

**University of Guelph
Numeracy Project**

About Graphing Quadratics: Examples



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About Graphing Quadratics: Examples

Circle

- ▶ What is the equation of a circle with centre (0, 3) and radius 1?
 $x^2 + (y-3)^2 = 1$
- ▶ What is the equation of a circle with centre (2,-5) and radius 5?
 $(x - 2)^2 + (y + 5)^2 = 25$
- ▶ What are the centre and radius of the circle $(x - 13)^2 + (y+7)^2 = 121$?
centre = (13,-7), radius = 11
- ▶ What are the centre and radius of the circle $(x - 4)^2 + (y - 4)^2 = 64$?
centre = (4, 4), radius = 8
- ▶ Given the following equation, convert between General and Standard Form:
 $6x^2 + 6y^2 - 1x + 4y - 2 = 0$

Step 1: $x^2 + y^2 - (1/6)x + (2/3)y - 1/3 = 0$

Step 2: $(x^2 - (1/6)x) + (y^2 + (2/3)y) = 1/3$

Step 3: $(x^2 - (1/6)x + (1/144)) + (y^2 + (2/3)y + 1/9) = 3/2 + 1/144 + 1/9$

Step 4: $(x - (1/12))^2 + (y + (1/3))^2 = 216/144 + 1/144 + 16/144 = 233/144$

Step 5: centre = (1/12, -1/3), radius = $\sqrt{233/144}$

- Occasionally, instead of a circle, you are given a degenerate case.

For example, $x^2 + y^2 - 14x - 10y + 74 = 0$

Step 1: $A \neq 0$

Step 2: $(x^2 - 14x) + (y^2 - 10y) = -74$

Step 3: $(x^2 - 14x + 49) + (y^2 - 10y + 25) = -74 + 49 + 25$

Step 4: $(x - 7)^2 + (y - 5)^2 = 0$

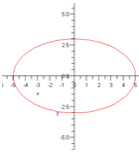
Step 5: This is a degenerate case, as a circle cannot have radius = 0. It is, in fact, the single point (7,5). Likewise, you might arrive at a radius with a value less than zero, which is degenerate as well, as circles cannot have negative radii.

- What is $11x^2 - 2y^2 = 8$ in General Form? $11x^2 - 2y^2 - 8 = 0$.
- What is $(x + 2)^2 - (y + 7)^2 = 169$ in General Form? Multiply out to get $x^2 + 4x + 4 - y^2 - 14y + 49 = 169$. Collect and rearrange to get $x^2 - y^2 + 4x - 14y + 4 + 49 - 169 = x^2 - y^2 + 4x - 14y - 116 = 0$.

Ellipse

Horizontal:

- $\frac{x^2}{25} + \frac{y^2}{9} = 1$



Vertical:

- $\frac{(x-2)^2}{4} + \frac{(y-1)^2}{49} = 1$



► $\frac{x^2}{81} + \frac{y^2}{25} = 1$ centre = (0,0); major axis $2\sqrt{81} = 18$;
 minor axis $2\sqrt{25} = 10$

► $\frac{(x+7)^2}{25} + \frac{(y-3)^2}{4} = 1$ centre = (-7,3); major axis 10;
 minor axis 4

► What is $5x^2 - 7y^2 + 2x + 10y - 3 = 0$ in standard form?

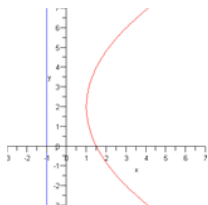
$(5x^2 + 2x) - (7y^2 - 10y) = 3$ gather the x and y terms
 $5(x^2 + (2/5)x) - 7(y^2 - (10/7)y) = 3$ factor out the first coefficient
 $5(x^2 + (2/5)x + 1/25) - 7(y^2 - (10/7)y + 100/196) = 9 + 1/5 - 700/196$
 add constants to both sides
 $5(x + 1/5)^2 - 7(y - (25/7))^2 = 197/35$ express as squares
 $\frac{(x+(1/5))^2}{175/197} - \frac{(y-(25/7))^2}{245/197} = 1$ divide by 197/35 to put in standard
 form
 centre = (-1/5, -25/7); major axis = $2\sqrt{(245/197)}$; minor axis = $2\sqrt{(175/197)}$

► What is $\frac{(x-18)^2}{9} + \frac{(y+27)^2}{121} = 1$ in General Form?

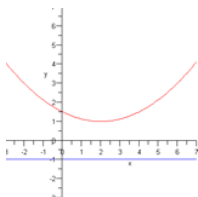
$121(x-18)^2 + 9(y+27)^2 = 9 \times 121$ get rid of fractions
 $121(x^2 - 36x + 324) + 9(y^2 + 54y + 729) = 1089$
 multiply squares
 $121x^2 - 4356x + 39204 + 9y^2 + 486y + 6561 - 1089 = 0$
 multiply out and collect
 $121x^2 + 9y^2 - 4356x + 486y + 44676 = 0$
 rearrange and collect constants

Parabola

- ▶ $(y - 2)^2 = 8(x - 1)$: vertex = (1,2);
 focus= $(1 + 8/4, 2) = (3, 2)$;
 directrix $x = 1 - 8/4$ or $x = -1$;
 $e = 1$



- ▶ $(x - 2)^2 = 8(y - 1)$: vertex = (2,1);
 focus= $(2, 1 + 8/4) = (2, 3)$;
 directrix $y = 1 - 8/4$ or $y = -1$;
 $e = 1$



- ▶ What is $y^2 + 9x - 3y + 7 = 0$ in Standard Form?

$$y^2 - 3y = -9x - 7$$

$$y^2 - 3y + (9/4) = -9x - 7 + (9/4)$$

$$(y - (3/2))^2 = -9x - (19/4)$$

$$(y - (3/2))^2 = -9(x + 19/36)$$

$$\text{vertex} = (19/36, -3/2); \text{focus} = (19/36, -(15/4)); \text{directrix } y = 3/4$$

- ▶ What is $3y^2 + 4x + 2 = 0$ in Standard Form?

$$3y^2 = -4x - 2$$

$$y^2 = -(4/3)x - (2/3)$$

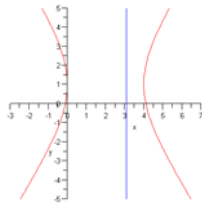
$$y^2 = -(4/3)(x + 1/2)$$

$$\text{vertex} = (-1/2, 0); \text{focus} = (-5/6, 0); \text{directrix } x = -1/6$$

Hyperbola

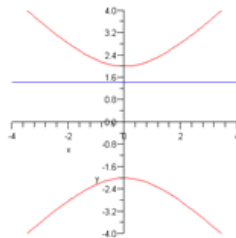
► $\frac{(x - 2)^2}{4} - \frac{(y - 1)^2}{9} = 1:$ centre = (2,1);

asymptotes: slope = $\pm 3/2$,
 point = (2,1), so $(y - 1) = 3/2 (x - 2)$
 and $(y - 1) = -3/2 (x - 2)$ are the
 asymptotes



► $\frac{y^2}{4} - \frac{x^2}{4} = 1:$ centre = (0,0);

asymptotes: slope = $\pm 2/2 = \pm 1$,
 point = (0,0) so $y = x$ and $y = -x$ are
 the asymptotes



► What is $7x^2 - 11y^2 + 6x + 2y - 144 = 0$ in Standard Form?

$$(7x^2 + 6x) - (11y^2 - 2y) = 144$$

$$7(x^2 + (6/7)x) - 11(y^2 - (2/11)y) = 144$$

$$7(x^2 + (6/7)x + (9/49)) - 11(y^2 - (2/11)y + (1/121)) = 144 + 9/7 - 1/11$$

$$7(x + (3/7))^2 - 11(y - (1/11))^2 = 11110/77$$

$$\frac{539(x + (3/7))^2}{11110} - \frac{847(y - (1/11))^2}{11110} = 1$$

- What is $8x^2 + 2y^2 - 6x + 3y - 9 = 0$ in Standard Form?

$$(8x^2 - 6x) + (2y^2 + 3y) = 9$$

$$8(x^2 - (3/4)x) + 2(y^2 + (3/2)y) = 9$$

$$8(x^2 - (3/4)x + 9/64) + 2(y^2 + (3/2)y + (9/16)) = 9 + (9/8) + (9/8)$$

$$8(x - (3/8))^2 + 2(y + (3/4))^2 = 90/8$$

$$\frac{64(x - 1)^2}{90} + \frac{16(y - 2)^2}{90} = 1$$

- What is $\frac{(x + 3)^2}{4} + \frac{(y - 2)^2}{9} = 1$ in General Form?

$$9(x + 3)^2 + 4(y - 2)^2 = 36$$

$$9(x^2 + 6x + 9) + 4(y^2 - 4y + 4) = 36$$

$$9x^2 + 54x + 81 + 4y^2 - 16y + 16 = 36$$

$$9x^2 + 4y^2 + 54x - 16y + 61 = 0$$

- What is $\frac{(x - 7)^2}{25} - \frac{(y + 5)^2}{121} = 1$ in General Form?

$$121(x - 7)^2 - 25(y + 5)^2 = 3025$$

$$121(x^2 - 14x + 49) - 25(y^2 + 10y + 25) = 3025$$

$$121x^2 - 1694x + 5929 - 25y^2 - 250y - 625 = 3025$$

$$121x^2 - 25y^2 - 1694x - 250y + 2279 = 0$$

Identification

Conic Sections

- ▶ Which conic is it?

(Recall $AC > 0$ implies ellipse or circle; $A = C$ implies circle; $AC = 0$ (either $A = 0$, or $C = 0$, but not both) implies parabola; $AC < 0$ implies hyperbola.)

$$7x^2 + 7y^2 - 5x + 11y = 0 \text{ circle}$$

$$2x^2 + y^2 + 4x - 7y + 25 = 0 \text{ ellipse}$$

$$3x^2 + 4x + 9y + 121 = 0 \text{ parabola}$$

$$2x^2 - 3y^2 + 8x - 7y - 81 = 0 \text{ hyperbola}$$