University of Guelph
Numeracy Project

About CRF Design

UNIVERSITY of GUELPH

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About CRF Design

What is a COMPLETELY RANDOMIZED FACTORIAL?

- A completely randomized factorial design (CRF) is a variation of ANOVA, where there is one dependent variable and two or more factors (independent variables), with 2 or more levels in each factor.

CRF

Completely Randomized Factorial

- There are times when a researcher wants to examine the effect of more than one variable on a dependent variable.

  The CRF design allows researchers to investigate the effects of 2 or more factors and to assess any interactions between them.

  This is one of the main differences that make conducting a CRF different than conducting 2 or more one-way ANOVAs. If you conduct two or more one-way ANOVAs, you cannot examine the interaction of the factors. If an interaction is found, the main effects of each single factor are rendered less important.

- It is easiest to conceptualize the main effects and interactions by making a table:

	FACTOR 2 (IV_2)	
FACTOR 1 (IV_1)	Level 1	Level 2
Level 1		
Level 2		
Level 3		

Each box in the table is called a cell.

Effects

- There are two effects that are being examined:

  1) Main effects
  2) Interactions

  Main Effects

- This is the single effect of a factor on the dependent variable, while ignoring the other factor(s).

- A main effect is calculated for each factor.

- Main effects can be misleading.

  Interactions

- This is the combined effect of the factors on the dependent variable.

- An interaction indicates that the means do not act consistently across Factor2 for every level of Factor1 (or vice versa), where a significant interaction renders significant main effects less important.

- For example, in a CRF design with two factors, you can examine the main effect of Factor1, the main effect of Factor2, and the combined effect of Factor1 and Factor2.

Assumptions

- There are 3 assumptions associated with calculating a CRF analysis:

  1) Random Sampling
  2) Normality
  3) Homogeneity of Variances

Independent Random Sampling

- This means that the participants were randomly sampled from the populations and were randomly assigned to one of the groups.

  In other words, each participant had an equal chance of belonging to any of the groups.

- Sometimes random sampling is not possible. This assumption can be violated if the sample size is large.

Normality

- The populations from which the samples were drawn were normally distributed.

- This assumption can also be violated if the sample size is large enough.

Homogeneity of variances

- This means that the populations from which the samples are drawn have equal variances.

- This assumption cannot be violated.

- Homogeneity of variance assumption is tested with Levene's test of homogeneity of variances.

  If Levene's test is non-significant, meaning that the variances are the same, then further analyses can be made.

Calculating a CRF

Step 1: State the Hypotheses

- The null hypothesis is H_0: $\mu_1 = \mu_2 = \mu_3$

  This means that all the population means are the same. The number of population means that are used corresponds to the number of levels in the independent variable.

- The alternate hypothesis is: H_1: at least one of the population means is different.

  The alternate hypothesis cannot be written as H_1: $\mu_1 \neq \mu_2 \neq \mu_3$

  This means that population mean 1 is not the same as population mean 2, which is not the same as population mean 3. This is very restrictive. It is much easier to write out the alternate hypothesis in words, rather than symbols.

Step 2: Calculate the Variance

Part A: Calculate the variability for the entire data set

- Variability is calculated using the Sum of Squares (SS):

$$SStot = \Sigma X^2_{tot} - \frac{(\Sigma X_{tot})^2}{N}$$

  Where N is the total sample size (i.e. the combined sample size from each level).

- Because this is the total variance for the experiment, neither the between-groups variance nor the within-groups variance can be larger than this value.

- We want the between-groups variance to account for a greater proportion of the total variance than the within-groups variance, as this means that the IV had an effect on the DV.

Part B: Calculate the Between-groups Variability

- The between-groups variability contains the effect variability and some error.

  The effect variability tells us if there is an effect of the IV on the DV.

  The error is the result of individual differences.

- Between-groups variability is calculated using Sum of Squares (SS):

$$SS_{BG} = \left[\frac{(\Sigma X_1)^2}{n_1} + \frac{(\Sigma X_2)^2}{n_2} + \frac{(\Sigma X_3)^2}{n_3} + \ldots + \frac{(\Sigma X_x)^2}{n_x}\right] - \frac{(\Sigma X_{tot})^2}{N}$$

The number of $(\Sigma X)^2/n$ in the equation depends on the number of cells.

Part C: Calculate the Within-groups Variability

- The within-groups variability is a pure measure of error.

- Within-groups variability is calculated by:

$$SS_{WG} = SS_{tot} - SS_{BG}$$

Step 3: Calculate the Mean Square Values

PART A: Calculate the Within-groups Mean Square Value

- This is a pure error term.

It is calculated with:

$$MS_{WG} = \frac{SS_{WG}}{N-k}$$

Where N is the total sample size (i.e. the combined sample size from each level) and k is the number of levels.

PART B: Calculate the Between-groups Mean Square Value

- The between-groups variability needs to be divided into parts. The number of parts depends on the number of factors. This is where the calculation of the CRF differs from the calculation of the one-way ANOVA.

- $$SS_{IV1} = \left[\frac{(\Sigma\ level_1)}{n_1} + \frac{(\Sigma\ level_2)}{n_2}\right]$$
$$SS_{IV2} = \left[\frac{(\Sigma\ level_1)}{n_1} + \frac{(\Sigma\ level_2)}{n_2} + \frac{(\Sigma\ level_3)}{n_3}\right]$$

- The next step is to calculate the Mean Square value for each of the factors:

$$MS_{IV1} = \frac{SS_{IV1}}{k_1-1}$$
$$MS_{IV2} = \frac{SS_{IV2}}{k_2-1}$$

Where k_1 is the number of levels in IV_1 and k_2 is the number of levels in IV_2.

PART C: Calculate the Interaction Mean Square Value

- This is the combined variability of the two factors.

  $SS_{IV1 \times IV2} = SS_{BG} – SS_{IV1} – SS_{IV2}$

- The Mean Square value is found by dividing the value $SS_{IV1 \times IV2}$ by the degrees of freedom for IV_1 and IV_2:

  $df_{IV1 \times IV2} = (df_{IV1})(df_{IV2}) = (k_1–1)(k_2-1)$

STEP 4: Calculate the F-ratio

- The number of F-ratios depends on the number of between-groups measures (i.e. IV_1, IV_2, $IV_1 x IV_2$). It is calculated with:

  $F = \frac{MS_{IV1}}{MS_{WG}}$ Main effect of IV_1

  $F = \frac{MS_{IV2}}{MS_{WG}}$ Main effect of IV_2

  $F = \frac{MS_{IV1xIV2}}{MS_{WG}}$ Interaction effect of IV_1 and IV_2

  These are the obtained F terms and need to be compared to the critical F term, in order to determine if the factor had a significant effect on the DV. The critical F value is found in a table.

Post hoc Testing

- Post hoc tests are assessments that are performed after a significant result has been found.

  A post hoc test determines which levels are significantly different. Because of this, post hoc testing is only done when there are more than two levels.

- Post hoc testing for CRF designs is done in the same way as for one-way ANOVAs.

  If there is a significant interaction, it is not important to perform post hoc tests for any significant main effects.

  If there is no significant interaction, post hoc tests should be done for any significant main effects.

- For interactions, a decision needs to be made as to how to compare them. We could examine the changes in IV_1 over the levels of IV_2, or vice versa.

  The option that is chosen usually depends on how the research question is measured. Sometimes it makes more sense to compare the effects one way, rather than the other.

- There are two types of post hoc tests that could be done. The type of post hoc test that is chosen depends on if there is an equal sample size at each level of the factor, or if the sample sizes differ.

- The test that is performed when there is an equal sample size is called Tukey's Honestly Significant Differences (HSD).

  HSD signifies that the means being compared are truly different from one another.

Tukey's HSD

Step 1: State the Hypotheses

- We have to state null and alternate hypotheses for each of the comparisons.

H_0: $\mu_1 = \mu_2$; H_1: $\mu_1 \neq \mu_2$
H_0: $\mu_1 = \mu_3$; H_1: $\mu_1 \neq \mu_3$
H_0: $\mu_2 = \mu_3$; H_1: $\mu_2 \neq \mu_3$

NOTE: because we are only focusing on 2 population means at a time, we can use symbols to write that one mean is not equal to another mean.

Step 2: Calculate the q value

- The formula for calculating q-values is:

$$q = \frac{X_1 - X_2}{MS_{error} / n}$$

Where X_1 - X_2 is the mean difference, MS_{error} is the error term and n is the sample size.

- The q-value is calculated for each pair of means in each level.

The error term that is chosen depends on the effect being examined. For interaction and within-groups main effects, the within-groups MS_{error} is chosen. For between-groups main effects, the between-groups MS_{error} is chosen.

Step 3: Compare q_{obt} to q_{crit}

- The obtained q-values are compared to the critical q-value to determine which levels are significantly different.

  If the obtained q-value is larger than the critical q-value, then there is a significant difference.

- The critical q-value is found in a statistical table.

Fisher's protected t-test

- The test that is performed when there are unequal sample sizes is called Fisher's protected t-test.
- This test is called protected, as it is designed to remain conservative when looking for significant differences between pairs of means.

- The equation for calculating Fisher's protected t-test is:

$$t = \frac{X_1 - X_2}{MS_{WG}\,(1/n_1 + 1/n_2)}$$

  This formula changes slightly for every pair of means. For example, you would use X_1 and X_3 and n_1 and n_3 to find the mean differences between level 1 and level 3.

- The t values are compared to the means of each level. If the means are greater than the critical t-value, then the relationship between the levels is significant.

Glossary

ANOVA: Analysis of variance tests are a method for testing hypotheses when there are more than two levels of the independent variable.

Between-groups: a design that uses a separate sample of individuals for each treatment condition (level).

Dependent variable: a response variable that changes when exposed to the independent variable.

Factor (Independent variable): a variable that is manipulated to show an effect on the dependent variable.

Fisher's protected t-test: a type of post hoc test that is used when there are unequal sample sizes.

Interaction effect: the effect of the factors, in combination, on the dependent variable.

Main effect: the effect of a single factor on the dependent variable.

Post hoc testing: a type of test that is only performed after a significant result has been found and when there are more than two levels in the independent variable. It determines which levels are significantly different.

Tukey's HSD: a type of post hoc test that is used when there are equal sample sizes.

Within-groups: a design that uses the same sample of individuals for each treatment condition (level).

Variability: measures how observed values in the data set are distributed across a variety of categories.

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