University of Guelph Numeracy Project

About Single Factor ANOVAs

UNIVERSITY of GUELPH

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About Single Factor ANOVAs

What is a SINGLE FACTOR ANOVA

- ANOVA stands for analysis of variance. A single-factor (or one way) ANOVA is a flexible method for testing hypotheses about means when there is one independent variable (IV), with two or more levels, and one dependent variable (DV).

Single Factor ANOVA

- A single-factor ANOVA is similar to a t-test; however, unlike t-tests, it can have more than 2 levels in the independent variable.

  Limiting a study to only 2 levels is not always the best choice because, often, there are 3 or more levels that need to be examined in order to understand the effect a factor has on a dependent variable.

  Having more levels of the factor allows you to account for more variability in that variable.

- There are two types of variance that are important in calculating a single-factor ANOVA.

  The first is between-groups variance, in which participants in one level are treated independently of the participants in another.

  The second is within-groups variance, in which participants are exposed to every level of the factor.

Calculating Single Factor ANOVAs

- There are 5 steps involved in calculating a single factor ANOVA:

    1) State hypotheses
    2) Calculate Variance
        a) Total variance
        b) Between-groups
        c) Within-groups
    3) Calculate Mean Squares
    4) Calculate F-ratio
    5) Calculate effect size

STEP 1: State the hypotheses

- The null hypothesis is: H_0: $\mu_1 = \mu_2 = \mu_3$

    This means that all the population means are the same. The number of population means that are used corresponds to the number of levels in the independent variable.

- The alternate hypothesis is: H_1: at least one of the population means is different.

- The alternate hypothesis cannot be written as H_1: $\mu_1 \neq \mu_2 \neq \mu_3$

    This means that population mean 1 is not the same as population mean 2, which is not the same as population mean 3. This is very restrictive. It is much easier to write out the alternate hypothesis in words, rather than symbols.

Step 2: Calculate Variance

Total Variance

- Calculate the variability for the entire data set.

- Variability is calculated using the Sum of Squares (SS):

$$SS_{tot} = \Sigma X^2_{tot} - \frac{(\Sigma X_{tot})^2}{N}$$

Where N is the total sample size (i.e. the combined sample size from each level).

- Because this is the total variance for the experiment, neither the between-groups variance nor the within groups variance can be larger than this value.

  Ultimately, we want the between-groups variance to account for a greater proportion of the total variance than the within-groups variance, because this means that the IV had an effect on the DV.

Between-groups Variance

- Calculate the between-groups variability.

- The between-groups variability contains the effect variability and some error.

  The effect variability tells us whether there is an effect of the IV on the DV.

  The error is the result of individual differences.

- Between-groups variability is calculated using Sum of Squares (SS):

$$SS_{BG} = \left[\frac{(\Sigma X_1)^2}{n_1} + \frac{(\Sigma X_2)^2}{n_2} + \frac{(\Sigma X_3)^2}{n_3} + \ldots + \frac{(\Sigma X_x)^2}{n_x}\right] - \frac{(\Sigma X_{tot})^2}{N}$$

Within-groups Variance

- Calculate the within-groups variability.
- The within-groups variability is a pure measure of error.

Within-groups variability is calculated by:

$SS_{WG} = SS_{tot} - SS_{BG}$

Step 3: Calculate Mean Squares

- Mean squares estimate the population variance on the basis of variability of a given set of measures.

- There are two mean square values that need to be calculated: the between-groups mean square and the within-groups mean square.

PART A: Calculate the between-groups mean square value

- The between-groups mean square value represents the variability between the levels of the factor.

- It is calculated with:

$$MS_{BG} = \frac{SS_{BG}}{k-1}$$

Where k is the number of levels.

PART B: Calculate the within-groups mean square value

- This is a pure error term.

- It is calculated with:

$$MS_{WG} = \frac{SS_{WG}}{N-k}$$

Where N is the total sample size (i.e. the combined sample size from each level) and k is the number of levels.

STEP 4: Calculate the F-ratio

- The F-ratio is the value of the MS_{BG} divided by the MS_{WG}. In other words, the F-ratio is the variability between the levels of the factor divided by the error.

- It is calculated with:

$$F = \frac{MS_{BG}}{MS_{WG}}$$

This is the obtained F term, and it needs to be compared with the critical F term, in order to determine if the factor had a significant effect on the DV. The critical F value is found in a table.

STEP 5: Calculate the effect size

- If we find a significant result, we need to calculate the effect size. This tells us how significant a significant result is. Where the larger the effect size, the greater the significance.

- In single-factor ANOVAs, the effect size is calculated using η^2 (eta squared).

$$\eta^2 = \frac{SS_{BG}}{SS_{tot}}$$

This shows the proportion of the total variance that is accounted for by the between-groups variance.

- The values for η^2 range from 0-1:

A small effect: 0.1
A medium effect: 0.4
A large effect: 0.8

Post hoc Testing

- A post hoc test is done after a significant result has been found. It is not a test that is planned for at the beginning of the study, but rather it is a reaction to a significant result.

- A post hoc test determines which levels are significantly different. Because of this, post hoc testing is only done when there are more than two levels.

- There are two types of post hoc tests that could be done. The type of post hoc test that is chosen depends on if there is an equal sample size at each level of the IV, or if the sample sizes differ.

Tukey's Honestly Significant Differences (HSD)

- Tukey's is performed when there is an equal sample size. It is represented by the letter q.

- HSD signifies that the means being compared are truly different from one another.

STEP 1: State the hypotheses

- We have to state null and alternate hypotheses for each of the comparisons:

H_0: $\mu_1 = \mu_2$; H_1: $\mu_1 \neq \mu_2$
H_0: $\mu_1 = \mu_3$; H_1: $\mu_1 \neq \mu_3$
H_0: $\mu_2 = \mu_3$; H_1: $\mu_2 \neq \mu_3$

NOTE: because we are only focusing on 2 population means at a time, we can use symbols to write that one mean is not equal to another mean.

STEP 2: Calculate the q value

- The formula for calculating q-values is:

$$q = \frac{X_1 - X_2}{\sqrt{MS_{WG}/n}}$$

Where X_1 - X_2 is the mean difference, MS_{error} is the error term and n is the sample size.

- The q-value is calculated for each pair of means in each level.

- The error term that is chosen depends on the effect being examined. For interaction and within-groups main effects, the within-groups MS_{error} is chosen. For between-groups main effects, the between-groups MS_{error} is chosen.

STEP 3: Compare q_{obt} to q_{crit}

- The obtained q-values are compared to the critical q-value, to determine which levels are significantly different.

  If the obtained q-value is larger than the critical q-value, then there is a significant difference.

- The critical q-value is found in a statistical table.

Fisher's protected t-test

- This post hoc test is performed when there are unequal sample sizes.
- This test is called protected, because it is designed to remain conservative when looking for significant differences between pairs of means.

  The equation for calculating Fisher's protected t-test is:

$$t = \frac{X_1 - X_2}{\sqrt{MS_{WG}\,(1/n_1 + 1/n_2)}}$$

  This formula changes slightly for every pair of means. For example, you would use X_1 and X_3 and n_1 and n_3 to find the mean differences between level 1 and level 3.

- The t values are compared to the means of each level. If the means are greater than the critical t-value, then the relationship between the levels is significant.

Glossary

Between-groups:	a design that uses different groups of subjects for each treatment condition.
Dependent variable:	a variable that changes in response to the independent variable.
Effect size:	measures how significant a significant result is.
Factor (independent variable):	a variable that is manipulated to show an effect on the dependent variable.
Fisher's protected t-test:	a type of post hoc test that is used when there are unequal sample sizes.
Post-hoc testing:	a type of test that is performed only after a significant effect has been found. When there are more than two levels in the factor, it determines which levels are significantly different.
t-test:	used to test hypotheses about a population when the standard deviation of that population is unknown.
Tukeys' HSD:	a type of post hoc test that is used when there are equal sample sizes. HSD stands for "Honestly significant differences" and signifies that the means being compared are truly different.

Within-groups: a design that uses the same sample of individuals for each treatment condition.

Variability: a measure of how various observations differ within and across given categories.

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