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About Mathematical Equations: Examples

A mathematical equation must contain the following three essential components:

1) An equal sign ( = )
2) Two or more variables (i.e., x or y)
3) One or more algebraic operations (i.e., addition, subtraction, multiplication, division)

For example:

Perimeter of a rectangle (P) = (2 x length (l)) + (2 x width (w))

\[ P = 2l + 2w \]

Rules for Rearranging Equations

Order of Operations

Evaluate \[ [(3 - 5) / (7 x 2)] + 2 \]

\[ [(3 - 9) / (3 x 2)] + 2 \] Evaluate the expressions within the brackets first

= \[ [(-6) / (6)] + 2 \] Evaluate the division

= -1 + 2 Evaluate the addition last

= 1
Evaluate $8 - 2 + 7 \times 3 - (2 \times 6)$

$8 - 2 + 7 \times 3 - (2 \times 6)$  Evaluate the expressions within the brackets first

$= 8 - 2 + 7 \times 3 - (12)$  Evaluate the multiplication

$= 8 - 2 + 21 - 12$  Evaluate signs of equal priority level from left-to-right

$= 6 + 21 - 12$

$= 27 - 12$

$= 15$

**Rearranging Equations**

Suppose you are given the equation: $y = 7x^3 - 14$ and you are asked to solve for $x$.

To solve for $x$, you need to isolate $x$ on one side of the equation. To do so, you need to perform the order of operations in reverse (i.e., as opposed to BEDMAS, it becomes SAMDEB). In addition, you need to apply the inverse operations to eliminate other variables and/or constants.

$y = 7x^3 - 14$  add 14 to both sides

$y + 14 = 7x^3 - 14 + 14$  simplify

$y + 14 = 7x^3$  divide by 7 on both sides

$(y + 14) / 7 = 7x^3 / 7$  simplify

$(y + 14) / 7 = x^3$  take the cube root of both sides

cube root $[(y + 14) / 7] = $ cube root $[x^3]$  simplify

cube root $[(y + 14) / 7] = x$
► Suppose you are given the formula for density: 
density \( D \) = mass \( M \) / volume \( V \), and you are asked to solve for volume \( V \)

1) Write the equation as given:

\[ D = \frac{M}{V} \]

2) To isolate for the volume \( V \), divide by the mass \( M \) on both sides of the equation. (The equation will still stand true because you divided both sides by the same variable). This will eliminate the mass \( M \) on the right-hand side.

\[ \frac{D}{M} = \frac{M}{(M \times V)} \quad \text{Simplify} \]
\[ \frac{D}{M} = \frac{1}{V} \quad \text{Take the inverse of both sides} \]
\[ \frac{M}{D} = V \quad \text{The equation will now be rearranged as required} \]
\[ \frac{M}{D} = V \quad \text{or} \quad V = \frac{M}{D} \]

**Chemistry Applications**

► Given that a 0.805 mol sample of a gas has a temperature of 300 K, a volume of 22.7 L, and a pressure of 0.872 atm, rewrite the ideal gas law in terms of these values. The constant \( R \) has a value of 0.08206 L atm / mol K

\[ PV = nRT \]

\[ (0.872 \text{ atm}) (22.7 \text{ L}) = (0.805 \text{ mol}) (0.08206 \text{ L atm / mol K}) (300 \text{ K}) \]

Looking at this equation, you will notice that some of the units cancel out

\[ (0.872 \text{ atm}) (22.7 \text{ L}) = (0.805 \text{ mol}) (0.08206 \text{ L atm / mol K}) (300 \text{ K}) \]

Since the units “L” and “atm” are in the numerator on both sides of the equation, they cancel out algebraically as well
\[(0.872 \text{ atm}) (22.7 \text{ L}) = (0.805) (0.08206 \text{ L atm}) (300)\]

\[(0.872) (22.7) = (0.805) (0.08206) (300)\]

After evaluating the expressions on both sides of the equation, we are left with a consistent equation where the left side equals the right side:

\[19.8 = 19.8\]

As shown through this example, the units of each type of quantity (i.e., volume, pressure, temperature, and amount of moles) are all consistent with each other. If you happen to be given values where the units do not match with the other variables given, you must convert the units so that they cancel in the end. For instance, if the volume was given in mL instead of L, we would then have to convert the volume into L to match with the molar gas constant’s units. (Refer to the learning object on “Unit Conversions” for more detail.)

*Using Formulas to Solve for Unknown Variables*

- Consider the following formula:
  \[\Delta G = \Delta G^\circ + RT \ln Q\]
  \[\Delta G = \text{Gibbs free energy at any condition}\]
  \[\Delta G^\circ = \text{Gibbs free energy at standard conditions}\]
  \[Q = \text{reaction quotient}\]
  \[R = \text{molar gas constant}\]
  \[T = \text{temperature}\]

Solving for \(\Delta G\) is simple; but solving for \(Q\) is a bit more challenging. Let’s look at how we can rearrange this formula so that \(Q\) is isolated on one side of the equation.
\[ \Delta G = \Delta G^\circ + RT \ln Q \]

subtract both sides by \( \Delta G^\circ \)

\[ \Delta G - \Delta G^\circ = \Delta G^\circ + RT \ln Q - \Delta G^\circ \]

simplify

\[ \Delta G - \Delta G^\circ = RT \ln Q \]

divide both sides by RT

\[ \frac{\Delta G - \Delta G^\circ}{RT} = \frac{RT \ln Q}{RT} \]

simplify

\[ \frac{\Delta G - \Delta G^\circ}{RT} = \ln Q \]

inverse operation of \( \ln \) is \( e \)

take \( e \) of both sides

\[ e \left\{ \frac{\Delta G - \Delta G^\circ}{RT} \right\} = e \{ \ln Q \} \]

simplify (note: \( e^{\ln X} = X \))

\[ e \left\{ \frac{\Delta G - \Delta G^\circ}{RT} \right\} = Q \]

On your own, make sure that the units are consistent in all parts of the formula.