University of Guelph Numeracy Project

About CRF Design: Examples

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Example

► As part of a "year in review," the shooting accuracy of 4 basketball teams in the NBA were measured at the 32-game mark, the 57-game mark, and the 82-game mark. The sports analysts wanted to know if there was a difference in the accuracy achieved by the different teams. They randomly sampled 15 players on each team and examined their accuracy. Their findings are found in the table below:

	Point in Season			
Team	**32-Games**	**57-Games**	**82-Games**	**Mean**
1	53	68	72	66.7
	58	58	68	
	46	68	68	
	46	58	72	
	83	90	92	
2	48	92	49	66.5
	46	68	72	
	90	76	52	
	93	46	90	
	52	48	76	
3	48	60	70	64.7
	45	56	84	
	54	58	75	
	54	57	98	
	86	50	76	
4	86	92	74	62.8
	50	45	54	
	84	44	76	
	55	62	58	
	57	56	49	
Mean	61.7	62.6	71.3	

STEP 1: State the hypotheses

STEP 2: Calculate the variance

PART A: Calculate the Total variance

► It is easier to conceptualize this if we put the data in a table:

x_1	x_1^2	x_2	x_2^2	x_3	x_3^2	x_4	x_4^2	x_5	x_5^2	x_6	x_6^2
53	2809	68	4624	72	5184	48	2304	92	8464	49	2401
58	3364	58	3364	68	4624	46	2116	68	4624	72	5184
46	2116	68	4624	68	4624	90	8100	76	5776	52	2704
46	2116	58	3364	72	5184	93	8649	46	2116	90	8100
83	6889	90	8100	92	8464	52	2704	48	2304	76	5776
286	17294	342	24076	372	28080	329	23873	330	23284	339	24165

x_7	x_7^2	x_8	x_8^2	x_9	x_9^2	x_{10}	x_{10}^2	x_{11}	x_{11}^2	x_{12}	x_{12}^2
48	2304	60	3600	70	4900	86	7396	92	8464	74	5476
45	2025	56	3136	84	7056	50	2500	45	2025	54	2916
54	2916	58	3364	75	5625	84	7056	44	1936	76	5776
54	2916	57	3249	98	9406	55	3025	62	3844	58	3364
86	7396	50	2500	76	5776	57	3249	56	3136	49	2401
287	17557	281	15849	403	32763	332	23226	299	19405	311	19933

$$SS_{tot} = \Sigma X^2_{tot} - \frac{(\Sigma X_{tot})^2}{N}$$

$$SS_{tot} = 269505 - \frac{(3911)^2}{60}$$

$$SS_{tot} = 269505 - \frac{15295921}{60}$$

$$SS_{tot} = 269505 - 254932.02$$

$$SS_{tot} = 14572.98$$

PART B: Calculate the between-groups variance

$SS_{BG} = [(\Sigma X_1)^2 / n_1 + (\Sigma X_2)^2 / n_2 + (\Sigma X_3)^2 / n_3 + (\Sigma X_4)^2 / n_4 + (\Sigma X_5)^2 / n_5 + (\Sigma X_6)^2 / n_6 + (\Sigma X_7)^2 / n_7 + (\Sigma X_8)^2 / n_8 + (\Sigma X_9)^2 / n_9 + (\Sigma X_{10})^2 / n_{10} + (\Sigma X_{11})^2 / n_{11} + (\Sigma X_{12})^2 / n_{12}] - (\Sigma X_{tot})^2 / N$

$SS_{BG} = [(286)^2 /5 + (342)^2 /5 + (372)^2 /5 + (329)^2 /5 + (330)^2 /5 + (339)^2 /5 + (287)^2 /5 + (281)^2 /5 + (403)^2 /5 + (332)^2 /5 + (299)^2 /5 + (311)^2 /5] - (3911)^2 /60$

$SS_{BG} = [81796/ 5 + 116964/ 5 + 138384/ 5 + 108241/ 5 + 108900/ 5 + 114921/ 5 + 82369/ 5 + 78961/ 5 + 162409/ 5 + 110224/ 5 + 89401/ 5 + 96721/ 5] - 15295921/60$

$SS_{BG} = [16359.2 + 23392.8 + 27676.8 + 21648.2 + 21780 + 22984.2 + 16473.8 + 15792.2 + 32481.8 + 22044.8 + 17880.2 + 19344.2] – 254932.0$

$SS_{BG} = 257858.2 – 254932.0$

$SS_{BG} = 2926.2$

PART C: Calculate the within-groups variance

$SS_{WG} = SS_{tot} – SS_{BG}$

$SS_{WG} = 14572.98 - 2926.2$

$SS_{WG} = 11646.78$

STEP 3: Calculate the Mean square values

PART A: Calculate the within-groups mean square value

$$MS_{WG} = \frac{SS_{WG}}{N-k}$$

$$MS_{WG} = \frac{11646.78}{60-12}$$

$$MS_{WG} = \frac{11646.78}{48}$$

$$MS_{WG} = 242.64$$

PART B: Calculate the between-groups mean square value

This is where the calculations in the CRF design differ from the one-way ANOVA design, as we have to separate the between-groups variance into 3 parts (test, section, text x section).

The mean square value for Section is:

$$SS_{sec} = \frac{(\Sigma\ level_1)^2}{n_1} + \frac{(\Sigma\ level_2)^2}{n_2} + \frac{(\Sigma\ level_3)^2}{n_3} + \frac{(\Sigma\ level_4)^2}{n_4} - \frac{(\Sigma X_{tot})^2}{N}$$

$$SS_{sec} = \frac{(1000)^2}{15} + \frac{(998)^2}{15} + \frac{(971)^2}{15} + \frac{(942)^2}{15} - \frac{(3911)^2}{60}$$

$$SS_{sec} = \frac{1000000}{15} + \frac{996004}{15} + \frac{942841}{15} + \frac{887364}{15} - \frac{15295921}{60}$$

$$SS_{sec} = 66666.67 + 66400.27 + 62856.07 + 59157.6 - 254932.02$$

$$SS_{sec} = 148.59$$

$$MS_{sec} = \frac{SS_{sec}}{k-1}$$

$$MS_{sec} = \frac{148.59}{4-1}$$

$$MS_{sec} = \frac{148.59}{3}$$

$$MS_{sec} = 49.53$$

The mean square value for Test is:

$$SS_{test} = \frac{(\Sigma\ level_1)^2}{n_1} + \frac{(\Sigma\ level_2)^2}{n_2} + \frac{(\Sigma\ level_3)^2}{n_3} - \frac{(\Sigma\ X_{tot})^2}{N}$$

$$SS_{test} = \frac{(1234)^2}{20} + \frac{(1252)^2}{20} + \frac{(1425)^2}{20} - \frac{(3911)^2}{60}$$

$$SS_{test} = \frac{1522756}{20} + \frac{1567504}{20} + \frac{2030625}{20} - \frac{15295921}{60}$$

$$SS_{test} = 76137.8 + 78375.2 + 101531.25 - 254932.02$$

$$SS_{test} = 1112.23$$

$$MS_{test} = \frac{SS_{test}}{k\text{-}1}$$

$$MS_{test} = \frac{1112.23}{3\text{-}1}$$

$$MS_{test} = \frac{1112.23}{2}$$

$$MS_{test} = 556.12$$

The mean square value for the interaction of Section and Test is:

$$SS_{sec\ x\ test} = SS_{BG} - SS_{sec} - SS_{test}$$

$$SS_{sec\ x\ test} = 2926.2 - 148.59 - 1112.23$$

$$SS_{sec\ x\ test} = 1665.38$$

$$MS_{sec\ x\ test} = \frac{SS_{sec\ x\ test}}{(df_{sec})(df_{test})}$$

$$MS_{sec\ x\ test} = \frac{1665.38}{(4\text{-}1)(3\text{-}1)}$$

$$MS_{sec\ x\ test} = \frac{1665.38}{(3)(2)}$$

$$MS_{sec\ x\ test} = \frac{1665.38}{6}$$

$$MS_{sec\ x\ test} = 277.56$$

STEP 4: Calculate the F-ratios

There are 3 F-ratios that need to be calculated (Section, Test and Section x Test).

PART A: Calculate the F-ratio for the main effect of Section

$$F_{sec} = \frac{MS_{sec}}{MS_{WG}}$$

$$F_{sec} = \frac{49.53}{242.64}$$

$$F_{sec} = 0.20$$

The F_{crit} is 3.17. Therefore, there is not a significant main effect of Section.

PART B: Calculate the F-ratio for the main effect of Test

$$F_{test} = \frac{MS_{test}}{MS_{WG}}$$

$$F_{test} = \frac{556.12}{242.64}$$

$$F_{test} = 2.29$$

The F_{crit} is 2.78. Therefore, there is not a significant main effect of Test.

PART C: Calculate the F-ratio for the interaction effect of Section x Test

$$F_{sec\ x\ test} = \frac{MS_{sec\ x\ test}}{MS_{WG}}$$

$$F_{sec\ x\ test} = \frac{277.56}{242.64}$$

$$F_{sec\ x\ test} = 1.14$$

The F_{crit} is 2.27. Therefore, there is not a significant interaction effect of Section and Test.

STEP 5: Conduct Post hoc tests

- Due to the lack of significant results, post hoc tests are not necessary.