University of Guelph Numeracy Project

About Single Factor ANOVAs: Examples

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About Single Factor ANOVAs: Examples

Example

► Your friend has a habit of growing a multi-purpose beard (i.e. exam beard, playoff beard, etc.), which only tends to come off during the summer months. You try to explain to him that not only is the beard unappealing, but it adds years to his age. He claims the beard is not unappealing, in fact, he considers it to be his wingman. As such, you determine that a study is in order. To conduct this study, you ask some of your other friends to rate the bearded wonder's looks on a scale of 1 to 5, where 1 indicates good-looking and 5 indicates caveman. You start the study near the end of the summer, just before he stops shaving, and continue for 6 weeks. You ask 10 different friends to take a gander on week 1, week 4 and week 6. Results were:

Week		
Week 1	**Week 4**	**Week 6**
1	2	4
2	2	2
2	3	4
1	4	5
3	5	5
2	2	3
1	3	5
2	2	3
2	3	4
1	1	2

STEP 1: State the hypotheses

► The null hypothesis is: H_0: $\mu_1 = \mu_2 = \mu_3$

► The alternate hypothesis is: H_1: at least one of the population means is different.

STEP 2: Calculate the variance

PART A: Calculate the total variance

► $$SS_{tot} = \Sigma X^2_{tot} - \frac{(\Sigma X_{tot})^2}{N}$$

It is easier to conceptualize this if we put the data in a table:

X_1	X_1^2	X_2	X_2^2	X_3	X_3^2
Week 1		**Week 4**		**Week 6**	
1	1	2	4	4	16
2	4	2	4	2	4
2	4	3	9	4	16
1	1	4	16	5	25
3	9	5	25	5	25
2	4	2	4	3	9
1	1	3	9	5	25
2	4	2	4	3	9
2	4	3	9	4	16
1	1	1	1	2	4
17	**33**	**27**	**85**	**37**	**149**

► $SS_{tot} = \Sigma X^2_{tot} - \frac{(\Sigma X_{tot})^2}{N}$

$SS_{tot} = (33 + 85 + 149) - \frac{(17 + 27 + 37)^2}{30}$

$SS_{tot} = 267 - \frac{(81)^2}{30}$

$SS_{tot} = 267 - \frac{6561}{30}$

$SS_{tot} = 267 - 218.70$

$SS_{tot} = 48.30$

PART B: Calculate the between groups variance

► $SS_{BG} = \left[\frac{(\Sigma X_1)^2}{n_1} + \frac{(\Sigma X_2)^2}{n_2} + \frac{(\Sigma X_3)^2}{n_3}\right] - \frac{(\Sigma Xtot)^2}{N}$

$SS_{BG} = \left[\frac{(17)^2}{10} + \frac{(27)^2}{10} + \frac{(37)^2}{10}\right] - \frac{(81)^2}{30}$

$SS_{BG} = \left[\frac{289}{10} + \frac{729}{10} + \frac{1369}{10}\right] - \frac{6561}{30}$

$SS_{BG} = 28.90 + 72.90 + 136.90 - 218.70$

$SS_{BG} = 20.00$

PART C: Calculate the within groups variance

► $SS_{WG} = SS_{tot} - SS_{BG}$

$SS_{WG} = 48.30 - 20.00$

$SS_{WG} = 28.30$

Step 3: Calculate the Mean square values

Part A: Calculate the between-groups Mean Square value

► $MS_{BG} = \frac{SS_{BG}}{k-1}$

$MS_{BG} = \frac{20.00}{3-1}$

$MS_{BG} = \frac{20.00}{2}$

$MS_{BG} = 10.00$

PART B: Calculate the within-groups Mean Square value

► $MS_{WG} = \frac{SS_{WG}}{N-k}$

$MS_{WG} = \frac{28.30}{30-3}$

$MS_{WG} = \frac{28.30}{27}$

$MS_{WG} = 1.05$

STEP 4: Calculate the F-ratio

► $F = \frac{MS_{BG}}{MS_{WG}}$

$F = \frac{10.00}{1.05}$

$F = 9.52$

Our obtained F value is $F(2, 27) = 9.52$. We need to compare this value to the critical F value, which is found in a table.

► The critical F value for an alpha level of $\alpha = 0.05$, $df_{BG} = 2$ and $df_{WG} = 27$ is 3.35. The obtained F value is greater than the critical F value; therefore, we can conclude that week had a significant effect on looks.

STEP 5: Calculate the effect size

► We found a significant result, but we don't know just how different that result is than what would be expected by chance. To determine this, we need to calculate the effect size.

$$\eta^2 = \frac{SS_{BG}}{SS_{tot}}$$

$$\eta^2 = \frac{20.00}{48.30}$$

$$\eta^2 = 0.41$$

The effect size is 0.41. This is a medium effect size.

Therefore, we can conclude that the between-groups variance (the days that looks are judged) accounts for 41% of the total variance in looks.

STEP 6: Post hoc testing

► Because we found a significant effect of week on looks, we now have to conduct a post hoc test to determine which of the levels are significantly different.

► We are going to conduct Tukey's HSD because there is an equal sample size among the levels.

The null and alternate hypotheses are:

H_0: $\mu_1 = \mu_2$; H_1: $\mu_1 \neq \mu_2$
H_0: $\mu_1 = \mu_3$; H_1: $\mu_1 \neq \mu_3$
H_0: $\mu_2 = \mu_3$; H_1: $\mu_2 \neq \mu_3$

► The first step to conducting Tukey's is to calculate the q-value:

$$q = \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{MS_{WG} / n}} \qquad q = \frac{\overline{X}_1 - \overline{X}_3}{\sqrt{MS_{WG} / n}} \qquad q = \frac{\overline{X}_2 - \overline{X}_3}{\sqrt{MS_{WG} / n}}$$

$$q = \frac{1.7 - 2.7}{\sqrt{1.05 / 10}} \qquad q = \frac{1.7 - 3.7}{\sqrt{1.05 / 10}} \qquad q = \frac{2.7 - 3.7}{\sqrt{1.05 / 10}}$$

$$q = \frac{1.0}{\sqrt{0.105}} \qquad q = \frac{2.0}{\sqrt{0.105}} \qquad q = \frac{1.0}{\sqrt{0.105}}$$

$$q = \frac{1.0}{0.324} \qquad q = \frac{2.0}{0.324} \qquad q = \frac{1.0}{0.324}$$

$$q = 3.09 \qquad q = 6.17 \qquad q = 3.09$$

The next step is to compare the obtained q-values to the critical q value, $q_{crit} = 3.49$.

We can conclude that your friend looks significantly worse on Week 4 in comparison to Week 1.