Nuclear Structure Study of Cd-110 through Internal Conversion Electrons

by

Badamsambuu Jigmeddorj

A Thesis
Presented to
The University of Guelph

In partial fulfilment of requirements for the degree of Master of Science in Physics

Guelph, Ontario, Canada
©Badamsambuu Jigmeddorj, August, 2012
ABSTRACT

Nuclear Structure Study of Cd-110 through Internal Conversion Electrons

Badamsambuu Jigmeddorj
University of Guelph, 2012

Advisors:
Professor P.E. Garrett

For many years, $^{110}$Cd has been considered the classic example of a vibrational nucleus within both the Collective Model and the Interacting Boson Model (IBM). As a vibrational nucleus, $^{110}$Cd exhibits multi-phonon states. Recent studies on other cadmium isotopes have shown a breakdown of vibrational motion at the three-phonon level. The latest study on $^{112}$Cd suggests that some of these excitations may instead result from intruder bands. The study through internal conversion electrons is important to investigate intruder structures in $^{110}$Cd, using the enhanced E0 transitions between intruder states and spherical phonon states as a signature.

The nuclear structure of $^{110}$Cd has been studied with $^{110}$In beta decay through internal conversion electrons performed at TRIUMF using the 8π spectrometer. The level scheme of $^{110}$Cd through internal conversion electron transitions was constructed using the $e^- - \gamma$-coincidence matrix. The sub-shell ratios and multipolarities are determined and compared with the evaluated data set. The absolute internal conversion coefficients for some mixed transitions were determined using an internal calibration efficiency for Si(Li) detector and relative efficiency for HPGe detector.

The 396 keV and 708 keV E0 transitions have been observed between intruder and spherical phonon states. The E0 transition strength, $\rho^2(E0)$, of $0.115^{+0.077}_{-0.066}$ was determined for 708.21 keV line.
To the memory of my father, Gotov Jigmeddorj.
Hairt aavinhaa gegeen dursgald zoriulav.
Acknowledgements

I would like to thank each and every person who has contributed to making this project possible and to whom I am deeply indebted.

First of all, it is a honor for me to thank my supervisor, Dr. Paul Garrett, who guided me throughout my research with patience, understanding and much encouragement. I also want to thank him for not only giving me the rare opportunity to achieve my dream of studying in Canada, but for his support in allowing me to travel and attend many research activities.

The experiment this analysis is based on could not have been performed without the contribution of many people from different places to whom I owe a great debt. I would especially like to thank: Jack Bangay, Kathryn Green, Chandana Sumithrarachchi, Smarajit Triambak, Adam Garnsworthy, Gordon Ball, Greg Hackman, David Cross, W.D. Kulp, John Wood, and Steve Yates.

I am very lucky and will always be grateful for being part of the unique nuclear physics group at the University of Guelph which is composed by intelligent and helpful graduate students, creative fellowships and excellent supervisors. I want to thank all members, past and present, specifically: Alejandra Diaz Varela, Alex Laffoley, Carl Svensson, Julian Michetti-Wilson, Ryan Dunlop, Sophie Chagnon-Lessard, Baharak Hadinia, Vinzenz Bildstein, Kyle Leach, Evan Rand, Greg Demand, James Wong,
Laura Bianco, Drew Jamieson, Paul Finlay, Allison Radich and Zachary Fairchild. I have really enjoyed my time with all of you and greatly appreciate all the time you each took to help me.

This part of my acknowledgment is to my collaborators from Mongolia. I would like to thank Prof. G. Khuukhenkhuu for his guidance and long support, Profs. N. Ganbaatar, S. Davaa, N. Norov and P. Zuzaan for giving me opportunity to study and work throughout my career, and to rest members of the Nuclear Research Center of Mongolia for their many enlightening discussions.

It was great opportunity to work in Flerov Laboratory of Nuclear Reactions at the Joint Institute for Nuclear Research in Dubna, Russia. I am deeply thankful to Dr. Yu.P. Gangrsky, V.I. Zhemenik, D.V. Karaivanov, G.V. Mishinsky, K.P. Marinova, B.N. Markov, and S.G. Zemlyanoi for hosting my research in Dubna.

I owe the biggest debt to my family for their love and limitless support. (In Mongolian) Namaig turuulj, minii huliig duruund, gariig ganzgand hurgesen, hairt eej D. Dolgorsuren taniihaa achrald murgue. Mun hadam aav N. Byambaakhuu, hadam eej G. Doljinsuren hoyortoo namaig tursun ur shigee hairlaj, demijj baidagt tani gun talarhal hurgue. Ulmaar tursun ah, egch nartaa, buh duu nartaa, mun hurgen ah, bergen egch, hadam ah, hadam egch, hadam duu nartaa bolon busad buh hamaatan sadanguuddaa ner duridalguigeer bugded ni talarhalin ug hurgej baina.

Finally, my deepest gratitude goes to my wife B. Tumurtogoo and our loving children B. Khanzolboo and B. Khatanzolboo for their love, support and patience, and especially for always putting a smile on my face and filling my heart with happiness. Without your love and encouragement, this thesis could not has been completed.
Contents

Acknowledgements iv

1 Motivation 1

2 Theoretical Background for Nuclear Structure 3

2.1 Introduction 3

2.2 The Shell Model 4

2.3 The Collective Model 6

2.3.1 Vibrational Motion in Nuclei 7

2.3.2 Intruder States 12

2.4 The Interacting Boson Model 13

2.4.1 U(5) Symmetry Limit 16

3 Nuclear Decay 18

3.1 Beta Decay 18

3.1.1 Beta Decay Energy 19

3.1.2 Fermi Theory of Beta Decay 21

3.1.3 Allowed Beta Decays 24

3.1.4 Forbidden Beta Decays 25

3.2 Gamma Decay 26
3.2.1 Gamma Decay Energy ........................................ 27
3.2.2 Gamma Decay Transition Probability ......................... 28
3.2.3 M1 Transitions ............................................. 30
3.2.4 E2 Transitions of a Collective Nature ....................... 31
3.2.5 E1 Transitions ............................................. 32
3.3 Internal Conversion Process .................................... 33
3.3.1 Weisskopf Estimation of Transition Probabilities .......... 38
3.3.2 E0 Transitions ............................................. 39
3.3.3 Mixing Ratios ............................................. 40

4 Detecting Nuclear Radiations .......................... 42

4.1 Detecting Gamma-Rays ....................................... 42
4.1.1 Photoelectric Absorption .................................. 43
4.1.2 Compton Scattering ....................................... 44
4.1.3 Pair Production ........................................... 46
4.2 Germanium Detectors ......................................... 48
4.3 Detecting Electrons ........................................... 50
4.3.1 Energy Loss ............................................... 50
4.3.2 Backscattering of Electrons ................................. 51
4.4 Lithium-Drifted Silicon Detectors ........................... 51

5 Experiment and Detector Array ......................... 53

5.1 The Complete Gamma-ray Spectrometer ...................... 54
5.2 Measurements ................................................ 57
5.3 Electron-Gamma Coincidence Method ......................... 58
5.4 Germanium Detector Efficiency ............................... 60
5.4.1 Relative Efficiency ........................................... 61
5.4.2 Dead Time Corrections ....................................... 65
5.4.3 Absolute Efficiency ........................................... 65
5.5 Si(Li) Detector Efficiency ...................................... 67

6 Data Analysis ....................................................... 71
6.1 Analyzing Cascades .............................................. 78
6.2 Doublet Peaks ..................................................... 81
6.3 Compton-Scattered Peaks ....................................... 84

7 Results and Conclusions ........................................ 88
7.1 Identified Transitions ........................................... 88
7.2 Sub-shell Ratios .................................................. 89
  7.2.1 Mixing Ratio from Sub-shell Ratio ......................... 91
  7.2.2 113.71(4) keV Transition .................................. 91
  7.2.3 187.56(8) keV Transition .................................. 92
  7.2.4 195.04(4) keV Transition .................................. 93
  7.2.5 244.66(2) keV Transition .................................. 94
  7.2.6 460.53(19) keV Transition ................................. 95
  7.2.7 469.99(16) keV Transition ................................. 96
7.3 Level Scheme of Cd-110 ....................................... 97
7.4 Internal Conversion Coefficient .............................. 98
  7.4.1 Mixing Ratio from K-conversion Coefficient .......... 101
  7.4.2 396.61(13) and 708.21(30) keV Transitions .......... 101
  7.4.3 583.79(6), 677.01(9) and 759.77(12) keV Transitions 101
  7.4.4 626.18(8) keV Transition ................................. 102
<table>
<thead>
<tr>
<th>Section</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.4.5</td>
<td>641.05(2) keV Transition</td>
<td>103</td>
</tr>
<tr>
<td>7.4.6</td>
<td>817.23(11) keV Transition</td>
<td>103</td>
</tr>
<tr>
<td>7.4.7</td>
<td>900.69(9) keV Transition</td>
<td>104</td>
</tr>
<tr>
<td>7.4.8</td>
<td>997.28(10) keV Transition</td>
<td>104</td>
</tr>
<tr>
<td>7.4.9</td>
<td>1332.59(12) keV Transition</td>
<td>105</td>
</tr>
<tr>
<td>7.4.10</td>
<td>E0 Transitions</td>
<td>106</td>
</tr>
<tr>
<td>7.4.11</td>
<td>396.61(13) keV E0 Transition</td>
<td>108</td>
</tr>
<tr>
<td>7.4.12</td>
<td>708.21(30) keV E0 Transition</td>
<td>108</td>
</tr>
<tr>
<td>7.4.13</td>
<td>E0 Transition Strength</td>
<td>108</td>
</tr>
<tr>
<td>7.5</td>
<td>Conclusions</td>
<td>109</td>
</tr>
</tbody>
</table>

Bibliography

ix
### List of Tables

2.1 All combinations of two quadrupole phonons .......................... 10
2.2 Symmetric combinations of two quadrupole phonons ................. 10

5.1 The energies and relative intensities of $^{133}\text{Ba}$ .................. 62
5.2 The energies and relative intensities of $^{56}\text{Co}$ ..................... 63
5.3 The energies and relative intensities of $^{152}\text{Eu}$ .................... 63
5.4 The relative efficiency calibration parameters for our HPGe detector. 63
5.5 The decay rate of $^{60}\text{Co}$ source. .................................... 66
5.6 The absolute $\gamma$-ray efficiency for all detectors. .................... 67
5.7 Normalization factors for the $\gamma$-ray efficiency for all detectors. .... 67
5.8 Calculated relative efficiencies for conversion electron detector. ....... 67
5.9 The $\alpha_K$ values for 120 keV and 245 keV lines. ...................... 68

7.1 All identified conversion electron transitions of $^{110}\text{Cd}$. .............. 89
7.2 Sub-shell ratios for some intensive transitions. .......................... 90
7.3 Mixing ratio from $\alpha_K/\alpha_L$. ....................................... 91
7.4 Internal conversion coefficients for some identified transitions of $^{110}\text{Cd}$. 98
7.5 Mixing ratio from $\alpha_K$. ............................................. 101
List of Figures

2.1 The lowest four vibrational modes of a nucleus [13]. .................. 9
2.2 Low-lying levels of the harmonic vibrator phonon model [12]. ...... 11
2.3 Relative $B(E2)$ transition strengths between phonon states in the vi-
brational model [10]. ...................................................... 11
2.4 Schematic illustration of intruder excitations and normal states in Cd
[23]. .......................................................... 13
2.5 The energy states for $U(5)$ symmetry limit of IMB [24]. ............ 17

4.1 Sketch of the photoelectric absorption process. ......................... 43
4.2 Sketch of Compton scattering. ........................................ 44
4.3 General shape of the Compton electron energy distribution. .......... 45
4.4 Sketch of pair production. ............................................. 46
4.5 The relative importance of the three processes [29]. ................. 47
4.6 Three interaction mechanism behaviors for small detector. .......... 48
4.7 Basic configuration of a lithium-drifted p-i-n junction detector [29]. 52

5.1 Closed $8\pi$ complete spectrometer. .................................. 54
5.2 One hemisphere of $8\pi$ spectrometer (left) and SCEPTAR (right). . 55
5.3 Moving tape collector (left) and the mylar tape (right). .......... 56
5.4 Packed up SCEPTAR (left) and PACES (right). ....................... 56
5.5 A level scheme showing transitions on which gates taken from below . 59
5.6 The energies and peak area of $^{133}$Ba. ................................. 61
5.7 The energies and peak area of $^{56}$Co. ................................. 61
5.8 The energies and peak area of $^{152}$Eu. ................................. 62
5.9 The relative efficiency for a HPGe detector. ............................... 64
5.10 The absolute efficiency for all HPGe detectors. ............................ 66
5.11 Internal relative efficiency calibration for Si(Li) detector using $\alpha_K$(E2)
for 121 keV and 245 keV lines. ............................................... 69
5.12 Simulated efficiency for Si(Li) detector [35]. ............................... 69
5.13 Internal relative efficiency calibration for Si(Li) detector using $\alpha_K$(M1)
for 121 keV and 245 keV lines. ............................................... 70
5.14 Internal relative efficiency calibration for Si(Li) detector excluding two
values from 560 keV and 582 keV lines. ..................................... 70
6.1 Window selection on the 657 keV $\gamma$-transition. .......................... 72
6.2 Total projection spectrum of $\gamma$-rays. ................................... 73
6.3 Total projection spectrum of conversion electrons. .......................... 73
6.4 The initial estimated parameters of the peak in $^{60}$Co source spectrum. 75
6.5 Three components of the peak. ............................................. 75
6.6 Fixed parameters of the peak in $^{60}$Co source spectrum. ................. 76
6.7 Fitted total peak shape. .................................................... 76
6.8 Fitted sample peaks of spectrum gated on the 885 keV $\gamma$-transition. .... 77
6.9 A partial level scheme with identified transitions from a spectrum gated
on the 885 keV $\gamma$-transition. ............................................. 79
6.10 A partial level scheme with expected (blue) and doublet (yellow) trans-
sitions from a spectrum gated on the 885 keV $\gamma$-transition. .............. 79
6.11 A partial level scheme with identified transitions from a spectrum gated on the 678 keV $\gamma$-transition. ........................................ 80
6.12 A partial level scheme with identified transitions from a spectrum gated on the 245 keV $\gamma$-transition. ........................................ 80
6.13 A partial level scheme with identified transitions from a spectrum gated on the 937 keV $\gamma$-transition. ........................................ 82
6.14 A partial level scheme with identified transitions from a spectrum gated on the 626 keV $\gamma$-transition. ........................................ 82
6.15 A partial level scheme showing a doublet (yellow) peak transitions. . 83
6.16 A partial level scheme with identified transitions from a spectrum gated on 1117 keV $\gamma$-transition................................. 83
6.17 A spectrum gated on the 245 keV $\gamma$-transition showing Compton-scattered peaks (red) from many intense transitions. .............. 85
6.18 A spectrum gated on the 626 keV $\gamma$-transition showing Compton-scattered peaks (red) from many intense transitions. .............. 85
6.19 A spectrum created by gating on the 245 keV $\gamma$ transition and without background subtraction. ................................. 86
6.20 Electron spectra, created from main peak and its background slices, gated on the 245 keV $\gamma$-transition................................. 86
6.21 A spectrum gated on the 120 keV $\gamma$-transition showing Compton-scattered peaks (red) from many intense transitions. .............. 87

7.1 Spectrum gated on the 844 keV $\gamma$-transition showing K and L conversion electron peaks of the 113 keV transition and Compton-scattered peak (red) ........................................ 92
7.2 Spectrum gated on the 626 keV $\gamma$-transition showing K conversion electron peak of the 188 keV transition and K and L conversion electron peaks of the 245 keV transition, and Compton-scattered peaks (red). 93
7.3 Spectrum gated on the 1383 keV $\gamma$-transition showing K, L and M conversion electron peaks of the 195 keV transition. 94
7.4 Spectrum gated on the 1334 keV $\gamma$-transition showing K, L and M conversion electron peaks of the 245 keV transition. 95
7.5 A partial level scheme showing a doublet transitions of 582 keV and 461 keV lines. 96
7.6 A partial level scheme showing a doublet transitions of 468 keV line. 97
7.7 Experimental band level scheme of $^{110}$Cd. 99
7.8 Experimental phonon level scheme of $^{110}$Cd. 100
7.9 Spectrum gated on the 245 keV $\gamma$-transition showing K-conversion electron peak of the 626 keV transition (black) and Compton-scattered peaks (red) 102
7.10 Spectrum gated on the 677 keV $\gamma$-transition showing K-conversion electron peak of the 901 keV transition. 103
7.11 Spectrum gated on the 120 keV $\gamma$-transition showing K-conversion electron peak of the 997 keV transition (black) and Compton-scattered peaks (red). 104
7.12 Spectrum gated on the 120 keV $\gamma$-transition showing K-conversion electron peak of the 997 keV transition (black) and Compton-scattered peaks (red). 105
7.13 Possible E0 transitions between intruder and phonon states. 106
7.14 Spectrum gated on the 245 keV $\gamma$-transition showing K-conversion electron peak of the 397 keV E0 transition and Compton-scattered peaks (red).

7.15 Spectrum gated on the 626 keV $\gamma$-transition showing K-conversion electron peak of the 708 keV E0 transition.
Chapter 1

Motivation

In general, due to the complexities of nuclear physics, we must create a simplified nuclear model, then apply perturbations to it in order to understand nuclei. Further improvements to models involve incorporating additional degrees of freedom, which enlarge the basis and greatly add to the complexity. When the required basis becomes too large, one looks for a new physical picture that allows one to approach the problem from an alternate basis. This approach is universal.

The shell model is generally considered the fundamental nuclear model. Unfortunately, its use is, in practice, limited to nuclei close to a closed shells. Beyond the shell model, the development of nuclear structure in medium and heavy nuclei has followed two related paths, one, usually called microscopic models-like the Nilsson model, emphasizes the single-particle motion of nucleons in nuclei, while the other, generally known as the Collective models, like that of Bohr and Mottelson [1], treats the macroscopic motions and excitations of the even-even nuclei.

In 1974, a new approach to collective behavior in nuclei was proposed by Iachello and Arima [2]. It exploits the dynamical symmetry structure of nuclei, and utilizes
powerful group theory techniques to obtain many nuclear properties by simple algebraic techniques. There are many such algebraic models in use today. However, beyond a doubt, the most popular, successful, and widely tested one is the Interacting Boson Model (IBM) of Iachello and Arima.

Physicists have been doing experimental studies to describe each type of nuclear phenomenon that is predicted by nuclear models since Rutherford’s first experiment discovered the atomic nucleus in 1909 [3]. In conjunction, nuclear models have been developed to interpret experimental results.

We have been experimentally studying the even-even $^{110}_{48}$Cd nucleus, as it has been considered as the classic example of a vibrational nucleus within both the Collective model [1] and the IBM [2] for many years. As a vibrational nucleus, $^{110}$Cd exhibits multi-phonon states in its levels scheme which were identified in experiments such as the ($\alpha$,2n) reaction [4], $\beta$ decay measurements [5] and an (n,n'$\gamma$) study [6]. Many other studies have greatly expanded the Cd level scheme and suggested intruder structures that are based on more deformed 2p-4h proton excitations [7, 8, 9, 10].

There are the systematic deviations at the three-phonon level that occur across the Cd isotopic chain, revealing the breakdown of vibrational motion in low-spin states [11]. Moreover, recent work on $^{112}$Cd has suggested an alternative interpretation [12], where the three-phonon 0$^+$ state is assigned as an intruder excitation.

The study through internal conversion is important to investigate intruder structures in $^{110}$Cd, using the enhanced E0 transitions between intruder states and spherical phonon states.
Chapter 2

Theoretical Background for Nuclear Structure

2.1 Introduction

The nucleus is extremely small and may contain up to a several hundred individual protons and neutrons. Nuclear systems may seem so complex that one might think it is impossible to understand its fine details. However, we have achieved a remarkable understanding of nuclei and their fascinating phenomena. We have basic models that provide a framework for our understanding and that are quite simple.

Nuclear models generally can be divided into independent particle models (IPM) in which the nucleons are assumed to move nearly independently in a common nuclear potential and strong interaction models (SIM) in which the nucleons are strongly coupled to each other.

The simplest SIM is the liquid drop model in which the nuclei are considered to be nearly incompressible liquid droplets of extremely high density. This model gives a good physical picture of the fission process. This kind of study is important in
the investigation of very heavy artificial elements and in the treatment of nuclear explosions.

The simplest IPM is the Fermi gas model in which the nucleus is treated as a degenerate Fermi gas of nucleons. The nucleus has many properties that can be explained more simply in terms of independent-particle behavior rather than in terms of the strong-interaction picture implied by the liquid drop model. In this model, the nucleons are assumed to move freely, except for effects of the exclusion principle, throughout a sphere of radius $R = R_0 A^{1/3}$, $R_0 = 1.3 \, fm$.

2.2 The Shell Model

The liquid drop and the Fermi gas models represent the nucleus in crude terms. They cannot explain specific properties of excited nuclear states. A successful model must reasonably well account for previously measured nuclear properties and must predict additional properties that can be measured in new experiments.

Atomic theory based on the shell model has provided a remarkable clarification of the complicated details in atomic structure. Nuclear physicists attempted to use a similar theory to solve the problem of nuclear structure, in the hope of similar success in clarifying the properties of nuclei. Indeed, the existence of nuclear shells is confirmed by many experiments showing the separation energy increases gradually with $N$ or $Z$ except for a few sharp drops that occur at the same neutron and proton numbers. We guess that the sharp discontinuities in the separation energy correspond to the filling of major shells. The numbers at which the discontinuous behavior occurs are called magic numbers ($Z$ or $N=2, 8, 20, 28, 50, 82, \text{and } 126$) and represent the effects of filled major shells. When we try to carry this model over to the nuclear
subject, we immediately encounter two major inconsistencies. These are the existence of nuclear potential and the existence of nuclear spatial orbits.

The question of the nuclear potential existence is dealt with by the fundamental assumption of the shell model. We assume that the motion of a single nucleon is governed by a potential caused by all of the other nucleons. If we treat each individual nucleon in this way, then we can allow the nucleons in turn to occupy the energy levels of a series of sub-shells.

The existence of definite spatial orbits depends on the Pauli principle. Let’s consider a collision, occurring in a heavy nucleus, between two nucleons in a state near the very bottom of the potential well. When the nucleons collide they will transfer energy to one another. If all of energy levels are filled, up to the level of the valence nucleons, there is no way for one of the nucleons to gain energy except to move up to the valence level. The other levels near the original level are filled and cannot accept an additional nucleon. This kind of transfer, from a low-lying level to the valence band, requires more energy than the nucleons transfer in a collision. Because of this the collisions cannot occur, and indeed nucleons can orbit as though they were transparent to one another.

The first step to develop the shell model is the choice of the potential. We know that there are two popular potentials which are the infinite square well and the harmonic oscillator. These potentials are not a good approximation to the nuclear potential. To improve this model we need a more realistic potential for which we choose an intermediate form [13]:

\[ V(r) = \frac{-V_0}{1 + exp [(r - R)/a]} \]  

(2.1)

where \( V_0 \) is the well depth, and \( R \) and \( a \) are the mean radius and skin thickness, respectively. Unfortunately this potential cannot give all proper magic numbers. In
1949, several physicists showed that the inclusion of a spin-orbit potential could give the proper separation of the sub-shells [14].

The energy levels of the nuclear shell model are calculated based on the assumption that the nuclear potential is spherical. We know that this is not true for nuclei in the range $150 \leq A \leq 190$ and $A > 230$. For these nuclei we should use a shell-model potential that approximates the actual nuclear shape. The non-spherical, or deformed, shell model approach is usually called the Nilsson model.

The shell model has considerable success in accounting for the spins and parities of odd-$A$ ground states, and is satisfactory in accounting for magnetic dipole and electric quadrupole moments. This particular application of the shell model is known as the extreme independent particle model. The basic assumption of this model is that all nucleons but one are paired, and the nuclear properties arise from the motion of the single unpaired nucleon. However, especially for even-even nuclei, with an increasing number of valence nucleons, this model becomes less successful due to the large number of interactions between the valence nucleons.

### 2.3 The Collective Model

Although the shell model describes the magic numbers and the properties of many levels very well, it has a number of failures for even-even nuclei. The most outstanding one is the fact that many quadrupole moments are much larger than those predicted by the shell model [15]. It was shown by Rainwater that such large quadrupole moments can be explained within the concept of a shell model if the closed-shell core is assumed to be deformed[16]. Indeed, if the core is ellipsoidal it acquires a quadrupole moment proportional to the deformation. A deformation of the core is evidence for
many-body effects, and collective modes of excitation are possible. The appearance of such modes are not surprising. The idea of collective effects was introduced earlier in several investigations [17, 18]. A more classical example of such collective effects is provided by plasma oscillations [19]. The existence of large nuclear quadrupole moments provides evidence for strong collective effects in nuclei. Beginning about 1950, Aage Bohr and Ben Mottelson started a systematic study of collective motions in nuclei [1]. This macroscopic model is generally known as the collective model of Bohr and Mottelson.

In this model, many nucleons contribute cooperatively to the nuclear properties. The collective properties vary smoothly and gradually with mass number and are mostly independent of the number and kind of valence nucleons outside of filled sub-shells.

Closed shell nuclei are spherically symmetric and not deformed. The primary collective motions of such nuclei are surface vibrations, like the surface wave on a liquid drop. The nuclei with $A < 150$ are generally treated in terms of collective model based on vibrations about a spherical equilibrium shape.

Far away from closed shells, the nucleons outside the core cause polarization of the core, and the nucleus can acquire a permanent deformation. The entire deformed nucleus can rotate. Nuclei with $A$ between 150 and 190 show rotation structures most characteristic of a nonspherical system.

Vibrations and rotations are the two major types of collective nuclear motion.

2.3.1 Vibrational Motion in Nuclei

The idea of an incompressible system which can perform shape oscillations without change of density was first treated by Rayleigh [17]. The investigations of nuclear
vibrations use much of the mathematical approach developed by Rayleigh but, of course, the oscillations are quantized. Imagining a liquid drop vibrating at high frequency we can get a good idea of the physics of nuclear vibrations. Although the average shape is spherical, the instantaneous shape is not. The instantaneous coordinate of a point on the nuclear surface is described in terms of the spherical harmonics [13]:

\[ R(\theta, \phi, t) = R_0 + \sum_{\lambda \geq 1} \sum_{\mu=-\lambda}^{+\lambda} \alpha_{\lambda \mu}(t) Y_{\lambda \mu}(\theta, \phi) \]  

(2.2)

where the \( \alpha_{\lambda \mu}(t) \) are the time-dependent amplitudes of spherical harmonics and are not completely arbitrary, and \( Y_{\lambda \mu}(\theta, \phi) \) are spherical harmonics. The lowest four vibrational modes of a nucleus are shown in Figure 2.1. The constant \( \lambda = 0 \) term is incorporated into the radius of the spherical nucleus \( R_0 \), which is just \( R_0 A^{1/3} \). We ignore this term because the energies of the compressibility of nuclei are extremely high. A typical \( \lambda = 1 \) term, known as a dipole vibration, for small amplitudes, gives a displacement of the nuclear mass center and cannot result from the action of internal nuclear forces. It is known as "Giant Dipole Resonance (GDR)" which exists in the absolute majority of the photoabsorption. It corresponds to the fundamental frequency for absorption of electric dipole radiation by the nucleus acting as a whole, and is most simply understood as the oscillation of all neutrons against all protons in the nucleus. The GDR is placed in the region of 13-15 MeV for heavy nuclei and in the region of 20-24 MeV for light ones. This term is not allowed, since no external force are acting on the system. We need to consider the next lowest mode (\( \lambda = 2 \)) which corresponds to quadrupole vibrations. We restrict this work to these terms. Then we can reduce Equation 2.2 to

\[ R(\theta, \phi) = R_0 + \sum_{\mu=-2}^{+2} \alpha_{2\mu} Y_{2\mu}(\theta, \phi) \]  

(2.3)
The Hamiltonian for an oscillating liquid drop, for quadrupole and small deformation, has the form of [20]
\[ H = \frac{1}{2} B \sum_{\mu} |\alpha_{2\mu}|^2 + \frac{1}{2} C \sum_{\mu} |\alpha_{2\mu}|^2. \] (2.4)
where \( B \) is the parameter corresponding to the mass and \( C \) is the potential energy parameter. This Hamiltonian describes a five-dimensional harmonic oscillator because there are five independent variables of \( \alpha_{2\mu} \). In analogy to the solution \( E_N = (N + \frac{3}{2}) \hbar \omega \) of the Schrödinger equation for a three-dimensional harmonic oscillator, the energies of the quantized oscillator are given by
\[ E_N = \left( N + \frac{5}{2} \right) \hbar \omega, \quad \hbar \omega = \left( \frac{C}{B} \right)^{1/2}. \] (2.5)

In analogy with the quantum theory of electromagnetism a quantum of vibrational energy is called a phonon. Whenever we produce mechanical vibrations, we can equivalently say that we are producing vibrational phonon. A single unit of nuclear quadrupole vibration is thus a quadrupole phonon.

The \((\lambda = 2)\) quadrupole phonon carries 2 units of angular momentum and even parity, since the parity of \( Y_{\lambda \mu} \) is \((-1)^{\lambda}\). If we add two units of angular momentum to a \( 0^+ \) state, it gives only \( 2^+ \) state. If we add a second quadrupole phonon, there are 5 possible components of \( \mu \) for each phonon and 25 possible combinations of the \( \lambda \mu \) for the two phonon, as calculated in Table 2.1 [13].
Table 2.1: All combinations of two quadrupole phonons

<table>
<thead>
<tr>
<th></th>
<th>$\mu_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_1$</td>
<td>-2 -1 0 +1 +2</td>
</tr>
<tr>
<td>-2</td>
<td>-4 -3 -2 -1 0</td>
</tr>
<tr>
<td>-1</td>
<td>-3 -2 -1 0 1</td>
</tr>
<tr>
<td>0</td>
<td>-2 -1 0 +1 +2</td>
</tr>
<tr>
<td>+1</td>
<td>-1 0 +1 +2 +3</td>
</tr>
<tr>
<td>+2</td>
<td>0 +1 +2 +3 +4</td>
</tr>
</tbody>
</table>

Table 2.2: Symmetric combinations of two quadrupole phonons

<table>
<thead>
<tr>
<th></th>
<th>$\mu_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_1$</td>
<td>-2 -1 0 +1 +2</td>
</tr>
<tr>
<td>-2</td>
<td>-4 - - - -</td>
</tr>
<tr>
<td>-1</td>
<td>-3 -2 - - -</td>
</tr>
<tr>
<td>0</td>
<td>-2 -1 0 - -</td>
</tr>
<tr>
<td>+1</td>
<td>-1 0 +1 +2 -</td>
</tr>
<tr>
<td>+2</td>
<td>0 +1 +2 +3 +4</td>
</tr>
</tbody>
</table>

Considering the total wave functions for phonons and making the proper symmetric combinations, we find not 25 but 15 possible combinations, as shown in Table 2.2.

We can group the symmetric combinations in the following way:

- $l = 4$  $\mu = +4, +3, +2, +1, 0, -1, -2, -3, -4$
- $l = 2$  $\mu = +2, +1, 0, -1, -2$
- $l = 0$  $\mu = 0$

As we expected, there is a triplet of states with spins $0^+, 2^+, 4^+$ at twice the energy of the first $2^+$ state. This triplet is a common feature of vibrational nuclei and gives strong support to this model. A similar calculation for three quadrupole phonon gives states $0^+, 2^+, 3^+, 4^+, 6^+$. This harmonic picture of single and multi-phonon excitations is illustrated in Fig. 2.2 [12].

The $\sum_J$ term is the sum of the E2 transition strengths. Relative energies for each state are included on the left. The B(E2) strength between the 1-phonon state
Figure 2.2: Low-lying levels of the harmonic vibrator phonon model [12].

Figure 2.3: Relative B(E2) transition strengths between phonon states in the vibrational model [10].

and the ground state can be used to normalized all other B(E2) transition strengths relative to this. If a state has more than one decay path the B(E2) strength must be divided between those paths.

The relative B(E2) strengths for each decay are determined by the squares of the coefficients of fractional parentage (CFP) [10]. It is given in Figure 2.3 for the harmonic vibrator model. The multi-phonon states whose energies look the same as in Figures 2.2 and 2.3 are never exactly at the same energy. This is due to anharmonicities in the vibrational model assuming two-body residual interactions.
The next highest mode of vibration is the $\lambda = 3$ octupole mode, which carries three units of angular momentum and negative parity. Adding a single octupole phonon to $0^+$ ground state gives a $3^-$ state, which is also commonly found in vibrational nuclei. The energy of this state is usually above the two-phonon triplet. As we go higher in energy the excitations are very complicated to handle and may not a part of the collective structure of nuclei.

### 2.3.2 Intruder States

The cadmium nucleus has $Z = 48$ and there are two proton holes relative to the closed $Z = 50$ shell which interact with the valence neutrons. We can view that this configuration gives the normal states. If two protons are promoted to the next higher shell $Z = (50 \rightarrow 82)$, there are now 2 valence protons and 4 valence proton-holes which may interact with the valence neutrons (Figure 2.4). This configuration gives rise to another set of states which are identified as intruder states. In this configuration there are six valence protons that are able to interact with the valence neutrons, where the normal states only had two valence protons available to interact with the same number of valence neutrons. The additional number of valence protons increases the attractive forces between the nucleons (p-n interactions). Because of this, intruder levels should be more deformed than the normal levels and the intruder state excitation energies are lowered. The varying deformation shapes of the intruder and normal levels are due to the different mean square radii associated with each configuration. The states with different deformations can co-exist in one nucleus at the same time [21]. Shape co-existence is now known to be a widespread occurrence across the nuclear chart [22].

Although the collective model is a general picture used to describe the structure
of the nucleus, there are anharmonicities and residual interactions which may cause mixing of levels that modify the level energies and transition rates beyond this model. Additional states in the collective model may arise from the coupling of the first quadrupole state \( (2^+)^1 \) with the first octupole state \( (3^-)^1 \). Since these are two different phonon states, angular momentum coupling yields a quintuplet of negative-parity states with \( J^\pi = (1^- - 5^-) \).

### 2.4 The Interacting Boson Model

In 1974, a new approach to collective behavior in nuclei was proposed by Iachello and Arima [2]. It exploits the dynamical symmetry structure of a Hamiltonian and utilizes powerful group techniques to obtain many nuclear properties by simple algebraic techniques. There are many such algebraic models in use today. Beyond a doubt the most popular, successful, and widely tested one is the Interacting Boson Model (IBM) of Iachello and Arima.
The basic idea of the IBM is to assume that the valence fermions couple in pairs only to angular momenta 0 and 2 and that the low-lying collective excitations of medium-heavy and heavy nuclei can be described in terms of the energies and interactions of such pairs. Formally, it means that nuclear structure is reduced to solving the problem of the number of interacting $s$ and $d$ bosons. These fermion pairs, having integer spin, are treated as bosons which are called $s$ ($J = 0$) and $d$ ($J = 2$) bosons for obvious reasons. There are several assumptions behind this model:

- Closed shells of either protons or neutrons, and excitations out of them, are neglected.

- Low-lying excitations of even-even nuclei depend only on the valence space.

- There is no distinction made between proton and neutron bosons. A given nucleus with $N_p + N_n$ valence protons and neutrons has $N = (N_p + N_n)/2$ numbers of $s$ and $d$ bosons.

- The states of this boson system result from the distribution of the fermions in $s$ and $d$ pairs, and thus depend only on the $s$ and $d$ boson energies and on interactions between bosons.

We can consider IBM Hamiltonians composed of creation and destruction operators for $s$ and $d$ bosons limited to a maximum of two-boson interactions, $s$, $s^\dagger$, $d$, $d^\dagger$, and their combinations

$$
H = \epsilon' n_d + \frac{1}{2} \sum_J \mathcal{C}_J \left( d^\dagger d^\dagger \right)^{(J)} \cdot \left( \tilde{d}\tilde{d} \right)^{(J)} + \frac{\nu_2}{\sqrt{10}} \left[ \left( d^\dagger d^\dagger \right)^{(2)} \cdot \left( \tilde{d}s \right) + H.c. \right] \\
+ \frac{\nu_0}{2\sqrt{5}} \left[ \left( d^\dagger d^\dagger \right) \cdot \left( ss \right) + H.c. \right]
$$

(2.6)

where $\epsilon'$, $\mathcal{C}_J$ ($J = 0, 2, 4$), $\nu_2$ and $\nu_0$ are six free parameters. The first term simply counts the number of $d$ bosons and multiplies it by a $d$ boson energy. This gives the
unperturbed energy of a state with $n_d$ noninteracting $d$ bosons. The second group of three terms introduces interactions between pairs of $d$ bosons that depend on the angular momentum to which they are coupled. This interaction does not change the relative numbers of $s$ and $d$ bosons and does not mix the basis states as well. The other terms have the property of changing the number of $d$ bosons by $\Delta n_d = \pm 1$, $\pm 2$. These terms mix different basis states of a given $J$ and this leads to collective like behaviour. Collective behavior still occurs for $d$-boson conserving Hamiltonians i.e, pure U(5) limit $n_d$ is a good quantum number. The $d$ operator is defined as $d_m = (-1)^m d_{-m}$ and introduces a phase factor upon conjugation. The term, $H.c$, represents the Hermitian conjugates of the corresponding operators.

There is another equivalent form often used that can be related to a simple physical interpretation:

$$H = \epsilon'' n_d + a_0 P^\dagger P + a_1 J^2 + a_2 Q^2 + a_3 T_3^2 + a_4 T_4^2$$  \hfill (2.7) 

where

$$n_d = \sqrt{5} \left( d^\dagger \tilde{d} \right),$$
$$P = \frac{1}{2} \left( d \tilde{d} - s s \right),$$
$$J = \sqrt{10} \left( d^\dagger \tilde{d} \right)^{(1)},$$
$$Q = \left( d^\dagger s + s^\dagger \tilde{d} \right) - \frac{\sqrt{7}}{2} \left( d^\dagger \tilde{d} \right)^{(2)},$$
$$T_3 = \left( d^\dagger \tilde{d} \right)^{(3)}, \text{ and}$$
$$T_4 = \left( d^\dagger \tilde{d} \right)^{(4)}. \hfill (2.8)$$

The most important point to note in Equation 2.8 is the $\Delta n_d$ character of the various terms; those in $n_d$, $J^2$, $T_3^2$ and $T_4^2$ have $\Delta n_d = 0$, $P^\dagger P$ has $\Delta n_d = 0$, $\pm 2$ contributions, while $Q^\dagger$ has $\Delta n_d = 0$, $\pm 1$, $\pm 2$ parts.
As we mentioned above, a fundamental feature of the IBM is its group theoretical structure. Since $s$ ($J = 0$) boson has, in accordance to $(2J + 1)$, only one magnetic sub-state and a $d$ ($J = 2$) boson has five magnetic sub-states, then $s - d$ boson system can be looked at mathematically as a six dimensional space. The basis states span that space. The spectrum generating algebra for the nucleus is $U(6)$. All physical observables of a system, such as Hamiltonian and transition operators, can be described in terms of the algebraic group structure $U(6)$. There are three solvable limits of symmetry that are physically interesting, $U(5)$, $SU(3)$ and $O(6)$ which exist as decompositions of $U(6)$. Each has specific, characteristic properties and a definite geometric analogue.

### 2.4.1 $U(5)$ Symmetry Limit

The $U(5)$ symmetry is the IBM version of a vibrator. Some of the best examples of vibrational nuclei are the Cd isotopes, and they have been identified as an actual example of nuclei manifesting $U(5)$ symmetry [23]. The Hamiltonian for $U(5)$ limit of IBM can be expressed in terms of the operators of the multipole expansion [24]

$$H = \epsilon n_d + a_1 J^2 + a_2 T_3^2 + a_4 T_4^2,$$

and its eigenvalues are

$$E = \alpha n_d + \beta n_d (n_d + 4) + 2\gamma \nu (\nu + 3) + 2\delta L (L + 1).$$

We can get the energy spectrum in the harmonic $U(5)$ symmetry limit where $H = \epsilon n_d$ that corresponds to a harmonic vibrator in a geometrical framework. The energy states for $U(5)$ symmetry limit of IBM are shown in Figure 2.5 which presents their spins and parities and where $n_d$ is number of $d$ bosons, $n_\beta$ the number of $d$
bosons coupled pairwise to J=0 and $n_\Delta$ is the number of d bosons coupled tripletwise to J=0 [24].

The characteristic features of the corresponding wave functions can be deduced from the terms appearing in $H$, all of which are diagonal with respect to the $n_d$ quantum number. The $U(5)$ limit is a very general an-harmonic vibrator, and levels with different $J$ but the same $n_d$ need not be degenerate.

Although the IBM is intimately connected with concept of dynamical symmetry, searches for $U(5)$ like nuclei naturally focus on those regions where the geometric vibrational model is also appropriate. The nuclei of even Cd have been proposed as a near harmonic empirical manifestation of $U(5)$ [25].
Chapter 3

Nuclear Decay

The radioactive decay of naturally occurring uranium and thorium are in large part responsible for the birth of the nuclear physics. We have the capability to produce radioactive nuclei in the laboratory through nuclear reactions and can use those artificial radioactive nuclei in various experiments.

As part of a program detailed study of the $^{110}$Cd isotope, we have observed the $\beta$ decay of $^{110}$In which is produced in the TRI-University Meson Facility (TRIUMF) through nuclear spallation reactions. We have observed both the $\gamma$ decay and an internal conversion process which follow the $\beta$ decay.

3.1 Beta Decay

The emission of electrons from the nucleus, the electron capture by a nucleus and the emission of positrons from the nucleus are grouped under the common name beta ($\beta$) decay.

The basic $\beta$ decay process is the conversion of a proton to a neutron through the emission of positron ($\beta^+$ decay) or of a neutron into a proton through the emission
of electron ($\beta^-$ decay). The continuous energy distribution of the $\beta$ decay process led Pauli to propose the emission of a second particle during the process. Later this particle was named by Fermi the neutrino. Experiment shows that there are two different kinds of neutrinos emitted in $\beta$ decay which are called the neutrino and the antineutrino. These particles are nearly massless and have a spin of $\frac{1}{2}$. The antineutrino is emitted in $\beta^-$ decay, while the neutrino is emitted in $\beta^+$ decay and electron capture. The basic decay processes for free particles are:

- $n \rightarrow p + e^- + \bar{\nu}$ negative beta decay ($\beta^-$)
- $p \rightarrow n + e^+ + \nu$ positive beta decay ($\beta^+$)
- $p + e^- \rightarrow n + \nu$ orbital electron capture ($\epsilon$)

$\beta$ decay is mediated by the weak interaction that causes a transition of a nucleus into its next lower or higher neighbor on the periodic table with changes to both $Z$ and $N$ by one unit while $A$ remains constant.

\[
\begin{align*}
\frac{A}{Z} X \rightarrow & \frac{A}{Z+1} Y + e^- + \bar{\nu} \\
\frac{A}{Z} X \rightarrow & \frac{A}{Z-1} Y + e^+ + \nu \\
\frac{A}{Z} X + e^- \rightarrow & \frac{A}{Z-1} Y + \bar{\nu} 
\end{align*}
\]  

(3.1)

In our case, $\beta^+$ decay of $^{110}\text{In}$ competes with its electron capture:

- $^{110}_{49}\text{In} \rightarrow^{110}_{48}\text{Cd} + e^+ + \nu_e$ \hspace{1cm} (3.2)
- $^{110}_{49}\text{In} + e^- \rightarrow^{110}_{48}\text{Cd} + \nu_e$

### 3.1.1 Beta Decay Energy

For $\beta^+$ decay of $^{110}\text{In}$ neglecting the recoil energy and the neutrino mass, the net energy released in the decay, Q value, is equal to:

\[
Q_{\beta^+} = \left[ m_N \left( ^{110}_{49}\text{In} \right) - m_N \left( ^{110}_{48}\text{Cd} \right) - m_{e^+} \right] c^2 \]  

(3.3)
where $m_N$ indicates nuclear mass. To convert nuclear masses into the tabulated neutral atomic masses, we use

$$m^{(110\text{In})}c^2 = m_N^{(110\text{In})}c^2 + 49m_e c^2 - \sum_{i=1}^{49} B_i$$

$$m^{(110\text{Cd})}c^2 = m_N^{(110\text{Cd})}c^2 + 48m_e c^2 - \sum_{i=1}^{48} B_i$$

(3.4)

where $B_i$ represents the binding energy of the $i$th electron. Neglecting the differences in electron binding energy, we find the $Q$ value from Equation 3.3 and 3.4:

$$Q_{\beta^+} = [m^{(110\text{In})} - m^{(110\text{Cd})} - m_e^- - m_e^+] c^2$$

$$= (109.907165u - 109.903002u - 2 \times 0.000548u)(931.502\text{MeV/u})$$

$$= 2.856\text{MeV}$$

(3.5)

From above expression, to permit $\beta^+$ decay, the atomic mass energy difference must be at least $2m_e c^2 = 1.022\text{MeV}$.

For electron capture processes, the calculation of the $Q$ value must take into account that the atom is in a atomic excited state immediately after the capture. That is, if the capture takes place from an inner shell, there is an electronic vacancy in that shell. The vacancy is quickly filled by electrons from higher shells and make downward transitions with the emission of characteristic $X$ rays. The total $X$-ray energy is equal to the binding energy of the captured electron. The $Q$ value is then

$$Q_e = \left[m^{(110\text{In})} - m^{(110\text{Cd})}\right] c^2 - B_n$$

$$= (109.907165u - 109.903002u)(931.502\text{MeV/u}) - 0.0267\text{MeV}$$

$$= 3.851\text{MeV}$$

(3.6)

where $B_n$ is the binding energy of the captured $n$-shell electron ($n = K, L, \ldots$).

Also, all of the above expressions refer to decay between nuclear ground states. If the final nuclear state is an excited state, then the $Q$ value must be accordingly
decreased by the excitation energy of the state:

\[ Q_{ex} = Q_{\text{ground}} - E_{ex}. \]  \hfill (3.7)

On the other side, the maximum excitation energy of final nuclei is equal to the \( Q \) value.

### 3.1.2 Fermi Theory of Beta Decay

In 1934, Fermi developed a successful theory of \( \beta \) decay based on Pauli’s neutrino hypothesis. Using Fermi’s Golden Rule, the transition probability caused by a weak interaction compared is [13]:

\[ \lambda = \frac{2\pi}{\hbar} |M_{fi}|^2 \rho(E_f) \]  \hfill (3.8)

where \( M_{fi} \) is the transition matrix element which is the integral of the interaction \( M \) between the initial and final quasi-stationary states of the system:

\[ M_{fi} = \int \psi_f^* M \psi_i d\nu \]  \hfill (3.9)

and the factor \( \rho(E_f) \) is the density of final states, which can also be written as \( dn/dE_f \), the number \( dn \) of final states in the energy interval \( dE_f \).

The final state wave function must include not only the nucleus but also the electron and neutrino. For \( \beta \) decay, the interaction matrix element has the form

\[ M_{fi} = g \int [\varphi_f^* \varphi_e^* \varphi_\nu^*] O_X \psi_i d\nu \]  \hfill (3.10)

where \( \varphi_f \) refers only to the final nuclear wave function and \( \varphi_e \) and \( \varphi_\nu \) give the wave functions of the electron and neutrino, and \( O_X \) is the mathematical operators of \( M \), where \( X \) gives the form of the operator \( O \). The quantity in square brackets represents
the entire final system after the decay. The value of the constant $g$ determines the strength of the interaction.

The number of final states which have simultaneously an electron and a neutrino with the proper momenta is

$$d^2n = dn_e dn_\nu = \frac{4\pi p^2 dp V}{h^3} \frac{4\pi q^2 dq V}{h^3}$$

(3.11)

where $|p| = p$ is the momentum of an electron (or positron), $|q| = q$ is the momentum of a neutrino (or antineutrino), $4\pi p^2 dp$ is the spherical volume which represents momenta in the range $dp$ at $p$ and $V$ is the volume where is an electron confined. Each quantum state occupies $h^3$ in phase space.

The electron and neutrino wave functions have the usual free-particle form, normalized within the volume $V$ and approximated as $pr \ll 1$ (for an electron with 1 MeV kinetic energy, $p=1.4$ MeV/c and $p/h=0.007$ fm$^{-1}$):

$$\varphi_e(r) = \frac{1}{\sqrt{V}} e^{ip \cdot r/h} = \frac{1}{\sqrt{V}} \left[ 1 + \frac{ip \cdot r}{h} + \cdots \right] \approx \frac{1}{\sqrt{V}}$$

$$\varphi_\nu(r) = \frac{1}{\sqrt{V}} e^{iq \cdot r/h} = \frac{1}{\sqrt{V}} \left[ 1 + \frac{iq \cdot r}{h} + \cdots \right] \approx \frac{1}{\sqrt{V}}$$

(3.12)

Now we can have the partial decay rate for electrons and neutrinos with the proper momenta as:

$$d\lambda = \frac{2\pi}{h} g^2 |M_{fi}'|^2 \frac{p^2 dp q^2 dq}{h^6} \frac{dE_f}{dE_f}$$

(3.13)

where $M_{fi}' = \int \psi^*_f O_X \psi_i dv$ is the nuclear matrix element.

The final energy is $E_f = E_e + E_\nu = E_e + qc$, and so $dq/dE_f = 1/c$ at fixed $E_e$. Concerning the shape of electron spectrum, the partial decay rate gives the number of electrons with momentum between $p$ and $dp$ can be written as

$$N(p)dp = C p^2 q^2 dp$$

(3.14)
where $C$ is the constant which includes all factors that do not involve the momentum assuming nuclear matrix element to be independent of $p$. The number of electrons with momentum $p$, the spectrum shape, neglecting nuclear recoil energy and assuming $q = \frac{Q - T_e}{c}$, is given by

$$N(p) = \frac{C}{c^2} p^2 (Q - T_e)^2$$

$$= \frac{C}{c^2} p^2 \left[ Q - \sqrt{p^2 c^2 + m_e^2 c^4 + m_e c^2} \right]^2$$

where $Q$ is the decay energy and $T_e$ is the kinetic energy of electrons. There are systematic differences between theory and experiment of electron distributions caused by the final state interactions between the $\beta$ particle and the daughter nucleus. This leaded Fermi to introduce an additional factor, the Fermi function $F(Z', p)$ or $F(Z', T_e)$, where $Z'$ is the atomic number of the daughter nucleus. Finally, we must consider the effect of the nuclear matrix element, $M_{fi}$, which we have up to now assumed not to influence the shape of the distribution spectrum. The approximation, which is known as the allowed approximation and what we made in Equation 3.12, is often found to be a very good one, but there are some cases in which it is very bad, in fact, there are cases in which $M_{fi}$ vanishes in the allowed approximation, giving no spectrum at all. In such cases we must take the next terms of the plane wave expansion. Such cases are called forbidden decays. These decays are not absolutely forbidden, but they are far less likely to occur than allowed decays when the latter are not prevented by angular momentum considerations.

The complete $\beta$ spectrum includes three factors, statistical factor, Fermi function and the nuclear matrix element which may include an additional electron and neutrino momentum dependence $S(p, q)$ from forbidden terms:

$$N(p) = p^2 (Q - T_e)^2 F(Z', p)|M_{fi}|^2 S(p, q).$$

(3.16)
The total decay rate for allowed decays can be written as
\[
d\lambda = \frac{g^2 |M_{fi}|^2}{2\pi^3\hbar^7c^5} \int_0^{p_{max}} F(Z', p)p^2 (Q - T_e)^2 dp
\] (3.17)

This integral depends only on \(Z'\) and on the maximum electron total energy \(E_0\) \((cp_{max} = \sqrt{E_0^2 - m_e^2c^4})\). We can represent it as function as
\[
f(Z', E_0) = \frac{1}{(m_e c)^3(m_e c^2)^2} \int_0^{p_{max}} F(Z', p)p^2 (Q - E_0)^2 dp
\] (3.18)

where the constants have been included to make \(f\) dimensionless. The function \(f(Z', E_0)\) is known as the Fermi integral. With \(\lambda = \frac{0.693}{t_{1/2}}\), we have
\[
ft_{1/2} = 0.693 \frac{2\pi^3\hbar^7}{g^2m_e^5c^4|M_{fi}|^2}
\] (3.19)

The quantity on the left side of Equation 3.19 is called the comparative half-life or \(ft\) value. The \(ft\) values of \(\beta\) decay is in range from about \(10^3\) to \(10^{20}\) s.

### 3.1.3 Allowed Beta Decays

We have not considered the role of spin, orbital angular momenta and parity in the beta transitions so far. In the allowed approximation, the electron and neutrino cannot carry any orbital angular momentum and there is no parity change because we replaced the electron and neutrino wave functions with their values at the origin \((r = 0)\). So, the only change in the angular momentum of the nucleus must result from the spins of the electron and neutrino, each of which has the value \(S = 1/2\). These two spins can be parallel, total \(S = 1\), or antiparallel, total \(S = 0\). The transitions having \(S = 0\) are called Fermi transitions and the transitions having \(S = 1\) are called Gamow-Teller transitions [13]. For Fermi decay in allowed approximation \((l = 0)\), there can be no change in the nuclear spin: \(\Delta I = |I_i - I_f| = 0\). For Gamow-Teller decay in allowed approximation, they carry one unit of a total angular momentum.
and $I_i$ and $I_f$ must be coupled through a vector length $1$: $I_i = I_f + 1$. This is possible only if $\Delta I = 0$ or $1$ except for $I_i = 0$ and $I_f = 0$, in which case only the Fermi transition can contribute.

Now we have selection rules for allowed $\beta$ decays, which are

$$\Delta I = 0, 1 \quad \Delta \pi \ (parity \ change \ (-1)^l) = \text{no} \quad (3.20)$$

It is convenient to define the ratio of the Fermi and Gamow-Teller amplitudes:

$$y = \frac{g_F M_F}{g_{GT} M_{GT}} \quad (3.21)$$

where $M_F$ and $M_{GT}$ are the actual Fermi and Gamow-Teller nuclear matrix elements.

### 3.1.4 Forbidden Beta Decays

The most usual occurrence of a forbidden decay is when initial and final states have opposite parities, and thus the selection rule for allowed decay is violated. The maximum angular momentum of electrons, which have around $1 \ MeV$ energy, that can be carried relative to nucleus is $pR/\hbar = 0.04$ in unit of $\hbar$. It is not zero but it is less likely to have $l = 1$ decays relative to $l = 0$ and is extremely unlikely to have decays with $l = 3, 5, 7, \ldots$ Those forbidden decays with $l = 1$ are called first-forbidden decays. As in the allowed decays, they have Fermi types and Gamow-Teller types [13].

The selection rules for first-forbidden decays are

$$\Delta I = 0, 1, 2 \quad \Delta \pi \ (parity \ change \ (-1)^l) = \text{yes} \quad (3.22)$$
We can cite some examples of many first-forbidden decays:

\[ ^{17}\text{N} \rightarrow ^{17}\text{O} \quad (\frac{1}{2}^{-} \rightarrow \frac{5}{2}^{+}) \]

\[ ^{76}\text{Br} \rightarrow ^{76}\text{Se} \quad (1^{-} \rightarrow 0^{+}) \]  \( (3.23) \)

\[ ^{122}\text{Sb} \rightarrow ^{122}\text{Sn}^* \quad (2^{-} \rightarrow 2^{+}) \]

Transitions with \( \Delta I \geq 2 \), but with no change in parity, are permitted by neither the allowed nor the first-forbidden selection rules. For these transitions we must look to the \( l = 2 \) decays, and these are known as second-forbidden decays. The selection rules for second-forbidden decays are

\[ \Delta I = 2, 3, \quad \Delta \pi \text{ (parity change } (-1)^l \text{) } = \text{no} \]  \( (3.24) \)

Examples of second-forbidden decays are:

\[ ^{22}\text{Na} \rightarrow ^{22}\text{Ne} (3^{+} \rightarrow 0^{+}) \]  \( (3.25) \)

\[ ^{137}\text{Cs} \rightarrow ^{137}\text{Ba} (\frac{7}{2}^{+} \rightarrow \frac{3}{2}^{+}) \]

Moreover, we would find third-forbidden decays \( (l = 3) \) and even fourth-forbidden decays \( (l = 4) \), which are generally too weak to observe. Only when no other decay mode is possible we can observe these extremely rare third- and fourth-forbidden decays.

### 3.2 Gamma Decay

Most nuclear reactions and decays leave the final nucleus in an excited state. The excited states decay rapidly to the ground or isomeric state through the emission of \( \gamma \)-rays, which are photons of electromagnetic radiation. Gamma rays have energies typically in range of 0.1 to 10 MeV depending on the energy difference between nuclear states and corresponding wavelengths between \( 10^{4} \) and \( 100 \text{ fm} \), which are far shorter...
than the other types of electromagnetic radiations. The studies of γ-ray emission have become the standard technique of nuclear spectroscopy.

Since each nuclear state has a definite angular momentum or spin \( j \), its component \( m \) and parity \( \pi \), a photon must take out angular momentum \( J \), its eigenvalue \( L \) and component \( \mu \), and parity \( \pi \) in accordance with the conservation laws.

\[
j_i = J + j_f, \\
\pi_i \times \pi = \pi_f.
\]

(3.26)
The angular momentum of the photon, \( L \), is called the multipole order of radiation.

In nuclei as well as in atoms, the size of the radiating system is much smaller than the wave length of that radiation. It follows that the probability of emission of radiation decreases rapidly with increasing multi-pole order. The parity of the photon radiation field is

\[
\pi(ML) = (-)^{L+1} \\
\pi(EL) = (-)^L
\]

(3.27)

3.2.1 Gamma Decay Energy

Let us consider the decay of a nucleus of mass \( M \) at rest, from initial excited state \( E_i \) to a final state \( E_f \). The final nucleus will not be at rest but must have a recoil momentum \( p_R \) and corresponding recoil kinetic energy \( T_R \), which we assume to be non-relativistic \( (T_R = p_R^2/2M) \). Conservation of total energy and momentum give:

\[
E_i = E_f + E_\gamma + T_R \\
0 = p_R + p_\gamma
\]

(3.28)

Using the relativistic relationship \( E_\gamma = cp_\gamma \) and \( p_R = p_\gamma \), we can define the energy differences:

\[
\Delta E = E_i - E_f = E_\gamma + \frac{E_\gamma^2}{2Mc^2}
\]

(3.29)
which has the solution
\[ E_\gamma = M c^2 \left[ -1 \pm \left( 1 + \frac{2 \Delta E}{M c^2} \right)^{1/2} \right] \tag{3.30} \]
Because of \( \Delta E \ll M c^2 \), we can keep only the first three terms in the expansion of the square root:
\[ E_\gamma \approx \Delta E - \frac{(\Delta E)^2}{2 M c^2}. \tag{3.31} \]
The actual \( \gamma \)-ray energy is thus diminished from the maximum available decay energy \( \Delta E \). Except one circumstance in which the recoil plays an important role and which is known as the Mössbauer effect, we can assume that \( E_\gamma = \Delta E \).

### 3.2.2 Gamma Decay Transition Probability

Generally, electromagnetic radiation can be treated either as a classical wave phenomenon or a quantum phenomenon. Let us recall the classical description of multipole radiation.

- The angular distribution of multipole (2\( ^L \)) radiation, relative to a properly chosen direction, is governed by the Legendre polynomial \( P_{2L}(\cos \theta) \).

- The parity of the radiation field is
  \[
  \pi(ML) = (-)^{L+1} \\
  \pi(EL) = (-)^L
  \tag{3.32}
  \]

- The radiated power is
  \[
  P(\sigma L) = \frac{2(L+1)c}{\epsilon_0 L[(2L+1)!!]^2} \left( \frac{\omega}{c} \right)^{2L+2} [m(\sigma L)]^2
  \tag{3.33}
  \]
  where \( m(\sigma L) \) is the amplitude of the varying electric and magnetic multipole moment, and where the double factorial indicates \( (2L + 1) \cdot (2L - 1) \cdots 3 \cdot 1 \). \( \sigma \) represents electric \( E \) or magnetic \( M \) radiation.
We can carry the classical theory to quantum theory by quantizing the sources of the radiation field and multipole moments. Replacing the multipole moments in Equation 3.33 by appropriate multipole operators that can change the nucleus from its initial state to the final state. The decay probability is governed by the matrix element of the multipole operator

$$M_{fi}(\sigma L) = \int \Psi_f^* M(\sigma L) \Psi_i d\nu$$

(3.34)

and the total transition probability per unit time for photon emission, the decay constant, can be expressed as [26]

$$\lambda(\sigma L) = \frac{P(\sigma L)}{\hbar \omega} = \frac{8\pi (L + 1)}{\hbar L[(2L + 1)!!]^2} \left( \frac{E_\gamma}{\hbar c} \right)^{2L+1} B(\sigma L, j_i \rightarrow j_f)$$

(3.35)

where $E_\gamma$ is electromagnetic transition energy and the quantity

$$B(\sigma L, j_i \rightarrow j_f) \equiv \sum_{\mu, m_f} | < j_f m_f | M(\sigma L, \mu) | j_i m_i > |^2$$

(3.36)

is called the reduced transition probability.

The reduced transition probability is often estimated in single-particle or Weisskopf units. This Weisskopf unit is a rough estimate of the reduced transition probability value for a single-particle (proton or neutron). These estimates use the assumption that the transition involves only a single nucleon moving from one shell to another in the spherical shell basis and approximates the partial wave function as a constant.

In the electric case, the $EL$ transition probability is estimated for single particle to be

$$B^{s.p.}(EL) \simeq \frac{(1.2)^{2L}}{4\pi} \left( \frac{3}{L + 3} \right)^2 A^{2L/3} [e^2(fm)^{2L}] .$$

(3.37)

In the magnetic case, Weisskopf estimation for $ML$ transition probability is

$$B^{s.p.}(ML) \simeq \frac{10(1.2)^{2L-2}}{\pi} \left( \frac{3}{L + 3} \right)^2 A^{(2L-2)/3} [\mu_N^2(fm)^{2L-2}] .$$

(3.38)
We can rewrite total electric and magnetic transition probabilities for single particle:

\[ \lambda^{s.p.}(EL) = \frac{2(1.2)^2 L^2 A^{L/3}(L + 1)}{h L [(2L + 1)!!]^2} \left( \frac{3}{L + 3} \right)^2 \left( \frac{E_L}{\hbar c} \right)^{2L+1} \left[ e^2 f m^{2L} \right] \] (3.39)

and

\[ \lambda^{s.p.}(ML) = \frac{80(1.2)^2 L^2 A^{(2L-2)/3}(L + 1)}{h L [(2L + 1)!!]^2} \left( \frac{3}{L + 3} \right)^2 \left( \frac{E_\gamma}{\hbar c} \right)^{2L+1} \left[ \mu^2 f m^{2L-2} \right] \] (3.40)

From Equation 3.35, it also becomes clear that the transition probability \( T \) increases rapidly with the transition energy. This is the reason why transitions with small energy differences are sometimes harder to observe. Internal conversion process become important in such cases.

### 3.2.3 M1 Transitions

In general, measurements of \( B(M1) \) are very difficult because they often involve short lifetimes, and are only weakly involved in Coulomb excitation. A collective state is composed of a number of particle excited states. The M1 transition operator in the collective coordinates is simply [26]

\[ \mu = g_R I \] (3.41)

where \( g_R \) factor is often referred to as the rotational \( g \) factor (the random moment) and \( I \) is the total rotational angular momentum.

Actually, most of the \( 2^+_2 \rightarrow 2^+_1 \) transitions are found to involve only a small M1 component, indicating large suppression of the M1 strength. Empirical values of \( B(M1) \) are as low as 1/100, and only in the closed-shell region do they become high. Magnetic radiation is weaker than the electric radiation. Therefore, we often have a competition between \( M1 \) and \( E2 \).
3.2.4 E2 Transitions of a Collective Nature

Nuclei far from the closed-shell region exhibit collective properties. There are two extreme limits of motion in collective model. One is the quadrupole vibration of a nucleus with spherical equilibrium. In this case, the excited states are characterized by the phonon quantum number \( n \) and its energy spacing \( \hbar \omega = \hbar \sqrt{C/B} \), where \( B \) is the mass parameter and \( C \) is the restoring force constant. The ground band \((0^+, 2^+, 4^+, \ldots)\) consists of equidistant energy levels. When the nucleus changes its energy state from \(|n>\) to \(|n'>\), a current is induced due to the change in surface density.

An important ingredient for the collective model is the electric quadrupole operator expressed in the collective coordinates \( \alpha_{2\mu} \). Specifying the nuclear shape and charge distribution, the quadrupole operator may be written as

\[
\hat{Q}_{2\mu} = \frac{3}{4\pi} Z e R_0^2 \left( \hat{\alpha}_{2\mu} - \frac{10}{\sqrt{70\pi}} [\hat{\alpha} \times \hat{\alpha}]^2_\mu \right).
\]

where \( R_0 \) is the nuclear average radius. It shows that the operator allows a change in the phonon number by 1 or 2, so that typically E2 transitions will strongly link states differing by one phonon.

The B(E2) values are relatively easily obtained if the quadratic term in the quadrupole operator is neglected. In this case, the \( 2^+ \rightarrow 0^+ \) reduced transition probability is [26]

\[
B(E2, n = 1 \rightarrow 0) = \left( \frac{3}{4\pi} Z e R_0^2 \right)^2 \frac{\hbar \omega}{2C} = \frac{1}{5} \left( \frac{3}{4\pi} Z e R_0^2 \right)^2 \beta_{00}^2
\]

where

\[
\beta_{00}^2 = \frac{5\hbar^2}{2B} \cdot \frac{1}{\hbar \omega}
\]
is the square of the zero-point amplitude. The vibrational amplitude increases with $n$ number and we have a simple relation

$$B(E2, n, I \rightarrow n - 1, I - 2) = n \cdot B(E2, n = 1, I = 2 \rightarrow n = 0, I = 0). \quad (3.45)$$

### 3.2.5 E1 Transitions

It can be proved that any nuclear state with definite parity does not process a static E1 moment. In nuclei, there is no typical single-particle E1 transition. Most E1 transitions are strongly suppressed and all the low energy transition strengths are much weaker than the Weisskopf estimate. The highest suppression factor is around $10^{15}$, and usually suppression factors of $10^3$ to $10^5$ are encountered.

Since the E1 transitions are the fastest of all except for very low energy transitions, which could appear only at the end of the cascade, it is necessary to know the effective energy dependence of the E1 transition probabilities as a function of $\gamma$-ray energy. The intrinsic dependence is $E^3$ for the E1 transition probabilities.

While there can be no static moment if parity is conserved, there may be oscillations of nucleons that can cause a dynamic E1 moment. An oscillating proton of amplitude $r$ should be accompanied by oscillation of the remaining $A$ nucleons to the opposite direction with amplitude $r/A$. Then, the total induced E1 moment becomes

$$Q'(p) = Q + Z(-Q/A) = \left(1 - \frac{Z}{A}\right)Q = \frac{N}{A}Q \quad (3.46)$$

We might say that the center of mass effect acts as if the charge of the protons were effectively reduced to

$$e_{c.m.}(p) = \frac{N}{A}e. \quad (3.47)$$
In the same way, an oscillating neutron induces an E1 moment through the recoiling protons, that is,

\[ Q'(n) = Z(-Q/A) = -\frac{N}{A}Q. \]  

(3.48)

Therefore, the neutron has an effective charge of

\[ e_{c.m.}(n) = -\frac{N}{A}e. \]  

(3.49)

This effect can be expressed by using the isospin as follows:

\[ e_{c.m.} = \frac{T}{A}e - t \cdot e \]  

(3.50)

where \( t \) is +1/2 for neutrons and -1/2 for protons, and \( T \) is isospin which is

\[ T = \frac{1}{2}(N - Z). \]  

(3.51)

Therefore, the E1 transition is forbidden between \( T = 0 \) states in a self-conjugate nucleus. More generally, the E1 transition is forbidden between states of the same \( T \). This is called isospin selection rule for E1 transitions. The observed suppression factors over the allowed \( (T = 1) \) E1 transitions are

\[ \frac{B(E1)(\Delta T = 1)}{B(E1)(\Delta T = 0)} > 10. \]  

(3.52)

### 3.3 Internal Conversion Process

Internal conversion process is a radioactive decay process where an excited nucleus interacts with an electron in one of the lower atomic orbitals, causing the electron to be emitted from the atom. Thus, in an internal conversion process, a high-energy electron is emitted from the excited nucleus.
In the internal conversion process, the wave function of an inner shell electron penetrates the nucleus. The electron may couple to the excited state and take the energy of the nuclear transition directly, without an intermediate $\gamma$ ray being produced first. The kinetic energy of the emitted electron is equal to the transition energy in the nucleus, minus the binding energy of the electron [13].

$$T_e = (E_i - E_f) - E_B$$  \hspace{1cm} (3.53)

where $E_i$ and $E_f$ are the energies of the nucleus in its initial and final states, respectively, while $T_e$ and $E_B$ are the kinetic energy and the binding energy of the electron, respectively. Equation 3.53 tells us that the internal conversion process has a threshold energy equal to the electron binding energy in a particular shell. The conversion electrons are labeled according to the electronic shell from which they come: K, L, M, and so on, corresponding to principal atomic quantum numbers $n=1,2,3,...$ . If we can observe electron spectra at very high resolution, we can even see the substructure corresponding to the individual electrons in the shell. For example, the L ($n=2$) shell has atomic orbitals $2s_{1/2}$, $2p_{1/2}$, and $2p_{3/2}$; electrons originating from these shells are called $L_I$, $L_{II}$, and $L_{III}$ conversion electrons, respectively.

Most internal conversion electrons come from the K shell as these two electrons, which are in 1s state of atom have the highest probability of being found inside the nucleus. After the electron has been emitted, the atom is left with a vacancy in one of the inner electron shells. This hole will be filled with an electron from one of the higher shells and subsequently a characteristic x-ray or Auger electron will be emitted.

Internal conversion is favored when the energy gap between nuclear levels is small and is also the primary mode of de-excitation for E0 transitions. The tendency towards internal conversion can be determined by the internal conversion coefficient,
which is defined as the ratio of de-excitations that go by the emission of electrons to those that go by γ emission. In some cases the internal conversion process competes strongly with γ decay, in others it may be completely negligible. This competition is quantified in the form of the internal conversion coefficient which is [13].

\[ \alpha = \frac{I_e}{I_\gamma} = \frac{\lambda_e}{\lambda_\gamma} \]  \hfill (3.54)

where \( I_e \) is the intensity of conversion electrons and \( I_\gamma \) is the intensity of γ-ray emission observed from a decaying nucleus, while \( \lambda_e \) is the decay probability by the conversion electrons and \( \lambda_\gamma \) is the decay probability by γ emission. We assume that \( \alpha \) represents the total internal conversion coefficient and can define partial coefficients representing the individual atomic shells:

\[ \alpha = \alpha_K + \alpha_L + \alpha_M + \cdots \]  \hfill (3.55)

We can extract partial coefficients for the sub-shells, such as

\[ \alpha_L = \alpha_{L_1} + \alpha_{L_{11}} + \alpha_{L_{111}} \]  \hfill (3.56)

and similarly for other shells.

The calculation of the internal conversion coefficient is the similar to γ emission. Because the process is electromagnetic in origin, the matrix element of the process is described by the same expression used for γ emission [13]

\[ \mathcal{M}_{f_i}(\sigma L) = \int \Psi_f^* \mathcal{M}(\sigma L) \Psi_i d\nu \]  \hfill (3.57)

where \( \mathcal{M}(\sigma L) \) is the multipole operator, \( \sigma = E \) or \( M \), and \( L \) is multipole index. The initial state includes a bound electron, so that \( \Psi_i = \Psi_{i,N} \Psi_{i,e} \) where \( N \) indicates the nuclear wave function and \( e \) indicates the electron wave function. Similarly, \( \Psi_f = \Psi_{f,N} \Psi_{f,e} \) where \( \Psi_{f,e} \) is the free-particle wave function. The nuclear part of
the matrix element of Equation 3.57 is identical for both $\gamma$ emission and internal conversion processes:

$$\lambda_{\gamma}(\sigma L) \propto |M_{fi}(\sigma L)|^2$$

and

$$\lambda_e(\sigma L) \propto |M_{fi}(\sigma L)|^2$$

and the internal conversion coefficient $\alpha$, the ratio of $T_e$ and $T_\gamma$, is independent of the details of nuclear structure. The coefficient $\alpha$ will depend on the atomic number of the atom in which the process occurs, and on the energy of the transition and its multipolarity.

A non-relativistic calculation gives the following instructive results for electric (E) and magnetic (M) multipoles [13]:

$$\alpha(EL) \approx \frac{Z^3}{n^3} \left( \frac{L}{L+1} \right) \left( \frac{e^2}{4\pi\epsilon_0\hbar c} \right)^4 \left( \frac{2m_e c^2}{E} \right)^{L+5/2}$$

$$\alpha(ML) \approx \frac{Z^3}{n^3} \left( \frac{e^2}{4\pi\epsilon_0\hbar c} \right)^4 \left( \frac{2m_e c^2}{E} \right)^{L+3/2}$$

In these expressions, $Z$ is the atomic number of the atom in which the conversion electron takes place, $n$ is the principal quantum number of the bound electron wave function, the factor $(Z/n)^2$ comes from the term $|\Psi_{ie}(0)|^2$ that appears in the conversion rate and the dimensionless factor $(e^2/4\pi\epsilon_0\hbar c)$ is the fine structure constant with a value close to $\frac{1}{137}$.

Even though these expressions for the conversion coefficients are only approximate they illustrate a number of feature of the conversion coefficients:

1. They increase as $Z^3$, and so the conversion process is more important for heavy nuclei than for light nuclei.

2. The conversion coefficient decreases rapidly with increasing transition energy.

3. The conversion coefficients increase rapidly as the multipole order increases.
4. The conversion coefficients for higher atomic shells \((n > 1)\) decrease as \(\frac{1}{n^3}\).

We therefore expect relatively large K-shell conversion coefficients for low-energy, high-multipolarity transitions in heavy nuclei, with smaller values in other cases. A measurement of \(\alpha\) allows us to determine the relative parities of nuclear states because conversion coefficients differ considerably for \(EL\) and \(ML\) transitions.

The relationship between the experimentally measured partial half-life and the actual half life connected by an internal conversion mode \(\gamma_K\) is given with [27]

\[
T_{1/2}^{exp} = T_{1/2}(\sigma L)\epsilon(\gamma_K)
\]  

(3.60)

where \(T_{1/2}\) is half-life and \(\epsilon(\gamma_K)\) is the fraction of the decays proceeding via the K shell internal conversion mode \(\gamma_K\) and is given by [27]

\[
\epsilon(\gamma_K) = \frac{\lambda_K^i}{(1 + \alpha_i)\lambda_i^q} = \frac{BR(\gamma_K)}{(1 + \alpha_K)}
\]  

(3.61)

where \(\lambda_i^q\) is the relative partial decay constant for \(i\)th gamma transition, \(\alpha_i\) is the total conversion coefficient for \(i\)th transition, and \(BR(\gamma_K)\) is the total branching ratio \((\gamma\) ray + conversion \(e^-)\) for \(K\) transition. Substituting Equation 3.61 into 3.60 we have expression for the experimentally measured partial half-life

\[
T_{1/2}^{exp} = T_{1/2}(\sigma L)\frac{BR(\gamma_K)}{(1 + \alpha_K)}
\]  

(3.62)

We can specifically express a reduced transition probabilities in Weisskopf units as following

\[
B(\sigma L)(W.u.) = \frac{B(\sigma L)}{B_{sp}(\sigma L)} = \frac{T_{1/2}^{sp}(\sigma L)}{T_{1/2}(\sigma L)}
\]  

(3.63)

where

\[
T_{1/2}^{sp}(\sigma L) = \frac{Ln2}{T_{sp}(\sigma L)}.
\]  

(3.64)
Substituting Equations 3.62 and 6.64 into 3.63, we have \( B(\sigma L) \)

\[
B(\sigma L)(W.u.) = \frac{BR(\gamma_K) Ln2}{T^{exp}_{1/2} (1 + \alpha_K) T^{s.p.}(\sigma L)}. 
\]  

(3.65)

Finally, we have a general expressions for electric and magnetic reduced transition probabilities in Weisskopf units

\[
B(EL)(W.u.) = \frac{BR(\gamma_K) Ln(2)\hbar L[(2L + 1)!!]^2}{2(1.2)^{2L}(L + 1)A^{2L/3}} \left( \frac{\hbar c}{E_{\gamma}} \right)^{2L+1} \left( \frac{L + 3}{3} \right)^2 
\]

and

\[
B(ML)(W.u.) = \frac{BR(\gamma_K) Ln(2)\hbar L[(2L + 1)!!]^2}{80(1.2)^{2L-2}(L + 1)A^{(2L-2)/3}} \left( \frac{\hbar c}{E_{\gamma}} \right)^{2L+1} \left( \frac{L + 3}{3} \right)^2 
\]

(3.66)

(3.67)

Internal conversion is an essential tool for observation of \( E0 \) transitions, which are forbidden to go by \( \gamma \) radiation because the phonon spin is 1. The \( E0 \) transition is particularly important in decays from \( 0^+ \) initial states to \( 0^+ \) final states, which cannot occur by any other first-order process.

### 3.3.1 Weisskopf Estimation of Transition Probabilities

We can specifically express M1, E1 and E2 transition probabilities in Weisskopf unit from Equation 3.66 and 3.67.

\[
B(M1) = \frac{2.202 \times 10^{-5} BR(\gamma)}{E_{\gamma}^3 T^{exp}_{1/2} (1 + \alpha)} [W.u.] . 
\]

(3.68)

\[
B(E1) = \frac{6.764 \times 10^{-6} BR(\gamma)}{E_{\gamma}^3 A^{2/3} T^{exp}_{1/2} (1 + \alpha)} [W.u.] . 
\]

(3.69)

\[
B(E2) = \frac{9.527 \times 10^{6} BR(\gamma)}{E_{\gamma}^2 A^{4/3} T^{exp}_{1/2} (1 + \alpha)} [W.u.] . 
\]

(3.70)
3.3.2 E0 Transitions

Between two $0^+$ states only the E0 transition takes place. It occurs only through the internal conversion or the internal pair creation process since the $\gamma$ decay cannot have zero units of angular momentum, as mentioned above. The E0 transition is associated with a change of nuclear surface (shape) without changing the angular momentum. The E0 transition probability for the $i^{th}$ conversion process is expressed as [26]

$$\mathcal{W}_i(E0) = \Omega_i \rho^2,$$  \hspace{1cm} (3.71)

where $\Omega_i$ parameter depends on the transition energy and the electronic state and

$$\rho \equiv < f | \mathcal{M}(E0) | i >$$ \hspace{1cm} (3.72)

is the nuclear matrix element called the strength parameter.

We can evaluate the strength of K-conversion electron E0 transition as following

$$\frac{1}{\tau_K(E0)} = \rho^2 \Omega_K$$ \hspace{1cm} (3.73)

where $\tau_K(E0)$ is the partial lifetime for the K-conversion of the E0 transition. The partial life time is

$$\tau_K(E0) = \frac{\tau}{BR_{K}(E0)}$$ \hspace{1cm} (3.74)

where $\tau$ is the level lifetime, and $BR$ is the branching fraction for the transition. The branching fraction for a particular transition is

$$BR_{K}(E0) = \frac{I^K_e(E0)}{\sum_i I^i_\gamma + \sum_i I^i_e(E0)} = \frac{I^K_e(E0)}{\sum_i (1 + \alpha_i) I^i_\gamma + \sum_i I^i_e(E0)}$$ \hspace{1cm} (3.75)
Using the approximation that $\sum_i I_i(E0) \ll \sum_i I_i$ and further that $\alpha_i \ll 1$, we have the result

$$BR_K(E0) = \frac{I^K_e(E0)}{\sum_i I^\gamma_i} = \frac{\alpha_K(E0)I^\gamma}{\sum_i I^\gamma_i} = \alpha_K(E0)BR_{\gamma}$$

where $BR_{\gamma}$ is the branching ratio of the corresponding $I_i \rightarrow I_f \gamma$ ray to which the $\alpha$ coefficient was referenced. Substituting Equations 3.74 and 3.76 into 3.73, we have an expression for transition strength

$$\rho^2 = \frac{\alpha_K(E0)BR_{\gamma}}{\tau\Omega K}$$

An E0 transition may also proceed between states of the same spin and parity in competition with M1 and E2 transitions. Observation of the E0 transition is greatly facilitated in measurements of conversion electrons, because the conversion coefficients for high-energy M1 and E2 transitions are usually very small.

### 3.3.3 Mixing Ratios

The mixing ratio of the mixed E2/M1 transition can be written as

$$\delta^2 = \frac{I_\gamma(E2)}{I_\gamma(M1)}.$$  

(3.78)

The total intensity of conversion electrons is

$$I_e = I_e(M1) + I_e(E2) = I_\gamma(M1)(\alpha(M1) + \alpha(E2)\delta^2)$$

(3.79)

and the total intensity of $\gamma$-ray emission is

$$I_\gamma = I_\gamma(M1) + I_\gamma(E2) = I_\gamma(M1)(1 + \delta^2).$$

(3.80)

Using Equations 3.85 and 3.86, the K-conversion coefficient is

$$\alpha_K = \frac{I^K_e}{I_\gamma} = \frac{\alpha_K(M1) + \alpha_K(E2)\delta^2}{1 + \delta^2}$$

(3.81)
and the L-conversion coefficient is

\[
\alpha_L = \frac{I_L}{I_\gamma} = \frac{\alpha_L(M1) + \alpha_L(E2)\delta^2}{1 + \delta^2}.
\] (3.82)

Solving \(\delta^2\) from Equations 3.81 and 3.82, we have a mixing (E2/M1) ratio is

\[
\delta^2 = \frac{\alpha_K(M1) - \frac{\alpha_K}{\alpha_L} \alpha_L(M1)}{\frac{\alpha_K}{\alpha_L} \alpha_L(E2) - \alpha_K(E2)}
\] (3.83)

from K-to-L conversion coefficient ratio and

\[
\delta^2 = \frac{\alpha_K(M1) - \alpha_K}{\alpha_K - \alpha_K(E2)}
\] (3.84)

from absolute K-conversion coefficient.
Chapter 4

Detecting Nuclear Radiations

Many detectors of nuclear radiations are similar in their basic principles of operation except scintillation detectors. The radiation enters the detector, interacts with the atoms of the detector material that loose part of or all of its energy, and releases a large number of relatively low-energy electrons from their atomic orbits. These electrons are then collected and formed into a voltage or current pulse for analysis.

4.1 Detecting Gamma-Rays

The detection of $\gamma$ rays is critically dependent on causing the $\gamma$-ray photon to undergo an interaction that transfers all or part of the photon energy to an electron in the absorbing material. The created fast electrons can provide clues regarding the nature of the incident gamma rays. These electrons have a maximum energy equal to the energy of the incident $\gamma$-ray photon and will lose their energy in the same manner as a $\beta$ particle. The electron lose energy through ionization and excitation of atoms within the absorber material and through bremsstrahlung emission. Gamma-rays interact with matter primarily through three processes: *photoelectric absorption*,
Compton scattering, and pair production.

4.1.1 Photoelectric Absorption

In the photoelectric absorption process, a photon is absorbed by an atom of the detector material in which the photon completely disappears. In its place, an energetic photo-electron is ejected by the atom from one of its bound shells (Figure 4.1).

![Figure 4.1: Sketch of the photoelectric absorption process.](image)

The photoelectron appears with an energy given by \[ E_e = h\nu - E_b \]

where \( E_b \) represents the binding energy of the photoelectron in its original shell.

The photoelectric process is the predominant mode of interaction for \( \gamma \) rays of relatively low energy. The process is also enhanced for absorber materials of high atomic number \( Z \). There is no single analytic expression valid for the probability of photoelectric absorption per atom over all ranges of \( E_\gamma \) and \( Z \), but a rough approximation is

\[ \tau \approx C \times \frac{Z^n}{E_\gamma^{3.5}} \]

\[ (4.2) \]
where $C$ is constant and the exponent $n$ varies between 4 and 5 over the $\gamma$-ray energy region of interest. This dependence is primary reason why $\gamma$-ray shields are from high $Z$ materials.

### 4.1.2 Compton Scattering

The interaction process of Compton scattering takes place between the incident $\gamma$-ray photon and an electron in the detector material. In Compton scattering, the incoming $\gamma$-ray photon is deflected through an angle $\theta$ with respect to its original direction Figure 4.2. The photon transfers a portion of its energy to the electron which is assumed initially to be rest, and undergoes recoil. The energy transferred to the electron can vary from zero to a large fraction of the $\gamma$-ray energy [29]. We can
determine the scattered photon energy assuming the target electrons are free and at rest and using the conservation of linear momentum and total energy [29]:

$$h\nu' = \frac{h\nu}{1 + \frac{h\nu}{m_0c^2}(1 - \cos\theta)}$$

(4.3)

where $m_0c^2$ is the rest-mass energy of electron ($0.511 MeV$) and $\theta$ is a scattered angle.

The kinetic energy of the recoil electron is therefore

$$E_e = h\nu - h\nu' = h\nu \left(1 + \frac{h\nu}{m_0c^2(1 - \cos\theta)}\right)$$

(4.4)
We can identify two extreme cases from this scattering:

- In the case of $\theta \cong 0$, the recoil Compton electron has very little energy and the scattered $\gamma$ ray has nearly the same energy as the incident $\gamma$ ray.

- In a head-on collision ($\theta \cong \pi$), the incident $\gamma$ ray is backscattered toward its direction of origin and maximum energy can be transferred to an electron.

Normally, a continuum of energies can be transferred to the electron, ranging from zero up to the maximum. The general shape of the electron energy distribution for any specific $\gamma$-ray energy is shown in Figure 4.3 [29]. The gap between the maximum Compton recoil electron energy and the incident $\gamma$-ray energy is given by

$$E_C = h\nu - E_e \bigg|_{\theta=\pi} = \frac{h\nu}{1 + \frac{2h\nu}{m_0c^2}}$$  \hspace{1cm} (4.5)

In the limit that the incident $\gamma$-ray energy is large, or $h\nu \gg m_0c^2/2$, this energy difference tends toward a constant value given by

$$E_C \cong \frac{m_0c^2}{2} = 0.256 \ \text{MeV}$$  \hspace{1cm} (4.6)

In actual detector materials where electrons are bound, the binding energy of the electron prior to the scattering process can effect the shape of the Compton continuum particularly for low-energy incident $\gamma$ rays.
In addition to Compton scattering, another type of scattering can occur in which the $\gamma$-ray photon interacts coherently with all the electrons of a detector atom. This coherent or Rayleigh scattering process neither excites or ionizes the atom, and the $\gamma$-ray photon retains its original energy after the scattering event. The probability of coherent scattering is significant only for low photon energies and is most dominant in high-$Z$ materials.

### 4.1.3 Pair Production

The third interaction process is pair production, in which a photon creates an electron-positron pair and then the photon disappears in the progress (Figure 4.4). We need the energy of $2m_0c^2$ to create the electron-positron pair and a minimum $\gamma$-ray energy of 1.02$MeV$ is required to make the process energetically possible. The excess energy appears in the form of kinetic energy shared by the electron-positron pair. The energy balance is [29]

$$E_{e^-} + E_{e^+} = h\nu - 2m_0c^2$$  \hspace{1cm} (4.7)
Both the electron and positron travel a few millimeters at most before all their kinetic energy is lost to the detector material. The positron produced in the pair production may be annihilated after it stops in the material, and depending on the crystal size, one or both of the 511 keV photons may subsequently escape. This process generates the single and double escape peaks observed in the spectra. Practically, the probabil-

![Figure 4.5: The relative importance of the three processes [29].](image)

ity of this process remains very low until the $\gamma$-ray energy approaches several ($\approx 5$) MeV and pair production is predominantly confined to high-energy $\gamma$ rays. The relative importance of the three processes described above for different detector materials and $\gamma$-ray energies is illustrated in Figure 4.5.

The behavior of the electron energy deposition spectra depends on the size of detectors. Three interaction mechanism behaviors for small detector are given Figure 4.6. In case of small size detectors compared with the mean free path of the secondary $\gamma$ radiations, the processes of photoelectric absorption and single Compton scattering give rise to the low-energy spectrum at the left. At higher energies, the pair
production process adds a double escape peak shown in the spectrum at the right.

### 4.2 Germanium Detectors

The simple junction and surface barrier semiconductor detectors are not easily adaptable for applications that involve more penetrating radiation such as gamma rays. Their major limitation is the maximum depletion depth or active volume that can be created. The thickness of the depletion region is given by [29]

\[
d = \left( \frac{2eV}{eN} \right)^{1/2}
\]

(4.8)

where \( V \) is the reverse bias voltage, \( N \) is the net impurity concentration of the bulk semi-conductor material, \( \epsilon \) is the dielectric constant and \( e \) is the electronic charge. At given applied voltage, greater depletion depths can only be achieved by lowering the net impurity concentration.
Techniques have been developed to achieve this goal in germanium. Since impurities tend to be more soluble in the molten germanium than in the solid, impurities are preferentially transferred to the molten zone and swept from the sample. Large single crystals of germanium are then slowly grown from this purified feedstock. Detectors that are manufactured from this ultra pure germanium (impurity level of $10^9$ atoms/cm$^3$) are usually called high-purity germanium (HPGe) detectors.

$\gamma$ rays enter the depletion region of semiconductor detectors and creates electron-hole pairs. Electrons flow in one direction while holes flow in the other direction. Total number of electrons collected can form a electronic pulse whose amplitude is proportional to the energy of the radiation.

**Energy Resolution**

The dominant characteristic of germanium detectors is their excellent energy resolution when applied to $\gamma$-ray spectroscopy. The overall energy resolution achieved in a germanium system is normally determined by a combination of three factors: the inherent statistical spread in the number of charge carries, variations in the charge collection efficiency, and contributions of electronic noise. Representative full width at half maximum (FWHM) values for a small planar germanium detector are about 150-250 eV at 5.9 keV, increasing to 400-600 eV at 122 keV. Larger coaxial detectors will produce FWHM values of 800-1200 eV at 122 keV, rising to 1.7-2.3 keV at 1333 keV.
4.3 Detecting Electrons

As soon as fast electrons enter the detector material, they immediately interact simultaneously with many atomic electrons. Depending on the energy of the fast electrons, the atom of detector materials can be excited or ionized. Also, fast electrons can be resulted the emission of radiation, which is called bremsstrahlung radiation, from detector material.

4.3.1 Energy Loss

Fast electrons lose their energy at a lower rate and follow a much more zigzag path through detector materials comparing with heavy charged particles. The specific collisional energy loss due to ionization and excitation for fast electrons is given by Bethe [29]

\[
- \left( \frac{dE}{dx} \right)_c = \frac{2\pi e^4 N Z}{m_0 \nu^2} \left( \ln \frac{m_0 \nu^2 E}{2I^2 \alpha} - \ln 2 (2 \sqrt{\alpha} - \alpha) + \alpha + \frac{1}{8} (1 - \sqrt{\alpha})^2 \right) \quad (4.9)
\]

where \( \nu \) is velocity of the fast electron, \( N \) and \( Z \) are the number density and atomic number of the detector atoms, \( m_0 \) is electron rest mass, \( e \) is the electronic charge and \( \alpha = 1 - \beta^2 \) where \( \beta = \nu/c \). The parameter \( I \) represents the average excitation and ionization potential of the detector material and is from experiment.

The specific energy loss through the bremsstrahlung radiation process is

\[
- \left( \frac{dE}{dx} \right)_r = \frac{e^4 N E Z (Z + 1)}{137 m_0^2 c^4} \left( 4 \ln \frac{2E}{m_0 c^2} - \frac{4}{3} \right) \quad (4.10)
\]

We can see that radiative losses are most important for high electron energies (\( E \)) and for detector materials of large atomic number (\( Z^2 \)) from Equation 4.10. So, only fast electrons can have a significant yield of bremsstrahlung.
The total energy losses for electrons is the sum of the collisional and radiative losses and the ratio of the specific energy losses is given by [29]

\[
\frac{(dE/dx)_r}{(dE/dx)_c} \approx \frac{EZ}{700}
\]

where \(E\) is in units of \(\text{MeV}\).

### 4.3.2 Backscattering of Electrons

The large-angle deflections of electrons lead them to the phenomenon of backscattering. The backscattered electrons do not deposit all their energy in the detector material and can have a significant effect on the response of detectors designed to measure the energy of externally incident electrons. Backscattering is most pronounced for electrons with low incident energy and detector material with high atomic number. Therefore Si is preferred to Ge or other high Z materials.

### 4.4 Lithium-Drifted Silicon Detectors

The depletion depth of the highest purity semiconductor silicon can be achieved to 1-2 \(\text{mm}\) by reverse biasing. If thicker silicon detectors are required, a different approach must be used to fabricate the device. This approach is to create a compensated material. The process of lithium drifting can be applied to create a region of compensated (intrinsic) silicon in which the concentration of acceptor and donor impurities are exactly balanced over thicknesses up to 5-10 \(\text{mm}\). When compensated silicon provided with non injecting electrodes, this region serves as the active volume of detector. Such detectors are called \textit{lithium-drifted silicon detectors}, Si(Li). Once the drifting process is completed, the resulting detector has the simplified configuration shown in Figure 4.7 [29].
In silicon, the ion mobility is low enough at room temperature to permit temporary storage of Si(Li) detectors without cooling. However, the compensated region is sufficiently thick so the leakage current fluctuations can be a significant noise source at room temperature. Therefore, to achieve good resolution the Si(Li) needs to be cooled, preferably to liquid nitrogen temperature.

The $\gamma$-ray full-energy peak efficiency for silicon detectors is very low, and consequently they are not widely used in general $\gamma$-ray spectroscopy. The first common application area of Si(Li) detector is the detection of very-low-energy $\gamma$ rays or X-rays, where the probability for photoelectric absorption can be reasonably high. The second common application area of Si(Li) detector is the detection and spectroscopy of beta particle or other externally incident electrons. The relatively low $Z$-value of silicon ensures that backscattering of electrons incident on the face of the detector will be minimized. A Si(Li) detector whose intrinsic thickness exceeds that of the maximum penetration distance of electrons makes a very suitable electron spectrometer.
Chapter 5

Experiment and Detector Array

This experiment was done at the TRI-University Meson Facility (TRIUMF), Van-
couver, B.C., Canada, where the world’s largest cyclotron has the capability of accel-
erating protons to energies of 500 MeV. The beam of protons, up to a 100 µA [30],
is delivered to the Isotope Separator and ACcelerator (ISAC) facility, and bombards
the thick target material. Fragmentation, spallation, and fission reactions produce a
variety of isotopes within the target. The thick targets are operated at high temper-
ature ($\gg 2200^0C$) [30] to enhance the diffusion and effusion of the product nuclei out
of the target. A variety of ion-sources are used to ionize the atoms, including surface
ion source and a laser ion source. For the experiment described here, the surface ion
source was used. An ion source is closely coupled to the target to decrease delays
between production and ion beam generation. The ionized atoms that are deflected
by the magnets of the mass separator are chosen according to their mass-to-charge
ratio with a resolution of $\sim 1/1000$. However, there always exists the possibility that
contaminants from isobaric nuclei with the same A/q are still included in the beam.
For this experiment, the mass separator selected A=110 products with a resultant
mixed beam of $1.2 \times 10^7$ ions/s of $^{110}$In in the 7$^+$ ground state with a 4.9 hr half life,
and $1.7 \times 10^6$ ions/s of $^{110}$In in the $2^+$ isomeric state with a 69 min half life. The $7^+$ ground state provided us with access to the high-spin states that are the focus of the present study. The high-intensity ion beam of $^{110}$In was delivered to the center of the $8\pi \gamma$-ray spectrometer and implanted onto the moving tape collector.

### 5.1 The Complete Gamma-ray Spectrometer

In order to perform as complete spectroscopy as possible, a variety of detector systems need to be used.

The $8\pi$ spectrometer consists of four detector systems (Figure 5.1). The first one is a $\gamma$-ray spectrometer comprising a close-packed array of 20 Compton-suppressed HPGe detectors with relative efficiency of 23% [31] and a total photo-peak efficiency of 1.4% at 1.33 MeV [32]. Each of 20 HPGe detectors is surrounded by bismuth-

![Figure 5.1: Closed 8\pi complete spectrometer.](image)

germanate-oxide (BGO) shield for active Compton suppression. Metal collimators are placed directly in front of the suppression shields to protect the shields from
direct radiation from the beam-implantation site. The free inner volume (left of Figure 5.2) has a diameter of 19.8 cm and can receive a variety of auxiliary detection systems.

![Figure 5.2: One hemisphere of 8π spectrometer (left) and SCEPTAR (right).](image)

Second detection system is SCintillating Electron Positron Tagging ARray (SCEPTAR) (right in Figure 5.2), a compact array of 20 BC404 plastic scintillator detectors for a β spectroscopy. SCEPTAR is mounted inside a spherical Delrin vacuum chamber [32] divided into two hemispheres for easy access.

An integral part of SCEPTAR is a moving tape collector system to remove long-lived daughter activity from the focus of the array (Figure 5.3). The low-energy beams from ISAC are focused at the center of the SCEPTAR chamber and deposited onto a 12.7 mm wide collector tape (right in Figure 5.3) that is fed from a large aluminum storage chamber (left in Figure 5.3) connected to the vacuum chamber containing the downstream half of SCEPTAR. A 5 cm thick lead shielding wall located immediately in front of the storage chamber shields the HPGe detectors from long-lived activity remaining on the tape.

The upstream half of SCEPTAR (left in Figure 5.4) can be replaced by a third
detection system, Pentagonal Array of Conversion Electron Spectrometers (PACES), consists of an array of 5 liquid nitrogen cooled Si(Li) conversion electron detectors (right in Figure 5.4) that surround the implantation point of the radioactive ion beam. Each of 5 Si(Li) detectors has a thickness of 5 mm and an area of 250\(\text{mm}^2\) and is placed 3 cm from the beam implantation.

Last one is Dipentagonal Array for Nuclear Timing Experiments (DANTE), an array of 10 BaF$_2$ \(\gamma\)-ray detectors. The BaF$_2$ scintillators have an extremely fast
time response and allow the half-lives of excited nuclear states that exist for only picoseconds to be measured directly by fast electronic timing methods.

5.2 Measurements

The experiment was done by repeated cycle measurement. In each cycle, the beam was implanted onto the tape of the moving tape collector at the center of $8\pi$ spectrometer. Each cycle consisted of four main stages. The first stage was the tape move which caused the remaining radioactivity on the tape from the previous cycle to be removed from the center of the array. The second stage was used to acquire a sample of the background activity. The third stage was the deposition of the in-beam ions to the tape (beam on), with the final stage the decay of the radioactive species on the tape (beam off).

The electronic signal produced by $\gamma$ ray or conversion electrons hitting the semiconductor detectors in $8\pi$ $\gamma$-ray spectrometer are amplified by a preamplifier. The electronic signal is a simple electrical pulse of which the amplitude is proportional to the energy of $\gamma$ ray or conversion electrons. There are two electrical signals from the preamplifier on the Ge detectors, but only one which is split to 2 for the Si(Li) detectors.

One signal is used to define the energy that is processed by a spectroscopy amplifier and converted by an analog-to-digital converter (ADC) before being binned in the raw histogram. The spectroscopy amplifier integrates the electrical pulses and smooths the voltage pulse. The ADC determines the pulse height and converts this into digital channel.

The other signal is used to control a time and is processed by a timing filter
and fed to the constant-fraction-discriminator (CFD) before fed into a time-to-digital converter (TDC). TDC provides time difference between two rays in coincidence with each other.

The $8\pi$ spectrometer has a four individual data streams that correspond to the four different systems of HPGe detectors, PACES, SCEPTAR, and DANTE. Each of these detector types also have their own trigger. The frame work of this thesis, only the $e^{-}\gamma$ coincidence data were analyzed.

**Energy calibration**

Precise energies are from the $\gamma$-ray data. We only have an approximate energy calibration by taking the $\gamma$-ray energies and subtracting the binding energies. We are not really measuring the energies so much as just making sure we have the right peaks.

### 5.3 Electron-Gamma Coincidence Method

When we study conversion electron transitions, it may be that conversion electrons follow $\gamma$-rays, or $\gamma$-rays follow electrons in a decay cascade. It is important that we determine whether the second radiation is in true time coincidence with the first one. If two radiations come close enough together in time, they are said to be ‘prompt’.

A time coincidence of two radiations from the same nucleus is called a true or real coincidence. It is also possible for radiations from different nuclei to trigger the timing circuits; this produces an unwanted chance or accidental coincidence. In principle, discriminating between true and chance coincidences is not hard. After accepting first radiation in the start channel of the time-to-amplitude converter (TAC), we wait
only a very short time (nanoseconds) for the second radiation to arrive and trigger the stop channel. True prompt coincidences with from a peak in the TAC spectrum centered at 0 ns, whereas the chance coincidences, being random in time, will be spread out and form a nearly flat background.

The numbers of \((e^-\gamma)\) coincidence counts that we measure can be constructed by the following expression:

\[
N_{e^-\gamma} = N \left[ I_{e^-} \epsilon_{e^-} \cdot BR_{\gamma} \epsilon_{\gamma} \cdot \epsilon_{e^-\gamma} \cdot \eta(\theta_{e^-\gamma}) \right] \tag{5.1}
\]

where \(N\) is overall normalization factor, \(I_{e^-}\) is the conversion electron intensity, \(\epsilon_{e^-}\) is the detector efficiency for conversion electrons, \(\epsilon_{\gamma}\) is the detector efficiency for \(\gamma\)-rays, \(BR\) is the \(\gamma\)-ray branching ratio, \(\epsilon_{c}\) is the coincidence efficiency and \(\eta(\theta_{e^-\gamma})\) is angular correlation factor. This expression assumes that we are gating on a \(\gamma\)-ray transition from below, and observing the \(e^-\) spectrum (Figure 5.5). Assuming

\[
\epsilon_{e^-\gamma} \approx 1 \text{ since timing gates are wide so low-energy pulses are not discriminated against}
\]

and \(\eta(\theta_{e^-\gamma}) \approx 1 \text{ since the combinations of all the } 8\pi \text{ HPGe detectors at the various angles wash out } \eta \text{ effects, we can solve for the intensity of conversion electrons from Equation 5.1 so that}
\]

\[
I_{e^-}^{(1)} = \frac{N_{e^-\gamma}}{N \left[ \epsilon_{e^-}^{(1)} \cdot BR_{\gamma}^{(2)} \epsilon_{\gamma}^{(2)} \right]} \tag{5.2}
\]
We can write down the similar expressions for \((\gamma\gamma)\) coincidence:

\[
N_{\gamma\gamma}^c = N \left[ I_\gamma^{(1)} \cdot \epsilon_\gamma^{(1)} \cdot B R_\gamma^{(2)} \cdot \epsilon_\gamma^{(2)} \cdot \eta(\theta_{\gamma\gamma}) \right]
\]

where \(I_\gamma\) is the \(\gamma\)-ray intensity, \(\epsilon_\gamma^{(1)}\) and \(\epsilon_\gamma^{(2)}\) are the individual detector efficiencies for first and second \(\gamma\)-rays, respectively. With the same assumptions \(\epsilon_c^{(\gamma\gamma)} \approx 1\) and \(\eta(\theta_{\gamma\gamma}) \approx 1\) as before, we have the intensity of the \(\gamma\)-rays:

\[
I_\gamma^{(1)} = \frac{N_{\gamma\gamma}^c}{N \left[ \epsilon_\gamma^{(1)} \cdot B R_\gamma^{(2)} \cdot \epsilon_\gamma^{(2)} \right]}
\]

In accordance to Equation 3.54 the conversion coefficients can be computed in the following way:

\[
\alpha = \frac{I_{\gamma^e}^{(1)}}{I_\gamma^{(1)}} = \frac{N_{\gamma^e}^c \epsilon_\gamma^{(1)}}{N_{\gamma\gamma}^c \epsilon_c^{(1)}}
\]

It is not easy to determine efficiencies for conversion electron detectors because of unpredictable backgrounds. Considering the conversion coefficients for all pure \(E2\) transitions, which are well known, we will be able to determine efficiencies for conversion electron detectors. Then, the remaining conversion coefficients can be calculated.

## 5.4 Germanium Detector Efficiency

The knowledge of the detector efficiency becomes necessary to have precise measurement in order to relate the number of pulses counted to the number of radiations incident on the detector. There are two classes of efficiencies: absolute and relative. Absolute efficiency is defined as

\[
\epsilon_{\text{abs}} = \frac{N_r}{N_e}
\]

where \(N_r\) is number of pulses recorded, \(N_e\) is number of radiation quanta emitted by source, and are dependent not only on detector properties but also on the details of

60
the counting geometry. The relative efficiency is defined as

\[ \epsilon_{\text{rel}} = \frac{N_r}{R_i} \]  

(5.7)

where \( R_i \) is the relative number of radiation quanta incident on detector, and no longer includes the absolute source strength.

### 5.4.1 Relative Efficiency

The programs SOURCE and EFFIT of Radware package [33] are employed for making the efficiency calibration. The SOURCE program combines a ‘sto’ file, from

![Figure 5.6: The energies and peak area of \(^{133}\text{Ba}\).](ba133.sto)

![Figure 5.7: The energies and peak area of \(^{56}\text{Co}\).](co56.sto)

the analysis of a source spectrum, with a data file containing the energies and relative intensities of the source \( \gamma \) rays. Before running SOURCE program, ‘sto’ file should be
edited so that the lines match one-to-one with the lines in the 'source' data file. The relative efficiency of Ge detectors was determined using sealed sources of $^{152}$Eu, $^{56}$Co, and $^{133}$Ba. These sources were placed at the center of $8\pi$ and their data were collected.

Table 5.1: The energies and relative intensities of $^{133}$Ba.

<table>
<thead>
<tr>
<th>Energy (keV)</th>
<th>Relative Intensity</th>
<th>Energy (keV)</th>
<th>Relative Intensity</th>
</tr>
</thead>
<tbody>
<tr>
<td>53.1622(6)</td>
<td>2140(30)</td>
<td>276.3989(12)</td>
<td>7160(50)</td>
</tr>
<tr>
<td>80.9979(11)</td>
<td>32900(300)</td>
<td>302.8508(5)</td>
<td>18340(130)</td>
</tr>
<tr>
<td>160.6120(16)</td>
<td>638(5)</td>
<td>356.0129(7)</td>
<td>62050(0)</td>
</tr>
<tr>
<td>223.2369(13)</td>
<td>453(3)</td>
<td>383.8485(12)</td>
<td>8940(60)</td>
</tr>
</tbody>
</table>

Figure 5.8: The energies and peak area of $^{152}$Eu.

The information from the analysis of a source spectrum was saved to the STO file (Figure 5.6, 5.7 and 5.8). The gf3 program of Radware package is employed for fitting peaks. Also, the energies and relative intensities of the three calibration sources were taken from the NNDC [34] and the source file was made from that information. All 'source' files are given in Table 5.1, 5.2 and 5.3. Once we had 'sto' and 'source' files, we can easily combine 'sto' and 'source' files, and make resultant 'sin' file. Input data are taken from 'sin' files generated by SOURCE, and several such files may be
read and combined using normalization factors. To fit the data from three sources simultaneously, the first file is read upon entering the program and data fitted as best as we can. Then the second file is added. The efficiencies calculated from the data

<table>
<thead>
<tr>
<th>Energy (keV)</th>
<th>Relative Intensity</th>
</tr>
</thead>
<tbody>
<tr>
<td>846.770(20)</td>
<td>99939.9(0)</td>
</tr>
<tr>
<td>977.372(50)</td>
<td>1421(6)</td>
</tr>
<tr>
<td>1037.8430(40)</td>
<td>2252(6)</td>
</tr>
<tr>
<td>1175.1010(40)</td>
<td>2252(6)</td>
</tr>
<tr>
<td>1238.2880(30)</td>
<td>66640(120)</td>
</tr>
<tr>
<td>1360.2120(40)</td>
<td>4283(12)</td>
</tr>
<tr>
<td>1771.3570(40)</td>
<td>15410(60)</td>
</tr>
</tbody>
</table>

Table 5.2: The energies and relative intensities of $^{56}$Co.

<table>
<thead>
<tr>
<th>Energy (keV)</th>
<th>Relative Intensity</th>
</tr>
</thead>
<tbody>
<tr>
<td>2015.2150(50)</td>
<td>3016(12)</td>
</tr>
<tr>
<td>2034.7910(50)</td>
<td>7770(30)</td>
</tr>
<tr>
<td>2598.5000(40)</td>
<td>16970(40)</td>
</tr>
<tr>
<td>3202.0290(80)</td>
<td>3209(12)</td>
</tr>
<tr>
<td>3253.5030(40)</td>
<td>7923(21)</td>
</tr>
<tr>
<td>3272.0790(40)</td>
<td>1875.9(2)</td>
</tr>
</tbody>
</table>

in this second file will be listed, together with the ratio of the present fit to these new values. The program will then prompt for a normalization factor. We take the average listed ratios, for energies which overlap well with the data in the first file, to obtain an empirical normalization. It creates a file to be used for input to the program

<table>
<thead>
<tr>
<th>Energy (keV)</th>
<th>Relative Intensity</th>
</tr>
</thead>
<tbody>
<tr>
<td>121.7817(3)</td>
<td>28670(150)</td>
</tr>
<tr>
<td>244.6975(8)</td>
<td>7610(40)</td>
</tr>
<tr>
<td>295.9392(17)</td>
<td>448(6)</td>
</tr>
<tr>
<td>344.2785(12)</td>
<td>26600(500)</td>
</tr>
<tr>
<td>367.7887(16)</td>
<td>882(10)</td>
</tr>
<tr>
<td>411.1163(11)</td>
<td>2237(25)</td>
</tr>
<tr>
<td>443.9650(3)</td>
<td>3158(42)</td>
</tr>
<tr>
<td>488.6792(20)</td>
<td>420(4)</td>
</tr>
<tr>
<td>678.6230(50)</td>
<td>472(7)</td>
</tr>
<tr>
<td>688.6700(50)</td>
<td>859(9)</td>
</tr>
</tbody>
</table>

Table 5.3: The energies and relative intensities of $^{152}$Eu.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>4.739</td>
<td>1.578</td>
<td>0.000</td>
<td>3.814</td>
<td>-0.848</td>
<td>-0.041</td>
<td>10.711</td>
</tr>
</tbody>
</table>

Table 5.4: The relative efficiency calibration parameters for our HPGe detector.
EFFIT. The program EFFIT fits an efficiency calibration. Then we fit the combined data of the two files and add a third data set. The best fitted relative efficiency curve is given in Figure 5.9. We determine seven parameters of the calibration which are labeled A, B, C, D, E, F, and G (Table 5.4). The functional form for the efficiency described by those parameters is

\[
\epsilon \ = \ \exp \left[ (A + Bx + Cx^2)^{-G} + (D + Ey + Fy^2)^{-G} \right]^{-1/G} \tag{5.8}
\]

where \(x = \log(E_\gamma/E_1)\) and \(y = \log(E_\gamma/E_2)\). Here \(E_\gamma\) is the \(\gamma\)-ray energy, and \(E_1\) and \(E_2\) have the values 100 keV and 1000 keV, respectively. The parameters \(A, B\) and \(C\) describe efficiency at low energies while \(D, E\) and \(F\) describe efficiency at high energies. \(G\) parameter is an interaction parameter between the two regions of
energies. If the efficiency turns over gently, G will be small. The parameter C should usually be left fixed to zero.

5.4.2 Dead Time Corrections

However powerful and precise a detection system for nuclear radiation is, there is a nonzero probability the system will miss some events during the measurement. The reasons can be associated to detector processing and auxiliary electronics. There will be a minimum amount of time that must separate two events as two separate pulses. This minimum time separation is usually called the dead time of detector system. To correct for dead time, a sort code with a dead time correction was used. An average dead time per event was found by the sort code. The dead time correction factor can be calculated as following

\[ F_t = \frac{1}{1 - \frac{t_{\text{ave}}}{t_{\text{bin}}}} \]  

(5.9)

where \( t_{\text{ave}} \) is an average dead time per event and \( t_{\text{bin}} \) is a bin size in time unit. The dead time correction factor was determined as 1.097104(18) for HPGe detectors.

5.4.3 Absolute Efficiency

Absolute efficiency is defined by ratio of the number of \( \gamma \) rays measured in detectors, \( N_m \), to the number of \( \gamma \) rays emitted from the source, \( N_e \),

\[ \epsilon_{\text{abs}} = \frac{N_m}{N_e} \]  

(5.10)

The absolute efficiency of the HPGe detector was measured using \(^{60}\text{Co}\) source which has well known decay rate (Table 5.5). The half life of \(^{60}\text{Co}\) is 1925.28 days and there were 538.04 days from date initial rate given to date the measurement was performed.
The number of γ rays emitted from source was corrected for both the γ ray intensities and decay time. The absolute efficiencies (November 2008) were determined for the

Table 5.5: The decay rate of $^{60}$Co source.

<table>
<thead>
<tr>
<th>Date</th>
<th>June 1, 2007</th>
<th>Nov. 20, 2008</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decay rate (microCi)</td>
<td>1.04(1)</td>
<td>0.857(9)</td>
</tr>
<tr>
<td>Decay rate (kBq)</td>
<td>38.48(39)</td>
<td>31.71(33)</td>
</tr>
</tbody>
</table>

Figure 5.10: The absolute efficiency for all HPGe detectors.

two γ rays emitted by the $^{60}$Co source and are shown in Table 5.6. The absolute efficiencies were applied to relative efficiencies with an average normalization factor (Table 5.7) and resultant plot are shown in Figure 5.10.
Table 5.6: The absolute $\gamma$-ray efficiency for all detectors.

<table>
<thead>
<tr>
<th>Lines</th>
<th>$I_{\gamma}$</th>
<th>$\epsilon_{abs}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1173.23 keV</td>
<td>99.85%</td>
<td>1.224(13)%</td>
</tr>
<tr>
<td>1332.49 keV</td>
<td>99.98%</td>
<td>1.107(11)%</td>
</tr>
</tbody>
</table>

Table 5.7: Normalization factors for the $\gamma$-ray efficiency for all detectors.

<table>
<thead>
<tr>
<th>Energy</th>
<th>$\epsilon_{abs}$</th>
<th>$\epsilon_{rel}$</th>
<th>Norm.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1173.23 keV</td>
<td>$1.224(13) \times 10^{-2}$</td>
<td>39.91</td>
<td>$3.07(3) \times 10^{-4}$</td>
</tr>
<tr>
<td>1332.49 keV</td>
<td>$1.107(11) \times 10^{-2}$</td>
<td>35.78</td>
<td>$3.09(3) \times 10^{-4}$</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td></td>
<td></td>
<td>$3.08(4) \times 10^{-4}$</td>
</tr>
</tbody>
</table>

5.5 Si(Li) Detector Efficiency

It is not easy to determine efficiencies for conversion electron detectors because of unpredictable backgrounds and of insufficient conversion electron sources. Since the PACES efficiency also appears to change with time, we adopted a purely internal calibration. Considering the conversion coefficients for pure $E2$ and $E1$ transitions,

Table 5.8: Calculated relative efficiencies for conversion electron detector.

<table>
<thead>
<tr>
<th>Transition multi-polarities</th>
<th>Transition energies (keV)</th>
<th>$\alpha_K$-coefficient</th>
<th>$\epsilon_{e^-}$ efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>120.59(1)</td>
<td>0.237(4)</td>
<td>10.93(22)</td>
</tr>
<tr>
<td>M1</td>
<td>244.66(2)</td>
<td>0.0351(5)</td>
<td>13.43(63)</td>
</tr>
<tr>
<td>E1</td>
<td>461.25(8)</td>
<td>0.00226(4)</td>
<td>14.38(103)</td>
</tr>
<tr>
<td>E2</td>
<td>559.56(8)</td>
<td>0.00418(6)</td>
<td>11.56(95)</td>
</tr>
<tr>
<td>E1</td>
<td>581.61(8)</td>
<td>0.001313(19)</td>
<td>12.54(88)</td>
</tr>
<tr>
<td>E2</td>
<td>657.24(1)</td>
<td>0.00272(4)</td>
<td>14.91(26)</td>
</tr>
<tr>
<td>E2</td>
<td>706.78(3)</td>
<td>0.00226(4)</td>
<td>14.41(43)</td>
</tr>
<tr>
<td>E2</td>
<td>743.59(12)</td>
<td>0.00200(3)</td>
<td>14.78(170)</td>
</tr>
<tr>
<td>E2</td>
<td>843.39(7)</td>
<td>0.001468(21)</td>
<td>14.66(114)</td>
</tr>
<tr>
<td>E2</td>
<td>884.08(2)</td>
<td>0.001315(19)</td>
<td>13.45(29)</td>
</tr>
<tr>
<td>E2</td>
<td>936.81(2)</td>
<td>0.001151(17)</td>
<td>13.11(30)</td>
</tr>
<tr>
<td>E2</td>
<td>1084.95(15)</td>
<td>0.000831(12)</td>
<td>12.65(189)</td>
</tr>
</tbody>
</table>

which are well known, we will be able to determine relative efficiencies for conversion electron detectors using HPGe detector efficiency, $e^- - \gamma$ and $\gamma - \gamma$ coincidence data.
Si(Li) detector efficiency can be calculated from Equation 5.5 as

$$\epsilon_{e^-} = \frac{\epsilon_\gamma N_e^{e^-\gamma}}{\alpha N_c^{\gamma\gamma}}$$

(5.11)

where $\epsilon_\gamma$ is the detector efficiency for $\gamma$-rays, $\alpha$ is conversion coefficient, and $N_c$ is a number of coincidence counts. As mentioned above, pure E1 and E2 transitions are well known and their conversion coefficients are accurate. However, these transitions are not enough to get a full behavior of relative efficiency for the PACES detector, especially in the low energy region. Therefore, we considered more two transitions at 121 keV and 245 keV. In principle, these transitions could be of mixed M1/E2 nature. The values of $\alpha_K$ are listed in Table 5.9 for the 121 keV and 245 keV lines. Using the $\alpha_K$(E2) for 121 keV and 245 keV lines, relative efficiency calibration for Si(Li) detectors would result in the plot shown in Figure 5.11. Comparing with the simulated absolute efficiency of Si(Li) (Figure 5.12) [35], we consider this plot unlikely since it results in a rapid and step decline in the efficiency at low energy. Assuming M1 multipolarity for 121 keV and 245 keV lines results in the plot are shown in Figure 5.13. The sub-shell ratio of the 120 keV transition is also in agreement with a nearly pure M1 multipolarity. There are two points of 560 keV and 582 keV lines way off. Those two points were excluded in our final internal efficiency calibration curve which is shown in Figure 5.14.

<table>
<thead>
<tr>
<th>Energy</th>
<th>M1</th>
<th>E2</th>
</tr>
</thead>
<tbody>
<tr>
<td>120.59(1) keV</td>
<td>0.237(4)</td>
<td>0.599(9)</td>
</tr>
<tr>
<td>244.66(2) keV</td>
<td>0.0351(5)</td>
<td>0.0539(8)</td>
</tr>
</tbody>
</table>
Figure 5.11: Internal relative efficiency calibration for Si(Li) detector using $\alpha_K(E2)$ for 121 keV and 245 keV lines.

Figure 5.12: Simulated efficiency for Si(Li) detector [35].
Figure 5.13: Internal relative efficiency calibration for Si(Li) detector using $\alpha_K(M1)$ for 121 keV and 245 keV lines.

Figure 5.14: Internal relative efficiency calibration for Si(Li) detector excluding two values from 560 keV and 582 keV lines.
Chapter 6

Data Analysis

Using the sort code, $e^- - \gamma$ or $\gamma - \gamma$ coincidence matrices were generated from the collected raw data. The energy and time were corrected during the sorting. The time-random background was subtracted by a program submat, a modification of addmat, and the gates were taken based on the projection spectra using slice program. These programs are included in Radware package [33]. To take a gate, the total projection spectra of $\gamma$ rays and electrons were created from the two dimensional matrix file and the windows for peak (A in Figure 6.1) and for the (largely) Compton background (B and C in Figure 6.1) were selected on desired peak. Total projected sample spectra are shown in Figure 6.2 and 6.3. We have analyzed more than 60 spectra gated from $e^- - \gamma$ and $\gamma - \gamma$ coincidence data sets.

To fit peaks in spectra, the program gf3 of the Radware package [33] was used. The gf3 is a peak-fitting program designed primarily for use in analyzing $\gamma$-ray spectra from Germanium detectors but it can also be used to analyze spectra from Si(Li) electron detectors.

The peak is fitted by three components in gf3 program. The main component of the peak is usually Gaussian in Ge detectors, physically arising from complete
Figure 6.1: Window selection on the 657 keV $\gamma$-transition.
Figure 6.2: Total projection spectrum of $\gamma$-rays.

Figure 6.3: Total projection spectrum of conversion electrons.
charge collection of a photoelectric event in the detector (1 in Figure 6.5). There is an exponential tail on the low-energy side of a peak, arising from incomplete charge collection, often due to trapping of charge at dislocations in the crystal lattice caused by impurities (2 in Figure 6.5). This tail component is called a skewed Gaussian. Last component arises mainly from multiple Compton scattering of photons into the detector and from escape of photoelectrons from the Ge crystal, which result in a slightly higher background on the low-energy side of the peak (3 in Figure 6.5). The total peak shape made by these three components and fixed parameters are shown in Figure 6.6 and 6.7. The initial estimated parameters of peak are shown in Figure 6.4.

The fitted parameters $A$, $B$ and $C$ define a background (Figure 6.4) [33],

$$Background = A + Bx + Cx^2$$  \hfill (6.1)

where $x$ is the channel number. The parameters $R$, $Beta$ and $Step$ define the shape of the peaks (Figure 6.4) [33],

$$TotalPeak = H \left( 1 - \frac{R}{100} \right) + H \frac{R}{100}$$  \hfill (6.2)

where first part is a Gaussian height and second part is a skew Gaussian height. $Beta$ is the decay constant of skew Gaussian, in channels while $Step$ is the relative height of a smoothed step function.

The parameters, $R$ and $Beta$, are allowed to vary during a fit, the skewed Gaussian component will usually greatly increase. If $Beta$ is unknown, the relative height of the skewed Gaussian is not generally well-determined by the fit. However, at least one of the parameters $R$ and $Beta$ can be fixed at some reasonable or previously-determined value, then the quoted error on the area is usually reasonable and reliable.
Figure 6.4: The initial estimated parameters of the peak in $^{60}$Co source spectrum.

Figure 6.5: Three components of the peak.
Figure 6.6: Fixed parameters of the peak in $^{60}$Co source spectrum.

Figure 6.7: Fitted total peak shape.
Figure 6.8: Fitted sample peaks of spectrum gated on the 885 keV $\gamma$-transition.
The program gf3 estimates a peak width as [33]

\[ FWHM = \sqrt{F^2 + G^2 x + H^2 x^2} \]  \hspace{1cm} (6.3)

where \( FWHM \) is the full width at half maximum of the peak and \( F, G \) and \( H \) are constraint parameters having default values of 3, 2, and 0, respectively. In order to fit a peak well and to identify possible doublet peaks in spectra, the widths of the known, single-line peaks can be measured as a function of energy. An example of a fit to some transitions of conversion electrons using gf3 is given in Figure 6.8.

### 6.1 Analyzing Cascades

The study of conversion electrons and \( \gamma \) rays emitted by nuclei is the good way to learn about the structure of excited states. One could expect that all non-ground state transitions should be observed in the coincidence spectrum on the lowest lying transition in a cascade. However, we have never seen all transitions in one gated spectrum due to weak intensities, small branching ratios, and contaminant background. A partial level scheme with all identified and expected transitions from a spectrum gated on the 885 keV \( \gamma \) ray are shown in Figure 6.9 and 6.10, respectively. The transition in red specifies the gate was taken on it, and blue colored transitions indicate peaks which could be identified in the gate. Green colored marking indicates a new transition identified. To identify missing transitions, the thing we need to do is to take gates on transitions from below or from above the missing transitions.

For example, the level scheme from a spectrum gated on the 678 keV transition is shown in Figure 6.11. There are two transitions identified which were missed in a spectrum gated on the 885 keV transition and new transition as well in that level scheme.
Figure 6.9: A partial level scheme with identified transitions from a spectrum gated on the 885 keV $\gamma$-transition.

Figure 6.10: A partial level scheme with expected (blue) and doublet (yellow) transitions from a spectrum gated on the 885 keV $\gamma$-transition.
Figure 6.11: A partial level scheme with identified transitions from a spectrum gated on the 678 keV $\gamma$-transition.

Figure 6.12: A partial level scheme with identified transitions from a spectrum gated on the 245 keV $\gamma$-transition.
The transitions of 1334 keV, 708 keV, 626 keV and 397 keV were identified by a spectrum gated on the 245 keV transition from above (Figure 6.12). The 188 keV transition was identified by spectra gated on 626 keV and 1334 keV transitions. Finally, all expected transitions through a gate on the 884 keV transition were identified by two more transitions. A total of 28 gates taken on desired transitions of the projected γ-ray spectrum and the same idea and process as mentioned above were applied for analyzing those gates.

6.2 Doublet Peaks

Some peaks in the spectra are a result of doublet of transitions. As long as we look at level scheme from a spectrum gated on the 885 keV transition, the yellow colored 707 keV transition is a doublet peak with black framed and the yellow colored 708 keV transition (Figure 6.10). The 707 keV transition can be isolated clearly by taking a gate on 937 keV transition (Figure 6.13) while only the gate on 626 keV transition will be able to show 708 keV transition cleanly (Figure 6.14).

The next doublet to be considered can be counted when we take a gate on the 120 keV transition (Figure 6.15). Doublet peaks are composed by 462 keV transition from 3121.6 keV $6^+$ to 2659.8 keV $5^-$ and 461 keV transition from 2539.7 keV $5^-$ to 2078.8 keV $3^-$. So, 462 keV transition was defined clearly from a spectrum gated on the 1117 keV transition from 2659.8 keV $5^-$ to 1542.5 keV $4^+$ (Figure 6.16).

For the 461 keV transition there was not a good way to see this transition separated from a doublet due to the 582 keV transition from above it was shown as one of doublet peaks with 585 keV transition (Figure 6.15). There is a 1420 keV transition from below the 461 keV transition but useless because those transitions are in the...
Figure 6.13: A partial level scheme with identified transitions from a spectrum gated on the 937 keV $\gamma$-transition.

Figure 6.14: A partial level scheme with identified transitions from a spectrum gated on the 626 keV $\gamma$-transition.
Figure 6.15: A partial level scheme showing a doublet (yellow) peak transitions.

Figure 6.16: A partial level scheme with identified transitions from a spectrum gated on 1117 keV γ-transition.
same cascade. There is one more 461 keV transition from 3525.2 $6^+$ to 3064.6 keV $6^+$ levels which makes more trouble (Figure 6.15) and can only been seen from $\gamma - \gamma$ coincidences.

Another 585 keV and 582 keV doublet peaks can easily be separated taking a gate on 997 keV and 937 keV transitions (Figure 6.15).

### 6.3 Compton-Scattered Peaks

There is always counts from $\gamma$-ray Compton scattering on Si(Li) and Ge detectors in our spectra. We observe huge Compton edges from intense $\gamma$-ray peaks (Figure 6.17 and 6.18). When we take a gate on 245 keV $\gamma$ ray transition, there can be Compton-scattered $\gamma$ rays from every intense transition whose energies are higher than that of the gate energy in the spectrum. Compton-scattered $\gamma$ rays make a coincidence with (657-245) keV, (884-245) keV, (707-245) keV, (937-245) keV and so on, energies in the Si detector (Figure 6.19).

When we take a gate the windows are selected in the $\gamma$ ray spectrum (in small spectrum in Figure 6.20). The black colored part indicates the main window for the gate, and green and red colored parts indicate peak backgrounds. Three spectra are created using 'slice' program which takes a slice on two dimensional coincidence matrix by windows of $\gamma$ side and projecting onto the electron axis (Figure 6.20).

Compton-scattered peaks are always shifted when we change a gated energy little bit because $E_C = E_{\text{initial}} - E_{\text{gate}}$ (red and green colored peaks in Figure 6.20). Two background spectra are subtracted from the main spectrum and gated spectrum will be created (Figure 6.17). So, Compton-scattered $\gamma$-ray peaks can be distinguished by their shape having two wells on either side of a peak. Some times it makes the
Figure 6.17: A spectrum gated on the 245 keV $\gamma$-transition showing Compton-scattered peaks (red) from many intense transitions.

Figure 6.18: A spectrum gated on the 626 keV $\gamma$-transition showing Compton-scattered peaks (red) from many intense transitions.
Figure 6.19: A spectrum created by gating on the 245 keV $\gamma$ transition and without background subtraction.

Figure 6.20: Electron spectra, created from main peak and its background slices, gated on the 245 keV $\gamma$-transition.
Figure 6.21: A spectrum gated on the 120 keV $\gamma$-transition showing Compton-scattered peaks (red) from many intense transitions.

spectrum ugly as shown in Figure 6.17, 6.18 and 6.21.
Chapter 7

Results and Conclusions

The nuclear structure of $^{110}$Cd was studied by its internal conversion electrons through $\beta^+$ decay of $^{110}$In. The nuclear structure of $^{110}$Cd exhibits states assigned as spherical phonon states and as intruder states. The intruder states are assumed to be more deformed 2p-4h proton excitation [13]. This thesis was focused on enhanced E0 transitions between intruder states and spherical phonon states. The E0 transition arises when there: a change in nuclear shape and does not take any angular momentum out. The information of E0 transitions is a valuable clue that the intruder states are from more deformed shape in $^{110}$Cd. The level scheme of $^{110}$Cd from conversion electron transitions was extended greatly over previous works, despite the large Compton background.

7.1 Identified Transitions

All identified transitions of $^{110}$Cd through conversion electrons are shown in Table 7.1. In total, 28 conversion $e^-$ transitions were identified from the spectra gated on 28 $\gamma$ transitions. We have seen K, L and M shell electrons of 13 transitions and K
and L shell electrons of one transition. There were identified 13 conversion electron transitions by only K shell electrons.

Table 7.1: All identified conversion electron transitions of $^{110}$Cd.

<table>
<thead>
<tr>
<th>$E_i$ (keV)</th>
<th>$j_i$</th>
<th>$E_f$ (keV)</th>
<th>$j_f$</th>
<th>$E_\gamma$ (keV)</th>
<th>Known multipolarity</th>
<th>Identified e$^-$ transition</th>
</tr>
</thead>
<tbody>
<tr>
<td>3121.6</td>
<td>6$^+$</td>
<td>3008.4</td>
<td>(5$^+$)</td>
<td>113.7</td>
<td>K/L/M</td>
<td>K/L/M</td>
</tr>
<tr>
<td>2659.8</td>
<td>5$^-$</td>
<td>2539.7</td>
<td>5$^-$</td>
<td>120.1</td>
<td>M1/E2</td>
<td>K/L/M</td>
</tr>
<tr>
<td>3064.6</td>
<td>6$^+$</td>
<td>2876.8</td>
<td>6$^+$</td>
<td>187.8</td>
<td>-</td>
<td>K/L/M</td>
</tr>
<tr>
<td>3121.6</td>
<td>6$^+$</td>
<td>2926.7</td>
<td>5$^+$</td>
<td>194.9</td>
<td>-</td>
<td>K/L/M</td>
</tr>
<tr>
<td>3121.6</td>
<td>6$^+$</td>
<td>2876.8</td>
<td>6$^+$</td>
<td>244.8</td>
<td>-</td>
<td>K/L/M</td>
</tr>
<tr>
<td>2876.8</td>
<td>6$^+$</td>
<td>2480.0</td>
<td>6$^+$</td>
<td>396.8</td>
<td>M1/E2</td>
<td>K/L/M</td>
</tr>
<tr>
<td>2539.7</td>
<td>5$^-$</td>
<td>2078.8</td>
<td>3$^-$</td>
<td>400.9</td>
<td>E2</td>
<td>K/L/M</td>
</tr>
<tr>
<td>3121.6</td>
<td>6$^+$</td>
<td>2659.9</td>
<td>5$^-$</td>
<td>461.7</td>
<td>-</td>
<td>K/L/M</td>
</tr>
<tr>
<td>2250.6</td>
<td>4$^+$</td>
<td>1783.5</td>
<td>2$^+$</td>
<td>467.1</td>
<td>E2</td>
<td>K/L/M</td>
</tr>
<tr>
<td>3121.6</td>
<td>6$^+$</td>
<td>2561.3</td>
<td>4$^+$</td>
<td>560.3</td>
<td>E2</td>
<td>K/L/M</td>
</tr>
<tr>
<td>3121.6</td>
<td>6$^+$</td>
<td>2539.7</td>
<td>5$^-$</td>
<td>581.9</td>
<td>E1</td>
<td>K/L/M</td>
</tr>
<tr>
<td>3064.6</td>
<td>6$^+$</td>
<td>2479.9</td>
<td>6$^+$</td>
<td>584.7</td>
<td>M1/E2</td>
<td>K/L/M</td>
</tr>
<tr>
<td>2876.8</td>
<td>6$^+$</td>
<td>2250.6</td>
<td>4$^+$</td>
<td>626.2</td>
<td>E2</td>
<td>K/L/M</td>
</tr>
<tr>
<td>3121.6</td>
<td>6$^+$</td>
<td>2479.9</td>
<td>6$^+$</td>
<td>641.7</td>
<td>M1/E2</td>
<td>K/L/M</td>
</tr>
<tr>
<td>657.8</td>
<td>2$^+$</td>
<td>0.0</td>
<td>0$^+$</td>
<td>657.8</td>
<td>E2</td>
<td>K/L/M</td>
</tr>
<tr>
<td>2220.1</td>
<td>4$^+$</td>
<td>1542.5</td>
<td>4$^+$</td>
<td>677.6</td>
<td>M1/E2</td>
<td>K/L/M</td>
</tr>
<tr>
<td>3187.3</td>
<td>8$^+$</td>
<td>2479.9</td>
<td>6$^+$</td>
<td>707.4</td>
<td>E2</td>
<td>K/L/M</td>
</tr>
<tr>
<td>2250.6</td>
<td>4$^+$</td>
<td>1542.5</td>
<td>4$^+$</td>
<td>708.1</td>
<td>M1/E2</td>
<td>K/L/M</td>
</tr>
<tr>
<td>2220.1</td>
<td>4$^+$</td>
<td>1475.8</td>
<td>2$^+$</td>
<td>744.3</td>
<td>E2</td>
<td>K/L/M</td>
</tr>
<tr>
<td>3239.8</td>
<td>6$^+$</td>
<td>2479.9</td>
<td>6$^+$</td>
<td>759.9</td>
<td>M1/E2</td>
<td>K/L/M</td>
</tr>
<tr>
<td>1475.8</td>
<td>2$^+$</td>
<td>657.8</td>
<td>2$^+$</td>
<td>818.0</td>
<td>M1/E2</td>
<td>K/L/M</td>
</tr>
<tr>
<td>3064.6</td>
<td>6$^+$</td>
<td>2220.1</td>
<td>4$^+$</td>
<td>844.5</td>
<td>E2</td>
<td>K/L/M</td>
</tr>
<tr>
<td>1542.5</td>
<td>4$^+$</td>
<td>657.8</td>
<td>2$^+$</td>
<td>884.7</td>
<td>E2</td>
<td>K/L/M</td>
</tr>
<tr>
<td>3121.6</td>
<td>6$^+$</td>
<td>2220.1</td>
<td>4$^+$</td>
<td>901.5</td>
<td>-</td>
<td>K/L/M</td>
</tr>
<tr>
<td>2479.9</td>
<td>6$^+$</td>
<td>1542.5</td>
<td>4$^+$</td>
<td>937.4</td>
<td>E2</td>
<td>K/L/M</td>
</tr>
<tr>
<td>2539.7</td>
<td>5$^-$</td>
<td>1542.5</td>
<td>4$^+$</td>
<td>997.2</td>
<td>E1</td>
<td>K/L/M</td>
</tr>
<tr>
<td>2561.3</td>
<td>4$^+$</td>
<td>1475.8</td>
<td>2$^+$</td>
<td>1085.5</td>
<td>E2</td>
<td>K/L/M</td>
</tr>
<tr>
<td>2876.8</td>
<td>6$^+$</td>
<td>1542.5</td>
<td>4$^+$</td>
<td>1334.3</td>
<td>E2</td>
<td>K/L/M</td>
</tr>
</tbody>
</table>

7.2 Sub-shell Ratios

The information from sub-shell ratios are important to obtain transition multipo-larities. Sub-shell ratios for some intensive transitions are shown in Table 7.2. There are some sub-shell ratios that were not able to be determined by internal conversion electrons through this study.

There are many sub-shell ratios determined lower than expected that may reflect
Table 7.2: Sub-shell ratios for some intensive transitions.

<table>
<thead>
<tr>
<th>$E_i$ (keV)</th>
<th>$j_i$</th>
<th>$E_f$ (keV)</th>
<th>$j_f$</th>
<th>$E_\gamma$ (keV)</th>
<th>Multipolarity</th>
<th>$\alpha_K/\alpha_L$</th>
<th>$\alpha_K/\alpha_L$</th>
<th>$\alpha_K/\alpha_L$</th>
<th>$\alpha_K/\alpha_L$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Total$_{exp}$</td>
<td>M1$_{theo}$</td>
<td>E2$_{theo}$</td>
<td>E3$_{theo}$</td>
</tr>
<tr>
<td>3121.6</td>
<td>6+</td>
<td>3008.4</td>
<td>(5⁹)</td>
<td>113.7</td>
<td>E2/M1</td>
<td>4.79(102)</td>
<td>8.00(16)</td>
<td>4.15(9)</td>
<td>8.14(17)</td>
</tr>
<tr>
<td>2659.8</td>
<td>5−</td>
<td>2539.7</td>
<td>5−</td>
<td>120.1</td>
<td>E2/M1</td>
<td>7.08(26)</td>
<td>8.01(16)</td>
<td>4.35(9)</td>
<td>8.16(17)</td>
</tr>
<tr>
<td>3064.6</td>
<td>6+</td>
<td>2876.8</td>
<td>6+</td>
<td>187.8</td>
<td>-</td>
<td>-</td>
<td>8.10(16)</td>
<td>5.75(12)</td>
<td>8.30(17)</td>
</tr>
<tr>
<td>3121.6</td>
<td>6+</td>
<td>2926.7</td>
<td>5+</td>
<td>194.9</td>
<td>E2/M1</td>
<td>6.11(106)</td>
<td>8.11(16)</td>
<td>5.85(12)</td>
<td>8.31(17)</td>
</tr>
<tr>
<td>3121.6</td>
<td>6+</td>
<td>2876.8</td>
<td>6+</td>
<td>244.8</td>
<td>E2/M1</td>
<td>6.44(56)</td>
<td>8.17(17)</td>
<td>6.41(13)</td>
<td>8.36(17)</td>
</tr>
<tr>
<td>2876.8</td>
<td>6+</td>
<td>2480.0</td>
<td>6+</td>
<td>396.8</td>
<td>E2/M1</td>
<td>-</td>
<td>8.30(17)</td>
<td>7.31(15)</td>
<td>8.45(17)</td>
</tr>
<tr>
<td>2539.7</td>
<td>5−</td>
<td>2078.8</td>
<td>5−</td>
<td>120.1</td>
<td>E2/M1</td>
<td>7.08(26)</td>
<td>8.01(16)</td>
<td>4.35(9)</td>
<td>8.16(17)</td>
</tr>
<tr>
<td>3121.6</td>
<td>6+</td>
<td>2659.9</td>
<td>5−</td>
<td>461.7</td>
<td>E1</td>
<td>4.54(122)</td>
<td>8.34(17)</td>
<td>7.53(15)</td>
<td>8.49(17)</td>
</tr>
<tr>
<td>2250.6</td>
<td>4+</td>
<td>1783.5</td>
<td>2+</td>
<td>467.1</td>
<td>E2</td>
<td>-</td>
<td>8.34(17)</td>
<td>7.53(15)</td>
<td>8.49(17)</td>
</tr>
<tr>
<td>3121.6</td>
<td>6+</td>
<td>2561.3</td>
<td>4+</td>
<td>560.3</td>
<td>E2</td>
<td>4.85(116)</td>
<td>8.39(17)</td>
<td>7.77(16)</td>
<td>8.53(17)</td>
</tr>
<tr>
<td>3121.6</td>
<td>6+</td>
<td>2539.7</td>
<td>5−</td>
<td>581.9</td>
<td>E1</td>
<td>4.00(80)</td>
<td>8.40(17)</td>
<td>7.82(16)</td>
<td>8.54(17)</td>
</tr>
<tr>
<td>3064.6</td>
<td>6+</td>
<td>2479.9</td>
<td>6+</td>
<td>584.7</td>
<td>E2/M1</td>
<td>4.23(85)</td>
<td>8.40(17)</td>
<td>7.82(16)</td>
<td>8.54(17)</td>
</tr>
<tr>
<td>2876.8</td>
<td>6+</td>
<td>2250.6</td>
<td>4+</td>
<td>626.2</td>
<td>E2</td>
<td>-</td>
<td>8.41(17)</td>
<td>7.90(16)</td>
<td>8.55(17)</td>
</tr>
<tr>
<td>3121.6</td>
<td>6+</td>
<td>2479.9</td>
<td>6+</td>
<td>641.7</td>
<td>E2/M1</td>
<td>6.55(64)</td>
<td>8.42(17)</td>
<td>7.92(16)</td>
<td>8.56(17)</td>
</tr>
<tr>
<td>657.8</td>
<td>2+</td>
<td>0.0</td>
<td>0+</td>
<td>657.8</td>
<td>E2</td>
<td>7.41(32)</td>
<td>8.43(17)</td>
<td>7.95(16)</td>
<td>8.56(17)</td>
</tr>
<tr>
<td>2220.1</td>
<td>4+</td>
<td>1542.5</td>
<td>4+</td>
<td>677.6</td>
<td>E2/M1</td>
<td>2.66(36)</td>
<td>8.43(17)</td>
<td>7.98(16)</td>
<td>8.57(17)</td>
</tr>
<tr>
<td>3187.3</td>
<td>8+</td>
<td>2479.9</td>
<td>6+</td>
<td>707.4</td>
<td>E2</td>
<td>7.83(72)</td>
<td>8.44(17)</td>
<td>8.02(16)</td>
<td>8.58(17)</td>
</tr>
<tr>
<td>2250.6</td>
<td>4+</td>
<td>1542.5</td>
<td>4+</td>
<td>708.1</td>
<td>E2/M1</td>
<td>-</td>
<td>8.44(17)</td>
<td>8.02(16)</td>
<td>8.58(17)</td>
</tr>
<tr>
<td>2220.1</td>
<td>4+</td>
<td>1475.8</td>
<td>2+</td>
<td>744.3</td>
<td>E2</td>
<td>-</td>
<td>8.36(17)</td>
<td>8.07(16)</td>
<td>8.59(17)</td>
</tr>
<tr>
<td>3239.8</td>
<td>6+</td>
<td>2479.9</td>
<td>6+</td>
<td>759.9</td>
<td>E2/M1</td>
<td>-</td>
<td>8.46(17)</td>
<td>8.09(16)</td>
<td>8.60(17)</td>
</tr>
<tr>
<td>1475.8</td>
<td>2+</td>
<td>657.8</td>
<td>2+</td>
<td>818.0</td>
<td>E2/M1</td>
<td>-</td>
<td>8.48(17)</td>
<td>8.15(17)</td>
<td>8.61(17)</td>
</tr>
<tr>
<td>3064.6</td>
<td>6+</td>
<td>2220.1</td>
<td>4+</td>
<td>844.5</td>
<td>E2</td>
<td>-</td>
<td>8.49(17)</td>
<td>8.18(17)</td>
<td>8.62(17)</td>
</tr>
<tr>
<td>1542.5</td>
<td>4+</td>
<td>657.8</td>
<td>2+</td>
<td>884.7</td>
<td>E2</td>
<td>7.69(60)</td>
<td>8.50(17)</td>
<td>8.22(17)</td>
<td>8.63(17)</td>
</tr>
<tr>
<td>3121.6</td>
<td>6+</td>
<td>2220.1</td>
<td>4+</td>
<td>901.5</td>
<td>-</td>
<td>-</td>
<td>8.50(17)</td>
<td>8.23(17)</td>
<td>8.63(17)</td>
</tr>
<tr>
<td>2479.9</td>
<td>6+</td>
<td>1542.5</td>
<td>4+</td>
<td>937.4</td>
<td>E2</td>
<td>8.23(67)</td>
<td>8.51(17)</td>
<td>8.24(17)</td>
<td>8.64(17)</td>
</tr>
<tr>
<td>2539.7</td>
<td>5−</td>
<td>1542.5</td>
<td>4+</td>
<td>997.2</td>
<td>E1</td>
<td>-</td>
<td>8.52(17)</td>
<td>8.31(17)</td>
<td>8.66(17)</td>
</tr>
<tr>
<td>2561.3</td>
<td>4+</td>
<td>1475.8</td>
<td>2+</td>
<td>1085.5</td>
<td>E2</td>
<td>-</td>
<td>8.54(17)</td>
<td>8.36(17)</td>
<td>8.67(17)</td>
</tr>
<tr>
<td>2876.8</td>
<td>6+</td>
<td>1542.5</td>
<td>4+</td>
<td>1334.3</td>
<td>E2</td>
<td>-</td>
<td>8.58(17)</td>
<td>8.48(17)</td>
<td>8.71(17)</td>
</tr>
</tbody>
</table>

Contributions from Compton-scattered $\gamma$ rays made coincident with L shell electrons. There are some doublets (581.9 keV and 584.7 keV) and triplets (467.1 keV, 468 keV and 469 keV) which make the extraction of sub-shell ratios impossible. Most identified transitions are well known except four transitions of 113.7 keV, 187.8 keV, 194.9 keV and 244.8 keV. The multipolarities of the 461 keV and 901 keV transitions were not assigned in NNDC site.
7.2.1 Mixing Ratio from Sub-shell Ratio

Mixing ratio $\delta^2$ was calculated from an experimental sub-shell $\frac{\alpha_{exp}^K}{\alpha_{exp}^L}$ ratio using Equation 3.84 for mixed E2/M1 transitions

$$\delta^2 = \frac{\alpha_K(M1) - \frac{\alpha_{exp}^K}{\alpha_{exp}^L}\alpha_L(M1)}{\frac{\alpha_{exp}^K}{\alpha_{exp}^L}\alpha_L(E2) - \alpha_K(E2)}$$

and presented in Table 7.3.

<table>
<thead>
<tr>
<th>$E_i$ (keV)</th>
<th>$j_i^\pi$</th>
<th>$E_f$ (keV)</th>
<th>$j_f^\pi$</th>
<th>$E_{\gamma}$ (keV)</th>
<th>Multi-polarity</th>
<th>Mixing ratio $\delta^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3121.6</td>
<td>6$^+$</td>
<td>3008.4</td>
<td>(5$^+$)</td>
<td>113.7</td>
<td>E2/M1</td>
<td>1.16(25)</td>
</tr>
<tr>
<td>2659.8</td>
<td>5$^-$</td>
<td>2539.7</td>
<td>5$^-$</td>
<td>120.1</td>
<td>E2/M1</td>
<td>0.073(3)</td>
</tr>
<tr>
<td>3121.6</td>
<td>6$^+$</td>
<td>2926.7</td>
<td>5$^-$</td>
<td>194.9</td>
<td>E2/M1</td>
<td>3.23(57)</td>
</tr>
<tr>
<td>3121.6</td>
<td>6$^+$</td>
<td>2876.8</td>
<td>6$^+$</td>
<td>244.8</td>
<td>E2/M1</td>
<td>25.24(231)</td>
</tr>
</tbody>
</table>

7.2.2 113.71(4) keV Transition

As an example of the obtained data quality, a new 113 keV transition was identified. This transition can been seen in spectra gated on 657 keV, 884 keV, 677 keV and 844 keV $\gamma$-ray transitions (Figure 7.2). K and L electrons of that new transition can been seen only in a spectrum gated on the 844 keV transition. The 844 keV transition between the 3064.6 keV and 2220.1 keV levels is a doublet with the 845 keV transition between 3008.4 keV and 2162.6 keV levels. The $\gamma - \gamma$ coincidences are absolutely clear that the 113 keV line comes from the 3121.6 keV to the 3008.4 keV levels. The K/L sub-shell ratio of this transition indicates mixing of E2 and M1 transitions with an extracted $\delta^2$ value of 1.16(25). However, the conversion coefficient of this transition indicates a pure M1 transition (see Table 7.4). Thus, the spin of the 3008.4 keV level can be 5$^+$ to make M1 transition from 3121.6 keV 6$^+$ level.
Figure 7.1: Spectrum gated on the 844 keV $\gamma$-transition showing K and L conversion electron peaks of the 113 keV transition and Compton-scattered peak (red).

### 7.2.3 187.56(8) keV Transition

This is one of three transitions for which no information is presented at the data site of National Nuclear Data Center (NNDC) [34]. K shell electrons of 187.56 keV transition can been seen clearly only in spectra gated on 626 keV (Figure 7.2) and 1334 keV $\gamma$-ray transitions both of which are from the 2876.8 keV level. The final level of this transition was firmly assigned as 2876.8 $6^+$ and the initial level is the 3064.6 keV $6^+$. Thus, multipolarity of this transition between 3064.6 keV $6^+$ and 2876.8 $6^+$ levels can be mixed M1+E2. Therefore, the conversion coefficient of this transition favors mixed E2+M1 multipolarity with evaluated $\delta^2$ value of 0.039(4) (Table 7.5).
Figure 7.2: Spectrum gated on the 626 keV $\gamma$-transition showing K conversion electron peak of the 188 keV transition and K and L conversion electron peaks of the 245 keV transition, and Compton-scattered peaks (red).

7.2.4 195.04(4) keV Transition

This is second transition for which no information is presented at the data site of NNDC. K, L and possibly M shell electrons of this transition can been seen clearly only in a spectrum gated on the 1383 keV $\gamma$-ray transition from 2926.7 keV to 1542.4 keV levels (Figure 7.3). The final level of this transition is the firmly assigned 2926.7 keV $5^+$ and initial level is the well-known 3121.6 keV $6^+$ level. The K/L sub-shell ratio of this transition indicates mixing of E2 and M1 transitions with an extracted $\delta^2$ value of 3.23(57). Therefore, the conversion coefficient of this transition favoring mixed E2+M1 too with evaluated $\delta^2$ value of 4.30(26) (Table 7.5). Thus, multipolarity of this transition between 3121.6 keV $6^+$ and 2926.7 keV $5^+$ can be mixed E2+M1.
Figure 7.3: Spectrum gated on the 1383 keV $\gamma$-transition showing K, L and M conversion electron peaks of the 195 keV transition.

### 7.2.5 244.66(2) keV Transition

This is third transition for which no information is presented at the data site of NNDC. K and L shell electrons of 244.7 keV transition can be seen clearly in a spectrum gated on the 626 keV $\gamma$-ray transition (see Figure 7.2), and K,L and M shell electrons of the 244.7 keV transition can be seen clearly in a spectrum gated on the 1334 keV $\gamma$-ray transition (Figure 7.4). Those two transitions on which gates are taken are both arise from the 2876.8 keV level. The final level of this transition was firmly assigned as 2876.8 keV 6$^+$ and initial level is the well-known 3121.6 keV 6$^+$. The K/L sub-shell ratio of this transition indicates mixing of E2 and M1 transitions with an extracted $\delta^2$ value of 25.24(231). Thus, multipolarity of this transition can
be mixed E2+M1, but this transition was used for internal calibration as a pure M1 transition (see Chapter 5.5).

7.2.6 460.53(19) keV Transition

This transition is from 2539.7 keV 5− to 2078.8 keV 3− levels and it has a doublet of 462 keV transition which is in the same cascade (3122→2660) and a doublet of 461 keV transition which can been seen from a spectrum with a gate on 582 keV transition that can show this transition clearly without the other member of the doublet. The 582 keV transition, however, is a doublet itself with 584 keV transition that is from 3064.6 keV 5− level. But there is one more 461 keV transition from
3525.2 $6^+$ to 3064.6 keV $6^+$ levels (Figure 7.5). This transition is a weak one and can only been seen from $\gamma-\gamma$ coincidences. Therefore, the $\alpha$ coefficient of this transition is not reliable. Unfortunately, there is not able to have any information about its conversion coefficient from our data.

**7.2.7 469.99(16) keV Transition**

This transition is between intruder states from 2250.6 keV $4^+$ to 1783.5 keV $2^+$ but it is a doublet with another 468 keV transition which is to the 2876.8 keV $6^+$ level in the same cascade. This can clearly be seen from $\gamma-\gamma$ coincidences (Figure 7.6). A gate that could cleanly show this transition without the presence of the doublet could not be found.
7.3 Level Scheme of Cd-110

The sub-shell ratios and assigned multipolarities can be used to build the transition level scheme. The level scheme of $^{110}$Cd was built by internal conversion electron transitions (Figure 7.7). In total, 28 spectra were analyzed from gates taken from $e^- - \gamma$ coincident matrix and each spectrum of which includes at least one important peak. The level scheme of $^{110}$Cd through the conversion electrons was extended. Since the beam was dominantly composed of the high spin $^{110}$In, high-spin states in the daughter where populated with a large intensity.

The level scheme of $^{110}$Cd includes not only ground state band but also the intruder band, quasi-particle and octupole bands. There is an octupole band which could be based on the octupole ($\lambda=3$) phonon state. There are also some additional states in level scheme that may arise from the coupling of the first quadrupole state $2^+$ with the first octupole state.

Previously assigned phonon levels observed in $^{110}$Cd are shown in Figure 7.8. We were able to see the 657.8 keV $2^+$ state at the 1 phonon level, 1475.8 keV $2^+$ and 1542.5 keV $4^+$ states at the 2 phonon level, and 2220.1 keV $4^+$ and 2479.9 keV $6^+$ states at the 3 phonon level. Other possible multiphonon candidates were not
observed. Those levels at 1783.5 keV $2^+$, 2250.6 keV $4^+$ and 2876.8 keV $6^+$ states were previously assigned as intruder states. The $0^+$ band head was not observed. The negative parity states at 2078.8 keV $(3^-)$, 2539.7 keV $(5^-)$ and 2659.8 keV $(5^-)$ were observed. The 2079 keV $3^-$ state is the octupole phonon state, and the 2540 keV $5^-$ is a candidate for the quadrupole octupole coupled state. The remaining low-spin members of the quadrupole octupole coupling were not observed. There are some 3121.6 keV $6^+$ and 3239.8 $6^+$ higher spin states populated that cannot be given a specific configuration assignment based on the present data.

### 7.4 Internal Conversion Coefficient

The conversion coefficients for 14 transitions observed were calculated using Equation 5.5 and are shown in Table 7.4. The internal efficiency calibration of Si(Li) shown in Figure 5.12, was defined to determine the absolute conversion coefficient of some transitions. There were 12 transitions used for making internal efficiency calibration of Si(Li) detector. Conversion coefficients of the 461 and 467 keV transitions were
Figure 7.7: Experimental band level scheme of $^{110}$Cd.
Figure 7.8: Experimental phonon level scheme of $^{110}\text{Cd}$. 
not able to defined due to doublet peaks.

### 7.4.1 Mixing Ratio from K-conversion Coefficient

Mixing ratio $\delta^2$ was calculated from an experimental absolute K-conversion coefficient $\alpha_{K}^{exp}$ using Equation 3.85

$$\delta^2 = \frac{\alpha_K(M1) - \alpha_{K}^{exp}}{\alpha_{K}^{exp} - \alpha_K(E2)}$$

for some mixed E2/M1 transitions and presented in Table 7.5.

<table>
<thead>
<tr>
<th>$E_i$ (keV)</th>
<th>$j_i^\pi$</th>
<th>$E_f$ (keV)</th>
<th>$j_f^\pi$</th>
<th>$E_\gamma$ (keV)</th>
<th>Multipolarity</th>
<th>Mixing ratio $\delta^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3064.6</td>
<td>6$^+$</td>
<td>2876.8</td>
<td>6$^+$</td>
<td>187.8</td>
<td>E2/M1</td>
<td>0.039(4)</td>
</tr>
<tr>
<td>3121.6</td>
<td>6$^+$</td>
<td>2926.7</td>
<td>5$^-$</td>
<td>194.9</td>
<td>E2/M1</td>
<td>4.30(26)</td>
</tr>
<tr>
<td>1475.8</td>
<td>2$^+$</td>
<td>657.8</td>
<td>2$^+$</td>
<td>817.2</td>
<td>E2/M1</td>
<td>0.160(20)</td>
</tr>
</tbody>
</table>

### 7.4.2 396.61(13) and 708.21(30) keV Transitions

The 397 keV transition is from intruder 2876.8 keV 6$^+$ level to phonon 2479.9 keV 6$^+$ level. The 708 keV transition is from intruder 2250.6 keV 4$^+$ level to phonon 1542.5 keV 4$^+$ level. Their total K-conversion coefficients are much higher than that both E2’s and M1’s. Thus, there are possible E0 transitions.

### 7.4.3 583.79(6), 677.01(9) and 759.77(12) keV Transitions

These transitions are between 6$^+$ levels. The multipolarities of these transitions may be mixed M1+E2, although the conversion coefficients of these transitions favor pure E2 in their $\sigma$ region (see Table 7.4).
7.4.4 626.18(8) keV Transition

This transition is between intruder states from 2876.8 keV $6^+$ to 2250.6 keV $4^+$.

![Spectrum gated on the 245 keV transition](image)

Figure 7.9: Spectrum gated on the 245 keV $\gamma$-transition showing K-conversion electron peak of the 626 keV transition (black) and Compton-scattered peaks (red)

and its conversion coefficient was higher than expected for an E2 transition. There was only one gate with the 245 keV transition that shows this transition clearly. The 245 keV gated spectrum, however, has a large number of Compton-scattered peaks. There can be a Compton-scattered peak from the 844 keV transition at the same position of the K-conversion line of the 626 keV transition since 844-245=599 keV (Figure 7.9). For this reason, it is believed that the K-conversion coefficient is affected by Compton background and is not reliable.
7.4.5 641.05(2) keV Transition

This transition is from 3121.6 keV $6^+$ to 2479.9 keV $6^+$. Multipolarity of this transition may be mixed M1+E2, but the conversion coefficient of this transition indicates pure M1 in its $\sigma$ region (see Table 7.4).

7.4.6 817.23(11) keV Transition

This transition is from 1475.8 keV $2^+$ to 657.8 keV $2^+$. Its multipolarity can be mixed M1+E2. The conversion coefficient of this transition favors mixed E2+M1, too, with evaluated $\delta^2$ value of 0.160(20) (Table 7.5).

Figure 7.10: Spectrum gated on the 677 keV $\gamma$-transition showing K-conversion electron peak of the 901 keV transition.
7.4.7 900.69(9) keV Transition

This transition should have pure E2 multipolarity from 3121.6 keV $6^+$ to 2220.1 keV $4^+$ levels and is a weak for conversion electron transition. A spectrum showing this transition is given in Figure 7.10. Its conversion coefficient was found to be a larger than that expected for a pure E2 and even M1 transition, however, because of its large uncertainty it is within $2\sigma$ uncertainty range of $\alpha_K(E2)$.

7.4.8 997.28(10) keV Transition

This transition is known pure E1 from 2539.7 keV $5^-$ to 1542.5 keV $4^+$ levels and is quite weak for conversion electron transition. There were two possible transitions
to gate on to see this transition 120 keV and 884 keV. The peak of this transition was not distinguishable in a spectrum gated on the 884 keV due to huge backgrounds. The spectrum gated on the 120 keV transition was able to show the 997 keV K-conversion peak, but it was still not clear (Figure 7.11). Its conversion coefficient was found to be smaller than that expected for a pure E1 transition, however, it was still within its $2\sigma$ uncertainty region.

### 7.4.9 1332.59(12) keV Transition

This transition is known pure E2 between intruder and spherical phonon states from 2876.8 keV $^6+ \to 1542.5$ keV $^4+$ levels and was weak for conversion electron

![Spectrum gated on the 245 keV transition](image)

Figure 7.12: Spectrum gated on the 120 keV $\gamma$-transition showing K-conversion electron peak of the 997 keV transition (black) and Compton-scattered peaks (red).
transition. There were two possible transitions to be used as gates to see this transition: 245 keV and 884 keV. The peak of this transition was not distinguishable in a spectrum gated on the 884 keV due to huge backgrounds. The spectrum gated on the 245 keV transition was able to show this transition, however, the peak was still indistinguishable from background (Figure 7.12). Its conversion coefficient was too large compared with that expected for a pure E2 transition, and due to the background the results are not reliable.

7.4.10 E0 Transitions

We expected E0 transitions between intruder and spherical phonon states. Indeed,
Figure 7.14: Spectrum gated on the 245 keV $\gamma$-transition showing K-conversion electron peak of the 397 keV E0 transition and Compton-scattered peaks (red).

Figure 7.15: Spectrum gated on the 626 keV $\gamma$-transition showing K-conversion electron peak of the 708 keV E0 transition.
7.4.11 396.61(13) keV E0 Transition

The 396.61(13) keV transition is from 2876.8 keV 6+ intruder state to 2479.9 keV 4+ phonon state and has expected conversion coefficients of $\alpha_K(M1)=0.01023(15)$ and $\alpha_K(E2)=0.01137(16)$. The spectrum showing that the 396 keV E0 transition is given in Figure 7.14. The measured conversion coefficients was 0.0175(14) for 396.6 keV transition. Assuming that the measured transition is a mixed E2+M1+E0, we have evaluated that there is minimum of $\alpha_k(E0)=0.0061(15)$ E0 transition for 396.6 keV line.

7.4.12 708.21(30) keV E0 Transition

The 708.21(30) keV transition is from 2250.6 keV 4+ intruder state to 1542.5 keV 2+ phonon state and has expected conversion coefficients of $\alpha_K(M1)=0.00255(4)$ and $\alpha_K(E2)=0.00225(4)$. The spectrum showing that the 708 keV E0 transition is given in Figure 7.15. The measured conversion coefficients was 0.00296(22) for 708.21 keV transition. Assuming that the measured transition is a mixed E2+M1+E0, we have evaluated that there is minimum of $\alpha_k(E0)=0.00041(22)$ E0 transition for 708.21 keV line.

7.4.13 E0 Transition Strength

Using the Equation 3.78, the E0 transition strength was evaluated for 708 keV line as following

$$\rho^2 = \frac{\alpha_K(E0)BR_{\gamma}}{\tau \Omega_K}, \quad (7.1)$$

The 708 keV transition is from 2250.6 keV level. $BR_{\gamma}=0.813(5)$ for the 2250.6 keV level is from $\gamma$-ray analysis. $\tau = 1080^{+450}_{-250}$ fs of the 2250.6 keV level is from $(n,n'\gamma)$
analysis. $\Omega_K = 2.723 \times 10^9$ for E0 708 keV transition is found from BRICC. We have the transition strength $\rho^2$ value of $0.115^{+0.077}_{-0.066}$ for 708 keV E0 transition.

### 7.5 Conclusions

The nuclear structure of $^{110}$Cd has been studied first time by internal conversion electron transitions through $e^- - \gamma$ coincidence method. This is the first extensive analysis of $e^- - \gamma$ coincidences done with the $8\pi$ for conversion $e^-$ transition over a wide energy range.

As part of this study, an absolute and a relative efficiencies for HPGe detectors were determined. The absolute efficiency for 20 HPGe detectors was defined as $1.231(13)\%$ at 1173.23 keV energy and $1.112(11)\%$ at 1332.49 keV in November 2008. The absolute efficiency for 18 HPGe was defined in different study as $0.808(8)\%$ at 1173.23 keV energy and $0.746(8)\%$ at 1332.49 keV in November 2010 [35]. The absolute efficiency for 20 HPGe was defined in another study as $1.013(30)\%$ at 1173.23 keV energy and $0.926(23)\%$ at 1332.49 keV in 2007 [36].

Using well-known E2 and E1, and assumed M1 transitions, internal efficiency calibration for Si(Li) detectors were made and used for getting absolute K-conversion coefficient of some other transitions.

The level scheme of $^{110}$Cd was built by conversion electron transitions through $e^- - \gamma$ coincidences following the $\beta^+$ decay of $^{110}$In. The level scheme was extended by several states with high spins but not that many with lower spins.

A new 113.71(4) keV transition from 3178.4 keV $(5^+)$ to 3064.6 keV $6^+$ levels was observed, along with 3 new $187.56(8)$, $195.04(4)$ and $244.66(2)$ transitions.

The sub-shell ratios were obtained for some mixed E2+M1 transitions of which K
and L shell conversion electrons measured. Unfortunately, many subshell ratios could not be obtained because of doublets, triplets and weakness of peaks. A first guess about the multipolarity of transitions were made from these sub-shell ratios.

Internal conversion coefficients were determined for 14 transitions and multipolarity mixing of 6 transitions were determined. This information is very useful to extract B(M1) and B(E2) values for transitions. Second guess about the multipolarity of transitions were made from those conversion coefficients.

An E0 transition was detected for two 397 keV and 708 keV transitions between intruder and phonon states. The transition strength, \( \rho^2(E0) \), of \( 0.115^{+0.077}_{-0.066} \) was determined for 708.21 keV line. Revealing E0 transitions between intruder and normal states, an assumption that the intruder states are from a more deformed shape of the nucleus was confirmed by this study.

Future studies should concentrate on the lower spin states to see a E0 transitions between 0\(^+\) states.
Bibliography


