that Eq. (57) over-estimated breaking heights on most slope gradients, as well as a dependence of \( H_b \) on slope (contradicting both Eq. (55) and Eq. (57)).

Singamsetti and Wind [1980] investigated a plane emergent beach and suggest Eq. (58) might predict breaker depth index values well. Unfortunately, Eq. (58) has been shown to overestimate \( \gamma_b \) [Camenen and Larson, 2007].

\[
\gamma_b = 0.937m^{0.155} \left( \frac{H_o}{L_o} \right)^{-0.13}
\]  

(58)

Ogawa and Shuto [1984] analyzed the run-up of breaking and non-breaking periodic waves on various beach configurations of bars, slopes and steps. Ogawa and Shuto presented relationships for \( H_b/H_o \) and \( h_b/H_o \) and suggested their validity range was \( 0.01 < m < 0.1 \) and \( 0.02 < H_o/L_o < 0.065 \). The associated breaker index is calculated using:

\[
\gamma_b = 1.48m^{0.21} \left( \frac{H_o}{L_o} \right)^{-0.05}
\]  

(59)

Negatively, subsequent laboratory tests have shown that Eq. (59) displays increasing root-mean-square error (RMSE) as slope increases [Rattanapikitikon and Shibayama, 2000].

In 1985, Battjes and Stive [1985] revisited previous works and concluded:

\[
H_b = 0.14L_b \tanh \left( 0.5 + 0.4 \tanh \left[ 33 \left( \frac{H_o}{L_o} \right)^{2 \pi \frac{h_b}{0.88L_b}} \right] \right)
\]  

(60)

Rattanapikitikon, Vivattanasirisak and Shibayama [2003] compared Eq. (60) against 574 different cases and reported additional uncertainty when using Eq. (60) rather than Owaga and Shuto’s predictor. Eq. (59) also displayed increasing RMSE with slope.
In 1992, Gourlay [1992] reviewed seven sets of laboratory data and found the wave height amplification factor from offshore to breaking could be approximated using Eq. (61), which relies $H_b$ rather than $H_o$:

$$\frac{H_b}{H_o} = 0.478H_o \left(\frac{H_b}{L_o}\right)^{-0.28} \quad (61)$$

However, Gourlay notes that the “breaker height was found to be insensitive to the beach slope, [while] this is not the case for the depth”. When used to predicted breaking wave heights, Rattanapitikon and Shibayama [2000] found Eq. (61) to have poor performance, particularly on horizontal slopes.

In 2003, Rattanapitikon, Vivattanasirak and Shibayama [2003] found correlation between $H_b/L_b$ and $H_b/L_o$ on plane sloped laboratory experiments and proposed that $H_b$ could be calculated using Eq. (62) for $0 < m < 0.38$ and $H_o/L_o < 0.112$:

$$H_b = (-1.40m^2 + 0.57m + 0.23)L_b \left(\frac{H_o}{L_o}\right)^{0.35} \quad (62)$$

Qualifying their findings and experiments, they note that “it is difficult to measure wave height and water depth at the same time. [Therefore] The water depth is only approximated”. Positively, Eq. (62) still predicts a breaker height when the slope reduces to zero. In 2006, Rattanapitikon and Shibayama [2006] updated Eq. (62) as a result of recently released experimental data:

$$H_b = (-0.57m^2 + 0.31m + 0.58)L_o \left(\frac{H_o}{L_o}\right)^{0.83} \quad (63)$$

Eq. (63) has been found to show good correlation with experimental datasets[Rattanapitikon and Shibayama, 2006; Goda, 2010].

In 2005, Tsai et al. [2005] recognized that many empirical relationships were based mainly on gentle slopes, and investigated the breaking conditions on steep sloped beaches ($m = 1/3, 1/5$). Tsai et al. proposed a steep slope modification of Le Mehaute and Koh’s 1967 equation for situations when $m > 1/5$:

$$H_b = 0.79H_o \left(\frac{H_o}{L_o}\right)^{-0.19} m^{1/7} \quad (64)$$

Unfortunately, their experiments were completed on non-emergent plane beaches, in contrast to the majority of previous experiments. Non-emergent beaches feature an extended and submerged flat slope ($m = 0$) after the wave breaking position (see Figure 2-13), which has been shown to introduce different
return flows and breaking conditions [Seyama and Kimura, 1988; Smith and Kraus, 1991]. As a result, Eq. (64) and the data from Tsai et al. should not be compared directly with emergent slope studies.

Camenen and Larson [2007] analyzed and compared the outputs of Eq. (30), Eq. (57), Eq. (49), Eq. (50), Eq. (81), Eq. (82) and Eq. (73). Discovering that none of the relationships were able to achieve more than 50% accuracy (within 10% of experimental data) and were increasingly inaccurate on steep slopes \(0.2 > m > 0.1\), Camenen and Larson proposed to combine recent trigonometric and offshore steepness relationships:

\[
\gamma_b = 0.284 \left(\frac{H_o}{L_o}\right)^{-0.5} \tanh \left[0.87 + \left(0.32 + 14 \frac{H_o}{L_o}\right) \sin \left(\frac{\pi}{2} \left(\frac{m}{m_{max}}\right)^{\beta}\right)\right] \left(\frac{H_o}{L_o}\right)^{0.5} \tag{65}\]

where: \(m_{max} = 0.10 + 1.6 \left(\frac{H_o}{L_o}\right)\)

\[
\beta = 1 + 14 \left(\frac{H_o}{L_o}\right) \quad \text{if} \ m \leq m_{max} \tag{66}
\]

\[
\beta = -\left[1 + 20 \left(\frac{H_o}{L_o}\right)\right] \quad \text{if} \ m > m_{max} \tag{67}
\]

Camenen and Larson [2007] note the discrepancy between the maximum theoretical output \(\gamma_b\) of 1.15, in contrast to experimental values reaching 1.6, yet provide no reasoning for this discrepancy.

Yao et al. [2012] completed shallow water breaking studies using a non-emergent plane beach slope, much like those from Tsai et al. In order to find the best correlation with the collected data, Yao et al. found that the breaker index on the fore-reef slope \((\gamma_1)\) and on the horizontal post reef platform \((\gamma_2)\) were both required. While good correlation may have been found, the requirement for \textit{a priori} knowledge of these two characteristics limits the ease of use of Eq. (68):

\[
\gamma_b = \frac{\gamma_1 - \gamma_2}{2} \left\{ \tanh \left(\frac{\alpha}{1.4} \left(1.4 - \frac{h_x}{H_o}\right)\right) + \frac{\gamma_1 + \gamma_2}{\gamma_1 - \gamma_2} \right\} \tag{68}
\]

Figure 3-4 compares Eq. (54), Eq. (55), Eq. (56), Eq. (57) and Eq. (63) and illustrates that as the slope increases the relationships of Sunamura and Horikawa [1974] and Rattanapitikon and Shibayama [2006] find greater correlation. The relationship of Van Dorn [1978] overpredicts breaking wave heights on shallow slopes. Generally, increasing slopes result in increasing predicted \(\gamma_b\) values.
Figure 3-4: Response comparison of offshore wave steepness breaking height predictors

Figure 3-5 compares the breaker indices predicted by Eq. (58), Eq. (59) and Eq. (65) showing that shallower slopes result in lower breaking indices. Also illustrated is the fact that for low steepness waves ($H_o/L_o < 0.02$), Eq. (65) predicts decreasing breaker indices on steep slopes, in contrast to the other relationships.

### 3.1.1.6 Exponential Relationships

The final form of published breaker height relationship utilizes an exponential relationship. In 1974, Goda [1974] found an exponential wave height dependence on the water depth and seafloor slope. Based on laboratory tests with $0.05 < m < 0.2$ and $0.001 < H_o/L_o < 0.051$, Eq. (69) was the most cited breaker height predictor until Goda revised Eq. (69) in 2010 (see Eq. (76) below):

$$H_b = 0.17L_o \left(1 - e^{-\frac{1.5\pi h_b}{L_o} \left(1 + 15m^3\right)}\right)$$

(69)

Tsai et al. [2005] discovered that Goda’s relationship overestimated wave breaking height on steep slopes, while Muttray and Oumeraci [ASCE, 2000] found that a constant value of 0.167, rather than 0.17, resulted in better agreement for slopes over 1/30.
In 1972, Weggel [1972] used previously published laboratory data by Galvin [1968] and enveloped the published data using solitary wave theory. Weggel’s goal was not to find an exact solution for $\gamma_b$ but rather to provide a high estimate of the breaker height to be used for coastal design. He proposed using:

$$\gamma_b = K \left( b(m) - a(m) \frac{H_0}{gT^2} \right)$$  \hfill (70)

where $a(m) = 43.8(1.0 - e^{-19m})$, $b(m) = 1.56(1.0 + e^{-19.5m})^{-1}$ and $K = 1$. Camenen and Larson [2007] confirmed that Eq. (70) generally overestimates exact $\gamma_b$ values. Dean et al. [1985] attempted to include the effects of oblique wave incidence directly into the breaker height relationship. Dean et al. used $\gamma_b$ from Eq. (71) and represented the offshore wave direction with $\theta_0$:

$$H_b = \left( \frac{\gamma_b}{g} \right)^{1/5} \left[ \frac{(0.80 * H_0^2 \cos \theta_0)}{2} \right]^{2/5}$$  \hfill (71)

Unfortunately, using a constant value of $C_o = 0.80$ in Eq. (71) under predicted the wave height at breaking by 12% and Dean et al. suggested the need for an additional slope correction factor.

Smith and Kraus [1991] revisited Eq. (70) with the addition of newly collected data and recalculated the constant coefficients. They suggested:

$$\gamma_b = b(m) - a(m) \left( b(m) \right)$$  \hfill (72)

where $a(m) = 5.00(1.0 - e^{-43m})$ and $b(m) = 1.12/(1.0 + e^{-60m})$. Camenen and Larson [2007] found Eq. (72) to underestimate the breaker depth index despite the updated coefficients.

Komar [1998] noted that many breaker depth index equations intrinsically include both breaking height and depth resulting in difficulty extracting either value. Komar suggested using Eq. (73) and (74) to calculate the individual parameters.

$$H_b = 0.39g^{0.2}(TH_0^2)^{0.4}$$  \hfill (73)

$$d_b = H_b \left\{ 1.2 \left[ \frac{m}{(H_b/L_0)^{0.5}} \right]^{0.27} \right\}$$  \hfill (74)

The lack of slope dependence for the wave breaking height, and slope dependence for the breaking depth relationship have been independently confirmed [Gourlay, 1992; Rattanapitikon and Shibayama, 2006].
Rattanapitikon and Shibayama [2000] revisited Eq. (69), using 24 separate sources of experimental data, and theorized the best fit to the measured data was achieved using:

$$H_b = 0.17L_o \left( 1 - e^{\left( \frac{\pi h_b}{L_o} \left( 16.21m^2 - 7.07m - 1.55 \right) \right) } \right) \quad (75)$$

It should be noted that Rattanapitikon and Shibayama compared datasets from waves breaking over bars, plain slopes, stepped slopes and submerged reef profiles on par despite recommendations that should be considered as separate cases [Smith and Kraus, 1991; Camenen and Larson, 2007].

In light of the findings of Rattanapitikon and Shibayama, Goda [2010] revisited his 1972 predictor (Eq. (69) above) and revised the effect of slope to give:

$$H_b = 0.17L_o \left( 1 - e^{\left( \frac{1.5\pi h_b}{L_o} \left( 1+11m^3 \right) \right) } \right) \quad (76)$$

Figure 3-6 graphically illustrates the divergence of the proposed formulas as slopes increase. Eq. (75) shows decreasing wave heights when seafloor slopes exceed 0.25, while Eq. (69) and (76) predict exponentially increasing wave heights. The Smith and Kraus [1991] relationship flattens off at a breaker index of 1.087 and is unable to predict larger breaker indices.

Figure 3-6: Response comparison of exponential breaking height predictors
3.1.2 Regular Waves Breaking on Barred Slopes

The vast majority of wave tank and flume results presented thus far were conducted over plane sloped beach profiles. However, Larson and Kraus [1988] showed that waves breaking over barred slopes behave differently to those breaking on plane slopes. Building on the work of Sunamura [1981], Larson and Kraus suggested that the breaking wave height in offshore bar bathymetric configurations is independent of wave steepness and bar slope. Larson and Kraus predicted that only the bar crest depth plays a role in the breaking wave height and suggested using Eq. (77).

\[ H_b = \frac{0.66}{h_c} \]  

(77)

Cooker et al. [1990] created a numerical model and completed tank tests in order to investigate the correlation between breaking wave characteristics and submerged cylinder sizing. The study is a basic mimic of a barred beach profile. Cooker et al. proposed that breaking depended on two parameters: the wave amplitude and the submerged bar radius. Given that the breaking water depth is directly correlated to the cylinder radius, this corroborates Eq. (77).

Smith and Kraus [1991] performed detailed investigations into barred and plane sloped beaches separately to determine if it was possible for a single relationship to cover both situations. Smith and Kraus found that waves breaking over offshore bars are not affected by the same mechanisms as waves breaking on plane emergent sloped beaches and should be considered as separate cases. Smith and Kraus [1991] defined breaking as the instant when the front wave profile becomes vertical, and proposed that:

\[ \gamma_b = 0.41 + 0.98\xi_0 \]  

(78)

For \( 0.3 \leq \xi_0 \leq 0.85 \)

\[ \gamma_b = 1.45 - 0.22\xi_0 \]  

(79)

For \( 1.6 \leq \xi_0 \leq 3.5 \)

They noted too much scatter in \( \xi_0 \) between 0.85 and 1.6 to confidently predict a response. Figure 3-7 plots Eq. (48) against Smith and Kraus’ findings. For barred slopes, it was suggested that the decreasing value of \( H_b/h_b \) at larger values of \( \xi_0 \) is associated with the return flow of water over the bar in the wave flume causing the wave to break prematurely. This situation would not generally be mimicked in real world conditions and application of Eq. (79) should be done with caution.

Blenkinsopp and Chaplin [2008] experimented with waves breaking over a barred profile, and their results showed no discernible effect of offshore wave steepness or reef bar crest depth on the breaker depth index. Hence Blenkinsopp and Chaplin predicted:

\[ \gamma_b = 0.85 \]  

(80)
The entire presented barred slope relationships are outlined in Table A-2 in Appendix A. Figure 3-7 plots the response of the presented barred slope relationships with respect to increasing SSP values.

Figure 3-7: Response comparison of plane and barred slope profile breaker indices

3.1.3 Irregular Waves Breaking On Linear, Emergent Slopes

The initial attempt to predict the breaker indices and breaking heights of individual waves within irregular wave trains was completed by Seyama and Kimura [1988]. Seyama and Kimura analyzed individual waves, using a zero-down analysis, from laboratory irregular wave spectrums and proposed Eq. (81) to predict the breaker depth index:

\[
\gamma_b = \left(0.16 \frac{L_o}{h_b} \left(1 - \exp \left(-0.8\pi \frac{h_b}{L_o} \left[1 + 15m^4\right]\right)\right) - 0.96m + 0.2\right)
\]  

(81)

Seyama and Kimura discovered breaker depth index values approximately 30% lower for irregular waves than regular waves (confirmed by Goda [2010] – see Figure 3-8).

Figure 3-8: Comparison of predicted irregular breaking wave heights.
In order to increase correlation between Eq. (81) and measured data points, Seyama and Kimura defined the breaking depth as $h_b = h + \frac{(a_c - a_t)}{2}$. Figure 3-9 provides a graphical illustration of this new breaking depth definition. While altering the breaking depth definition dramatically reduced the variation in their plotted data, the new introduced definition is difficult to implement outside of a wave flume.

![Figure 3-9: Illustration of the alternate breaking depth definition used by Seyama and Kimura [1988]](image)

In 1990, Kamphuis [1991] investigated irregular waves on a sandy barred slope. Kamphuis was interested in altering regular wave prediction relationships to account for irregular waves. Defining breaking as the instant the crest particle velocity is equal to the wave celerity, Kamphuis completed a series of wave tank and flume experiments and updated constants in equations Eq. (69), Eq. (70), Eq. (50), Eq. (51) and Eq. (58). These are outlined in Table A-3 in Appendix A. Kamphuis also presented a new breaking wave height relationship, based on small amplitude theory:

$$H_{sb} = \left[0.095 \exp(4.0m)\right] L_b \tanh\left(\frac{2\pi h_b}{L_{pb}}\right)$$

Eq. (82) approaches $\gamma_b = 0.56$ as the beach slope approaches zero as per the findings of Battjes and Stive[1985], and Riedel and Byrne[1986]. Figure 3-10 compares the outputs of Eq. (50) and Eq. (51) for regular waves, against those from Eq. (82) for irregular waves. Eq. (50) and (51) were chosen as comparative relationships since Kamphuis calibrated Eq. (82) using values collected at the break point. Figure 3-10 illustrates the variation between the predicted wave heights using regular and irregular wave predictors. At low slope values, breaker heights for irregular waves are smaller than those for regular waves. However, the opposite is predicted for steeper slope values.
3.2 Geometric Descriptors for Breaking Waves

Waves do not all break in the same way; some plunge spectacularly, sending white spray high into the air, while others crumble slowly and quietly. In order to attempt to numerically categorize this visual difference, Iribarren and Nogales [1949] suggested the use of the SSP. The SSP is still the most cited and consistent method for categorizing breaking wave events despite numerous authors proving its limitations. This section reviews updated and alternate methods to categorize and predict the physical shape and ‘intensity’ of the breaking event.

3.2.1 Breaking Wave Categorization via Surf Similarity Parameter Variations

Iribarren and Nogales [1949], and subsequently Weggel [1972] and Gourlay [1992], suggested that all breaking wave events could be divided into three broad categories depending on the output of Eq. (30), or its breaking characteristics equivalent:

\[ \xi_b = \frac{\tan \alpha}{\sqrt{H_b/L_\alpha}} \]  

The offshore SSP (Eq. (30)) is designed to be used for waves with perpendicular approach angles to and emergent beach with consistent parallel seafloor contours. In many situations, Eq. (30) is not suitable and the breaking SSP should be used. Mead and Black [2001] and Smith and Kraus [1991] independently confirmed that the critical value for wave breaking depends both on the full beach profile (\(\alpha\)) and wave conditions (\(H_b, L_\alpha\)) but both versions of the SSP are still limited in their ability to correctly categorize breaking events. As an example, the maximum theoretical seafloor slope for plunging waves, using Eq. (30) and the assumption of a constant slope, is 12° or 1:4.7 [Camfield and Street, 1968]. However, in the
case of a barred beach profile or a shoreward bathymetric platform post-breaking, the seafloor slope can be larger than 12° and still create plunging waves [Mead, 2010]. In the case of barred beach profiles, Smith and Kraus [1991] found that the critical values for the SSP were different from those presented by Iribarren and Nogales for plane sloped beaches. Smith and Kraus proposed using the values in Table 3-1 for barred conditions. It should be noted that the offshore wave condition values used were based on reverse extrapolations from breaking wave data recorded in the wave tank [Smith and Kraus, 1991; Nelson, 1997; Haller and Catalan, 2005] and inherent theory based uncertainty is unavoidable.

Table 3-1: Smith and Kraus SSP Values for Barred Beaches

<table>
<thead>
<tr>
<th>Condition</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spilling</td>
<td>$0.4 &gt; \xi_o$</td>
</tr>
<tr>
<td>Plunging</td>
<td>$0.4 &lt; \xi_o &lt; 1.2$</td>
</tr>
<tr>
<td>Surging/Collapsing</td>
<td>$\xi_o &gt; 1.2$</td>
</tr>
</tbody>
</table>

Citing the work of Richken and Papanicoloau [1988], Grilli, Svendsen and Subramanya [1997] theorized that all waves initially break with a plunging vortex and only the scale of the breaking vortex changes between three classical breaker types. Vinje and Bevög’s [1980] numerical work corroborated this findings and confirmed that plunging waves can occur independent of seafloor slope values. Duncan’s [1994] experimental evidence shows that a turbulent roller can form without plunging but only in waves with sufficiently small size to allow surface tension forces to be dominant.

Grilli, Svendsen and Subramanya [1997] noted that as wavelength and wave period of solitary waves are theoretically infinite, therefore Eq. (30) and Eq. (83) cannot be used for solitary breaking wave models. They suggested using Eq. (84) and Table 3-2 to categorize breaking solitary waves on beaches. Khayyer et al.[2008] corroborated these solitary wave breaker classification methods, using a numerical Corrected Incompressible Smoothed Particle Hydrodynamics (CISPH) model.

$$S_o = \frac{L_o \tan \alpha}{h_o}$$

Table 3-2: Solitary Wave Breaker Type Separation Values

<table>
<thead>
<tr>
<th>Condition</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spilling</td>
<td>$0.025 &gt; S_o$</td>
</tr>
<tr>
<td>Plunging</td>
<td>$0.025 &lt; S_o &lt; 0.3$</td>
</tr>
<tr>
<td>Surging/Collapsing</td>
<td>$0.37 &gt; S_o &gt; 0.3$</td>
</tr>
</tbody>
</table>

Camenen and Larson [2007] plotted the breaker classification conditions from 23 previous studies and graphically analyzed the lines of separation between each breaker type (see Figure 3-11). Camenen and Larson postulated that each inflection point in the graph marked a breaker type boundary and suggested the values presented in Table 3-3 for the breaking wave category boundaries.
Figure 3-11: Breaker type separation by fitted line inflection points [Camenen and Larson 2007]

Table 3-3: Breaker Type Separation Lines

<table>
<thead>
<tr>
<th>Breaker Type Separation Lines</th>
<th>Equation</th>
<th>Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spilling/Plunging Boundary</td>
<td>$\xi_b = 1.5 - 1.25\gamma_b$</td>
<td>$0.6 &gt; \xi_0 &gt; 0.2$</td>
</tr>
<tr>
<td>Plunging/Collapsing Boundary</td>
<td>$\xi_b = 4.4 - 2.5\gamma_b$</td>
<td>$2.9 &gt; \xi_0 &gt; 0.8$</td>
</tr>
<tr>
<td>Collapsing/Surging Boundary</td>
<td>$\xi_b = 1.7 + 3.3\gamma_b$</td>
<td>$6 &gt; \xi_0 &gt; 3.6$</td>
</tr>
<tr>
<td>Non-Breaking Boundary</td>
<td>Equation (34) with $H_o/L_o = 0.14$</td>
<td></td>
</tr>
<tr>
<td>Unstable Wave Boundary</td>
<td>Equation (34) factored by 1.25</td>
<td></td>
</tr>
</tbody>
</table>

The most recent classification system, proposed by Yao et al. [2012], suggests that spilling, plunging and non-breaking waves are best classified by using the breaker depth to offshore wave height ratio ($h_b/H_o$). Table 3-4 details the relevant categorization values. Unfortunately, the need for a priori knowledge of breaking depth may limit the predictive ability of Yao et al.’s categorization.

Table 3-4: Yao et al. Breaking Classification System

<table>
<thead>
<tr>
<th>Classification</th>
<th>Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plunging</td>
<td>$h_b/H_o &lt; 1.8$</td>
</tr>
<tr>
<td>Spilling</td>
<td>$1.8 &lt; h_b/H_o &lt; 2.8$</td>
</tr>
<tr>
<td>Non-breaking</td>
<td>$h_b/H_o &gt; 2.8$</td>
</tr>
</tbody>
</table>
3.2.2 Breaking Wave Categorization via Wave Vortex Parameters

Longuet-Higgins [1982] suggested the wave vortex may be an additional method to categorize breaking waves and could be used as a measure of the “intensity” of the breaking event. Mead and Black [2001] built on this suggestion and analyzed the previously published images of wave vortex ratios at 23 different plane and barred location worldwide. Black and Mead found that Eq. (85) gave an $R^2$ value of 0.71 and, using only the bathymetric slope, predicted the vortex ratio well (see Figure 3-12).

\[
Y = 0.065m' + 0.821
\]  

(85)

where $m' = m \cos \varphi$, $m$ is the seafloor slope normal gradient and $\varphi$ is the peel angle of the wave [Walker, 1974]. The vortex ratios calculated using Eq. (85) predict the ‘intensity’ of the wave breaking event according to Table 3-5 [Mead and Black, 2001].

For their study, Mead and Black collected no direct quantitative wave data. As a consequence, Eq. (85) proposes that the vortex ratio is independent of all wave characteristics and incorrectly gives a positive vortex ratio when the slope is zero, contradicting the findings of Duncan [1994]. In order to calculate the slope for Eq. (85), Mead and Black assumed $y_b = 0.78$ and averaged the slope over a depth variation of 4 – 8 m from this calculated breaking depth. Mead [2010] maintains that this equation cannot be applied to all waves but rather only waves in the ‘surfable’ range, which covers waves with heights from 0.5 – 4 m and periods ranging from 9 – 18 sec.
Table 3-5: Black and Mead Intensity Values

<table>
<thead>
<tr>
<th>Intensity</th>
<th>Extreme</th>
<th>Very High</th>
<th>High</th>
<th>Medium High</th>
<th>Medium</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vortex Ratio (V)</td>
<td>1.6 – 1.9</td>
<td>1.9 – 2.2</td>
<td>2.2 – 2.5</td>
<td>2.5 – 2.8</td>
<td>2.8 – 3.1</td>
</tr>
</tbody>
</table>

Vinje and Brevig’s [1980] numerical work on horizontal seafloors illustrated that both deep and shallow water waves create plunging breakers of similar vortex ratios, thus contradicting the findings of Mead and Black [2001]. Geometrically, the major significant difference between the numerical shallow and deep water plunging waves of Vinje and Brevig was the location at which the wave “jet” impacted the wave front. In the case of shallower water waves, the jet impacted in the trough between the wave of interest and the preceding wave; while in the case of the deeper water waves, the jet impacted halfway down the wave face, as shown in Figure 3-13.

When investigating barred beach profiles, Sayce et al. [1999] suggested that the breaker type over submerged bars is more strongly dependent on wave height and water depth than wave steepness and beach gradient, the two factors included in the SSP and Eq. (85).

Using a laboratory barred beach profile, Blenkinsopp and Chaplin [2008] investigated the effect of bar crest submergence on the vortex parameters. Blenkinsopp and Chaplin defined breaking as the point when the front face became vertical, and the breaking depth was referenced against the average MWL over the entire experimental series. The calculated vortex ratios ranged from 1.46 to 2.28, on a constant slope of 1:10, indicating that slope is not the only factor effecting vortex parameters as predicted by Eq. (85). However, Blenkinsopp and Chaplin noted that the vortex area (Eq. (86)), normalized by the square of the deep water wave height, increased with decreasing reef crest depths and with decreasing offshore wave steepness values:

\[ A_c = 2 \left( \frac{\sqrt{3}}{5} \right) l_c w_c \]  (86)
On a plane emergent breach profile, Grilli and Svendsen [1997] used a Fully Non-Linear Potential Flow Mathematical Model (FNPM) to predict that “the size of the jet does not change relative to the wave height for waves of different incident wave height on a given slope”. Grilli and Svendsen defined the jet size as the vertical distance between the wave crest and the jet-tip just prior to impact with the trough or wave face (L₂ in Figure 3-14).

Newer models, such as the Smoothed Particle Hydrodynamics (SPH) model, are predicting realistic breaking wave profiles [Khayyer et al., 2008; Zheng et al., 2009]. Khayyer et al. [2008] qualitatively compared the physical characteristics of solitary breaking waves in Figure 3-15. A basic visual comparison between SPH results and laboratory images [Zheng et al., 2009] shows the excellent correlation (Figure 3-15).
Unfortunately, the corrected incompressible SPH methods underestimated the horizontal wave jet length, \( L_1 \) in Figure 3-14 and overestimated the vertical wave jet height \( L_2 \) yet shows great promise for better predictions than many previous methods [Khayyer et al., 2008].

In perhaps the most detailed study until this point, Johnson [2009] investigated trends between previous suggested vortex parameters and wave characteristics in a wave flume. Using four different slopes and 72 different wave steepness values, Johnson noted that the vortex ratio increased with wave height and is unaffected by wave period, contradicting the findings of Blenkinsopp [2008] and Mead and Black [2001]. In addition, Johnson’s data illustrated that lower seafloor slopes do not affect the vortex ratio (in contradiction to Mead and Black [2001]) and that the reef crest water depth plays a defining role in vortex shape (as suggested by Blenkinsopp [2008]). Johnson was unable to explicitly quantify any of these trends due to low correlation values.

Mead and Black [1992] recommended that Eq. (85) be revisited to quantify the effect of wave period, wave height and complex seafloor gradients on breaking vortex shape. This is re-enforced by Johnson [2009] who notes, “the combined effects of wave height, period, water level, reef crest width and seaward reef slope on breaking intensity through the breaking wave vortex shape parameters, needs to be assessed”.

3.2.3 Breaking Wave Categorization via Wave Plunge Distance Parameter

Another descriptor used to quantify geometric differences in plunging waves is the wave plunge distance \( X_p \). \( X_p \) is defined as the horizontal distance, in the direction of wave propagation, from the position when the wave face becomes vertical to the position where the falling jet of a plunging wave impacts the wave trough (see Figure 3-16 for an illustration). While the location of the plunge point is an important parameter for many surf zone process calculations, including longshore current generation, few published prediction equations could be found.

![Figure 3-16: Illustration of the wave plunge distance, Xp [Smith and Kraus, 1991]](image)
Galvin [1968] conducted laboratory experiments on 3 different emergent plane slope beaches \((m = 1/5, 1/10\) and 1/20\) and, using basic wave kinematics, estimated that the plunge distance could be calculated using:

\[
\frac{X_p}{H_b} = 4.0 - 9.25m
\]  

(87)

Weisher and Byrne [1979] found better correlation by eliminating the slope dependence entirely, and proposed Eq. (88):

\[
X_p = 4.0H_b
\]  

(88)

More recent authors [Weisher and Byrne, 1979; Singamsetti and Wind, 1980; Visser, 1982; Kraus and Larson, 1988] have found that both Eq. (87) and Eq. (88) underestimate \(X_p/H_b\). Smith and Kraus [1991] related the plunge distance to the SSP using data from Galvin [1968], Singamsetti and Wind [1980] and Visser [1982]. Smith and Kraus [1991] noted that the return flow of water over the barred profiles caused the incoming waves to “trip” and break prematurely. This resulted in plunge distance reductions of 60% - 70% over plane slope conditions, and suggested that additional parameters may be required for barred slopes. Smith and Kraus differentiated between plane slopes and barred profile relationships according to Eq. (89) and Eq. (90):

For plane slopes:

\[
\frac{X_p}{H_b} = 3.95\xi_o^{-0.25}
\]  

(89)

For barred slopes:

\[
\frac{X_p}{H_b} = 0.63\xi_o^{-1.00} + 1.81
\]  

(90)

Gourley [1994] noted that, “no consistent simple means of predicting this distance [plunge distance] is available. If scaled in terms of breaker height, \(x_p\) is very sensitive to the bottom slope and hence the shape of the outer reef-top and reef edge”. Gourley predicted that the plunge distance is approximately identical to the distance offshore in which the wave height decreases slightly, due to conversion of potential to kinetic energy (see Chapter 2).

An accurate prediction for the plunge distance is an important correction required for field work based on overview photographic images. Hutt et al. [1997], Walker [1974], Mead and Black [2001] and Scarfe et al. [2003] all extracted wave breaking positions from aerial photographs. Without correcting for plunge distances, the extracted positions do not represent the true breaking positions and will introduce additional uncertainty in field results.
3.3 Conclusions based Characteristic Predictor Review

A highly detailed review of breaker depth index and geometric predictors has been presented in Chapter 3 in order illustrate the continually improving understanding of breaking waves and the various methods used to predict and classify breaking waves. A second objective was to illustrate that the majority of traditionally cited predictors for breaking waves are inherently limited due to laboratory scaling issues, added uncertainty in the dataset due to inconsistent definitions of basic wave parameters, lack of experimental validation, or involve assumptions which inherently limit their accuracy and application.

Through the application of several novel measurement techniques presented later in this thesis, this study was the first able to collect a full suite of irregular wave breaking characteristics from the field, to analyse the collected data using a single set of parameter definitions and eliminate any unnecessary assumptions about the wave by directly measuring as many relevant characteristics as possible. Via a direct comparison of all the predictors presented in Chapter 3, using the collected dataset, this study was able to independently quantify the performance of each predictor and determine the best breaker depth index predictors, as well as investigate presented geometric classification systems.
4 Previous Breaking Wave Field Studies

The majority of the physical work in shallow water breaking wave studies has been completed in laboratory environments due to the complexities involved with field studies. Traditional field studies are cost-prohibitive, require suitable wave climates and often involve significant risk to equipment and researchers. Additionally, the resulting dataset often suffers from poor spatial resolution and accuracy.

Traditional field studies have generally employed photopoles or resistant-type wave gauges to measure breaking wave height and depth conditions. These two devices are static single point measurement devices and thus the data spatial resolution is directly correlated to the number and distribution of deployed gauges. The number of deployed gauges is, in turn, defined both by the project budget and study location. Many breaking wave studies have piggy backed on larger, better funded projects in order to allow the deployment of sufficient in-situ sensors to achieve reasonable spatial resolution. The cost prohibitive nature of such experiments has stifled additional field based research.

Additionally, study locations are often determined by local scientific establishments, rather than the suitability of the study location. Thus the collected data inherently includes many adverse, avoidable and complicating variables.

Detailed below are the most cited field studies investigating shallow water breaking waves. This list is not exhaustive and only details the most relevant previous efforts to investigate breaking waves in field conditions.

4.1 Ajigaura Beach, Japan (1978):

On July 5, 1978, sixty photopoles were installed at Ajigaura Beach, Japan [Hotta and Mizuguchi, 1980]. Ajigaura Beach is a sandy beach with an average 1 to 60 slope, which faces directly east towards the Pacific Ocean, and has a constantly changing, non-permanent seafloor profile. Poles were positioned and collected data at approximate 2m intervals, stretching from water’s edge to approximately 120m offshore.

A single observation period of 12.75 minutes was used to collect the wave data, and maximum significant wave heights of 0.7 m and periods ranging from 6 to 11 s were measured. Wave directional information was taken from a single current meter installed at the most seaward photopole. The pole was installed inshore from the breaking positions of the largest waves and hence measured wave direction information from within the surf zone, rather than as an incoming parameter.

It is undetermined whether the line of deployed photopoles was perpendicular to the incoming waves during the study period. Additional information on this study may be found in Hotta and Mizuguchi [1980].
4.2 Nearshore Sediment Transport Study, California (1980):

Thornton and Guza [1986] conducted a US Army and US Federal Government funded study at Torres Pines State Park and Leadbetter beaches, California from January 30th to February 23rd, 1980. The main objective of the study was to investigate the changes in spectral variance density of a wave as it travels through the surf zone and the associated sediment transport. Hence understanding and predicting factors at play in the highly non-linear wave breaking process was only a secondary goal.

A combination of wave staffs, current meters and pressure sensors were deployed, allowing for a good approximation for both incoming wave characteristics and surf zone characteristics. Incident wave heights measured at a 4m depth varied between 0.56 – 0.40 m while periods varied between 11 s and 16 s.

Unfortunately, seafloor slope changes were averaged over the entire study and the exact breaking positions were not collected, hence resulting in a limited dataset for breaking wave studies.

4.3 DUCK’85, North Carolina (1985):

On September 4 – 6, 1985, a detailed photopole study was conducted at the Field Research Facility in Duck, North Carolina. DUCK85 was the most detailed field study of shallow water waves at the time. Nine tests were completed over the 3 days, with an average duration of 12:40 minutes, resulting in 1 hour and 54 minutes of total study time. Wave periods varied from 10 to 13 s and the largest measured wave was 1.9m. A full description of the nine tests is provided by Ebersole and Hughes [1986].

DUCK85 used a single line of 14 photopoles, spread at 6 – 7 m intervals, over the entire surf zone from the shoreline seaward to beyond the breaker line. The final measured height and position of the breaking waves was spatially poor due to the large pole spacing. No directional information was collected due to the limitations of a single line of photopoles.

The seafloor at the Field Research Facility is non-static and dramatically changed during the study period, creating a situation where the exact depth at breaking could not be defined. Additionally, Daly [1992] analyzed the DUCK85 dataset and found that high-tide wave reflections affected breaking conditions, thus limiting the application of DUCK85 data for the study of breaking waves.
4.4 SUPERDUCK, North Carolina (1986):

Following up on the completion of DUCK85, SUPERDUCK was conducted from September 11 – 24, 1986 at the Field Research Facility in Duck, North Carolina. 51 tests were completed, with an average duration of 12:40 min, over 12 days. During the study period, wave periods varied from 4.3 – 13.5 s and the largest measured wave was 1.58 m [Rosati et al., 1990].

SUPERDUCK consisted of 22 photopoles spaced at regular 6m intervals, and built on the findings of DUCK85. As with the DUCK85, the 6 m spacing was insufficient to accurately measure exact breaking conditions of individual waves. Additionally, no directional wave information was collected.

Bathymetric surveys were completed daily yet large variations were found to occur between survey days, introducing uncertainty in the bathymetric depths.

4.5 DELILAH Nearshore Experiment, North Carolina (1990):

During October 1 – 21, 1990, the DELILAH experiment was conducted at the same location as the DUCK85 and SUPERDUCK experiments. Funded by the US Army Corps of Engineers and the Office of Naval Research, DELILAH featured 4 interdependent cross-shore and along-shore arrays of pressure sensors and pressure-based wave gauges. The main cross-shore array featured 9 combined pressure and current sensors covering approximately 250 m. Significantly more sensors would have been required to accurately measure the exact position and wave height at breaking [Birkemeier et al., 1997].

Favourably, the measured wave heights varied from 0.5 – 2.8 m and periods varied from 4 – 14 s. In contrast to the two previous studies, a full frequency-direction wave spectrum was collected using a permanent Field Research Facility pressure array at 8 m depth.

While the shifting seafloor slopes make this an excellent location to study sediment transport, the continually moving seafloor means bathymetric slopes considered for breaking were also varying. Only four surveys were completed over the 20 day period, and profiles showed considerable bathymetric alteration between surveys. Hence, the predicted seafloor slopes are likely not accurate.

4.6 Mead and Black Wave Breaking Intensity Study (2000):

Mead and Black[2001] were the first to extract wave vortex ratio and angle parameters, and attempt to predict breaking “intensity” through seafloor slope changes. Mead and Black collected bathymetric data at 22 different locations around the globe using a combination of nautical charts and single beam echo soundings. Wave measurements were extracted from published magazine and video images of breaking
waves, and qualitative knowledge of the breaking areas. No direct in-situ wave measurements were taken as part of this study. Wave breaking depths were calculated using McCowan’ breaker index constant, $\gamma_b = 0.78$, and wave heights extrapolated from published wave profile images.

### 4.7 Video Imagery Wave Height Analysis Experiments

Shand et al. [2012], Gal et al. [2011] and de Vries et al. [2010] have all preformed detailed analyses of breaking wave heights using remote video cameras and image analysis techniques.

Shand et al. used two cameras mounted at different elevations on the same vertical plane at Castlecliff Beach, New Zealand. Gal et al. used a single camera system to analyze waves breaking at Narrowneck beach, Australia, while de Vries et al. used a set of horizontal stereo cameras to analyze the breaking waves both in laboratory conditions and at Scheveningen beach, Netherlands. All presented methods extracted detailed wave height information using video camera image pixel locations and relative separations.

None of the mentioned studies collected water depth information and bathymetric profiles during their studies. This may be due to the fact that all study locations featured non-permanent seafloor slopes and constantly changing seafloor profiles. Wave height information was the only published parameter from the mentioned studies, it appears that these studies did not attempt to measure any further breaking wave characteristics.

### 4.8 Conclusions Based Previous Studies

As illustrated throughout this chapter, no previous studies have managed to collect a full suite high resolution breaking wave conditions, based on field collected irregular breaking waves. Each of the experiments discussed were limited for differing reasons: Lack of modern measurement techniques and equipment, poorly suited study locations, failure to collect all pertinent information or individual breaking wave measurements were simply not a primary study objective.

As a result, to the best of the author’s knowledge, the study presented in this thesis is the first to collect a high resolution database of all relevant breaking characteristics. The data suite is based on a wave-by-wave analysis of irregular waves and includes all wave characteristics at both the point of breaking and beyond the surf zone, as well as detailed analysis of relevant seafloor slopes over non-uniform bathymetry. This information collected in this study is without par and provides an excellent dataset to further our collective understanding of the most nonlinear of all wave transformation processes; shallow water wave breaking.
5 Experimental Overview

Shallow water breaking waves are affected by incoming wave characteristics, bathymetric conditions, and numerous controllable and uncontrollable variables. As a result, the most important step in designing an experiment to study shallow water breaking waves is to choose a location which minimizes the uncontrollable and complicating effects of localized winds, wave/wave interactions, wave reflection, and currents. In order to find the best possible locations for this study, numerous locations around the world were analyzed for wave climate, local geographic features, access to scientific facilities and ease of access. In addition, Northern Hemisphere locations were given priority due to increased swell height and consistency in the northern hemisphere during the North American Fall. Of all the final candidate locations, Santa Cruz California and Barbados were determined to be the best possible locations for this study.

In order to achieve a full understanding of the processes at play during shallow water wave breaking, a complete and detailed dataset of wave parameters, seafloor characteristics and environmental conditions was required. This dataset required a full bathymetric survey of the surf zone and surrounding seafloor, detailed measurements of incident wave characteristics pre-breaking and at the instant of breaking, as well as the associated geographic position and geometric profile of each wave.

Chapter 4 showed that it is extremely difficult to collect and extract data from the surf zone with sufficient high resolution accuracy, in a non cost-prohibitive manner. This study employs a novel low-cost technique to extract data with excellent spatial and temporal resolution, both before and during the breaking process.

An abridged version of Chapter 5, Chapter 6 and Chapter 7 was accepted for publishing in the peer reviewed Journal of Coastal Research on April 16th, 2013 [Robertson et al., 2013].

5.1 Study Locations

Field data collection was completed during the Fall of 2011 in Santa Cruz, California and on the Caribbean island of Barbados. September and October 2011 were spent collecting data at The Hook and Sewers Peak along Pleasure Point in Santa Cruz (36.95 ° N, 121.97 ° W). During November and December 2011, measurements were taken at Tropicana Beach, Barbados (13.22 ° N, 59.64 ° W). All three of these locations were carefully chosen in order to minimize the effects of local wind driven seas and multiple differing swell regimes.
5.1.1 Hook and Sewers Peak, Santa Cruz, California

The coastline of Santa Cruz provides an excellent location to study breaking waves. First, the west coast of North America is an extremely active area for ocean waves. Open to the vast Pacific Ocean, the Santa Cruz region sees significant incoming swell throughout the year and thus results in frequent opportunities to study breaking waves.

Offshore from Santa Cruz, Monterey Bay is dissected by the Cabrillo and Soquel Canyons (as shown by deep unlabelled channels in Figure 5-1) which allow waves to propagate far into Monterey Bay in their deep water form, before entering shallower water and refracting towards the coastline. As a result, seasonal offshore wave directions vary between 160 ° and 320 ° yet waves breaking at the Hook and Sewers Peak locations only vary between 185 ° and 215 °. Hence, the depth-based refraction causes a reduction in the incoming directional spread by up to 70 % and results in incoming waves of almost consistent direction and period at The Hook and Sewers Peak.

The coastline of Pleasure Point runs approximately southwest to northeast (bearing 235 ° to 45 °) and is protected from localized high frequency wind waves caused by the dominant northwest coastal air flow. Additional high frequency wave attenuation results from significant growths of bull kelp, *Nereocystis luetkeana*, just beyond the surf zone. The combined effect of the bull kelp dampening and favourable geographic orientation removes local wind waves from the breaking zone, allowing only the incoming low frequency waves arrive unaffected.
5.1.2 Tropicana, Barbados

The island of Barbados sits at 13 ° N, 59 ° W and is significantly further east of the rest of the Caribbean islands. As a result, Barbados is open to more open ocean, low frequency waves than other Caribbean Islands. The study location, Tropicana Beach, is located approximately 2/3 of the way up the west coast of the island (see Figure 5-2), and is only exposed to a small incoming swell direction window. Waves breaking on the reef at Tropicana are primarily formed by North Atlantic winter storm systems and require an incoming swell direction of less than 350 ° to propagate down the west coast. However, the islands of Guadeloupe and Barbuda block any incoming swells from less than 335 °, resulting in only a 15 ° directional window for breaking waves at Tropicana Beach.

The dominant northeast trade winds during the winter months blow directly offshore on the west coast and prevent the creation of incoming wind-generated waves in the study area. When choosing specific study days, it was equally important to choose days without significant offshore winds which would result in dramatically altered breaking conditions at Tropicana.

Given the small incoming wave direction window and the necessity of low wind activity, breaking waves on Tropicana Beach are infrequent. However when conditions do align, waves breaking over the reef at Tropicana Beach provide the perfect repeatable field conditions for scientific analysis. Over the 8 week study period, only 3 days of adequate wave conditions occurred.

Figure 5-2: Map of the Tropicana Beach study location
5.2 Bathymetric Data Collection

Seafloor slope has been proven to be a major determining factor in the breaking depth, height and geometric shape of waves[Grilli et al., 1997; Couriel et al., 1998]. The calculated slope is very sensitive to small variations in incoming wave direction and bathymetric shape. It was therefore necessary to obtain detailed bathymetric maps of the study locations in order to extract seafloor slopes. All three locations studied feature permanent coral or rock seafloor geology. This results in no temporal variation of seafloor profiles and depths during the study periods.

5.2.1 Hook and Sewers Peak, California

The seafloor along Pleasure Point was surveyed as part of the California Seafloor Mapping Program (CSMP). The United States Geological Survey (USGS) completed a full bathymetric survey from Ano Nuevo (37.1057 ° N, 122.3422 ° W) to Moss Landing (36.803 ° N, 121.791 ° W) in Monterey County, during the fall of 2009 as part of the CSMP. This survey covered The Hook and Sewers Peak study areas. A 234.5 kHz Seal Swath plus, M phase differencing, sidescan sonar mounted on the Research Vessel Parke Snavely produced a survey map from the shallowest possible water depths to approximately 100m depth. The Seal sidescan sonar featured a vertical depth accuracy of 0.1 m at 57 m depth and 0.3 m at 171 m depth.

Two shore-based Trimble Global Positioning System (GPS) base stations broadcasted continuous Real Time Kinematic (RTK) latitude and longitude positions to the R/V Snavely, via an Ultra High Frequency (UHF) radio link. These positions were then input into a CodaOctopus F180 measurement unit, achieving centimeter latitude and longitude accuracy. All the recorded depths were adjusted to the mean low lowest water (MLLW) level using a NAVD88 geoid datum.

Knowing that the large research vessel R/V Snavely would be unable to survey shallow surf zone areas, the USGS conducted a full shallow water bathymetric survey of the Pleasure Point area in March 2005 in preparation to the detailed multibeam R/V Snavely survey. Using a Trimble RTK GPS and 200 kHz single beam Flash Fire echosounder, mounted on a wave runner, the USGS surveyed from a minimum depth of 0.5 m out to deep water. Data points were collected along 0.6 x 0.6 m gridlines and feature sub decimeter accuracy. This data was post-processed to a NAVD88 MLLW datum and stitched together with the deep water swath data.

In order to assess possible offsets and uncertainties between the two datasets, overlapping data points were compared. Only minor errors (< 0.1 m at 10m depth or 1 %) were found between the datasets in overlapping areas. This lack of variation can be attributed to properly characterized, accurate equipment and the robust permanent nature of the seafloor in the Pleasure Point area.
This dataset was kindly provided to the author by the USGS as part of a collaborative effort to better understand the wave environment in the Pleasure Point area.

5.2.2 Tropicana, Barbados

The west coast of Barbados is littered with picture perfect white sand beaches. Hotel complexes dominate the seafront and rely on the perfect beaches to power the local economy. As a result, the Barbados Government Coastal Zone Management Units’ (CZMU) main directive is to better understand the movement of sediment along this overdeveloped coast. In order to get a baseline of bathymetry along the coast, CZMU contracted Terra Remote Ltd to perform a full Light Detection and Ranging (LIDAR) survey. CZMU was kind enough to provide the author with a 2000m x 2000m portion of this bathymetric data, centered around Tropicana Beach (13.217 ° N, 59.65 ° W).

Unfortunately, the LIDAR data suffers from 30m x 30m data resolution and was insufficient for the prediction of wave breaking characteristics. Additional shallow bathymetric surveys were completed on Nov 21st and Dec 13th 2011, using a Kongsberg 38/200 kHz single beam echosounder and a Hemisphere digital GPS. Bathymetric surveys followed a 1 m x 1 m grid pattern, covering a 150 m x 150 m area surrounding the surf zone, using on a research vessel loaned from McGill University’s Bellairs Research Institute.

Barbados has its own water level datum, the Lamont Datum [Griffith, S. Engineering, 1994; Miller et al., 2010]. All depths were tidally corrected to the Lamont datum since no consistent method for transferring from Lamont to MLLW was available.

5.3 ADCP Deployments and Data Collection Parameters

Detailed knowledge of the incoming wave conditions is mandatory for any investigation of breaking wave characteristics. The direct measurement of the incoming wave height and period, as well as the calculations of the full frequency-direction spectrum, eliminates the often used generalized assumptions that are required to predict necessary parameters, such as effective seafloor slopes, and their inherent unnecessary uncertainties.

A 1200 kHz RD Instruments Acoustic Doppler Current Profiler (ADCP) was mounted on the seabed just offshore of the surf zone, in 4 to 8 m of water, to collect continuous full raw binary dataset of the entire water column using 0.10 m vertical cells and a 0.5 s ping rate. This allowed complex post-processing analyses to be conducted on the raw data, rather than using an instrument which simply records and stores spectral parameters.

With a level deployment, the four acoustic beams from the ADCP will project upwards with a 20 ° offset from the vertical allowing the ADCP to calculate the directional aspects of the sea state. However, for this
study the ADCP was deployed on a slight angle to ensure that one of the beams project vertically and was able to record the highest quality time series of the sea surface for later zero-crossing analyses.

Appendix B details the ADCP set-up parameters used for the Santa Cruz and Barbados deployments.

5.3.1 Hook, California

Given the location and local deployment concerns, the ADCP was deployed for a single 3 week period at The Hook. In order to ensure the ADCP did not move or shift during the deployment, it was mounted to a heavy duty frame, borrowed from the USGS, and weighted with 80 lbs of lead weight (see Figure 5-3). The ADCP was deployed in 5.49 m MLLW of water and was additionally secured to the seafloor using two sand screws. The final deployed location (36.96 ° N, 109.97 ° W) was chosen due to a slight depression in the bathymetric profile, to reduce the wave induced shear forces.

Recording a full binary dataset requires substantial data storage space and battery power; hence it was not possible to run the ADCP continuously for the entire deployment. As a result, the ADCP was set to record three times daily for 1.5 hour periods: 8:00am – 9:30am, 4:00pm – 5:30pm and 0:00am – 1:30am PST daily. These time periods were chosen to capture the effect of all tidal depth changes on the breaking characteristics, and allow for convenient divisions of the 24 hour day.

5.3.2 Sewers Peak, California

On Oct 18th 2011, the ADCP was lifted from The Hook location and moved approximately 2km west to the Sewers Peak location. Redeployed at 36.95 ° N, 109.97 ° W in 7.00 m MLLW of water, the Sewers Peak location featured a considerable depth change (> 2 m) within 3 m from the ADCP location. This location was chosen to allow for shallow water recording depths yet maintain an instrument margin of safety from waves breaking directly above the instrument.
The data recording schedule at Sewers Peak followed the same guidelines as the Hook location. Figure 5-4 shows the ADCP being redeployed at the Sewers Peak location on Oct 18th, 2011.

Figure 5-4: Oct 18th, 2011 ADCP deployment from R/V Snavely at Santa Cruz

5.3.3 Tropicana, Barbados

Barbados has a long history of superhuman free divers and spear fisherman. As a result, Tropicana Beach and local beaches have a history of vandalism when scientific instruments are situated in shallow water environments. Hence the ADCP was only deployed and recovered whenever suitable wave conditions existed at Tropicana Beach. As with The Hook location, the ADCP was deployed in “pukas” or holes within the reef structure to lower wave induced shear stresses on the instrument. This also allowed to the ADCP to be positioned at the appropriate angle to ensure a single vertical measurement beam and the most accurate surface time series data collection possible.

Five deployments were conducted over the 8 week period yet only three days provided sufficient swell conditions to allow waves to break on the Tropicana reef. Given the manual placement of the ADCP for each swell event, the location of the ADCP changed each time. In order to minimize the effects of complicating variables (wind, tide, mixed swell conditions), data collection times changed on a day to day basis. Details for the three study days are given in Table 5-1.

<table>
<thead>
<tr>
<th>Date</th>
<th>Latitude</th>
<th>Longitude</th>
<th>Water Depth</th>
<th>Timing</th>
</tr>
</thead>
<tbody>
<tr>
<td>November 22nd, 2011</td>
<td>13.2199 °N</td>
<td>59.6438 °W</td>
<td>4.20 m</td>
<td>15:00 - 16:00</td>
</tr>
<tr>
<td>November 28th, 2011</td>
<td>13.2200 °N</td>
<td>59.6440 °W</td>
<td>4.80 m</td>
<td>10:30 - 13:00</td>
</tr>
<tr>
<td>December 7th, 2011</td>
<td>13.2198 °N</td>
<td>59.6441 °W</td>
<td>4.15 m</td>
<td>06:45 - 09:40</td>
</tr>
</tbody>
</table>
5.4 Wave Analysis Overview Camera

The extraction of high resolution breaking wave positions, and the associated breaking height, from irregular waves breaking in the field has plagued coastal engineers and scientists for generations. As previously discussed, traditional methods require a vast array of wave measurement devices (wave gauges, photopoles, etc) covering the entire area of interest. These methods are prohibitively expensive, result in low data resolution, necessitate substantial engineering design for surf zone deployment and entail significant risk to researchers.

In this study, breaking wave measurements were extracted using a low cost remote digital video camera. Overview cameras were mounted at an elevated location onshore, facing the breaking waves, and captured an oblique overview image of the entire study area. Figure 5-5 shows the field of view for the overview cameras at The Hook, Sewers Peak and Tropicana Beach locations respectively.

5.4.1 Hook and Sewers Peak, California

CoastalCOMS Ltd, the University of California Santa Cruz and the USGS have installed multiple in-situ high definition AXIS P5532-E Network cameras installed along Pleasure Point, as part of a long term coastal erosion monitoring project. These cameras feature a 720 X 480 pixel resolution and 53° field of view. For the present study, camera set points (pan, tilt and zoom) were determined to maximize the resolution and accuracy of extracted breaking wave characteristics. Cameras were set to automatically revert to the chosen set points and begin recording footage two minutes before ADCP recordings began.

The Hook camera was located at 39.9610 ° N, 109.9635 ° W at an elevation of 14.44 m above MLLW, while the Sewers Peak camera was located at 39.9568 ° N, 109.9710 ° W at an elevation of 14.55 m above MLLW. In order to coincide with the ADCP recording, the camera recorded video data from 8:00 am to 9:30 am and 4:00 pm to 5:30 pm PST daily.
5.4.2 Tropicana, Barbados

A Panasonic HDC-TM90P high definition video camera (1920 X 1080 pixel resolution, 40° field of view) was used for collecting data at Tropicana Beach. In order to achieve the necessary camera elevation, the congregation of St. Albans Church allowed the author to access their grounds and install a semi-permanent camera at 13.2194 ° N, 59.6421 ° W on a pole mounted to a beach retaining wall (see Figure 5-7). Final camera elevation was 4.54 m above Lamont zero water level.

5.5 Profile Image Camera

In order to study the geometric characteristics of the breaking wave vortex, a clear profile image of the wave is mandatory. Given the complexities of capturing a profile image from a raft, boat or other craft in the surf zone, a swimmer equipped with a waterproof video camera was initially thought to be the most
efficient method to capture the required image. However, initial efforts resulted in low wave capture success rates and alternate methods were investigated. With experience, it was discovered that choosing study locations that allowed land-based profile images to be captured resulted in the highest image capture success rates.

5.5.1 Hook and Sewers Peak, California

As the initial test sites, the author swum out into the surf zone with a Panasonic HDC-TM300 Full HD 1080i video camera mounted in an Equinox HD-6 Underwater housing. However the large spatial breaking zone of irregular waves, and the relatively slow speed at which the swimmer was able to relocate, resulted in relatively few wave vortex images being captured. For example, the author spent over 12 hours swimming with the video camera at The Hook yet was only able to capture 67 waves of sufficient quality.

At Sewers Peak, the author situated himself on a rock platform 200m west of the main breakpoint at 36.9545 ° N, 121.9759 ° W and captured profile images using a Nikon D200 SLR camera equipped with a 70-300 mm zoom lens. In contrast to the swimming method, the author was able to capture 35 excellent vortex profile images in just 2.5 hours using this remote method.

5.5.2 Tropicana, Barbados

Tropicana beach is a stunning white sand, crescent shaped beach. Waves break over the shallow reef located on the southern side of the bay, allowing the author to walk to the northern edge of the bay (13.2215 ° N, 59.6413 ° W) and capture vortex profile images directly from land (see Figure 5-8). The author was able to capture 85 excellent profile images in 3.5 study hours, spread over 2 days, using the same Nikon D200 SLR camera equipped with a 70-300 mm zoom lens.

![Figure 5-8: Example vortex profile image from Tropicana Beach, Barbados.](Image)
5.6 Review of Experimental Data Collection Events

As previously noted, the field data collection portion of this study was completed during the Fall of 2011 in Santa Cruz, California and on the Caribbean island of Barbados. Unlike laboratory experiments, field work requires all the local environmental conditions to align perfectly before data collections can begin. With this in mind, the author devoted four months entirely to collecting data on the best possible day at each study site. As shown in Table 5-2, September and October 2011 were spent collecting data at The Hook and Sewers Peak along Pleasure Point in Santa Cruz (36.95 ° N, 121.97 ° W). During November and December 2011, measurements were taken at Tropicana Beach, Barbados (13.22 ° N, 59.64 ° W).

<table>
<thead>
<tr>
<th>Location</th>
<th>Date</th>
<th>Study Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Hook, Santa Cruz</td>
<td>Oct 06th, 2012</td>
<td>08:10 - 09:15</td>
</tr>
<tr>
<td></td>
<td>Oct 07th, 2012</td>
<td>08:35 - 09:30</td>
</tr>
<tr>
<td></td>
<td>Oct 08th, 2012</td>
<td>08:10 - 08:50</td>
</tr>
<tr>
<td></td>
<td>Oct 09th, 2012</td>
<td>16:20 - 17:00</td>
</tr>
<tr>
<td></td>
<td>Oct 12th, 2012</td>
<td>08:30 - 09:10</td>
</tr>
<tr>
<td></td>
<td>Oct 12th, 2012</td>
<td>16:50 - 17:10</td>
</tr>
<tr>
<td>Sewers, Santa Cruz</td>
<td>Oct 25th, 2012</td>
<td>16:20 - 17:00</td>
</tr>
<tr>
<td></td>
<td>Oct 28th, 2012</td>
<td>16:20 - 16:50</td>
</tr>
<tr>
<td></td>
<td>Nov 01st, 2012</td>
<td>16:00 - 17:30</td>
</tr>
<tr>
<td>Tropicana, Barbados</td>
<td>Nov 28th, 2012</td>
<td>12:15 - 13:10</td>
</tr>
<tr>
<td></td>
<td>Dec 07th, 2012</td>
<td>08:10 - 10:30</td>
</tr>
</tbody>
</table>

While Table 5-2 details the days which featured suitable breaking conditions for data collection, numerous additional days of data collection were collected. These additional data sets were deemed unsuitable for detailed analysis due to rapid changes in the wave and environmental conditions, the influence of human foul play or corrupted data sets.

Interestingly, one of the biggest hurdles to determining the best study locations was social, not geographic. The best locations for this study were also extremely popular with surfers, whom were not always receptive to scientific studies occurring while they were surfing. For example, two initial datasets were collected at Jordan River, Canada during January 2012. Unfortunately, a local surfer vandalised the study equipment resulting in the need for a new study computer and loss of an entire dataset. This is despite detailed consultations with surfers and ocean users at all study locations.
6 Data Extraction and Processing Methods

The data collection portion of this study was completed over a four month period in the Fall of 2011. Over the four month period, only 11 days featured the consistent unidirectional swell regimes, calm atmospheric conditions and prerequisite tidal depths to allow for data collection. In total, 28 hours of video and ADCP data were collected and analyzed for all three locations. Each study day featured dramatically different swell heights and wave periods to allow for a statistically sufficient range of breaking conditions.

The data from the bathymetric surveys, overview video cameras, ADCP deployments and profile cameras was analyzed in order to extract all the necessary wave and study location characteristics. All extraction procedures and analysis techniques are detailed in the following sections.

An abridged version of Chapter 5, Chapter 6 and Chapter 7 was accepted for publishing in the peer reviewed Journal of Coastal Research on April 16th, 2013 [Robertson et al., 2013].

6.1 Bathymetric Data Processing

The first step for bathymetric processing in Santa Cruz was to correct all the USGS supplied x, y and z data for tidal variations and correct to a single depth datum. The x values corresponded to longitude positions, while y and z corresponded to latitude and depth values respectively. For The Hook and Sewers Peak, the bathymetric swath and single beam data was corrected to the MLLW datum, and exported to Matlab as a comma separated values (.csv) file. This data had previously been processed for tidal, pitch, roll and heave variations by USGS Geographic Information System (GIS) scientist Joshua Logan.

At Tropicana, the .xyz file exported from the 200kHz Kongsberg echosounder surveys were imported into Matlab and separated to isolate latitude, longitude and depth values. Depths were tidally corrected to the Lamont datum, using minute by minute tidal levels collected by a tide staff installed in Bridgetown Harbour. The different bathymetric depth datums between the Santa Cruz and Barbados have no effect on the final analysis since the final calculated depths are corrected using tidal heights recorded according to the local datum.

Three dimensional bathymetric surface models were then created using the grid data algorithm in Matlab. Grid points were based on a 1m x 1m grid and conflicting depth points were averaged. Areas of low raw data density were filled by linearly interpolating between the closest known depths. Once gridded, standard modeling techniques were employed to smooth data inconsistencies and create individual 500m x 500m bathymetric surface models for The Hook, Sewers Peak and Tropicana. Figure 6-1 shows these bathymetric surfaces superimposed on a Google Earth [Google, 2012] image.
Given that all subsequent modeling and depth-dependent calculations were based on depths extracted from the Matlab created bathymetric surface, it was important to ensure that the bathymetric surface models were true representations of the seafloor at each study location. The accuracy of the bathymetric surfaces were assessed by comparing the water depths measured using the ADCP against those extracted from the bathymetric model at the same location.

For The Hook, the ADCP reported a 5.49 m depth at the chosen deployment location, at an associated tidal depth of 0.92 m. Given that the ADCP sits approximately 0.30 m above the seafloor in the weighted mount, the true MLLW depth at the ADCP location was calculated to be 4.87 m. The bathymetric model at this location predicted a depth of 4.85 m, a difference of less than 1% uncertainty.

For Sewers Peak, the tidally corrected ADCP reported depth was 6.40 m while the bathymetric model reported a depth of only 6.1 m. This approximately 5% variation was higher than expected and further investigations at this location were therefore completed. In contrast to the other study locations, the seafloor at Sewers Peak features substantial kelp forests and low height seaweed habitats. Seaweed and seagrass is known to produce false under predicted seafloor depth readings when using an echosounder of approximately 200 kHz [RD Instruments, 2012; Stevens, 2012]. While the magnitude of this depth offset is undetermined, this known source of uncertainty was noted.

For the Tropicana Beach deployments it was expected that the model would predict lower depth values than those measured by the ADCP due to the deployment locations. For the Tropicana deployments, the weighted ADCP mount was lighter and more susceptible to wave induced forces. In order to reduce these wave loads, the ADCP was deployed in holes in the reef structure with minimum depths of approximately 0.3 m. For the November 28th deployment, the ADCP reported tidal corrected depths of 4.8 m Lamont while the surface model predicted depths of only 4.6m, resulting in a difference of approximately 4%.
The low relative errors between the measured water depths and the bathymetric model predicted depths at each location indicates good correlation and allows confidence in using the calculated depth values for subsequent calculations.

### 6.2 Incoming Wave Condition Data Processing

Incoming wave conditions and characteristics were measured via a shallow deployment ADCP. The ADCP was deployed within 100 m of the furthest seaward wave breaking positions and in 4 – 8 m of water.

The extraction of the necessary wave parameters was completed through two different processes: a surface time series analysis to determine individual wave heights \( (H_o) \) and periods \( (T_o) \), and a spectral analysis to determine statistical parameters of significant wave height \( (H_{sig}) \), peak frequency \( (F_p) \), and peak direction \( (\theta_p) \).

#### 6.2.1 Surface Time Series Analysis

A surface time series analysis was used to determine individual wave heights and periods using a zero-crossing analysis method. As noted previously, the ADCP deployments were tilted to ensure that one of the four beams projected vertically and was able to collect the most accurate time series of the water surface level possible.

For each deployment, the ADCP dataset was extracted from the device and exported as a raw file using RD Instruments’ WavesMon 10. The raw files were then processed according to the deployment depth, water temperature, and salinity profiles before exporting the final surface track and heading/pitch/roll (hpr) text files. The surface track text files detailed the independently collected millimeter (mm) depth measurements from each of the 4 ADCP beams collected at 4 Hz, while the heading/pitch/roll file detailed the time-associated heading, pitch and roll measurements made by the ADCP. Any changes in the heading, pitch or roll of the device greatly altered the reported surface track depths and required corrections based on these changes. Brandon Strong, RD Instruments Senior Engineer [Strong, 2011], provided Eq. (91) below in order to correct the final surface track for wave-induced ADCP motions.

\[
h = h_{acdp} \cos(J - R) \ast \cos(P)
\]

where \( h_{acdp} \) is the recorded depth, \( J \) is the beam janus angle (28°), and \( R \) and \( P \) are the instrument roll and pitch values respectively.
Analyses showed that these corrections were unnecessary at The Hook and Sewers Peak, due to the significant weight of the ADCP deployment, while the Tropicana deployments required only small corrections.

Next, missing data points (represented by zeros) were calculated by linearly interpolating between bounding measurements. Additionally, any reported depth values less than 3 m due to mid-water column interferences were also removed and a new linearly interpolated value was inserted.

Each 1.5 hr surface track time series was then run through a Fast Fourier Transform (FFT) to build a frequency-based variance spectrum diagram. In order to remove spurious noise and unwanted wave frequencies, a band pass filter was designed to ensure all relevant frequency peaks were included in the final analysis. Maze and Iwagaki [1982] recommend using $0.5 F_p$ for a low frequency cut-off and $6 F_p$ for a high frequency cut-off value. The low frequency cut-off removes the shallow water surf beat from the surface track data, which can create variations in subsequent zero crossing analyses results [Ippen and Kulin, 1954; Mizuguchi, 1982]. The high frequency band cut-off eliminates locally produced wind waves, boat wake and spurious high frequency waves. Additionally, the filtered high frequency waves do not “feel” the seafloor sufficiently early in order to refract to the required direction for breaking (noted in the Study Locations section of this thesis).

Noting that the longest measured peak wave period was 20 s, or 0.05 Hz, the initial band pass settings were set at 0.025 Hz and 0.333 Hz respectively. The choice of the initial high frequency cut-off is corroborated by the US National Ocean and Atmospheric (NOAA) data analysis procedure recommendations of a high frequency cut-off of 0.3 Hz [NOAA, 1996]. The presented bandpass set points were used as a starting point for designing the filter and then altered to ensure all peaks in the initial variance spectrum were included. Figure 6-2 shows an example filtered variance spectrum.
Figure 6-3 gives a short sample of the ADCP surface track time series for December 7th at Tropicana, Barbados. The green indicates the raw unfiltered surface track, while the blue lines indicate the post-processed water surface time series. The wave crests have been highlighted and numbered for analysis.

The surface track was then analyzed using both zero-up and zero-down crossing methods to extract the individual wave heights and periods for both methods, which were later compared to determine best practices. Zero-down crossing analysis was found to be more reliable and provide better individual wave descriptions by Mizuguchi [1982]. Mizuguchi found that the zero-up crossing method often failed to account for the bound harmonics that occur in true irregular wave trains. Figure 6-4 illustrates the difference between a zero-up and zero-down crossing analysis.
The extracted individual wave heights, periods and SWL crossing times were stored for later analysis. These values represent the incoming wave conditions and will be referred to as such for the remainder of this thesis.

In order to extract all possible parameters from the surface track time series, the Matlab Wave Analysis for Fatigue and Oceanography (WAFO) toolbox [Brodtkorb et al., 2000] was used to directly calculate wave moments ($m_1 - m_4$), $H_{m0}$ and $T_p$ for each 1.5 hour period. Table 6-1 details the WAFO outputs for each study period.

Table 6-1: WAFO spectral wave parameters

<table>
<thead>
<tr>
<th>Location</th>
<th>Dataset</th>
<th>Moments</th>
<th>Tp</th>
<th>Peak Frequencies</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Hm0</td>
<td>m1</td>
<td>m2</td>
</tr>
<tr>
<td>The Hook</td>
<td>Oct 6, 8:00</td>
<td>1.10</td>
<td>0.076</td>
<td>0.017</td>
</tr>
<tr>
<td></td>
<td>Oct 7, 8:00</td>
<td>0.95</td>
<td>0.056</td>
<td>0.012</td>
</tr>
<tr>
<td></td>
<td>Oct 8, 8:00</td>
<td>0.95</td>
<td>0.050</td>
<td>0.011</td>
</tr>
<tr>
<td></td>
<td>Oct 9, 16:00</td>
<td>0.74</td>
<td>0.034</td>
<td>0.008</td>
</tr>
<tr>
<td></td>
<td>Oct 12, 08:00</td>
<td>0.85</td>
<td>0.046</td>
<td>0.011</td>
</tr>
<tr>
<td></td>
<td>Oct 12, 16:00</td>
<td>0.82</td>
<td>0.042</td>
<td>0.012</td>
</tr>
<tr>
<td>Sewers Peak</td>
<td>Oct 25, 16:00</td>
<td>0.97</td>
<td>0.059</td>
<td>0.019</td>
</tr>
<tr>
<td></td>
<td>Oct 28, 16:00</td>
<td>0.74</td>
<td>0.034</td>
<td>0.008</td>
</tr>
<tr>
<td></td>
<td>Nov 1, 16:00</td>
<td>0.86</td>
<td>0.046</td>
<td>0.011</td>
</tr>
<tr>
<td>Barbados</td>
<td>Nov 28</td>
<td>0.54</td>
<td>0.019</td>
<td>0.007</td>
</tr>
<tr>
<td></td>
<td>Dec 07</td>
<td>0.54</td>
<td>0.017</td>
<td>0.007</td>
</tr>
</tbody>
</table>

6.2.2 Spectral Wave Analysis

In order to get incoming wave direction information, which is unavailable through a surface track analysis, a spectral analysis approach is required. RD Instruments provided the author with a copy of the spectral wave analysis software package, WavesView. Significant wave heights, periods, and directions were automatically calculated via WavesView, which was configured to output $H_{sig}$, $T_{sig}$ and $D_{sig}$ values at 5 minute intervals using overlapping 20 minute record lengths. The 5 minute interval measurements allowed for additional time domain resolution to be extracted from the statistically robust sliding 20 minute wave measurement interval.

A consistent source of uncertainty in many previous wave studies is an inability to differentiate between differing incoming swell regimes. WavesView is able to plot a 3D directional variance spectrum allowing for direction and frequency differentiation between incoming swell conditions. Figure 6-5 illustrates two different incoming swell regimes with similar wave periods (10 s / 12 s) yet different directions (335 ° / 305 °). A more traditional pure surface track study method would not allow for directional differentiation between these two swells and would inherently include unnecessary uncertainties.
Given the detailed and specific choice of study locations, the 3D spectral variance plots were predominantly unidirectional, allowing for easy extraction of the incoming wave direction. However, in situations with significantly different incoming wave periods, the spectral variance plot allowed for differing individual incoming wave directions to be assigned to the different measured wave periods.

### 6.3 Overview Image Processing for Breaking Wave Characteristics

The extraction of wave data at the instant of breaking has traditionally suffered from poor spatial resolution, been cost-prohibitive and featured significant researcher and equipment risk. Presented below is a novel optical technique to extract not only breaking wave heights and positions, but also the depth at the instant of breaking, from a single overview camera. To the best of the author’s knowledge, this study is the first to include extract all the relevant breaking wave characteristics at the instant of breaking, and to achieve this without significant cost or human safety risk.

#### 6.3.1 Breaking Wave Position and Height

The first step in extracting breaking wave heights and positions from overview video footage is rectifying the capture oblique image to the horizontal plan image. In order to rectify the oblique image, four ground control points are required. A single ground control point (GCP) features a video image pixel (x,y) location and the associated physical latitude and longitude coordinate position. In order to collect the GCP, a surfer equipped with a GPS paddled within the surf zone and marked a minimum of 8 different GCPs bounding the surf zone, all while keeping within the overview camera field of view. The pixel locations (x,y) of the surfer and the times of marking the GCPs within the overview image frame were recorded.
A Matlab created transformation matrix geographically corrected the oblique image and output a rectified overview image, with real world coordinate positions assigned to every pixel in the overview video image - see Step #1 in Figure 6-6 (Step #2 will be detailed in section 6.3.2). As part of the rectification process, the vertical effect of varying ocean tide levels required that all the GCPs and the bathymetric models be corrected on an individual wave basis. Tidal information was extracted from published NOAA tidal predictions for Santa Cruz rectifications while directly collected tide staff data from Bridgetown harbour were used for Tropicana corrections. Any effort to use image rectification for wave analysis without detailed tidal correction would be dramatically altered from the true conditions.

Automatic extraction of wave heights from remote video camera systems has been completed previously [de Vries et al., 2010; Gal et al., 2011; Almar et al., 2012; Shand et al., 2012], yet all published automatic methods are prone to sunlight, wind, wave type and localized obstruction based errors. These factors often result in falsely recorded wave heights and positions within automatic algorithms - see Figure 6-7 for two examples. As a result, manual methods were employed in this study to extract the necessary breaking wave height data.
For this study, breaking was defined as the instant when the wave face becomes vertical. The manual extraction of wave heights required the associated identification of the wave crest and trough locations at this instant. The crest location was identified by first noting the broken crest of the wave, indicated by the appearance of white water, and then visually following the wave crest until white water was no longer evident. The breaking wave crest pixel location was marked with an (X), shown in Figure 6-8. The pixel location at the associated wave trough was identified by assessing the trough impact location of the breaking wave jet (O). A line was drawn from (O) to a location along the wave front where the trough could be easily identified via pixel intensity value changes. The final wave height pixel count was measured from (X) to this trough line. Noting that wave jet impacts often do not occur in the wave trough, the locations of (O) were adjusted via the use of wave profile images captured simultaneously at the instant of breaking. These pixel locations were then transferred into real work coordinate positions via the use of the georectification transformation matrix.
The associated wave height could then be calculated using Eq. (92), the known camera height and the relative distances between the camera location and the crest/trough location. Figure 6-9 illustrates the basic geometry required for this calculation. It should be noted that the x-axis of Figure 6-9 changed for every studied wave and represents the direction/distance from the camera to each individual breaking wave. As a result, any measurements in the y-direction are not required. In order to account for tidal effects on individual waves, the camera height was corrected for on an individual wave basis. This effect is generally neglected in automatic algorithms, affecting their resulting accuracy.

\[ H_b = (x_c - x_t) \tan (90 - \theta_c) \]  

(92)

Figure 6-9: Geometric overview camera parameters for wave breaking position and height calculations.

The uncertainties associated with extracting wave height and position values from rectified images depends predominantly on the resolution of the video image and the height of the overview camera [Shand et al., 2012]. Higher elevation cameras will result in better resolution in breaking position, but a loss of resolution in wave height. Conversely, a lower elevation camera will result in lower resolution breaking position, but increased breaking wave height resolution. Figure 6-10 qualitatively illustrates the compromise between camera height and extracted parameter resolutions.

Figure 6-10: Optimization of overview camera height for wave height and location predictions (Altered version from [Shand et al., 2012])
In order to assess the total uncertainties associated with extracting wave breaking positions and heights via the presented remote video rectification method, the individual uncertainties associated with position rectification, height rectification and the manual choice of pixel locations needed to be quantified and totaled.

In order to assess the position accuracy of the rectification process, additional GCPs were compared against those calculated using pixel locations in the rectified image (see examples in Table 6-2). For the Hook location, the average offset variation was +/- 1.58 m, Sewers Peak had an average location variation of +/- 1.60 m, and Tropicana featured +/- 3.8 m. Given that the reported accuracy of the GPS was +/- 3 m, these errors were expected and understandable. The larger wave uncertainty values at Tropicana can be directly attributed to the lower overview camera elevation. It should be noted that some of the positions compared in the above analysis were outside the bounds created by the rectifying GCPs and hence an increase in variation error is expected.

Table 6-2: Overview of position uncertainties between known locations and those predicted from the pixel rectification process.

<table>
<thead>
<tr>
<th>Location</th>
<th>GPS Measured Position (UTM)</th>
<th>Tidally Corrected Rectified Pixel Location (UTM)</th>
<th>Uncertainty (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Northing</td>
<td>Easting</td>
<td>Northing</td>
</tr>
<tr>
<td>The Hook</td>
<td>592276</td>
<td>4090892</td>
<td>592275</td>
</tr>
<tr>
<td></td>
<td>592234</td>
<td>4090808</td>
<td>592232</td>
</tr>
<tr>
<td></td>
<td>592210</td>
<td>4090790</td>
<td>592209</td>
</tr>
<tr>
<td></td>
<td>592186</td>
<td>4090779</td>
<td>592185</td>
</tr>
<tr>
<td>Sewers Peak</td>
<td>591380</td>
<td>4090086</td>
<td>591381</td>
</tr>
<tr>
<td></td>
<td>591399</td>
<td>4090068</td>
<td>591399</td>
</tr>
<tr>
<td></td>
<td>591427</td>
<td>4090133</td>
<td>591426</td>
</tr>
<tr>
<td>Tropicana</td>
<td>213432</td>
<td>1462955</td>
<td>213430</td>
</tr>
<tr>
<td></td>
<td>213499</td>
<td>1462906</td>
<td>213494</td>
</tr>
<tr>
<td></td>
<td>213430</td>
<td>1462962</td>
<td>213423</td>
</tr>
<tr>
<td></td>
<td>213492</td>
<td>1462935</td>
<td>213491</td>
</tr>
</tbody>
</table>

In order to assess the height accuracy of the rectification process, a rectified height was compared against the known height of a videoed item. For The Hook and Sewers Peak, the height of a 1.75 m tall person was calculated from oblique video images and the resulting average calculated uncertainties were 2.3 % and 3.9 % at The Hook and Sewers Peak respectively. Working on the experience gained from The Hook and Sewers Peak deployment, a 12” buoy was deployed just north of the breaking waves, and within the video field of view, for all Tropicana deployments. The sole purpose of the buoy deployment was to provide assurance that the heights predicted from the rectified image agreed with physical dimensions. Calculated buoy height uncertainties were small and varied between 2.1 % and 3.3%.
The wave trough and crest pixel locations were manually selected and thus subject to the pixel resolution and human error. This uncertainty was quantified by manually picking crest and tough pixel positions for all waves on three different occasions. A detailed analysis showed manually chosen pixel positions were selected consistently +/- 1 pixel from the initial estimates and this value was thereafter used to quantify manual human uncertainties. Understanding that the quantitative effect of a single pixel error depends on the camera resolution and number of measured pixels in the wave height, the individual uncertainties varied dramatically between individual waves. The possible position and height uncertainties for all three locations, solely due to the manual choice of pixel locations, are detailed in Table 6-3. Note that the depth uncertainty is discussed in Section 6.3.2 below.

<table>
<thead>
<tr>
<th>Table 6-3: Pixel Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hook</td>
</tr>
<tr>
<td>Position Uncertainty (m)</td>
</tr>
<tr>
<td>Height Uncertainty (m)</td>
</tr>
<tr>
<td>Depth Uncertainty (m)</td>
</tr>
</tbody>
</table>

In order to assess the total uncertainty ($\sigma_{total}$) in the calculated wave height, the previously discussed independent uncertainties of position, height and manual pixel choice were added in quadrature, according to Eq. (93). The low average uncertainty of 4.7 % compares very favourably against previous work involving optical wave measurement techniques. For example, the technique used by Shand et al. resulted in substantially higher uncertainties in their wave height measurements (~7 %). The values presented in Table 6-4 and the comparative decrease in uncertainty when compared against previous work results in high levels of confidence in the outputted wave height measurements.

$$\sigma_{total} = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \ldots} \quad (93)$$

<table>
<thead>
<tr>
<th>Table 6-4: Low Total Breaker Height Uncertainties</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Hook</td>
</tr>
<tr>
<td>Mean</td>
</tr>
<tr>
<td>Minimum</td>
</tr>
<tr>
<td>Maximum</td>
</tr>
<tr>
<td>Std Dev</td>
</tr>
</tbody>
</table>

One constant source of uncertainty when using remote video systems to measure breaking wave characteristics is the impact of local atmospheric conditions, and particularly solar illumination. When the breaking waves were backlit, wave heights were generally measured too high, while the opposite was true when waves were front lit. Additionally, the effect of fog, heavy rain and localized winds all resulted in increased uncertainty of wave defining pixel positions. These effects were noted as sources of uncertainty during the data collection process and partially mitigated by collecting wave data only on days when these effects were minimised.
6.3.2 Breaking Wave Depth

Historically, water depth was commonly cited the only factor in determining the final breaking wave height [McCowan, 1894]. While subsequent research has indicated that the wave period and seafloor slope also play significant roles, water depth is still noted as the dominant determining factor. Unfortunately, the exact definition of the water depth at the instant of breaking varies between published authors and papers [Gourlay, 1994].

As shown in Step #2 of Figure 6-6, all pixel positions in the rectified image were associated with a bathymetric water depth. Therefore, breaking water depths were directly extracted from the bathymetric surface model via the tidally corrected breaking positions of individual waves. For this study, three different methods were employed and compared to identify which definition resulted in the best predictive fit:

- **Water depth at wave trough** ($h_t$): $h_t$ was directly extracted from the bathymetric model, using the pixel position at the optical wave trough position.

- **Corrected water depth at trough** ($h_c$): Laboratory experiments [Flick et al., 1981] and higher order non-linear numerical wave models [Le Mehaute et al., 1968] show the wave trough position to be below the SWL (see Figure 6-11). Additionally, Lin et al. [1998] completed a numerical study of shallow water breaking waves, using Cnoidal wave theory, and numerical analysis of their published breaking figures found the SWL breaking depth at ($h_t + 0.32 \times H_b$), which is extremely close to the ($h_t + 1/3 \times H_b$) predicted by Shand et al. [2012].

Assuming a similar vertical offset ratio of 1/3, the resulting horizontal distance offsets ($x_3$) were calculated using the camera wave trough angle ($\theta_{ct}$) and used to calculate the corrected depth at the true wave trough position using the local seafloor slope and measured bathymetric surface.

---

**Figure 6-11:** Schematic illustrating the difference between the trough depth, $h_t$, and the $h_c$ breaking depth. $H_c$ is referenced against the SWL.
- **Set-down corrected water depth \((h_{sd})\):** Longuet-Higgins [1963] discovered that individual shallow water waves induce shoreward currents and create a consistent water level set-down at the break point. He suggested using Eq. (34) to account for the wave set-down depth reduction at the point of breaking for regular waves. This correction was applied to \(h\) to investigate whether better data fitting could be achieved using a set-down corrected depth.

While the effect of wave set-down had previously been assumed negligible [Shand *et al.*, 2012], the output values for Eq. (34) were deemed significant enough to warrant further analysis. The mean calculated set-down correction at The Hook was calculated to be 0.120 m (standard deviation (SD=0.03 m), 0.15 m at Sewers Peak (SD=0.03 m), and 0.09 m at Tropicana (SD=0.01 m). Given that these values created up to 11.0 % variation in the final breaking depth, they were deemed significant enough to warrant further analysis.

In order to account for tidal variations on individual waves, it was imperative to have detailed tidal information. The US National Ocean and Atmospheric Administration (NOAA) publish daily high/low tide amplitude and timing for the Santa Cruz region. In order to accurately approximate minute by minute tidal depths, the timing and amplitude of NOAA published data was compared to those predicted by WXTide32 [Flater, 2012], using the location 36.5750 °N, -112.01 °E. Excellent correlation was found and minute resolution tidal data were exported from WXTide32 for use on The Hook and Sewers Peak datasets. For Tropicana, tidal information was recorded by a tidal staff in Bridgetown harbor, at location 13.096 °N, -59.619 °W, based on a Lamont datum, and kindly provided by CZMU.

As with the breaking position and wave height measurements discussed earlier, the breaker depth uncertainty associated with the manual collection of wave trough positions was quantified by a +/- 1 pixel uncertainty. This pixel uncertainty was transformed in meter-based depth uncertainties in Table 6-3 above.

The total breaker depth uncertainty from position uncertainties, bathymetric surface variations and manual extraction methods was calculated using Eq. (93). Table 6-5 gives a statistical overview of the total uncertainty levels at each location.

<table>
<thead>
<tr>
<th></th>
<th>The Hook</th>
<th>Sewers Peak</th>
<th>Tropicana</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td>4.9 %</td>
<td>7.3 %</td>
<td>10.0 %</td>
</tr>
<tr>
<td><strong>Minimum</strong></td>
<td>3.0 %</td>
<td>3.6 %</td>
<td>2.7 %</td>
</tr>
<tr>
<td><strong>Maximum</strong></td>
<td>10.0 %</td>
<td>31.0 %</td>
<td>23.0 %</td>
</tr>
<tr>
<td><strong>Std Dev</strong></td>
<td>1.4 %</td>
<td>5.7 %</td>
<td>5.1 %</td>
</tr>
</tbody>
</table>

Table 6-5 shows that the relative uncertainties determined for Sewers Peak and Tropicana were larger than those predicted for The Hook. These heightened values can be directly attributed to bathymetric conditions and overview camera elevations. When considering the novel depth measurement technique
presented in this thesis, the physical seafloor characteristics and theoretical complexity of attempting to
directly determine the effective seafloor depth datum, relative to breaking waves, the uncertainties
presented above are very good.

For Sewers Peak, additional uncertainty was introduced due to substantial kelp forests and seaweed on the
seafloor. When using a 234 kHz transducer, it is expected that the echosounder will record artificially
shallow depth values, due to acoustic reflections from the plants rather than from the true seafloor [RD
Instruments, 2012; Stevens, 2012]. The recordings were taken on a calm day at highest tide, and
kelp/seaweed height would have been maximized during the survey time, resulting in shallower reported
breaking wave depths for Sewers Peak than in reality. This correlates with the slightly higher variation
(~5 %) between the ADCP reported depth and surface models reported in the Chapter 6, Section 1.
However, Cox et al.[2003] found that when the seagrass height ($H_{sg}$) is small with respect to the depth
($H_{sg}/h_b > 0.7$), wave height attenuation will be less than 3%.

For Tropicana, the additional uncertainty was linked to the uneven nature of tropical coral reefs and the
lower overview camera elevation. As shown in Figure 6-10, the reduced camera elevation will increase
positional (and associated depth) uncertainty levels but will also result in lower wave height uncertainties
(see Table 6-4).

### 6.3.3 Breaking Wave Period

In order to analyze the effect of individual wave periods on the breaking height, depth and geometric
shape of waves, a standardized method of extracting breaking wave periods from the measured irregular
wave train was required.

Given the spatial and temporal variation in the breaking position and associated breaking wave period, the
breaking wave period was extracted using a consistent location in the video timestack. A recently
introduced method of analyzing irregular wave breaking in field conditions is via the creation of a video
timestack [de Vries et al., 2010; Gal et al., 2011; Almar et al., 2012; Shand et al., 2012]. A timestack is
created by extracting a single column of pixels from each video frame, and then concatenating them
according to the frame capture time. Using a modified excerpt of the automatic HbStacker code (kindly
provided by Shand et al.[2012]) and the Matlab mmread function, the 540th (of 720) column of pixels
was timestacked for The Hook and Sewers locations, while the 550th column was used for Tropicana
deployments.

An example timestack from Tropicana is shown in Figure 6-12. As shown, the x-axis represents time and
the y-axis represents distance offshore. Using the x-axis position when each individual wave passes a
fixed location, illustrated by the red line in Figure 6-12, individual breaking wave periods were extracted
by measuring the number of pixels between each wave crossing and multiplying by the frames per second
In order to compare differing breaking wave period definitions, the time between the preceding timestack wave and the wave of interest, as well as the time between the proceeding wave and the wave of interest were both extracted.

It is noted that the nomenclature of breaking wave period may be misleading. Given the fixed location used to calculate the wave periods, and the spatial variation in actual breaking conditions, the relative distance between individual breaking waves and the calculation point changes. The extracted wave periods vary between being recorded just prior to breaking, at the instant of breaking and post breaking (i.e. surf zone). Given the immediate proximity of extraction location to the break point, the term breaking wave period will be used throughout this thesis to define all periods calculated in this manner.

Figure 6-12: Excerpt of Tropicana timestack image. Red horizontal line is the known location used for period analyses, while the numbers which waves featured a full suite of breaking characteristics.

6.4 ADCP and Overview Image Wave Parameter Correlation

In order to compare the incoming wave characteristics against those measured at the point of breaking, a method of connecting individual waves measured by the ADCP and those measured by the overview video camera was required. Unfortunately, irregular waves do not feature many consistent parameters to ease the complexity of this task. Wave heights, periods and the geographic location of breaking varied considerably between individual irregular waves.

One of the major goals of this study was to determine the applicability of the vortex ratio as an indicator of breaking “intensity” and investigate the possibility of predicting the vortex ratio from wave characteristics. As a result, only waves with captured vortex profile images of sufficient quality were subjected to detailed incoming to breaking correlation investigations.

The first step in correlating incoming and breaking conditions was to identify individual waves with vortex profile images within the timestack image. The exact instant of capturing the wave vortex profile image was stored within the image metadata, while the overview video metadata stored both the video length and the instant the video stopped recording. A quick time subtraction calculation gave the time associated with the first column of pixels in the timestack image and the camera frames per second rate.
(fps) determined the all subsequent horizontal pixel time values. Individual waves with recorded vortex profile images were then easily identified in the timestack image. Figure 6-12 illustrates three numbered waves that have been identified.

It is important to remember that not all waves passing over the ADCP broke within the field of view of the overview camera. Waves below the critical amplitude will pass through the surf zone without breaking. In order to account for this effect, Mizuguchi [1982] recommended filtering the ADCP surface track data based on a minimum Primary Individual Wave amplitude value, $E_r$. This process removed all waves with amplitudes lower than $E_r$ and ensured a consistent number of waves passed over the ADCP and appeared in the video timestack, over a user defined time period. In order to provide a consistent geographic baseline location for this comparison, a horizontal breaking line was drawn in the timestack image (see Figure 6-12) and only waves that had broken prior to crossing the line were counted. The $E_r$ filter value was then determined to ensure the corresponding number of waves passed over the ADCP over the same time period.

It is important to note that the ADCP surface track data was bandpass filtered to remove the effects of surf beat while waves breaking in the timestack image were affected by surf beat and subject to inherent changes in breaking position. Minor variations between the timestack and ADCP wave counts were expected and the $E_r$ values were chosen to minimize this wave count difference.

Individual waves within an irregular wave train change height, relative position and shape between offshore and breaking locations making direct ADCP to timestack comparisons extremely difficult, if not impossible. In order to correlate individual waves from the timestack with those in the ADCP surface track, the associated wave travel times were required. The calculation of the travel time depends on the wave direction, wave period and bathymetric profile and was calculated according to the steps below:

1. Individual waves undergo significant transformations when approaching the surf zone, yet wave groups will maintain consistent numbers of waves and wave-to-wave periods. For The Hook and Sewers Peak, the approximate travel time was calculated by comparing wave groupings [List, 1991] in the timestack image and ADCP surface track. The Hook and Sewers Peak showed a 54 s and 30 s travel time respectively.

   During the Tropicana deployments, a 12” buoy was tethered to the ADCP seafloor mount. The wave travel time was directly calculated from watching the wave propagate from the ADCP to breaking in the video footage. An average 17 s travel time was measured.

2. In order to calculate the required individual wave travel time, rather than a wave group average, a regular Airy wave with oblique incidence on a plane sloped emergent beach was assumed. The intersection of the curved wave ray, extrapolated seaward from the breaking point, and the wave
crest, passing through the ADCP position was found using Snell’s Law of Refraction (Eq. (94)). Figure 6-13 provides a basic illustration of the wave ray, wave crest and intersection point. Snell’s constant was calculated using measured wave characteristics and celerities at the ADCP position.

\[
\frac{\sin \theta}{c} = \text{constant} \tag{94}
\]

Figure 6-13: Illustration of individual wave ray lines, wave crest lines and associated intersections.

3. The dispersion relationship (Eq. (95))[Holthuijsen, 2008] and bathymetric model were used to calculate the wave celerity and position along the curved wave ray, from the intersection point until breaking, for each individual wave at 1 second intervals.

\[
c = \left(\frac{gT}{2\pi}\right) \sqrt{\tanh \left(\frac{4\pi^2 h}{T^2 g}\right)} \tag{95}
\]

Unfortunately, calculation of the wave celerity using Eq. (95) relied on knowing the wave period a priori. Given that one of the goals of this correlation exercise was to calculate the respective individual wave periods, this value was still unknown and the method was abandoned.

4. Given that the water depth to wavelength ratios at the ADCP position were approximately 1/18, it was assumed that wave celerity was independent of wave period and only depended on the depth using the shallow water approximation [CERC-EW, 2008]:

\[
c = \sqrt{gh} \tag{96}
\]
Numerical analysis of the variation arising from the shallow water approximation led to a negligible wavelength variation of 0.01% from the dispersion relationship. This was due to the “almost” shallow water nature of the ADCP position, and the short distance between the ADCP and location of wave breaking. As a result, the shallow water approximation was considered valid.

It was noted that the wave period plays a very small role in the predicted positions, while small changes in wave depth and incoming direction create differences in theoretical travel times.

5. Theoretical travel times were calculated using the curved wave ray distance and individual wave celerities. Comparison of the calculated wave crest timing at the ADCP location and the directly measured wave crest timing showed the mean time difference at The Hook to be 1.7 s, with a standard deviation of 1.6 s. Recognizing that the average recorded wave period at The Hook was 18.5 s, the resulting 9.2% uncertainty was not sufficiently large to lead to incorrect identification of a single corresponding wave in the both the ADCP surface track and timestack image.

For Sewers Peak, the mean offset between calculated and measured wave crest timing was 3.0 sec, with a standard deviation of 2.1 s. For Tropicana, the mean offset between calculated and measured wave crest timing was just 0.8 sec, with a standard deviation of 0.6 s. When compared to the average dominant recorded wave period during the study times of 16.4 s and 10.9 s for Sewers Peak and Tropicana respectively, the associated 18.3% and 7.3% uncertainties were still small enough to ensure the correct identification of individual waves.

These levels of uncertainty were expected and could be due to numerous factors; manual extraction of timestack location pixel positions, individualized wave transformations, slight directional spread in individual wave directions and wave celerity changes due to tidal and wave set-up changes. However, as noted above, the resulting uncertainties were not large enough to result in incorrect wave pairings.

6. As a result of the presented travel time calculations, an individual wave from the ADCP surface track could be correlated to the associated breaking wave in the timestack image. Hence for every wave with a vortex profile image, both the breaking and incoming wave characteristics could be calculated.

Variations in the travel time offset was expected since the ADCP was not located in true shallow water wave conditions \((h < L_c/20)\), and the individual wave celerity \((c_b)\) was not consistent with the group velocity \((c_g)\).
A direct comparison between the group velocity and individual wave celerity, using the maximum and minimum recorded wave periods for each location, showed The Hook to have a 2.0% - 8.9% difference between group and individual wave celerity whereas Sewers Peak and Tropicana showed 1.6% - 10.0% and 3.8% - 7.9% respectively.

These variations in wave celerity explain why sequential waves found in the timestack were not always sequential in the ADCP surface time series. Waves in the front of wave groups at the ADCP position were found to be diminished or missing in the timestack image, while an additional wave was found at the end of the wave group. This is supported by standard deep water wave dispersion relationships [Holthuijsen, 2008]. It is noted that the dispersion relationship is based on regular wave trains, not irregular wave trains, and therefore additional variation could be expected.

6.4.1 Incoming Wave Period Analysis

As previously presented in the breaking wave height analysis, a standardized method of extracting incoming individual wave periods was required for further analyses. The need for a formalized method to extract breaking wave periods was further illustrated by the inconsistent values associated with the breaking and incoming wave period.

Differing methods of calculating both the breaking and incoming wave period are available. For the incoming wave period, the ADCP surface track data was analyzed using a zero-up crossing method, a zero-down crossing method, and crest-to-crest method. For the breaking wave period, the timestack image was analyzed to extract the period between the wave of interest and both the preceding and proceeding waves. The three incoming period measurement methods were then compared against the two breaking wave period techniques to determine which two methods found the highest correlation and lowest interval difference.

The results showed that a zero-up crossing analysis method was inferior to the zero-down crossing analysis method (Table 6-6) as predicted by Mizuguchi et al. [1982]. Mizuguchi et al. suggested that the zero-down crossing wave period best represented the breaking period, yet in this study shows that the individual crest-crest wave period was consistently better correlated with the breaking wave timestack than all other zero crossing methods. As shown in Table 6-6, the lowest variation was found using the individual crest-crest ADCP and the preceding wave timestack periods (3.1s). Thus for all proceeding calculations and breaking wave relationship development, the incoming wave period was defined by the ADCP-derived individual crest-crest wave period and the breaking wave period was determined by the preceding wave timestack period.
Table 6-6: Mean Wave Period Analysis Breakdown

<table>
<thead>
<tr>
<th>Standard Deviation (Percentage)</th>
<th>Preceding Wave</th>
<th>Proceeding Wave</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zero-Down Crossing (ZDC)</td>
<td>4.9 s (31 %)</td>
<td>5.4 s (30 %)</td>
</tr>
<tr>
<td>Crest – Crest</td>
<td>3.1 s (19 %)</td>
<td>4.6 s (26 %)</td>
</tr>
<tr>
<td>Zero-Up Crossing (ZUC)</td>
<td>5.7 s (36 %)</td>
<td>5.3 s (29 %)</td>
</tr>
<tr>
<td>Average (ZUC, ZDC)</td>
<td>4.7 s (30 %)</td>
<td>4.7 s (26 %)</td>
</tr>
</tbody>
</table>

6.4.2 Seafloor Slope Analysis

The effective bathymetric slope plays a determining role in governing breaking height, depth, and shape [Couriel et al., 1998], and the individual effective slope is a function of wave direction, wave ray method (curved or straight), and the relevant geographical distance. Beyond these basic tenets, the methods of calculating the effective seafloor slope vary considerably.

The effective slopes reported in published field studies vary between macro-scale features such as coastline approach slopes to meso-scale features such as instantaneous bar slopes. In laboratory tests, the included slope lengths are often based on the physical constraints of the constructed beach profile and flume size, rather than being based on the wave specific effective slope definitions. Most methods are based on averaging the slope of a defined geographic surface (x, y) distance. For example, the Coastal Engineering Manual (CEM)[CERC-EW, 2008] and Dally[1992] recommend that “for non-uniform beach slopes, the average bottom slope from the break point to a point one wavelength offshore should be used”.

Given that wave breaking is highly dependent on water depth and the effective seafloor slope should be dependent on incoming wave characteristics, a novel method of extracting wavelength dependent effective seafloor slopes was proposed and investigated to see if improved predictive performance could be achieved. Seafloor slopes were averaged from the breaking depth ($h_b$) to an individual 1/2 wavelength, 1/3 wavelength or 1/4 wavelength dependent depth. Figure 6-14 and Eqs. (97) – (99) illustrate and detail the novel effective seafloor slope calculations.

Figure 6-14: Illustration of suggested wavelength dependent effective seafloor slope configurations
\[ m_{1/2} = \left( \frac{h_b + \frac{L_b}{2}}{x_{1/2}} \right) \]  
\[ m_{1/3} = \left( \frac{h_b + \frac{L_b}{3}}{x_{1/3}} \right) \]  
\[ m_{1/4} = \left( \frac{h_b + \frac{L_b}{4}}{x_{1/4}} \right) \]

In order to determine if the newly presented definitions of effective seafloor slope improved prediction calculations, the CEM method was compared against \( m_{1/2} \), \( m_{1/3} \) and \( m_{1/4} \). The values for \( m_{1/2} \), \( m_{1/3} \) and \( m_{1/4} \) are calculated along the representative curved wave rays, in contrast to previous field studies using simple straight wave rays [Mead and Black, 2001]. Table C-2 presents a review of all extracted slope values.

Table 6-7: Example of differing effective slope values for five randomly chosen waves. CEM (Straight) indicates using the CEM method along a straight wave line, CEM (Curved) indicates using the CEM method along a curved wave ray.

<table>
<thead>
<tr>
<th>Wave #</th>
<th>CEM (Straight)</th>
<th>CEM (Curved)</th>
<th>Mead</th>
<th>( m_{1/2} )</th>
<th>( m_{1/3} )</th>
<th>( m_{1/4} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>114</td>
<td>0.064</td>
<td>0.069</td>
<td>0.018</td>
<td>0.026</td>
<td>0.047</td>
<td>0.064</td>
</tr>
<tr>
<td>115</td>
<td>0.061</td>
<td>0.064</td>
<td>0.024</td>
<td>0.027</td>
<td>0.044</td>
<td>0.067</td>
</tr>
<tr>
<td>116</td>
<td>0.037</td>
<td>0.041</td>
<td>0.036</td>
<td>0.027</td>
<td>0.045</td>
<td>0.059</td>
</tr>
<tr>
<td>117</td>
<td>0.035</td>
<td>0.036</td>
<td>0.038</td>
<td>0.027</td>
<td>0.049</td>
<td>0.061</td>
</tr>
<tr>
<td>118</td>
<td>0.045</td>
<td>0.059</td>
<td>0.051</td>
<td>0.026</td>
<td>0.047</td>
<td>0.067</td>
</tr>
</tbody>
</table>

Five randomly chosen waves and associated effective slopes were computed via the differing presented methods and are shown in Table 6-7 to illustrate the magnitude of slope changes. The effective seafloor slope varies considerably with incoming wave direction changes, which introduces uncertainty in the extracted values. Table 6-8 illustrates that by varying the incoming wave direction by a nominal 5°, the resulting changes in \( m_{1/3} \) and CEM slope are substantially larger. The value \( m_{1/3} \) was randomly chosen to represent the lower sensitivities of \( m_{1/2} \), \( m_{1/3} \) and \( m_{1/4} \) when compared against the CEM method. Given the amount of refraction occurring between breaking and the ADCP, and the directional spread of \( \theta_p \), it is realistic to assume the total possible variance in the incident wave directions to be as high as 30°. Hence a 5° variation is minimal yet illustrates the high sensitivity of the seafloor slope. The impact of this variation on the predicted wave heights is further discussed in subsequent sections of this thesis.

Table 6-8: Slope Sensitivity Analysis

<table>
<thead>
<tr>
<th>1.5 % Change (5 deg)</th>
<th>( m_{1/3} ) Slope</th>
<th>CEM Slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tropicana</td>
<td>6.5 % Change</td>
<td>18 % Change</td>
</tr>
<tr>
<td>Sewers Peak</td>
<td>4.5 % Change</td>
<td>18 % Change</td>
</tr>
<tr>
<td>Hook</td>
<td>12 % Change</td>
<td>16 % Change</td>
</tr>
</tbody>
</table>
6.5 Wave Profile and Vortex Processing

The final parameter to be extracted from the breaking wave data was the breaking vortex ratio and vortex angle. Capturing breaking wave profile images from irregular waves under field conditions proved to be more difficult than initially expected. However, through in-water videography and land-based photography sufficient high quality images were captured and used to extract vortex parameters.

Prior to extracting detailed breaking vortex parameters, each individual vortex profile image needed to be correlated to its corresponding wave in the timestack image. Using the time metadata from the vortex profile image and the overview camera, each individual wave vortex image was easily connected to timestacked wave. As part of the daily experiment setup procedure, both video cameras and the still camera were time synchronized to minimize time variance between measurement devices. However, the still camera metadata only recorded the instant the profile image was captured to a single second resolution. This resulted in a small amount of uncertainty with regards to the exact instant of correlation between the overview camera and profile camera. Understanding that this time uncertainty could not be reduced further via equipment manipulation, the author would study each of the 12 video frames collected every second to find the best visual correlation between the vortex and overview images. However, it should be noted that even in the worst case scenario with approximately 1 s variation, the measured breaking wave height and position variations are small and were calculated to be less than 2 %.

The captured vortex profile images were of differing quality and were manually binned into four different vortex quality bins by visual assessment. The quality of the vortex images were heavily affected by the quality of the wave vortex due to water users (surfers), local wind conditions, inconsistencies in the water flow in the wave face (“boils”) and wave reflections, as well as the quality of the captured image due to oblique capture angles, lens distortion and camera focus. Vortex image quality Bins #1 and #2 represented vortex profile images of excellent and good quality respectively, Bin #3 collected vortex profile images of dubious quality, while Bin #4 images were unusable and removed from the study. Profile images from Bin #1 - #3 were then imported into ImageJ [Rasband, 1997 - 2012] for vortex parameter processing. Using ImageJ, a cubic function was fitted using four defining vortex points: the point of intersection of the wave jet and wave trough (point #1), the vortex “turn-over” point (point #2), and the two points of maximum vortex width (point #3 & #4). Figure 6-15 gives a visual illustration of the four defining points.
ImageJ automatically measured length, width and angle of the fitted vortex length (see Figure 2-12 for a reminder of vortex length, width and angle definitions). The vortex ratio, $Y$, was calculated according to Eq. (33) and all the vortex angles were corrected to be lower than 90°.

In order to quantify the uncertainty in the pixel locations which define the fitted cubic functions, and the associated variations in the vortex lengths and widths, a maximum and minimum vortex length and width were also measured from the profile images. Analyses showed no discernible trend or skewedness when comparing the maximum and minimum vortex ratio values against the best fit line. Mean upper uncertainty was 36% while the mean lower uncertainty was 39%. While individual vortex ratios featured different upper and lower error values, the lack of a general trend indicates an unbiased measurement technique. Using the extracted vortex maximum and minimum values, a total percentage variation from the fitted cubic vortex ratio was computed according to Eq. (100):

$$
\%_{uncert} = 100 \left( \left| \frac{Y_{max} - Y}{Y} \right| + \left| \frac{Y_{min} - Y}{Y} \right| \right)
$$

(100)

In order to assess the validity of the manual quality binning system, the relative $\%_{uncert}$ values and the visual binned values for Bin #1, #2 and #3 were compared (Note: Vortex images in Bin #4 were not included in this comparison due their inherent lack of quantifiable vortex parameters). A correlation was found between the relative $\%_{uncert}$ values and the visual binned datasets, indicating that the visual system of rating vortex profile images is sufficient for rough quality analyses. Details of this comparison are given below:
- **The Hook:** The lowest 20% of waves by $\%_{\text{uncert}}$ had a mean rating of excellent (1.3) while the top 20% of waves by $\%_{\text{uncert}}$ featured a mean rating of dubious quality (2.6).

- **Sewers Peak:** The lowest 20% of waves by $\%_{\text{uncert}}$ had a mean rating of excellent (1.3) while the top 20% of waves by $\%_{\text{uncert}}$ featured a mean rating of dubious quality (2.7).

- **Tropicana:** The lowest 20% of waves by $\%_{\text{uncert}}$ had a mean rating of excellent (1.4) while the top 20% of waves by $\%_{\text{uncert}}$ featured a mean rating of dubious quality (2.6).

The $\%_{\text{uncert}}$ variations can be attributed to numerous complicating variables. Marginally oblique views will result in reported vortex ratios being slightly lower than a true perpendicular profile. Localized winds will cause the vortex cavity to collapse or expand depending on wind directions. Camera lens water droplet distortion, wave shoulder refraction (eliminating a perfect view of the tough/lip impact zone), water “boils” and reflected wave-wave interactions all complicate the extraction of a pure vortex profile. It should also be noted that every incoming wave did not plunge and create a vortex. Individual waves within the irregular wave train broke in differing manners and hence the individual properties of each wave played a role in whether they plunged or spilled. Additionally, the presented method is based on a ratio of pixel values, not physical measurements. Hence different camera resolutions and distances between the camera and breaking waves altered the relative importance of a single pixel and associated $\%_{\text{uncert}}$ levels.

To the best of the author’s knowledge, the dataset of breaking wave profiles and vortex parameters collected for this thesis is without peer and the most detailed ever collected from field conditions. Combined with quantified knowledge of all breaking wave characteristics, this dataset allowed for first detailed analysis of vortex prediction methods and possible “intensity” categorizations based on measured field data.
7 Investigation of Breaker Parameters collected via remote methods

In the previous chapter, individual wave-by-wave characteristics were extracted from an irregular wave train. Incoming wave heights, periods and directions were captured by an ADCP deployment, while breaking wave heights, periods, depths and effective seafloor slopes were calculated through detailed analyses of the overview video footage and bathymetric surfaces.

Chapter 7 details the analysis of the collected data to find trends and dependencies between the measured breaking and incoming wave characteristics. New breaking water depth and effective seafloor slope definitions were compared based on the collected dataset. Finally, the collected data was compared to the most cited and relevant published breaking wave height predictors in order to assess their correlation to the extracted dataset.

As a precursor to the detailed analyses to be completed, it is important to ensure analyses include all parameters that influence the final breaking wave height are tested and investigated. The wave height at breaking is a function of the breaker depth, seafloor slope, wavelength (or wave period) and angle of incidence.

\[ H_b = f(h_b, m, L_b, \theta) \]

Given that the pre-breaking wave height, measured by the ADCP, was not recorded in true deep water conditions and the extensive amount of refraction-based amplitude reduction that occurred before breaking at each location, this study scope did not include an analysis of the influence of \( H_0 \) on \( H_b \). In order to determine correlation between these two parameters, a full three dimensional time domain refraction/diffraction numerical analysis would be required for every individual wave at each location. This study follows a similar procedure to many others by determining the final breaking height based on breaking wave conditions (See Chapter 3.1.1.4 and Chapter 3.1.1.6). Conversely, the relationship between \( L_0 \) and \( L_b \) can easily be determined using the dispersion relations (Eq. (3)).

The novel effective slope measurement techniques suggested in Chapter 6.4.2 inherently includes the effects of both \( L_b \) and \( \theta \) due to the fact that the slope is calculated over a more realistic curved wave ray. This is considerably different to the majority of previous studies which simply calculated the effective slope along a beach perpendicular transect, despite the fact that the wave is known to follow a curved ray path. As a result, the effect of \( \theta \) is inherently included within the seafloor slope and not investigated as an independent variable.

An abridged version of Chapter 5, Chapter 6 and Chapter 7 was accepted for publishing in the peer reviewed Journal of Coastal Research on April 16th, 2013 [Robertson et al., 2013].
7.1 Data Overview and Error Analysis Methods

During the four month data collection period, only eleven days were considered suitable for wave data collection. In total, 28 hours of data were collected and analyzed. Thousands of waves broke during those 28 hours of analyzed video footage; however only those waves featuring a full suite of breaking characteristics were included in the final dataset. The required wave characteristics for inclusion in the analyzed set included wave height, water depth, wave period, wave vortex ratio and vortex angle. Table 7-1 gives a brief overview of the mean, bounding maximum and bounding minimum wave characteristics for the entire collected dataset. For a detailed wave-by-wave breakdown of all characteristics, refer to Table C-1 and Table C-2 in the Appendix C.

<table>
<thead>
<tr>
<th>Table 7-1: Study Data Overview</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hook, Santa Cruz</td>
</tr>
<tr>
<td>Number of Waves</td>
</tr>
<tr>
<td>Wave Heights (m)</td>
</tr>
<tr>
<td>Hc Depths (m)</td>
</tr>
<tr>
<td>Trough Depths (m)</td>
</tr>
<tr>
<td>Wave Periods (sec)</td>
</tr>
<tr>
<td>Slope - CEM</td>
</tr>
<tr>
<td>Slope - m_{1/3}</td>
</tr>
</tbody>
</table>

Through the following analyses, reference will be made to robust fit and direct comparison values. The robust fit values illustrate the best possible statistical correlation between the measured data and the regression fit, while the comparison data details the direct comparison of measured and predicted values. Robust fitting assigns a weighting to each data point, based on its distance from the fitted line, and thus ensures that individual outlying data points do not affect the general data trend and associated uncertainty values. First implemented by DuMouchel and O’Brien [1991], robust fitting has subsequently been used for image filtering, signal processing, co-operative purchasing assessments and a multitude of other fields.

Additionally used to evaluate the correlation between predicted and measured data, the verification results will be also be analysed using the direct comparison root mean square error (RMSE) and root mean square relative error (ER) values. ER presents the RMSE error as a relative error, according Eq. (101) [Rattanapitikon and Shibayama, 2000]:

\[
ER = 100 \sqrt{\frac{1}{n} \sum_{i=1}^{n} (H_{bc} - H_{bm})^2 / \sum_{i=1}^{n} H_{bm}^2}
\]

(101)
7.2 Breaker Depth and Seafloor Slope Definition and Optimization

A detailed analysis of the best breaking wave depth and effective seafloor slope definitions is required prior to analysis of the correlation between published predictors and the remotely collected breaking wave characteristics. The physical descriptions of the compared breaking depth definitions and effective seafloor slopes are included in Chapter 6.

7.2.1 Breaking Depth Optimization

In order to assess the best definition of breaking depth, three variations of breaking depths \( h_t, h_c, \) and a set down corrected \( h_{sd} \) were plotted against the breaker height for each location and the entire dataset separately. The best breaking wave water depth definition and measurement technique was then determined by analyzing the statistical uncertainty and variance measurements between a fitted linear robust regression line and the collected data.

Table 7-2 details the y-intersect, linear regression slope, root-mean-square error (RMSE) and robust \( R^2 \) values for all the locations and depth definitions.

<table>
<thead>
<tr>
<th>Location</th>
<th>Breaking Height vs. Breaking Depth</th>
<th>Constant (m)</th>
<th>Rgs Slope</th>
<th>RMSE (m)</th>
<th>Robust ( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Hook</td>
<td>Wave Trough</td>
<td>0.03</td>
<td>1.00</td>
<td>0.24</td>
<td>0.65</td>
</tr>
<tr>
<td></td>
<td>Corrected Depth</td>
<td>-0.10</td>
<td>1.02</td>
<td>0.23</td>
<td>0.71</td>
</tr>
<tr>
<td></td>
<td>Setdown Depth</td>
<td>1.20</td>
<td>0.46</td>
<td>0.36</td>
<td>0.29</td>
</tr>
<tr>
<td>Sewers Peak</td>
<td>Wave Trough</td>
<td>0.88</td>
<td>0.93</td>
<td>0.29</td>
<td>0.69</td>
</tr>
<tr>
<td></td>
<td>Corrected Depth</td>
<td>0.50</td>
<td>0.85</td>
<td>0.23</td>
<td>0.82</td>
</tr>
<tr>
<td></td>
<td>Setdown Depth</td>
<td>0.57</td>
<td>0.82</td>
<td>0.22</td>
<td>0.82</td>
</tr>
<tr>
<td>Tropicana</td>
<td>Wave Trough</td>
<td>0.57</td>
<td>0.96</td>
<td>0.15</td>
<td>0.48</td>
</tr>
<tr>
<td></td>
<td>Corrected Depth</td>
<td>0.32</td>
<td>0.73</td>
<td>0.10</td>
<td>0.80</td>
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<tr>
<td></td>
<td>Setdown Depth</td>
<td>0.68</td>
<td>0.56</td>
<td>0.12</td>
<td>0.67</td>
</tr>
<tr>
<td>All Waves</td>
<td>Wave Trough</td>
<td>0.56</td>
<td>0.96</td>
<td>0.27</td>
<td>0.75</td>
</tr>
<tr>
<td></td>
<td>Corrected Depth</td>
<td>-0.14</td>
<td>1.04</td>
<td>0.22</td>
<td>0.84</td>
</tr>
<tr>
<td></td>
<td>Setdown Depth</td>
<td>0.21</td>
<td>0.91</td>
<td>0.22</td>
<td>0.82</td>
</tr>
</tbody>
</table>

The parameter \( h_t \) consistently had the most variation indicated by the largest RMSE and lowest correlation values. Unfortunately, correcting for wave induced set down only increased the variation and resulted in lower correlation results than \( h_c \). Given that the set-down relationship is based on regular wave
experiments over a parallel bottom seafloor [Longuet - Higgins and Stewart, 1963], Eq. (34) may not be applicable in this situation.

The parameter $h_c$ displayed the lowest RMSE and highest correlation ($R^2 = 0.84$), across all individual datasets and the full combined dataset, suggesting that it may be the best descriptor for the breaking depth within the conditions tested. This is expected since $h_c$ is corrected to the SWL datum and accounted for in the optical variation in the wave trough location. It is also noted that $h_c$ agrees with the numerical theory of Lin et al. [1998] and wave flume experiments of Flick [1981].

![Figure 7-1: Wave face slope variation effect on breaking depth.](image)

Further analysis was conducted to see if better correlation could be found by correcting for the front wave face slope. Figure 7-1 illustrates how a breaking wave generally does not exhibit the classical theoretical saw tooth shape with the entire front face becoming vertical at the same instant, but rather only the crest becomes vertical and the wave face features a non-vertical overall slope. A basic sensitivity study was conducted by extracting breaking depths at various ratios of $x_j$ to determine if the wave face slope, and associated change in zero-crossing location, influenced the final correlations between breaking height and depth. As shown in Table 7-3, an optimized correlation was found using $1.3x_j$ to account for the wave face slope.

<table>
<thead>
<tr>
<th>Location</th>
<th>Breaking Height vs. Breaking Depth</th>
<th>Constant</th>
<th>Rgs Slope</th>
<th>RMSE</th>
<th>Robust $R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Waves</td>
<td>$1.0x_j$ Corrected Depth</td>
<td>-0.14</td>
<td>1.04</td>
<td>0.22</td>
<td>0.84</td>
</tr>
<tr>
<td></td>
<td>$1.3x_j$ Corrected Depth</td>
<td>0.16</td>
<td>0.88</td>
<td>0.20</td>
<td>0.85</td>
</tr>
<tr>
<td></td>
<td>$1.5x_j$ Corrected Depth</td>
<td>-0.25</td>
<td>0.98</td>
<td>0.26</td>
<td>0.75</td>
</tr>
</tbody>
</table>

The trend between breaker height and depth, using the $1.3x_j$ correction, is shown in Figure 7-2. Each individual site exhibits very similar trend lines to the final fitted regression line (Eq. (102)), hence reinforcing the relationship.
Given the known dependency of the breaker height on the wave period and seafloor slope, it is assumed that improved $R^2$ values will result from individual site specific regression fits. However, it is important to note the general trend in Figure 7-2 is similar between all three sites and the low y-intercept value of 0.16.

Goda [2010] recognized a 14% coefficient of variability (CoV) inherent within regular breaking waves, which are illustrated by the black dotted lines in Fig. 12. When investigating irregular waves, Goda analyzed the wave data of Seyama and Kimura [1988] finding 18% - 23% CoV for their irregular wave, laboratory based study. In the current irregular wave study, an 18% CoV captured more than 90% of all measured data points.

![Figure 7-2: Relationship between breaker height and breaker depth for the three field sites.](image)

Given that these parameters were extracted from field sites, using low cost, remote techniques, on irregular waves, the CoV for this study illustrates the comparative accuracy of using $h_c$ to define the breaking depth. Moving forward, it is concluded that the breaking wave water depth ($h_b$) is best represented by $h_c$ when using optical methods to remotely measure breaking waves. Given the trend towards increasing use of remote optical measurements systems, this novel method of the calculating the breaking depth enables far greater ability to analyse surf zone processes. Additionally, it is noted that individually measured irregular waves feature marginally larger CoV values, than regular waves, and may be treated as regular waves when investigated on an individual basis (as per the findings of Mizuguchi [1982]).

Noting the distribution of data points for each individual location, improved $R^2$ values are found when fitting exponential and logarithmic functions to the data. However, the goal of Figure 7-2 was to investigate a site independent trend between the breaker height and only the breaker depth for two
reasons: As a direct comparison against McCowan’s 1894 equation, and to isolate which depth definition found the best correlation.

7.2.2 Seafloor Slope Optimization

As described earlier, a consistent method and definition of the amount of seafloor slope responsible for the breaking conditions of waves is undetermined. In Chapter 6, a novel method of using a wavelength-dependent depth range was suggested to be more applicable than a simple wavelength dependent distance range.

In order to check the assumption that the effective seafloor slope for each breaking wave should be calculated over a wavelength dependent depth range, and not a wavelength dependent surface distance, two differing methods were used. Initially, $m_{1/2}$, $m_{1/3}$, and $m_{1/4}$ were compared to the slope extracted over a full wavelength, as suggested by the Coastal Engineering Manual [CERC-EW, 2008]. Each slope was plotted, in conjunction with the non-dimensionalized water depth, to predict the breaker height, and the robust $R^2$ values compared (see Figure 7-3). As shown in Table 7-4, the lowest RMSE values and highest $R^2$ values were found using $m_{1/3}$ and $m_{1/4}$, while the slope calculated according the CEM methods resulted in the lowest correlation with the measured data.

![Figure 7-3: Slope analysis comparison figures.](image-url)
The robust \( R^2 \) values in Table 7-4 are quite low, thus indicating a poor fit and weak direct correlation between the plotted variables. In order to provide additional evidence to the individual performance characteristics of each slope definition and to evaluate which effective slope definition provided the best correlation with the measured dataset, the four effective slope variations were used as inputs to Goda 2010 (Eq. (76)), Rattankipition and Shibayama 2000 (Eq. (75)), Kamphuis 1991 (Eq. (82)) and Smith and Kraus 1991 (Eq. (78)).

<table>
<thead>
<tr>
<th>Slope Definition</th>
<th>Constant</th>
<th>Rgs Slope</th>
<th>RMSE</th>
<th>Robust ( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/3 Slope</td>
<td>0.60</td>
<td>-0.11</td>
<td>0.093</td>
<td>0.20</td>
</tr>
<tr>
<td>1/2 Slope</td>
<td>0.64</td>
<td>-0.11</td>
<td>0.099</td>
<td>0.13</td>
</tr>
<tr>
<td>1/4 Slope</td>
<td>0.61</td>
<td>-0.10</td>
<td>0.093</td>
<td>0.20</td>
</tr>
<tr>
<td>Full Slope</td>
<td>0.86</td>
<td>-0.03</td>
<td>0.11</td>
<td>0.016</td>
</tr>
</tbody>
</table>

These four equations are often cited as the best predictors predicting breaking when investigating regular waves (Goda, Rattankipition and Shibayama), irregular waves (Kamphuis) and barred slopes (Smith and Kraus). As shown in Table 7-5, the robust \( R^2 \) values are greatly improved from Table 7-4. Considering the novel techniques introduced in this thesis and the randomness of waves breaking the field, the calculated \( R^2 \) values are very good.

Table 7-5 confirms that the breaking wave height was consistently better characterized using an effective breaking wavelength–dependent depth slope, rather than the published suggestion of a wavelength-dependent surface distance slope. Using \( m_{1/4} \) seems to consider a section of bathymetric profile that is too short to fully account for the breaking conditions, whereas \( m_{1/2} \) seems to include a slope that plays a less dominant role in breaking conditions.

<table>
<thead>
<tr>
<th>Location</th>
<th>( m_{1/4} ) ( R^2 )</th>
<th>( m_{1/3} ) ( R^2 )</th>
<th>( m_{1/2} ) ( R^2 )</th>
<th>CEM Slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hook</td>
<td>0.81</td>
<td>0.80</td>
<td>0.79</td>
<td>0.80</td>
</tr>
<tr>
<td>Sewers Peak</td>
<td>0.79</td>
<td>0.82</td>
<td>0.82</td>
<td>0.80</td>
</tr>
<tr>
<td>Tropicana</td>
<td>0.73</td>
<td>0.77</td>
<td>0.76</td>
<td>0.61</td>
</tr>
<tr>
<td>Average</td>
<td>0.77</td>
<td>0.80</td>
<td>0.79</td>
<td>0.74</td>
</tr>
</tbody>
</table>

It is concluded that \( m_{1/3} \) best represents the breaking wave effective slope for the conditions tested and it is suggested that Eq. (98) should be tested against a more diverse dataset before finding complete acceptance. For laboratory work, the physical dimensions of the desired slope and tank depth should determine the maximum possible created breaking wavelengths, while field studies should ensure sufficient bathymetric data is collected to allow for calculation of Eq. (98).
7.3 Traditional Wave Parameter Categorization

Given that all the relevant wave characteristics were extracted and defined by consistent optimized methods, it was possible to investigate the predictive ability of traditional non-dimensional wave parameters. The Ursell number quantifies the degree of non-linearity of a wave form from deep water into shallow water regions, and suggests which wave theory is most applicable (see Figure 2-2). The Ursell number was investigated using ADCP collected data prior to the surf zone. When the wave begin to breaks, the Ursell number is no longer applicable and the breaking SSP is the most commonly cited parameter for wave categorization.

7.3.1 Ursell Number

For each individual wave, the Ursell number was calculated using values extrapolated at the intersection of the ADCP wave crest line and the breaking wave ray (see Figure 6-13). The wavelength parameter required in Eq. (8) was calculated using the ADCP crest to crest wave period, due to its improved correlation to the breaking wave period (see Table 6-6). The depth was calculated at the wave crest / wave ray intersection point and the wave height was assumed constant along the wave crest.

The calculated mean Ursell number for the wave breaking at The Hook was 53.64, Sewers Peak was 21.58 and Tropicana was 62.93. These higher values are expected since the waves at the ADCP position are all well within the intermediate depth zone and almost reaching critical steepness values. The lower Ursell values for Sewers Peak can be directly attributed to the higher depth values at the intersection point.

Given that all Ursell numbers are greater or relatively close to 26 indicates that Cnoidal or Solitary Wave theories are the most valid to describe the wave form and associated particle paths at the intersection point. Additionally, this corroborates why $h_c$ may be the best descriptor for the breaking wave water depth.

7.3.2 Surf Similarity Parameter

The SSP, as defined by Eq. (30) and Eq. (83), was developed to provide a quantitative method of categorizing all shallow water wave breaking events into three broad categories. However, the SSP has been proven to be limited and unable to predict the breaking profiles of waves in non-ideal conditions.

Physically all the waves analyzed in this study featured an enclosed breaking vortex and are classified as plunging waves. The breaking conditions of all the individual waves were extracted, and it was possible to directly compute the breaking SSP using Eq. (83). As shown in Figure 7-4, Iribarren and Nogales [1949] SSP classification system was unable to accurately predict and categorize the breaker type of the
studied waves. Approximately half of all the breaking waves were incorrectly classified as spilling breakers. The lack of direct offshore wave height measurements made it impossible to directly calculate $\xi_0$ using Eq. (30), $S_o$ using Eq. (45) or Yaos’ [2012] $h_b/H_o$ values: breaking condition predictors were therefore used.

Figure 7-4: Calculated breaking SSP values comparison

Camenen and Larson[2007] predicted that Eq. (47) and the values presented in Table 3-3 would categorize breaking events better than the SSP. Figure 7-5 illustrates that while Camenen and Larson’s methods did provide better breaker type predictions, there was still disagreement with experimental results.

Figure 7-5: Camenen and Larson breaking condition comparison

The exact division between different categories of breaking waves is based on visual geometric wave profiles and hence is subject to individual interpretation. A breaking wave could be classified as spilling by one observer and plunging by another. Hence some variability within the results was expected. However, Figure 7-4 and Figure 7-5 illustrate that neither classification system is robust enough for predicting the breaking forms of irregular waves over complex seafloor profiles.
7.4 Breaker Index Parameter Prediction Methods

The non-dimensional breaker index \( (H_b/h_b) \) is one of the most frequently cited shallow water breaking wave characterization parameters. A detailed review of breaker index and breaker height predictors was included in Chapter 3. In this section, the effect of individual wave characteristics on the breaker index was investigated to extract basic trends and dependencies. Table 7-6 gives an overview of the linear regression fit characteristics for all the parameters investigated in this section and will be referenced throughout.

7.4.1 Breaker Index Depth Dependence

The breaking depth plays the dominant role in the breaking conditions of shallow water waves. In order to assess the effect of the breaking depth on breaker index values, a series of non-dimensionalised plots were prepared.

<table>
<thead>
<tr>
<th>Table 7-6: Overview of Breaker Index Parameter Dependencies</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>The Hook</strong></td>
</tr>
<tr>
<td>ND Depth</td>
</tr>
<tr>
<td>ND Wave Height</td>
</tr>
<tr>
<td>Seafloor Slope</td>
</tr>
<tr>
<td><strong>Sewers Peak</strong></td>
</tr>
<tr>
<td>ND Depth</td>
</tr>
<tr>
<td>ND Wave Height</td>
</tr>
<tr>
<td>Seafloor Slope</td>
</tr>
<tr>
<td><strong>Tropicana</strong></td>
</tr>
<tr>
<td>ND Depth</td>
</tr>
<tr>
<td>ND Wave Height</td>
</tr>
<tr>
<td>Seafloor Slope</td>
</tr>
<tr>
<td><strong>All Locations</strong></td>
</tr>
<tr>
<td>ND Depth</td>
</tr>
<tr>
<td>ND Wave Height</td>
</tr>
<tr>
<td>Seafloor Slope</td>
</tr>
</tbody>
</table>

The breaking depth was non-dimensionalised using gravity and the wave period squared \( (gT^2) \). The individual breaking wave period and the statistical incoming peak period were used as inputs in order to assess which resulted in better correlation. Analyses showed that using the individual wave period resulted in 33\% better correlation between the breaker index and breaker depth when compared against the peak period (see Figure 7-6). While logarithmic plots showed improved \( R^2 \) values against the linear fit shown in Figure 7-6, the performance comparison between spectral parameters and direction measured parameters did not. Understanding that peak period is calculated as a statistical descriptor of the spectral analysis rather than a direct reading of the sea level surface, the improved correlation using individual wave periods was expected. It is suggested that further irregular wave studies use individual wave period values, rather than spectral values, to increase the level of reliability of predicting relationships.
Three general trends emerged from the breaker index vs. non-dimensional depth analysis. Firstly, waves breaking in deeper water were generally found to have lower breaker index values – indicated by the negative reported slopes in Table 7-6. Secondly, waves that propagated over steeper effective slopes featured higher breaker indices. This slope trend was extracted by binning all data points with respect to 10 y-axis bins and extracting the average slope per bin. As shown in Figure 7-6, any attempts to quantify these trends would feature considerable uncertainty due to large amounts of scatter in the plots.

7.4.2 Breaker Index Height Dependence

As the only other parameter in the breaker index, the breaker height was investigated to determine its effect on the breaker index. As shown in Figure 7-7 and the low $R^2$ values in Table 7-6, no conclusive trends could be established about the direct effect of breaking wave height on the breaker index. Hence it was concluded that the breaker index is affected more by breaking depth variations than breaker height variations.

In contrast to Figure 7-6, no improved correlation due to the use of individual wave periods was evident.
7.4.3 Breaker Index Effective Slope Dependence

The effective seafloor slope was additionally investigated to isolate possible trends between the breaker index and effective slope. For each study location, an increase in the effective seafloor slope manifested itself with slightly higher breaker index values (illustrated by the positive slope of the regression fitted lines in Table 7-6). However, as shown in Figure 7-8, this slope dependency is not robustly quantifiable and is simply noted as a trend.

![Figure 7-8: The Hook breaker index vs. effective seafloor slope. The red line indicates a robust regression fit.](image)

Unfortunately, once all sites are plotted on the same axis, no trend was visible and scatter dominated the plot (see Figure 7-9). Evidently, the trends associated with each location were not site independent. Hence, it was concluded that the breaker index does not depend solely on the seafloor slope and cannot be accurately predicted without a detailed knowledge of the water depth, wave height and seafloor slope.

![Figure 7-9: Breaker index vs. effective seafloor slope at all locations. The red line indicates a robust regression fit.](image)
7.5 Breaker Steepness Prediction Methods

While the breaker index is the most published indicator for predicting wave breaking, numerous authors have found better predictive ability based on the breaking wave steepness parameter \((H_b/L_b)\). First completed by Miche [1944], who used orbital motions from linear wave theory to predict breaking, the breaker steepness has subsequently been used by Ostendorf and Madsen [1979], Kamphuis [1991] and Liu et al. [2011] to predict breaking.

7.5.1 Breaker Steepness Depth and Height Dependencies

Table 7-7 details the regression analysis outputs from looking at breaker steepness variables across all the study locations and the entire dataset, for the non-dimensionalised breaker depth \((h_b/(gT^2))\) and wave height \((H_b/(gT^2))\).

<table>
<thead>
<tr>
<th>Breaker Steepness VS ND Depth and ND Height</th>
<th>Constant</th>
<th>Slope</th>
<th>Linear RMSE</th>
<th>R-Square</th>
<th>ER</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hook (h_b/(gT^2))</td>
<td>0.02</td>
<td>9.10</td>
<td>0.003</td>
<td>0.25</td>
<td>30.0</td>
</tr>
<tr>
<td>(H_b/(gT^2))</td>
<td>0.02</td>
<td>16.0</td>
<td>0.002</td>
<td>0.69</td>
<td>25.0</td>
</tr>
<tr>
<td>Sewers Peak (h_b/(gT^2))</td>
<td>0.02</td>
<td>16.0</td>
<td>0.003</td>
<td>0.87</td>
<td>27.0</td>
</tr>
<tr>
<td>(H_b/(gT^2))</td>
<td>0.02</td>
<td>15.0</td>
<td>0.001</td>
<td>0.96</td>
<td>20.0</td>
</tr>
<tr>
<td>Tropicana (h_b/(gT^2))</td>
<td>0.03</td>
<td>5.00</td>
<td>0.003</td>
<td>0.27</td>
<td>16.0</td>
</tr>
<tr>
<td>(H_b/(gT^2))</td>
<td>0.02</td>
<td>11.0</td>
<td>0.001</td>
<td>0.70</td>
<td>13.0</td>
</tr>
<tr>
<td>All Waves (h_b/(gT^2))</td>
<td>0.02</td>
<td>12.0</td>
<td>0.003</td>
<td>0.67</td>
<td>13.0</td>
</tr>
<tr>
<td>(H_b/(gT^2))</td>
<td>0.02</td>
<td>15.0</td>
<td>0.002</td>
<td>0.91</td>
<td>9.70</td>
</tr>
</tbody>
</table>

Figure 7-10: Breaker steepness dependence on breaker depth
While excellent $R^2$ values were found, plots relating the breaker steepness to a non-dimensional depth or height are not as useful in prediction of breaking conditions as those relating to a breaker index. Figure 7-10 and Figure 7-11 show the breaker steepness dependencies on water depth and height.

In the non-dimensional water depth situation (see Figure 7-10), a basic dimensional analysis showed that the regression fitted line, indicating the dependence of the breaking steepness on the non-dimensional depth, depends on the breaker index. This is caused by the fact that the wavelength used to measure the breaking steepness was predicted using shallow water approximation of $L_b = \sqrt{gh_b T^2}$.

![Breaker Steepness Vs ND Breaker Height](image)

**Figure 7-11: Breaker steepness dependence on wave height**

When looking at non-dimensional wave height parameters (see Figure 7-11), a similar dimensional analysis showed the fitted line to be independent of wave height, due to algebraic cancellation, and explained the exceptionally high $R^2$ and low RMSE results shown in Table 7-7. As a result of these findings, it is suggested that the breaker index is a more appropriate parameter to investigate wave dependencies and data trends.

### 7.6 Published Breaking Height Relationship Correlations

Armed with the optimized effective seafloor slope and breaking depth definitions, it was possible to compare the remotely extracted values against those predicted by published relationships. By comparing the extracted data against the best and most often cited breaker height relationships the level of confidence in the presented low-cost remote wave measurement methods could be determined.

The vast majority published breaking predictors for regular waves can be broadly categorized as linear, SSP, trigonometric or exponential formulations. While Thompson and Vincent [1985] note that "reference
to monochromatic wave tests for irregular wave applications should be done with caution or avoided.

Additionally, the irregular wave based predictors of Seyama and Kimura[1988], and Kamphuis[1991]
were investigated to see if additional correlation was found. In the subsequent figures, the predicted $H_b$
values (crosses) are compared against those directly measured (filled diamonds). The colour fill of each
measured data point indicates the individual effective slope. A robustfit linear regression line has been
fitted to the measured data and plotted.

7.6.1 Linear Relationship Correlation

As shown in Figure 7-12 and Table 7-8, McCowan’s [1894] relationship under predicted the measured
breaking wave height values and featured a qualitative RMSE measurement of 0.41. The data offset may
be explained by that fact that all the waves measured in the current study were of the plunging variety,
while McCowan’s constant of 0.78 may find better agreement with spilling waves. The fitted linear
regression slope in Figure 7-12 is dimensionally consistent with the breaker index ($y_b$) and featured a
value of 0.88, which is close to $y_b = 0.83$ predicted by Yamada et al [1968] for solitary waves.

7.6.2 Surf Similarity Parameter Relationship Correlations

Battjes [1974] was the first to include the SSP parameter in predicting the breaking height of waves.
Unfortunately, Eq. (43) performed poorly against the measured dataset and featured a RMSE of 0.66 and
a linear $R^2$ value of zero. The scatter of measured data points in Figure 7-13 illustrates increased
variation and lower correlation, relative to Figure 7-12, yet the increasing trend is still immediately
visible.
7.6.3 Trigonometric Relationship Correlation

Using Miche’s linear wave-based theory, calibrated against laboratory results, Ostendorf and Madsen’s [1979] relationship under-predicted breaking wave heights, featuring a RMSE of 0.37 (see Figure 7-14). A binned plot of Figure 7-14 showed a curved relationship within the measured data, indicating that an exponential form of Eq. (50) may find better correlation.

Lui et al. ‘s [2011] predictive formula is complicated by the inclusion of the breaker height on both sides of the relationship and therefore needs to be solved iteratively. As shown in Figure 7-15, the predictive ability of Eq. (52) was inferior to previously published formulas and features high RMSE errors. Additionally, the predicted breaking wave heights are over predicted using Eq. (52). It is noted that due to the iterative solution method, a single x-axis label has not been noted in Figure 7-15.
Weggel [1972] attempted to predict the maximum wave height that may occur given the shallow flume results of Galvin [1968], who used regular waves over smooth plane beaches. Correlation between the predicted data and the fitted regression line was excellent, as shown in Figure 7-16. Weggel’s prediction finds the second best direct unoptimized correlation with the measured values ($R^2 = 0.76$). The illustrated variation could be due to flume-based wave reflection errors since active absorptive wave flumes were yet to be developed.

In 1991, Smith and Kraus [1991] updated the constants in Weggel’s prediction to achieve better correlation with their dataset. However, as shown in Figure 7-17, the more recent constants found lower correlation with the dataset and additional data scatter. As a result, the fitted regression line found a lower best fit $R^2$ value when compared to the original works of Weggel.
Figure 7-17: Comparison of measured data against predictions from Smith and Kraus’ relationship

Rattanapitikon and Shibayama’s [2000] predictor found excellent direct correlation with the measured dataset, with the predicted wave heights only slightly under-predicted when compared to the fitted linear regression line (Figure 7-18). The direct comparison between measured and predicted values features an RMSE of 0.24 and robust fit $R^2$ of 0.80, hence providing the best prediction.

Figure 7-18: Comparison of measured data against predictions from Rattanapitikon and Shibayama’s relationship

Goda’s [2010] relationship found very good correlation with the measured dataset and only a slight under-prediction of the final wave height values was noted - see Figure 7-19. The RMSE between the measured and predicted data was 0.30 and reduced from Goda’s 1974 breaker height approximation.
The superior predictive ability of the discussed exponential relationships corroborates the conclusions drawn in the individual parameter dependence testing. In Section 7.4, plots of the breaker index and the non-dimensionalised depth and slope showed better correlation using an exponential dependence.

### 7.6.5 Irregular Wave Relationship Correlation

All the previously discussed relationships were based on regular wave testing and empirical fitting. However, Seyama and Kimura [1988] and Kamphuis[1991] focused purely on providing predictors for irregular breaking waves.

The predicted breaker heights from Seyama and Kimura’s [1988] irregular wave relationship fall well below the measured data points (Figure 7-20). However, the measured values fall above the experimental bounds of Seyama and Kimura’s study parameters and may explain the lack of correlation. The validity of their equations is $0.01 < h_b / L_o < 0.3$, whereas the mean $h_b / L_o$ of the data presented here is 0.006. On average, the predicted wave heights were 43% lower than those measured, with the deviation increasing with increased wave height predictions. This indicates a substantial variation from the measured results.

However, it is noted that Seyama and Kimura used a different breaking water depth definition in finding Eq. (81), which may be a cause of some of the variation.

![Figure 7-19: Comparison of measured data against predictions from Goda’s relationship](image)
Kamphuis' [1991] irregular wave prediction was based on the maximum steepness of waves at the point of breaking, not on the breaker index value. The directly predicted breaking wave heights are considerably under-predicted (Figure 7-21) and $R^2 = 0$ against the measured breaker heights. The wave height predicted by the Kamphuis equation averaged 30% lower than those measured. This is in accordance with the postulation by Goda [2010] that “the incipient breaking height of the significant wave is about 30% lower than that of regular waves.” The fact that Kamphuis used $H_s$ in his analysis and that he did not investigate individual waves within the irregular wave train, may explain the considerable under-prediction of the final values.
Table 7-8 gives an overview of the linear robust regression line parameters and the values associated with the direct comparison. The robust $R^2$ values illustrate the best possible fits between the measured data and the regression fit.

### Table 7-8: Comparison of Published Relationships

<table>
<thead>
<tr>
<th>Methodologies</th>
<th>Statistics based on Best Fit (Robust) to Measured Data</th>
<th>Direct Comparison of Predicted and Measured Data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Constant</td>
<td>Rgs Slope</td>
</tr>
<tr>
<td>McCowan, 1894</td>
<td>0.16</td>
<td>0.88</td>
</tr>
<tr>
<td>Battjes, 1974</td>
<td>1.03</td>
<td>0.40</td>
</tr>
<tr>
<td>Weggel, 1972</td>
<td>-0.06</td>
<td>1.02</td>
</tr>
<tr>
<td>Ostendorf / Madsen, 1979</td>
<td>-0.04</td>
<td>0.17</td>
</tr>
<tr>
<td>Smith and Kraus, 1991</td>
<td>-0.03</td>
<td>1.04</td>
</tr>
<tr>
<td>Rattanapitikon and Shibayama, 2000</td>
<td>0.02</td>
<td>0.18</td>
</tr>
<tr>
<td>Goda, 2010</td>
<td>-0.01</td>
<td>0.19</td>
</tr>
<tr>
<td>Lui et al., 2011</td>
<td>-0.07</td>
<td>0.03</td>
</tr>
<tr>
<td>Seyama and Kimura, 1988</td>
<td>-0.06</td>
<td>0.35</td>
</tr>
<tr>
<td>Kamphuis, 1991</td>
<td>-0.01</td>
<td>0.14</td>
</tr>
</tbody>
</table>

The effect of seafloor slope sensitivity on predicted breaker height values was assessed by varying the slope values for The Hook by 7 %, 4.5 % for Sewers Peak and 12 % for Tropicana Beach, per the findings in Table 6-8. The resulting changes in direct comparison RMSE values averaged 0.01 (0.74 %), hence indicating negligible variations in breaker height, despite the noted sensitivity of the seafloor slope factor.

As shown in Table 7-8, the measured values extracted according to the presented low-cost remote study methods find excellent correlation with several published wave height predictors. In particular, excellent correlation was found with the exponential predictor from Rattanapitikon and Shibayama (Eq. (75)), which is often cited as the one of the best predictors for breaking height heights [Camenen and Larson, 2007; Lee and Mizutani, 2010].

The predicted values from the Kamphuis’ irregular wave formula showed an approximate 30% under prediction of final waves heights when compared against the measured data points, thus corroborating the prediction by Goda [2010]. The collected data confirms that when an irregular wave train is investigated on a wave-by-wave basis, rather than using statistical measurements, the breaking characteristics are well predicted and defined by regular wave-based formulas. This is in agreement with the predictions of Mizuguchi et al.[1982].
7.7 New Breaking Height Predictor

Analysis of the direction comparison ER and RMSE values in Table 7-8, and the functional forms of the equations in Table A-1, shows the regular wave exponential form used by Rattanapitikon and Shibayama, 2000 and Goda, 2010 best predicts the breaking wave heights for the conditions tested. Given the new effective slope measurement system, the slope based constants were investigated in order to access the possible need for a new optimized relationship.

The general form used by Goda, 2010 and Rattanapitikon and Shibayama, 2000 can be described as:

\[ \gamma_b = K_g \frac{L_o}{h_b} \left\{ 1 - \exp \left( \pi \frac{h_b}{L_o} K_{gm} \right) \right\} \]  \hspace{1cm} (103)

where \( K_g \) is an overall constant and \( K_{gm} \) is a slope based correction. The maximum linear RMSE and ER values for Rattanapitikon and Shibayama, 2000 and Goda, 2010 both showed good correlation with ER values of 1.8 % and 2.0 %, respectively. Given that both relationships are based on the same functional form, and only the effective slope function is varied, the similar ER values are expected.

Initially, using a quick basic linear regression fitting and the Rattanapitikon and Shibayama 2000 formula as the best possible predictor of the breaking wave height, Eq. (104) below found the best correlation and lowest variability to the dataset. This linear fit slope is within the range suggested by Kriebel et al. [2000], who noted \( K_g \) can vary between 0.09 and 0.18.

\[ H_b = 0.18L_o \left\{ 1 - \exp \left( \pi \frac{h_b}{L_o} \left[ 16.21 m^2 - 7.07 m - 1.55 \right] \right) \right\} - 0.02 \]  \hspace{1cm} (104)

However, given the newly presented method for extracting effective seafloor slopes from field tests, additional analyses on the slope constant were completed to determine if better correlation between measured and theoretical results could be found. Following the same optimization procedure as Rattanapitikon and Shibayama [2000], Eq. (103) can be rearranged to show:

\[ K_{gm} = \frac{L_o}{\pi h_b \ln \left( 1 - \frac{H_b}{0.17L_o} \right)} \]  \hspace{1cm} (105)

The measured \( K_{gm} \) is plotted against those calculated using Goda, 2010 and Rattanapitikon and Shibayama 2000 in Figure 7-22 A. Following a similar procedure to that of Goda, a linear best fit function is also plotted and can be expressed as:

\[ K_{gm} = 1.98m + 1.80 \]  \hspace{1cm} (106)
Substitution of Eq. (106) into the general functional form (Eq. (103)) results in:

$$\gamma_b = 0.17 \frac{L_{o}}{h_b} \left\{ 1 - \exp \left( \frac{h_b}{\tau_o} (1.98m - 1.80) \right) \right\}$$

(107)

When used to predict the final breaker height, the predicted values feature a RMSE of 0.20 and an ER value of 1.67 %, when compared against measured values (see Figure 7-22 B), showing that Eq. (107) finds better correlation with irregular wave data collected in this study than any previously published predictors. The previous predictions of Rattanapitikon and Shibayama (2000), and Goda (2010) were both based on the analysis of numerous datasets, within which differing depth and slope definitions were used to define the measurements. It is suggested that the additional correlation found in this study may be due to a single consistent depth measurement system and the novel effective slope definitions first presented in this thesis. Additional verification of Eq. (107) could possibly be achieved by the reanalysis of the data used by Rattanapitikon and Shibayama, using the presented standardized methods of measuring the effective seafloor slope and the breaking depth.

It is suggested that Eq. (107) be tested against a larger and more varied set of environmental conditions commonly occurring in Coastal Engineering practice prior to widespread use.
8 Vortex Parameters as Breaking Wave Intensity Predictor

Attempts to categorize and quantify the visual geometric differences of breaking waves have been ongoing since Iribarren and Nogales suggested the Surf Similarity Parameter in 1949. While still often quoted, the Surf Similarity Parameter has been shown to be unable to accurately distinguish between different wave types and breaking intensities in any non-ideal case.

In 1982, Longuet-Higgins[1982] suggested fitting a cubic function to the interior vortex of a plunging breaking and that the vortex ratio and vortex angle may be indicators of the breaking intensity. In doing so, Longuet-Higgins inspired a new area of breaking wave research. Thoroughly described in Chapter 3, numerous authors have investigated this possibility based on assumed or limited datasets and published results which are often contradictory [Mead and Black, 2001; Blenkinsopp, 2003; Fairley and Davidson, 2008; Johnson, 2009].

To the best knowledge of the author, the study presented in this thesis was the first complete field analysis of wave vortex ratios and angles. Every wave included in the study featured an individual effective seafloor slope, breaking wave height, period, and depth. As a defined goal of this study, all these parameters were used to determine if the wave vortex ratio and angle could be predicted and whether published methods of predicting breaker intensity were valid.

The majority of Chapter 8 was presented at the International Conference on Coastal and Ocean Engineering (Zurich, January 2013) and subsequently published in the peer reviewed World Academy of Science, Engineering and Technology in February, 2013 [Robertson and Hall, 2013].

8.1 Vortex Ratio and Vortex Angle Reliability and Suitability

Prior to detailed analyses and in order to test the reliability of the extraction method, two vortex profiles images with an excellent quality rating (Bin #1) were measured 10 times each and the resulting vortex ratios and angles were analyzed. These tests were completed at random periods over a single day to ensure the reviewer was not predisposed to continually pick the same point due to human conditioning. Following the procedure detailed in Chapter 6, the vortex ratios and angles were extracted.

<table>
<thead>
<tr>
<th></th>
<th>Vortex Ratio</th>
<th>Vortex Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Wave 1</td>
<td>Wave 2</td>
</tr>
<tr>
<td>Mean</td>
<td>2.2</td>
<td>2.2</td>
</tr>
<tr>
<td>Maximum</td>
<td>2.4</td>
<td>2.3</td>
</tr>
<tr>
<td>Minimum</td>
<td>2.1</td>
<td>2.0</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Uncertainty</td>
<td>5.3 %</td>
<td>5.1 %</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Wave 1</th>
<th>Wave 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>45.0</td>
<td>42.0</td>
</tr>
<tr>
<td>Maximum</td>
<td>48.0</td>
<td>46.0</td>
</tr>
<tr>
<td>Minimum</td>
<td>43.0</td>
<td>41.0</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>1.80</td>
<td>1.30</td>
</tr>
<tr>
<td>Uncertainty</td>
<td>3.9 %</td>
<td>3.1 %</td>
</tr>
</tbody>
</table>
As shown in Table 8-1, vortex ratio measurements were consistent within approximately 5% while the vortex angles were consistent within approximately 4%. The low relative uncertainties confirmed this method of data extraction was sufficiently accurate to analyze vortex ratio and angle trends. The percentage uncertainty shown in Table 8-1 was calculated by dividing the standard deviation by the mean value.

8.2 Vortex Ratio Relationship Analyses

As discussed in Section 6.5, all wave vortex profiles were categorized according to the clarity and perpendicularity of the captured image. In the following sections, only wave profile images of Excellent (Bin #1) and Good (Bin #2) quality ratings were used. When attempting to include profile images with Dubious (Bin #3) quality ratings, resultant plots suffered from considerable amounts of data scatter.

In all subsequent plots, the measured data points are noted as filled diamonds, the robust regression fitted line is illustrated by the red linear fitted line, and the vertical uncertainty bar plots indicate the maximum possible ($Y_{max}$) and minimum possible ($Y_{min}$) vortex. A complete overview of the statistical fit characteristics for all vortex ratio analyses is presented in Table 8-2.

Figure 8-1: Vortex ratio dependence on water depth for all three study locations.
8.2.1 Vortex Ratio Breaker Height and Depth Dependencies

Initial analyses to determine vortex ratio dependencies focused on the two most basic wave characteristics: the breaking wave height and the breaking water depth. As shown in Figure 8-1, the regression fitted dependencies for each individual site conflicted when comparing the vortex ratio and breaking water depth. A similar set of conflicting relationships were found using the breaker height.

In contrast, when the entire dataset was plotted, a general trend of increasing the non-dimensional depth and height resulted in reduced vortex ratios (see Figure 8-2). However, the resulting $R^2$ values for the non-dimensional depth and height were only 0.23 and 0.20 respectively, thus illustrating a weak correlation between the measured data and the regression fitted line. These trends contradict the findings of Johnson [2009], who found smaller waves created lower vortex ratios. Visual analysis of the error bars, indicating maximum and minimum possible vortex ratios, illustrate the large amount of scatter in the possible vortex ratios and the weakness of the regression fitted trend. Considering the irregular nature of ocean waves and the sensitivity of the vortex ratio to additional unplotted variables (local wind speed, wave/wave interactions, seafloor friction, etc) the poor correlations might have been expected.

8.2.2 Vortex Ratio Effective Seafloor Slope Dependence

The effective seafloor slope was found to play a determining role in the breaker index and was qualitatively predicted to play a similar role on the vortex ratio. The plot of the vortex ratio and seafloor slope (Figure 8-3) indicated the possibility that increasing seafloor slopes generally resulted in decreasing vortex ratios for this study. However, the large uncertainty error bars, considerable amounts of scatter and the $R^2$ value of only 0.27 indicate the trend was not definitive and may only be a function of the fitting algorithm.
Mead and Black [2001] found good correlation between seafloor gradients \(1/m\) and vortex ratios \(R^2 = 0.71\) using Eq. (85), yet subsequently published results have been unable to replicate these findings [Grilli et al., 1997; Blenkinsopp and Chaplin, 2008]. The vortex ratio vs. seafloor gradient for all locations was plotted in Figure 8-4a showing increasing gradients resulted in lower vortex ratio values.

Eq. (85) by Mead and Black [2001] was plotted in Figure 8-4 and found agreement with measured results at low gradient values. However, the general trend was considerably different and was best characterised using Eq. (108):

\[
Y = 0.01m' + 2.03
\]  

(108)

The lack of a consistent response of the vortex ratio to seafloor bathymetry is immediately evident when looking at the trends for individual sites. The trend line fitted for the data points collected at Tropicana Beach (Figure 8-4b) shows the opposite gradient dependence from Eq. (108) and Mead and Black’s prediction.
In Chapters 6 and 7, a new optimized definition of effective seafloor slope was presented and shown to improve the prediction of the breaker index. In Figure 8-5, the effective $m_{1/3}$ slope and the CEM single wavelength slope are plotted to determine if superior performance was achieved through the use of $m_{1/3}$. While both plots suffered from considerable data scatter, the CEM plot suggested slope independence while the novel $m_{1/3}$ plot did suggest a possible trend.

![Figure 8-5: Measured vortex ratios VS Mead predicted values for a) $M_{1/3}$ slope and b) CEM slope.](image)

### 8.2.3 Vortex Ratio Wave Period Dependence

Many ocean enthusiasts hold a firm qualitative belief that as the incoming wave period increases, the vortex ratio of the breaking waves will decrease. However, the regression line in Figure 8-6 indicates that increasing wave periods result in small increasing vortex ratios, yet the $R^2$ value of only 0.18 indicates substantial uncertainty in this trend. The associated vortex ratio uncertainty bars indicate no substantial dependence between the vortex ratio and wave period. No published studies have found correlation between period and vortex ratios.

![Figure 8-6: Vortex ratio dependence on breaking wave period](image)
8.2.4 Vortex Ratio Breaker Index and Breaker Steepness Dependencies

Breaking waves with higher breaker index values were expected to give smaller vortex ratios due to their larger wave heights to depth ratios. However, Figure 8-7 displays a weak trend contradicting this expectation. Noting the decreasing vortex ratio with both height and depth in Section 8.2.1, the lack of a high correlation trend in Figure 8-7 is expected.

![Vortex Ratio Vs Breaker Index](image1.png)

Figure 8-7: Vortex ratio dependence on breaker index

No defining trend was apparent between the breaking wave steepness and the vortex ratio (see Figure 8-8). A decreasing linear trend is shown by the regression fit yet with an $R^2$ value of 0.06, this trend could not be considered robust.

![Vortex Ratio Vs Breaker Steepness](image2.png)

Figure 8-8: Vortex ratio dependence on wave steepness
8.2.5 Surf Similarity Parameter and Modified Ursell Number

Given that no robust trends were extracted through the use of standard breaking wave parameters, the breaking Surf Similarity Parameter (SSP) and the Ursell number were investigated as possible predictors.

The breaking SSP differentiates between different breaking wave types and increasing SPP values are theorised to give lower vortex ratios. As shown in Figure 8-9, a corroborating trend was fitted to the measured data yet the fitted trend line features an $R^2$ of only 0.14. Additionally, the binned box plot in Figure 8-9 shows no definite trend with respect to the breaking SSP and parameter independence.

The Ursell number predicts the most accurate wave theory as waves move from deep water to shallow water. For this analysis, a modified Ursell number was calculated using breaking conditions to see if any dependence or trend could be extracted. When analysing the vortex ratio vs. Ursell number plots in Figure 8-10, a basic trend line is apparent: higher modified Ursell numbers result in higher vortex ratios. However, the $R^2$ value of 0.24 illustrates high variability and calls the validity of the fitted trend line into question.
### 8.2.6 Single Parameter Vortex Dependence Overview

As summarized in Table 8-2, the vortex ratio was rigorously analysed against all breaking wave and local bathymetric characteristics. No trend line found a correlation of determination above 0.28. As a result, no defining relationships could be drawn from the collected data, corroborating the findings of Johnson[2009], Blenkinsopp[2003] and Grilli et al.[1997]. These findings bring into doubt the suggestion that breaker vortex ratios may be good descriptor of breaking intensity.

#### Table 8-2: Vortex Ratio Relationships Overview

<table>
<thead>
<tr>
<th></th>
<th>Hook</th>
<th>Sewers Peak</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Constant</td>
<td>Slope</td>
</tr>
<tr>
<td>ND Depth</td>
<td>2.60</td>
<td>94.0</td>
</tr>
<tr>
<td>ND Wave Height</td>
<td>2.80</td>
<td>-84.0</td>
</tr>
<tr>
<td>Seafloor Slope</td>
<td>2.70</td>
<td>-1.80</td>
</tr>
<tr>
<td>Breaker Index</td>
<td>3.10</td>
<td>-0.33</td>
</tr>
<tr>
<td>Period</td>
<td>2.30</td>
<td>0.02</td>
</tr>
<tr>
<td>Breaker Steepness</td>
<td>3.00</td>
<td>-9.70</td>
</tr>
<tr>
<td>Breaking SSP</td>
<td>2.70</td>
<td>-0.02</td>
</tr>
<tr>
<td>Ursell Number</td>
<td>2.80</td>
<td>0.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Tropicana</th>
<th>All Waves</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Constant</td>
<td>Slope</td>
</tr>
<tr>
<td>ND Depth</td>
<td>2.50</td>
<td>-150</td>
</tr>
<tr>
<td>ND Wave Height</td>
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<td>-220</td>
</tr>
<tr>
<td>Seafloor Slope</td>
<td>1.60</td>
<td>14.0</td>
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<tr>
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<td>0.52</td>
</tr>
<tr>
<td>Period</td>
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<tr>
<td>Breaker Steepness</td>
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</tr>
<tr>
<td>Breaking SSP</td>
<td>1.70</td>
<td>1.10</td>
</tr>
<tr>
<td>Ursell Number</td>
<td>2.10</td>
<td>0.00</td>
</tr>
</tbody>
</table>

### 8.2.7 Multi-parameter Vortex Ratio Regression Analysis

Given the lack of dependence of the vortex ratio on individual wave characteristics, a non-dimensional multi-parameter approach was investigated. A simple linear regression analysis using the non-dimensional breaking wave height, water depth and effective seafloor slope was able to achieve an $R^2$ value of 0.31 using Eq. (109):

$$Y = 2.94 - 1.77 \frac{H_b}{gT^2} + 12.12 \frac{h_b}{gT^2} - 9.11m_{1/3}$$  \hspace{1cm} (109)
Eq. (109) does find better correlation with the measured vortex ratios yet considerable scatter was still evident in plotted results, as shown in Figure 8-11. The dashed lines indicate +/- 5% vortex ratio uncertainty due to the measurement technique uncertainties (see Table 8-1). Approximately 75% of all measured vortex ratios fall within +/- 5% of those predicted by Eq. (109). The improved multivariable prediction equation does show improved predictive abilities yet the coefficient of correlation is still comparatively low and confidence in the values predicted by presented relationship is correspondingly low.

![Vortex Ratio Prediction](image)

**Figure 8-11: Multivariable regression analysis for the vortex ratio**

### 8.3 Vortex Angle Relationships

In order to assess the possibility of predicting the vortex angle, a similar set of comparative plots were analysed. An overview of the correlations using the vortex angle is presented in Table 8-3. Unfortunately, none of the relationships featured an $R^2$ value above 0.02 and all featured large RMSE.

<table>
<thead>
<tr>
<th>Wave Vortex Angle Relationships</th>
<th>Constant</th>
<th>Slope</th>
<th>R-Square</th>
<th>Linear RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>ND Depth</td>
<td>47.0</td>
<td>-420</td>
<td>0.00</td>
<td>5.00</td>
</tr>
<tr>
<td>ND Wave Height</td>
<td>46.0</td>
<td>180</td>
<td>0.00</td>
<td>5.00</td>
</tr>
<tr>
<td>Seafloor Slope</td>
<td>45.0</td>
<td>54.0</td>
<td>0.02</td>
<td>4.90</td>
</tr>
<tr>
<td>Breaker Index</td>
<td>46.0</td>
<td>0.22</td>
<td>0.00</td>
<td>5.00</td>
</tr>
<tr>
<td>Period</td>
<td>48.0</td>
<td>-0.15</td>
<td>0.01</td>
<td>5.00</td>
</tr>
<tr>
<td>Breaker Steepness</td>
<td>43.0</td>
<td>84.0</td>
<td>0.01</td>
<td>5.00</td>
</tr>
<tr>
<td>Breaking SSP</td>
<td>44.0</td>
<td>5.60</td>
<td>0.02</td>
<td>4.90</td>
</tr>
</tbody>
</table>
The highest co-efficient of correlation values were found for seafloor slope and SSP plots, yet a basic visual analysis of Figure 8-12 (Seafloor Slope) and Figure 8-13 (Breaking SSP) show the high degree of data scatter and a lack of dependence with respect to the independent variables.

Figure 8-12: Vortex angle vs. effective slope relationship

These findings corroborate previous laboratory investigations into breaker vortex angles which found no dependence on individual wave characteristics [Mead and Black, 2001; Blenkinsopp and Chaplin, 2008; Johnson, 2009]. A multivariable regression analysis was unable to find better correlation with the measured vortex angles. An $R^2$ value of only 0.04 was achieved and the lack of rigorous trend is immediately evident in Figure 8-14.
Figure 8-14: Multivariable regression analysis for the vortex angle

It is suggested that one of the major contributing factors to the lack of angle dependence is a naturally increasing vortex angle, $\theta$, during the progression of the breaking event, as theoretically predicted by Vinje and Brevig [1980] and Longuet-Higgins [1982].

### 8.4 Sources of Variability in Vortex Parameters

No robust dependencies between the vortex parameters and the breaking wave characteristics were established or quantified. The number of possible complicating and negatively influencing variables is substantial and only a few are discussed in detail below.

First, it was noted that not every incident wave featured an enclosed vortex when breaking. At each location, individual subsequent waves would alternate between spilling wave profiles and plunging profiles. This could be due to the localized effect of wave induced water level set-up or set-down. Quantification of this effect would require a detailed time series of the water surface over the entire breaking area. Even if possible, this would be prohibitively expensive and the required number of in-situ devices would undoubtedly create substantial loading errors.

Wave groups create temporary increased water levels within the surf zone, which in turn creates seaward return flows between surf beats or rip currents. The effect of return water flow over the reef crest has been shown to affect the breaking shape and conditions of incoming waves [Smith and Kraus, 1991].

During the study periods, it was noted that individual waves would occasionally “catch up” to the waves preceding them in the wave train, interact constructively and create a much larger wave in a relatively
shallow water depth (colloquially known as a “double-up”). While each wave studied was visually analyzed by watching the overview video and eliminated if this effect could be noticed, there was no means to completely eliminate this effect from the dataset. There are undoubtedly some waves affected by wave-wave interactions within the extracted dataset.

Wind direction and speed greatly alters the vortex shape of breaking waves. Onshore winds will increase the vortex ratio by decreasing the relative vortex width, while offshore winds will increase the vortex width, thereby lowering the final vortex ratio. While the effects of local wind conditions were minimized by only collecting data on days with minimal to no wind (wind speed < 10 knots), isolated gusts of wind would have affected the extracted vortex ratio of the breaking waves.

8.5 Vortex Parameter Recommendations

Given that the only consistent trend between published papers and the data presented in this study was the lack of dependence between vortex parameters and breaking wave conditions, it is concluded that the suggested use of the vortex angle and vortex ratio as proxies for predicting the breaking wave intensity should be avoided.

Suggested below are a number of different geometric parameters which may provide better prediction of wave breaking intensity (see Figure 8-15 for visual explanation of suggested parameters). It is postulated that the geometric prediction of intensity will be multi-parameter dependent, rather than solely predicted by any single parameter.

- Jet Length / Breaker Height Ratio ($J_L/H_b$): The author’s analysis of Grilli et al.’s [1997] profile images shows the jet length/break height ratio changing dramatically with seafloor slope. Ratios of 0.50, 0.80 and 1 for 1/100, 1/35 and 1/15 slopes respectively were measured, illustrating that the jet length ratios may be a relevant parameter in breaking intensity.

- Rear Wave Height / Breaker Height Ratio ($H_r/H_b$): Often qualitatively noted by surfer and ocean enthusiasts, the rear wave height/breaker height may provide additional insight into the breaking intensity.

- Jet Thickness / Breaker Height Ratio ($J_t/H_b$) or Jet Area / Breaker Height $^2$ ($A/H_b^2$): Grilli et al.[1997] suggested the area of the jet at touchdown may be a good measure of intensity. These parameters will provide a non-dimensionalised indication of the amount of water with the overturning jet and breaking intensity.

- Jet Width / Breaker Height Ratio ($J_w/H_b$): Wave speed in shallow water is directly correlated to depth, and since the jet width is directly correlated to the speed differential between the wave crest and the main bulk of the wave, this may provide additional information about the breaking intensity.

125
Figure 8-15: Possible alternate geometric breaking wave description parameters

8.6 Vortex Parameter Discussion

Through a thorough investigation of all published studies involving vortex parameters and the data collected in this study, it is concluded that the use of the vortex ratio and vortex angle is insufficient to predict the breaking intensity of waves and should not be investigated further. With the sole exception of Mead and Black [2001], no published studies have found any substantial correlation between these parameters and breaking conditions, providing evidence to the inconclusive nature of pure vortex ratio or angle descriptions.

As a result, it is recommended that any further studies dealing with geometric characteristics include detailed measurements of the suggested novel geometric properties and investigate a multi-parameter approach. However, the difficulty of reliably extracting the recommended parameters from field conditions is high. Given the necessity of a perfect profile image to extract the recommended parameters and the complex nature of breaking conditions, it is possible that detailed trends will not be evident within the inherent uncertainty and scatter of this data collection method. Hence, the recommended parameters stand only as possible geometric descriptors of breaking intensity and it is suggested that purely geometric descriptors are both too variable and unstable for rigorous breaking wave intensity studies to be performed.
9 Conclusions

The analysis of the shallow water wave breaking phenomenon has been on-going for almost 150 years, and the level of understanding has increased greatly over that time. The collective understanding of wave transformations as they propagate from deep into shallow water, as well as their dissipation within the surf zone has improved steadily. However, the localized effects and processes that effect waves at the instant of breaking are still only vaguely understood. Hence, the measurement and prediction of wave characteristics at the instant of breaking is of paramount importance to any planning or design concerning the surf zone.

Wave breaking is extremely complex to mathematically model and hence much of the work detailing breaking wave characteristics is the result of empirically fitting relationships based on idealized regular wave conditions to laboratory experimental data. Laboratory studies are extremely valuable and allow researchers direct control over numerous wave parameters, yet are also inherently limited. Traditional field study techniques are plagued by low spatial resolution data, non-permanent seafloor profiles at chosen study locations, safety concerns for researchers and exorbitant equipment costs.

To the best of the author’s knowledge, no prior study has collected a more complete and detailed set of irregular breaking wave characteristics in the field. The final data sets from each of the three study locations feature excellent resolution and low uncertainty values for all wave characteristics. For example, breaking heights and positions extracted from the horizontally rectified and geo-corrected video images both featured less than 5% average uncertainty. Breaking wave periods extracted using a video timestamp method featured only 3% uncertainty. All breaking data was directly extracted from measurement devices, negating the need for broad assumptions about breaking conditions and the associated uncertainty.

Additionally, this thesis presents a novel, low cost method to extract accurate and precise breaking wave characteristics through unified seafloor bathymetric surfaces, directly measured shallow water incoming wave conditions and remote sensing operations. While additional parameters where collected and analysed in this study, the bulk of the required data point can be collected using a simple off-the-shelf camcorder (< $300 USD) and a handheld GPS (< $ 50 USD), making for an extremely low cost and readily deployed measurement system.

A detailed review of the published relationships predicting and approximating the characteristics of breaking waves illustrated the continually increasing understanding of the highly active and non-linear breaking wave event. Initial attempts dealt solely with wave breaking depth and wave height, while more recent predictors have expanded to included incoming wave characteristics, bathymetric slopes, vortex parameters and plunge distances in an effort to consistently and accurately predict breaking wave conditions over a wider range of allowable conditions. Unfortunately, the inconsistent definitions and
methods of extracting breaking wave characteristics within the published literature have resulted in unnecessary variation in predicted values.

Hence, within this thesis, numerous differing published and suggested definitions of breaking wave water depth, breaking period and effective slope were investigated. Three different methods of presenting the water depth at incipient breaking were compared and $h_c$, the water depth corrected for optical effects, was found to increase correlation with published predictors by up to 13%. The effective seafloor slope, or amount of seafloor slope which affects the breaking conditions of waves, has previously been overlooked thus far in published literature. Most studies have either been physically limited by laboratory size or have followed the advice of the Coastal Engineering Manual [2008], which suggests calculating effective slopes over a single horizontal wavelength. Having determined that wave breaking is more dependent on the water depth than on the geographic position, several novel effective slope definitions, calculated over a wavelength dependent depth basis, were suggested. Thorough analyses showed that $m_{1/3}$ reduced predicted wave height scatter, was more robust to directional uncertainty and improved final breaking wave height predictions by approximately 7.5%.

Analyses of wave periods at breaking and just seaward of the surf zone showed variation with regards to individual studied waves and hence would introduce additional uncertainty if not accounted for. This variation was attributed to the intermediate nature of the ADCP deployment and small variations between the individual wave celerity and the wave group speed. Comparison of two differing breaking and four different incoming wave period measurement methods showed the incoming crest-to-crest wave period and the breaking wave to proceeding wave period to have the highest correlation. Hence, the proceeding wave period is suggested for analyses using breaking wave periods, while the crest-to-crest period is suggested as the best incoming wave period parameter.

A detailed comparison of the performance of all breaking wave predictors, using the extracted irregular wave breaking parameters, illustrated that the exponential prediction form used by Rattanapitikom and Shibayama [2000] and Goda [2010] outperformed all other functional breaker prediction forms. Using the base exponential functional form, an optimized breaking wave height predictor using the newly defined breaker depth and effective seafloor slope parameters was found to improve $R^2$ correlations by 8% and 24% for Rattanapitikom and Shibayama’s 2000 equation and Goda’s 2010 predictor respectively. This increased correlation is attributed to the new depth and effective slope measurement techniques. However, it is noted that the dataset collected in this study does not cover all environmental conditions and the optimized breaker height prediction relationship should be tested against a more diverse dataset prior to complete adoption.

A detailed investigation of all published studies detailing breaking wave vortex ratios and angles illustrated the contradictory findings of several papers. With the sole exception of Mead and Black [2001], no published studies found any robust correlation between vortex ratio or vortex angle parameters
and breaking conditions. Given the fact that the collected dataset showed excellent correlation with breaker height predictors, and included detailed vortex measurements, a set of detailed analyses investigated the viability of predicting vortex parameters from incoming wave parameters was completed.

While no quantifiable or robust trends emerged from these analyses, corroborating the findings of Johnson [2009] and Blenkinsopp and Chaplin [2008], several generalized observations could be made basis on the plotted datasets. Increasing wave heights and breaking depths resulted in lower vortex ratios, which physically speaking correlated with a larger vortex width with respect to the vortex length. A decreasing effective seafloor slope, extracted using the $m_{1/3}$ method, resulted in a higher vortex ratios. While this agrees with qualitative knowledge and the findings of Mead et al. [2001], the trends did not follow the trends predicted by Black and Mead’s 2001 equation.

Through the detailed review of published works and the extracted experimental data, it is suggested that the sole use of the vortex ratio and vortex angle is insufficient to predict the breaking intensity of waves and should not be investigated further. It is recommended that any further studies dealing with geometric characteristics include detailed measurements of the suggested novel geometric properties, should investigate a multi parameter approach and be very realistic about the possibility of not being able to extract sufficiently precise measurements.

In conclusion, this thesis presents a novel, low cost, repeatable, accurate method to extract breaking wave conditions from the surf zone without significant researcher safety concerns and in many hostile environments. Final mean uncertainty values for breaking wave heights and depths were calculated to be only 4.7% and 7.5% respectively, showing the measurement methods able to accurately extract detailed information at the volatile instant when a wave breaks. Novel methods of calculating the breaking water depth from overview images allow researchers to now extract a full suite of breaking wave characteristics. Newly presented effective slope definitions better represent the amount of seafloor slope which affects breaking conditions and improved breaking wave height predictions by an average of 7.5%. Given the abundance of construction and pressure on our coastal areas, the methods, predictions and findings of this thesis open up new avenues to better explain, understand and predict the effect of irregular breaking ocean waves on our coastlines.
10 Recommendations

Numerous general recommendations based on the hindsight examination and analyses of the collected data are suggested; both for general coastal engineering practice and for improved remote measurement of waves at the instant of breaking. Firstly, the optimized breaking wave height predictor in Eq. (107) was optimized only within the data ranges collected in this study. However, these are not indicative of all wave and environmental conditions encountered in coastal engineering practice. It is recommended that Eq. (107) be tested against a larger and more diverse dataset before adoption.

Additionally, the effects of seafloor friction, swell/swell interactions, atmospheric effects and local complicating variables are inherently included in the measured dataset yet not investigated in detail. Additional analysis to extract the effect of these variables on the breaking conditions is recommended. These factors may play dominant roles in situations were which previously considered too energetic or dangerous for traditional methods of data collection. The presented novel method provides an excellent opportunity to begin to understand these environments.

The presented method may also allow coastal engineers to reanalyse historical video of destructive coastal events in order to better understand the wave climate and dynamics at play during these events. This sort of hindcast analysis may shed considerable light on individual waves within the storm and provide additional information to prevent future disasters.

In order to improve the performance characteristics of this study, a number of detailed recommendations can be made. Firstly, while significant effort was allocated to determining the best locations for this breaking wave study, the two Santa Cruz locations featured varying levels of seaweed growth in the surf zone. While this was unnoticeable during high tides, the kelp will have had some influence on the bathymetric profiling and possibly breaking conditions at lower tides. Hence, it is recommended that study locations should be free of significant seafloor seaweed or kelp. This will help reduce any bathymetric depth uncertainties due to false positive echo soundings, as well as any interactions with the breaking waves.

The pixel resolution of the overview camera played a considerable role in determining the final accuracy of the extracted height and position values. In order to minimize these effects it is recommended that only high resolution cameras (> 2 megapixel) continue be used for remote sensing studies. Unfortunately, the substitution of the overview cameras between Santa Cruz and Barbados deployments required rescripting of Matlab analysis code for the new camera; hence camera substitution should be avoided. Understanding that the overview camera height involves a trade-off between wave height and wave position accuracy (shown in Figure 6-10), it is suggested that a detailed examination determining the optimal camera height to breaking waves distance ratio would be extremely beneficial to this area of research.
For this study, the distance, $x_3$, in Figure 6-11, was used to correct for the optical trough depth and an optimised value of $1.3x_3$ used to correct for wave face slope effects. However, the best correction value will undoubtedly vary depending on the shape of each individual wave front (spilling, plunging, surging), the amount of localized water level set-up/set-down, the position of the studied wave within the wave group and host of other influences. A detailed examination into the effects of the suggested influences will help further improve the accuracy of remote optical wave measurement systems. Understanding the complexities and difficulty of extracting this data from a field site, it is recommended that these studies be performed in a deep water flume using waves with a minimum height of 100mm and minimum breaking depth of 100mm.

As shown in Chapter 6.4, the true wave ray path over complex seafloors plays a determining role in the effective seafloor slope. In order to improve on the presented calculation methods, further studies may involve numerous deployed ADCP units to extract incoming wave directions and include phase resolved numerical wave propagation models (such as REF/DIF [Kirby and Dalrymple, 1994]). However, it is noted that the added accuracy acquired via the suggested methods by result in negligible differences when calculating $m_{1/3}$.

With the sole exception of Mead and Black [2001], no quantifiable trends or robust dependencies between breaking vortex profile parameters and wave characteristics were found in this study or any published works. Hence, it is recommended that any persons wishing to further investigate geometric characteristics include detailed measurements of the suggested novel geometric properties and investigate a multi-parameter approach. However, they should be cognisant that the difficulty of reliably extracting the parameters recommend in Chapter 8 from field conditions is high. Given the necessity of a perfect profile image to extract the recommended parameters and the complex nature of breaking conditions, it is probable that detailed trends will not be evident within the inherent uncertainty and scatter of this data collection method. It is suggested that purely geometric descriptors are both too variable and unstable for rigorous breaking wave intensity studies to be performed.

Finally, and possibly most importantly, it is recommended that the definitions and measurement methods for calculating the breaking wave height, water depth, effective seafloor slope and wave period require uniformity across all coastal science and engineering disciplines. The methods and definitions used in this thesis are humbly recommended for all further work in detailing breaking wave characteristics.
11 References


## A. Appendix: Overview of Breaking Wave Relationships

<table>
<thead>
<tr>
<th>Primary Relationship</th>
<th>Secondary Relationship</th>
<th>Applicability</th>
<th>Study Conditions</th>
<th>Wave Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>McCowan, 1984</td>
<td>( y_b = 0.78 )</td>
<td></td>
<td>( m = \alpha, \eta_c = 1.78h_b )</td>
<td>Constant Depth</td>
</tr>
<tr>
<td>Miche, 1944</td>
<td>( H_b = 0.142L_b \tanh \left( \frac{2\eta_b}{L_b} \right) )</td>
<td>( m = \alpha )</td>
<td>Constant Depth</td>
<td>Small Amplitude</td>
</tr>
<tr>
<td>Le Mehaute and Koh, 1967</td>
<td>( H_b = 0.76H_b \left( \frac{H_b}{L_b} \right)^{-0.25} m^{1.7} )</td>
<td>( 0.50 &lt; m &lt; 1.5 ) ( 0.002 &lt; H_b/L_b &lt; 0.093 )</td>
<td>Laboratory and Field Data, Plane slope with converging width</td>
<td>Energy Flux Equations</td>
</tr>
<tr>
<td>Galvin, 1968</td>
<td>( y_b = \frac{1}{2} (1.40 - 6.85m) ) if ( m &lt; 0.07 ) ( y_b = 1.69 ) if ( m &gt; 0.07 )</td>
<td>( 0.001 &lt; H_b/L_b &lt; 0.051 )</td>
<td>Laboratory Data, Plane, emergent slope.</td>
<td>Regular Waves</td>
</tr>
<tr>
<td>Collins and Weir, 1969</td>
<td>( y_b = 0.72 + 5.6m )</td>
<td>Undetermined</td>
<td>Laboratory and historical data. Plane, emergent slopes</td>
<td>Linear Wave Theory and Empirical fitting</td>
</tr>
<tr>
<td>Cantfield and Street, 1968</td>
<td>( y_b = 0.75 + 25m - 112m^2 + 3870m^3 )</td>
<td>( \theta &lt; m &lt; 0.045 )</td>
<td>Laboratory and historical data. Multi-slope, non-emergent beach</td>
<td>Solitary Wave</td>
</tr>
<tr>
<td>Yamada et al., 1968</td>
<td>( y_b = 0.3261 )</td>
<td>( m = \alpha, \eta_c = 1.78h_b )</td>
<td>Plane slope</td>
<td>Solitary Wave</td>
</tr>
<tr>
<td>Goda and Morino, 1970</td>
<td>( y_b = 0.17 \frac{L_b}{h_b} \left( 1 - e^{-\frac{1.50(\eta_b + 1.2m^2)}{L_b}} \right) )</td>
<td>( 0.05 &lt; m &lt; 0.2 ) ( 0.001 &lt; H_b/L_p &lt; 0.051 )</td>
<td>Laboratory Data, Plane, emergent slope.</td>
<td>Regular Waves</td>
</tr>
<tr>
<td>Weggel, 1972</td>
<td>( y_b = \frac{b(m) - a(m)}{H_b gT^2} ) ( a(m) = 43.8(1.0 - e^{-19.0}) ) ( b(m) = 15.6(1.0 + e^{-19.5}) )</td>
<td>( 1/50 &lt; m &lt; 1/5 ) ( H_b/L_p \leq 0.06 ) ( \eta_c = 1.78h_b )</td>
<td>Laboratory Data, Plane Slopes</td>
<td>Solitary Wave</td>
</tr>
<tr>
<td>Komar and Gaughan, 1972</td>
<td>( H_b = 0.56h_b \left( \frac{H_b}{L_b} \right)^{1.5} )</td>
<td>Undetermined</td>
<td>Laboratory and Field data</td>
<td>Linear Wave</td>
</tr>
<tr>
<td>Battjes, 1974</td>
<td>( y_b = 1.062 + 0.137 \log(\xi_d) )</td>
<td>( 0.05 &lt; m &lt; 0.2 )</td>
<td>Plane, emergent slope.</td>
<td>Irregular Waves</td>
</tr>
<tr>
<td>Sunamura and Horikawa, 1974</td>
<td>( H_b = H_p m^{0.2} \left( \frac{H_b}{L_b} \right)^{-0.26} )</td>
<td>( 0.01 &lt; m &lt; 0.1 )</td>
<td>Laboratory Data, Plane, emergent slope.</td>
<td>Regular Waves</td>
</tr>
<tr>
<td>Madsen, 1976</td>
<td>( y_b = 0.72(1 + 6.4m) )</td>
<td>Undetermined</td>
<td>Laboratory and Field Measurements</td>
<td>Regular Waves</td>
</tr>
<tr>
<td>Van Dorem, 1979</td>
<td>( H_b = 0.4H_s \left( \frac{H_b}{L_b} \right)^{-1.7} ) ( H_d/L_o &lt; 0.07 )</td>
<td></td>
<td>Laboratory Data, Plane slope</td>
<td></td>
</tr>
<tr>
<td>Ostendorf and Madsen, 1979</td>
<td>( y_b = 0.14(h_b/h_d) \tanh \left( \frac{0.8 + 5m}{2} \right) \left( \frac{2.6h_b}{L_b} \right) )</td>
<td>( m &lt; 0.1 )</td>
<td>Laboratory Data, Barred slope</td>
<td>Regular Waves</td>
</tr>
<tr>
<td>Singamsetti and Wind, 1980</td>
<td>( y_b = 0.937m^{0.156} \left( \frac{H_b}{L_b} \right)^{-0.13} )</td>
<td>( 0.03 &lt; m &lt; 0.2 ) ( 0.018 &lt; H_b/L_p &lt; 0.079 )</td>
<td>Plane, emergent beach</td>
<td>Regular Waves</td>
</tr>
<tr>
<td>Sunamura, 1981</td>
<td>( y_b = 1.1 \left( \frac{m}{\sqrt{H_b/L_o}} \right)^{1/6} )</td>
<td>( 0.02 &lt; m &lt; 0.3 )</td>
<td>Laboratory and Field Data, Plane slope.</td>
<td>Regular and Irregular Waves</td>
</tr>
<tr>
<td>Ogawa and Shuto, 1984</td>
<td>( y_b = 1.48m^{0.21} \left( \frac{H_b}{L_b} \right)^{-0.25} ) ( h_b = 0.46h_b m^{-0.012} \left( \frac{H_b}{L_b} \right)^{-0.2} )</td>
<td>( 0.01 &lt; m &lt; 0.1 ) ( 0.02 &lt; H_b/L_p &lt; 0.065 )</td>
<td>Laboratory Data, Barred, plane and stepped beaches.</td>
<td>Regular Waves</td>
</tr>
<tr>
<td>Dean et al., 1985</td>
<td>( H_b = \left( \frac{T}{g} \right)^{1/5} \left( \frac{H_s L_c \cos \theta_b}{2} \right)^{1/2} ) ( h_b = 0.64h_b \left( \frac{H_s}{L_o} \right)^{-0.3} )</td>
<td>( 0.007 &lt; H_d/L_p &lt; 0.1 )</td>
<td>Laboratory Data, Plane and Stepped Beach.</td>
<td>Linear Theory and Energy Balance Eqns.</td>
</tr>
<tr>
<td>Battjes and Stive, 1985</td>
<td>( H_b = 0.14L_b \tanh \left( 0.5 + 0.4 \tanh \left( \frac{33H_b}{L_o} \right)^{2} \left( \frac{0.8L_b}{h_b} \right) \right) )</td>
<td>( m \leq 0.05 ) ( 0.01 &lt; H_d/L_p &lt; 0.032 )</td>
<td>Laboratory and Field Data, Plane and Barred Slopes.</td>
<td>Regular and Irregular Waves</td>
</tr>
<tr>
<td>Nelson, 1992</td>
<td>( y_{b,max} = 0.55 + \exp \left( -0.012 \cot \left( \alpha \right) \right) )</td>
<td>( m \leq 0.01 )</td>
<td>Laboratory and Field Data</td>
<td>Regular and Irregular Waves</td>
</tr>
<tr>
<td>Ref.</td>
<td>Equation</td>
<td>Applicability</td>
<td>Study Conditions</td>
<td>Wave Type</td>
</tr>
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<td>------</td>
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</tr>
<tr>
<td>Svendson, 1987</td>
<td>( y_b = 1.05 \left( \tan \beta / (h_b / L_b) \right)^{0.21} )</td>
<td>( 0.25 \leq \tan \beta / (h_b / L_b) \leq 1 )</td>
<td>Laboratory Data Plane slope</td>
<td>Regular waves</td>
</tr>
<tr>
<td>Smith and Kraus, 1991</td>
<td>( y_b = \left( h_b / L_b \right) m / a(m) ) where: ( a(m) = 5.00 \left( 1.0 + e^{-4.3m} \right) ) ( b(m) = 1.12 \left( 1.0 + e^{-4.3m} \right) )</td>
<td>( 0.0125 &lt; m &lt; 0.1 ) ( 0.0007 &lt; h_b / L_b ) ( &lt; 0.0921 )</td>
<td>Laboratory Data Plane slope</td>
<td>Regular Waves</td>
</tr>
<tr>
<td>Gourlay, 1992</td>
<td>( H_b = 0.478 H_c \left( H_c / L_c \right)^{-0.28} ) ( h_b = 0.259 \left[ \tan \gamma \left( H_c / L_c \right) \right]^{-0.17} )</td>
<td>( 0.022 \leq m \leq 0.15 ) ( 0.001 &lt; h_b / L_b ) ( &lt; 0.066 )</td>
<td>Laboratory Data Plane Slope</td>
<td>Regular Waves</td>
</tr>
<tr>
<td>Kaminsky and Kraus, 1993</td>
<td>( y_b = 1.204 \left( \frac{h_b}{L_b} \right)^{0.27} )</td>
<td>( 0.009 \leq m \leq 0.2 ) ( 0.001 &lt; h_b / L_b &lt; 0.092 )</td>
<td>Laboratory Data</td>
<td></td>
</tr>
<tr>
<td>Komar, 1998</td>
<td>( H_b = 0.399 \left( H_c / L_c \right)^{0.44} ) ( d_b = H_b \left { 1.2 \left[ \frac{H_c}{L_c} \left( H_c / L_c \right)^{0.50} \right]^{0.27} \right } )</td>
<td>( 0 \leq m \leq 0.44 ) ( 0.001 &lt; h_b / L_b &lt; 0.1 )</td>
<td>Laboratory Data Barred and Plane Slopes.</td>
<td>Linear Wave Theory – Regular Waves</td>
</tr>
<tr>
<td>Rattanapitikon Shibayama, 2000</td>
<td>( y_b = 0.175 \left( C_m / \sqrt{h_b} \right) \left[ 1 - e^{-4 \left( 0.21 \left( H_c / L_c \right)^{-1.51} \right)} \right] )</td>
<td>( 0 \leq m \leq 0.38 ) ( H_b / L_b &lt; 0.112 )</td>
<td>Laboratory Data Barred and Plane Slopes.</td>
<td></td>
</tr>
<tr>
<td>Rattanapitikon, et al. 2003</td>
<td>( H_b = (-1.40m^2 + 0.57m + 0.23)L_b \left( H_c / L_c \right)^{0.33} )</td>
<td>( 0 \leq m \leq 0.38 ) ( H_b / L_b &lt; 0.112 )</td>
<td>Laboratory Data Barred and Plane Slopes.</td>
<td></td>
</tr>
<tr>
<td>Tsai et al., 2005</td>
<td>( H_b = 0.79H_c \left( H_c / L_c \right)^{0.33} )</td>
<td>( m &gt; 0.2 )</td>
<td>Laboratory data – Plane, non-emergent beach</td>
<td></td>
</tr>
<tr>
<td>Rattanapitikon and Shibayama, 2006</td>
<td>( \frac{H_b}{H_c} = (-0.57m^2 + 0.31m + 0.58L_b \left( H_c / L_c \right)^{0.83} )</td>
<td>( h_b = (3.86m^2-1.98m+0.88)H_c \left( H_c / L_c \right)^{0.83} ) ( h_b = (3.86m^2-1.98m+0.88)H_c \left( H_c / L_c \right)^{0.83} )</td>
<td>( H_b / L_b \leq 0.1 ) ( H_b / L_b &gt; 0.1 )</td>
<td>Laboratory Data Barred and Plane Slopes.</td>
</tr>
<tr>
<td>Camenen and Larson, 2007</td>
<td>( y_b = 0.284 \left( \frac{H_b}{L_b} \right)^{0.35} \tanh \left( 0.87 + 0.14 \left( \frac{H_b}{L_b} \right)^{0.5} \left( \frac{m}{m_{\text{max}}} \right)^{0.5} \right) )</td>
<td>( 0 \leq m \leq 2 ) ( 0.001 &lt; h_b / L_b &lt; 0.117 )</td>
<td>Laboratory Data Barred, plane and stepped slope</td>
<td></td>
</tr>
<tr>
<td>Le Roux, 2007</td>
<td>( y_b = 0.835 + 0.0843m - 0.0036m^2 )</td>
<td>( m \leq 0.2 )</td>
<td>Laboratory Data.</td>
<td></td>
</tr>
<tr>
<td>Goda, 2010</td>
<td>( H_b = 0.17L_b \left[ 1 - e^{-2 \left( 1.5 \left( H_c / L_c \right)^{0.5} \right)} \right] )</td>
<td>( 0.005 \leq m \leq 0.11 )</td>
<td>Laboratory Data.</td>
<td>Regular Waves</td>
</tr>
<tr>
<td>Liu et al., 2011</td>
<td>( 0.69 = \left[ 1.21 - 3.30 \left( H_c / L_c \right) \right] \left[ 1.48 - 0.54 \left( H_c / L_c \right) \right] \left( H_c / L_c \right) \left( H_b / L_b \right) ) ( \sqrt{h_b / \zeta_b} )</td>
<td>( 0.005 \leq m \leq 0.38 )</td>
<td>Laboratory Data Barred, plane and stepped slope</td>
<td>Regular Waves</td>
</tr>
<tr>
<td>Yao et al., 2012</td>
<td>( y_b = \frac{y_1 - y_2}{2} \left( \tanh \left( \frac{a(1.4 - h_c)}{H_c} \right) - \frac{y_1 - y_2}{y_1 - y_2} \right) )</td>
<td>( m \leq 0.33 ) ( 0.003 &lt; h_b / L_b &lt; 0.0888 )</td>
<td>Laboratory Data.</td>
<td>Regular Waves</td>
</tr>
</tbody>
</table>

### Table A-2: Barred Slope Specific Relationships

<table>
<thead>
<tr>
<th>Primary Relationship</th>
<th>Secondary Relationship</th>
<th>Applicability</th>
<th>Study Conditions</th>
<th>Wave Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Larson and Kraus, 1988</td>
<td>( y_b = 1.14 \left( m / \sqrt{H_c / L_c} \right) )</td>
<td>( \frac{H_b}{H_c} = 0.66 )</td>
<td>( h_b / L_b \leq 0.3 ) ( 0.001 &lt; h_b / L_b &lt; 0.075 )</td>
<td>Laboratory Data. Barred Slopes.</td>
</tr>
<tr>
<td>Smith and Kraus, 1991</td>
<td>( y_b = 0.41 + 0.98 \zeta_0 ) ( y_b = 1.45 - 0.22 \zeta_0 )</td>
<td>( 0.3 \leq \zeta_0 \leq 0.85 ) ( 1.6 \leq \zeta_0 \leq 3.5 )</td>
<td>Laboratory Data Barred, Movable Slopes</td>
<td>Regular Waves</td>
</tr>
<tr>
<td>Bleilksopp and Chaplin, 2008</td>
<td>( y_b = 0.85 )</td>
<td></td>
<td></td>
<td>Laboratory Data. Barred Slopes.</td>
</tr>
<tr>
<td></td>
<td>Primary Relationship</td>
<td>Secondary Relationship</td>
<td>Applicability</td>
<td>Study Conditions</td>
</tr>
<tr>
<td>---</td>
<td>--------------------------------------------------------------------------------------</td>
<td>-----------------------------------------------------------------------------------------</td>
<td>---------------</td>
<td>---------------------------</td>
</tr>
<tr>
<td>Seyama and Kimura, 1988</td>
<td>$r_b = \left(0.16 \frac{L_a}{h_b} \left[1 - \exp\left(-0.88 \frac{h_b}{L_a} \left(1 + 15m^2\right)^{0.5}\right)\right] - 0.96m + 0.2\right)$</td>
<td>$H_b = \left[0.095 \exp(4.0m)\right] L_a \tanh\left(\frac{2\pi h_b}{L_{pl}}\right)$</td>
<td>$0.02 &lt; m &lt; 0.1$</td>
<td>Laboratory Data</td>
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<tr>
<td>Kamphuis, 1991</td>
<td>$r_b = 0.71m^{0.155} \left(\frac{H_a}{L_a}\right)^{-0.13}$</td>
<td>$H_b = 0.12L_a \left(1 - e^{-\frac{1.15h_b}{L_{pl}}} \left(1 + 15m^2\right)^{0.5}\right)$</td>
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Table A-3: Irregular Wave Specific Relationships
B. Appendix: ADCP Set-up Parameters

**Hook and Sewers Peak ADCP Deployment Set-Up:**

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**Tropicana ADCP Deployment Set-Up:**

| BP   | 000  | Bottom Track Pings per Ensemble |
| CF   | 11101| Flow Ctrl (EnsCyc; PngCyc; Binry; Ser; Rec) |
| CK   | | Keep Parameters as USER Defaults |
| CR #| | Retrieve Parameters (0 = USER, 1 = FACTORY) |
| CS   | | Start Deployment |
| EA   | +00000| Heading Alignment (1/100 deg) |
| EB   | +00000| Heading Bias (1/100 deg) |
| ED   | 00045| Transducer Depth (0 - 65535 dm) |
| ES   | 35 | Salinity (0-40 thousand) |
| EX   | 00000| Coord Transform (Xform: Type, Tilts, 3 Bm, Map) |
| EZ   | 1111101| Sensor Source (C,D,H,P,R,S,T) |
| TE   | 00:00:00.50| Time per Ensemble (hrs:min:sec,sec/100) |
| TF   | ***/***,**,**,**,** | Time of First Ping (yr/month/day, hour:min:sec) |
| TP   | 00:00:50 | Time per Ping (min:sec,sec/100) |
| TS   | 11/11/24,19:39:40 | Time Set (yr/month/day,hour:min:sec) |
| WD   | 111,000,000| Data Out (Vel, Cor, Amp; PG, St, P0; P1, P2, P3) |
| WF   | 0250 | Blank After Transmit (cm) |
| WN   | 070  | Number of depth cells (1-128) |
| WP   | 00001| Pings per Ensemble (0-16384) |
| WS   | 0005 | Depth Cell Size (cm) |
| WVV  | 175 | Mode 1 Ambiguity Vel (cm/s radial) |
| CK   | | Parameters saved as USER defaults |

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C. Appendix: Extracted Breaking Wave Characteristics

Table C-1 and Table C-2 below overview of the final extracted breaking wave characteristics used for all analyses. Readers interested in getting access to the entire raw data set, or portions thereof, should contact the author at bryson@uoguelph.ca. All data is publically available and can be transferred to interested persons via ftp site download or data storage hardware transfers. Note: Entire raw data set is approximately 50 gigabytes (GB) and requires the use of proprietary manufacturer software packages for initial analysis procedures.

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**Table注释：**
- 该表格显示了不同行和列的数据，数据格式为浮点数，可能表示某种测量或计算结果。
### Table C-2: Extracted slope and vortex properties (Blue = Hook, Red = Sewers, Green = Tropicana)

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D. Appendix: Robustfit Coefficient of Determination (R^2) Data Fit

The co-efficient of determination (R^2) is commonly used to describe the portion of the experimental data variability that can be explained by the regression model. In order to best approximate the correlation between a linear regression model, using a robust fitting analysis, and experimentally collected data, a robust R^2 parameter is required. First, robust R^2 allows for a weighting to be assigned to each data point depending on the individual distance from the regression line. This eliminates the need to continually remove outlying data points and provides a method to analyze the degree of correlation between observed and fitted values using Matlab’s robustfit() algorithm.

The following code demonstrates how one may compute a possible R^2 value for the robust fit:

```matlab
[b_rob, robust_stats] = robustfit(x,y);

sse = robust_stats.dfe * robust_stats.robust_s^2;
fitted = brob(1) + brob(2)*x;
ssr = norm(fitted-mean(fitted))^2;
rsquare_robustfit = 1 - sse / (sse + ssr);
```

As with all statistical qualities, the robust R^2 value can be misleading. For example if a significant portion of the data points are weighted with a zero weight, the robust fit R^2 may output a correlation when in reality, there is very little correlation. Hence it is important to confirm the robust R^2 by comparing against the standard R^2.