University of Guelph Numeracy Project

About Asymptotes

UNIVERSITY of GUELPH

TABLE OF CONTENTS

About Asymptotes

What is a ASYMPTOTE?

- Asymptotes describe the end behaviour of functions. In "About Limits and Continuity," we discussed what happens when functions approach certain values. Now, we concern ourselves with the behaviour of functions as x goes on forever. We call this the "end behaviour" of a function. How does the function behave at large values of x?

- Are there lines which the function approaches but never touches? Yes, these are known as asymptotes. The idea behind these lines is to allow accurate representation of a function, using only a pencil and paper.

Limits at Infinity

- You might be wondering what infinity actually means. Notice that when you look at graphs, the function is cut off, despite our knowing that the function goes on for an eternity. This is simply due to the fact that we do not have enough space to show the entire function. The idea here is to know the "end" behaviour of a function. That is to say, what does the function look like off our graph? Put simply, we are taking the limit as "x approaches infinity."

- The notation is the same as limit notation learned previously, written as follows:

$$\lim_{x \to \infty} f(x)$$

is read as "the limit of f(x) as x approaches infinity"

Horizontal Asymptotes

- Referring back to the previous topic, the example that was given was approaching positive 2 as x went to both positive and negative infinity. You might be tempted to think that it will never reach 2, and you would be correct! The line y=2 is a horizontal asymptote, which the function approaches, but never reaches.

- We find horizontal asymptotes by taking the limit of a function as it approaches positive and negative infinity. This can be done quickly, now that we have the idea of a limit evaluated at infinity.

Checking for Horizontal Asymptotes

- When trying to find whether a function has horizontal asymptotes, three cases can be considered.
- Consider the function:

$$f(x) = \frac{a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots + a_1 x + a_0}{b_m x^m + b_{m-1} x^{m-1} + b_{m-2} x^{m-2} + \ldots + b_1 x + b_0}$$

- Case I: $n<m$
  Then, we get that $y=0$ is a horizontal asymptote.
- Case II: $n = m$
  Then, we have $y = a_n/b_m$ as our horizontal asymptote.
- Case III: $n > m$
  Then, no horizontal asymptotes exist.

Vertical Asymptotes

- A vertical asymptote is an asymptote that is parallel to the y axis.

- To check for vertical asymptotes, we can find when the denominator of the function is zero. If the denominator is never zero, then we will not have any vertical asymptotes.

- We say the line x=a is a vertical asymptote if any of the following are true:

1) $\lim_{x \to a^+} f(x) = \pm\infty$

2) $\lim_{x \to a^-} f(x) = \pm\infty$

3) $\lim_{x \to a} f(x) = \pm\infty$

Oblique Asymptote

- When a linear asymptote is not parallel to the x- or y- axes, we call it an oblique asymptote (secant).

- The function f(x) is said to be asymptotic to the line y = mx + b (where m is the slope of the line and b is the y-intercept), if one of the following is true:

$$\lim_{x \to \infty} (f(x) - (mx + b)) = 0$$

$$\lim_{x \to -\infty} (f(x) - (mx + b)) = 0$$

Glossary

Asymptote: The line (or curve) a function approaches as x grows.

Degree: The highest exponent in a function
(e.g. f(x) = x^3 + x^2 + x =1 has a degree of 3).

Denominator: The bottom portion of a rational function, or any fraction
(e.g. f(x) = (x+1)/x has a denominator of x).

Horizontal Asymptote: An asymptote that is parallel to the x-axis.

Infinity: Mathematics answer to "forever" and "very large." Infinity is meant to represent a number we may never reach.

Limit: When we approach a certain x value from the left and right, plugging the values into f(x), we denote this as "the limit of f(x) as x approaches 'a'."

Numerator: The top portion of a rational function, or any fraction (e.g. f(x) = (x+1)/x has a numerator of x+1).

Secant (Oblique) Asymptote: A linear asymptote, not parallel to the x- or y-axes.

Vertical Asymptote: An asymptote that is parallel to the y-axis.

References

Goshaw, (2008). Concepts of Calculus With Applications: Pearson Education

Schiavone, P (1997). Calculus Solutions: How to succeed in Calculus From essential prerequisites to Practice Examinations. Canada: Prentice-Canada Inc.

Weiner, J (2005). The Mathematics SurvivalKit. Canada: Thomson Canada Limited.