About Area and Perimeter: Examples
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About Area and Perimeter: Examples

Square/Rectangle

Square

► s = 8 cm; A = 64 sq cm; P = 32 cm
► s = 5 km; A = 25 sq km; P = 20 km
► How many cm in a meter? 100
► How many sq cm in a sq meter? 10,000
► What is the length of one side of a square, given an area of 81 sq m? 9 m
► What is the length of one side of a square, given an area of 4 sq km? 2 km
► What is the length of one side of a square, given a perimeter of 24 cm? 6 cm
► What is the length of one side of a square, given a perimeter of 32 m? 8 m

Rectangle

► Suppose you have 120 m of fence for a rectangular plot (i.e., the perimeter is 120 m). What are some of the areas that can be enclosed?

Suppose the plot is square. Then, s = 120 / 4 = 30 m, and A = 900 sq m.

Suppose the plot is rectangular, with l = 40 m and w = 20 m. Then, the area is 800 sq m.

As it turns out, for any given perimeter, making the enclosure a square as opposed to a rectangle optimizes area.
Triangles

Right Triangle

- \(a = 5\) m, \(b = 12\) m, and \(c = 13\) m
  Area = \(30\) m\(^2\), Perimeter = \(30\) m

- \(a = 8\) km, \(b = 15\) km, and \(c = 17\) km
  Area = \(60\) km\(^2\), Perimeter = \(40\) km

Non-Right Triangles

- Area = \(3\) cm\(^2\)

- Area = \(6\) cm\(^2\)
In the figure below, you are given the length of two sides and one of the angles, but not the height h. You can find the height using the sin function.

\[ \sin(75°) = \frac{h}{2.1} \]

So, \( h = 2.1 \sin(75°) = 2.0 \text{cm} \), which means the area is

\[ A = \frac{1}{2} bh = \frac{1}{2} (3.0 \times 2.0) = 3.0 \text{cm}^2. \]

![Diagram](image1)

In this case, the right triangle is outside the triangle of which we are trying to find the area. The hypotenuse of the triangle is 3.7 cm. We can calculate the measure of one of the angles because it is supplementary to the 125° angle in the triangle:

\[ 180° - 125° = 55°; \sin(55°) = \frac{h}{3.7} \]

So, \( h = 3.7 \sin(55°) = 3.0 \text{cm} \), which means the area is

\[ A = \frac{1}{2} bh = \frac{1}{2} (4.0 \times 3.0) = 6.0 \text{cm}^2. \]

![Diagram](image2)

The perimeter of the triangle below is 5.1 + 4.9 + 6.6 = 16.6cm, so the semiperimeter \( s = 8.3 \text{cm} \).

\[ A = \sqrt{8.3(8.3 - 5.1)(8.3 - 4.9)(8.3 - 6.6)} \\
= \sqrt{8.3 \times 3.2 \times 3.4 \times 1.7} \\
= \sqrt{153.5} \approx 12.4 \text{cm}^2. \]

![Diagram](image3)
**Equilateral Triangles**

► Find the area and perimeter when:
\[ x = 3\text{cm} \text{ Area } = \sqrt{\frac{3}{4}} (3\text{cm})^2 \approx 3.90 \text{ cm}^2, \text{ Perimeter } = 9 \text{ cm} \]

► Find the area and perimeter when:
\[ x = 22\text{km} \text{ Area } = \sqrt{\frac{3}{4}} (22\text{km})^2 \approx 208.12 \text{ km}^2, \text{ Perimeter } = 66 \text{ cm} \]

► Suppose you have 144m of fence. How big an area can you enclose in a triangular plot?
\[ x = 48\text{m}, \text{ so Area } \approx 990.72\text{m}^2 \]

How big an area can you enclose in a square plot?
\[ x = 36\text{m}, \text{ so Area } = x^2 = 1296\text{m}^2 \]

Which is bigger? The square plot. Interestingly, this property always holds, as a square with perimeter equal to that of a triangle will encompass a greater area.

**Parallelogram**

► What is the area of a parallelogram with base 12 m and height 3 m?
\[ 36\text{m}^2 \]

► What is the area of a parallelogram with base 200 cm and height 14 cm?
\[ 2800\text{cm}^2 \]
► Suppose we have parallelograms with base $b = 9\text{cm}$ and height $h = 25\text{cm}$.

What is the area? $225\text{cm}^2$

What is the perimeter when $\theta = 10^\circ$? $P = 2(9) + 2(25/\sin(10)) \approx 305.9\text{cm}$

What is the perimeter when $\theta = 15^\circ$? $P = 2(9) + 2(25/\sin(15)) \approx 211.2\text{cm}$

What is the perimeter when $\theta = 30^\circ$? $P = 2(9) + 2(25/\sin(30)) = 118\text{cm}$

What is the perimeter when $\theta = 45^\circ$? $P = 2(9) + 2(25/\sin(45)) \approx 88.7\text{cm}$

What is the perimeter when $\theta = 60^\circ$? $P = 2(9) + 2(25/\sin(60)) \approx 75.7\text{cm}$

What is the perimeter when $\theta = 90^\circ$? $P = 2(9) + 2(25/\sin(90)) = 68\text{cm}$

► Area $= \frac{1}{2} 4(4 + 5) = 18\text{cm}^2$

Perimeter $= 5 + 4 + (4/\sin(80^\circ)) + (4/\sin(80^\circ))$

$\approx 17.1\text{cm}$

► Area $= \frac{1}{2} 4(3 + 5) = 16\text{cm}^2$

Perimeter $= 5 + 3 + (4/\sin(70^\circ)) + (4/\sin(80^\circ))$

$\approx 16.3\text{cm}$
**Circle**

- Find the area and circumference of the circle $r = 5\text{cm}$:
  
  \[
  \text{Area} = 25\pi \approx 78.5\text{cm}^2, \quad \text{Circumference} = 10\pi \approx 31.4\text{cm}
  \]

- Find the area and circumference of the circle $r = 10\text{cm}$:
  
  \[
  \text{Area} = 100\pi \approx 314.2\text{cm}^2, \quad \text{Circumference} = 20\pi \approx 62.8\text{cm}
  \]

- What is the radius of a circle with area $81\pi \text{cm}^2$?
  
  $9\text{cm}$

- What is the radius of a circle with circumference $144\pi \text{cm}$?
  
  $72 \text{cm}$

- What is the circumference of a circle with area $121\pi \text{m}^2$?
  
  The radius is $11 \text{m}$, so the circumference is $22\pi \text{m}$

- What is the area of a circle with circumference $64\pi \text{m}$?
  
  The radius is $32 \text{m}$, so the area is $1024\pi \text{m}^2$
**Ellipse**

► Consider the ellipse $x^2/16 + y^2/4 = 1$. What is its area?

\[ \text{Area} = 8\pi \approx 25.1 \text{ cm}^2 \]

► The ellipse given by $4x^2/9 + y^2/25 = 1$. What is its area?

\[ \text{Area} = 15\pi \approx 47.1 \text{ cm}^2 \]

**Complex Shapes**

► Considering the diagram below:

\[ 54 \text{ full squares} + 9 \text{ from partial squares} = 63 \text{ square units} \]