University of Guelph Numeracy Project

About t-tests: Examples

UNIVERSITY of GUELPH

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► A biochemistry professor noted that last semester, of 321 registered students, 153 attended lecture regularly by the time the third week had passed. The class average was 54%, while the average of those students who attended lecture was 78% with SS = 525.

In this example, the average of students who attend class is being compared against the overall class average.

There is only one sample and it is being compared to a population.

► Step 1: State the hypotheses and alpha level:

H_0: $\mu_D = 0$ (there is no difference in the averages)

H_1: $\mu_D \neq 0$ (there is a difference in the averages)

The level of significance is set at $\alpha = .05$, for a two-tailed test.

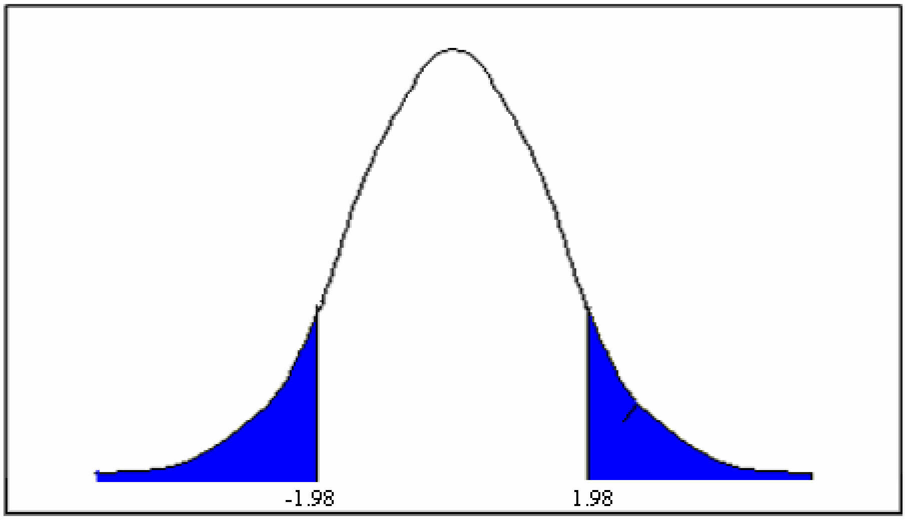
This means that if the obtained value of t falls in the bottom or top 2.5% of the distribution, we can conclude that our observations are significant.

► Step 2: Locate the critical region:

This is found by looking in a t-table. To find the degrees of freedom:

df = n - 1
df = 153 – 1
df = 152

In this case, the critical region is ± 1.98.



If the t value obtained is higher than + 1.98, or lower than –1.98, we can conclude that our observation is significant.

► Step 3: Calculate the obtained t value:

First, we calculate the variance so that it can be used to calculate the standard error:

$$s^2 = \frac{SS}{n-1}$$

$$s^2 = \frac{525}{152}$$

$$s^2 = 3.45$$

Now, we can calculate the standard error:

$$s_x = \frac{s^2}{n}$$

$$s_x = \frac{3.45}{153}$$

$$s_x = 0.02$$

Finally, we can calculate the obtained t-value:

$$t = \frac{\overline{X} - \mu}{s_x}$$

$$t = \frac{78 - 54}{0.02}$$

t = 1200

Therefore, the obtained t-value is +1200.

► Step 4: Make a decision:

The obtained t-value is greater than the critical t-value of +1.98.

Therefore, we can conclude that attending class significantly increased grades.

Independent Samples t-test

Independent Samples t-test

► A chaperone notes that some high school students at a theme park have a greater fear of roller coasters than others. She suspects this is due to the fact that some are significantly taller than their peers.

She notes the students' indicated comfort levels, and their heights, as listed below:

	Height (cm) for Students Comfortable on Roller Coasters	Height (cm) for Students Uncomfortable on Roller Coasters
n	12	34
$\overline{X}$	165	174
SS	1200	1150

- Step 1: State the hypotheses and select the alpha level:

  H_0: $\mu_D = 0$ (there is no difference in the students' scores)

  H_1: $\mu_D \neq 0$ (there is a difference in the students' scores)

  The level of significance is set at $\alpha = .05$, for a two-tailed test.

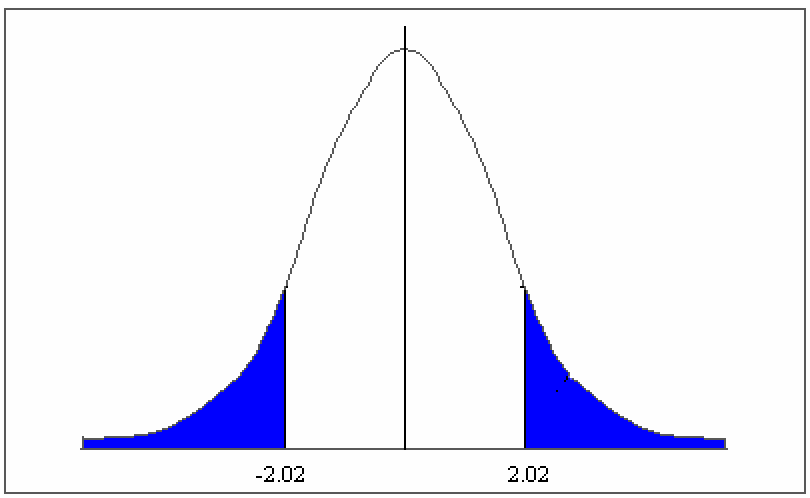
  This means that if the obtained value of t falls in the bottom or top 2.5% of the distribution, we can conclude that our observations are significant.

- Step 2: Locate the critical region:

  This is found by looking in a t-table. To find the degrees of freedom:

  $df = df_1 + df_2$
  $df = (n_1 - 1) + (n_2 - 1)$
  $df = (12 - 1) + (34 - 1)$
  $df = 44$

  In this case, the critical region is ± 2.02.



  If the t value obtained is higher than + 2.02, or lower than - 2.02, we can conclude that our observation is significant.

► Step 3: Calculate the obtained t value:

The equation for an independent samples t-test is:

$$t = \frac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{s_{(x1 - x2)}}$$

Next, we calculate the pooled variance. The pooled variance is used, rather than the sample variance, because there are unequal sample sizes:

$$s^2p = \frac{SS_1 + SS_2}{df_1 + df_2}$$

$$s^2p = \frac{1200 + 1150}{44}$$

$$s^2p = \frac{2350}{44}$$

$$s^2p = 53.41$$

Now, we can calculate the standard error:

$$s_{(x1 - x2)} = \frac{s^2p}{n_1} + \frac{s^2p}{n_2}$$

$$s_{(x1 - x2)} = \frac{53.41}{12} + \frac{53.41}{34}$$

$$s_{(x1 - x2)} = 6.02$$

Finally, we can calculate the obtained t-value:

$$t = \frac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{s_{(x1 - x2)}}$$

$$t = \frac{(165 - 174) - (0 - 0)}{6.02}$$

$$t = -1.50$$

Therefore, the obtained t-value is -1.50.

► Step 4: Make a decision:

The obtained t-value is not within the critical regions. Therefore, we can conclude that student height did not significantly affect roller coaster comfort levels.

Paired Samples t-test

Paired Samples t-test

► An example of a repeated measures t-test might occur at a Super Bowl party. A pool of money is put together, and each attendee (n = 37) weighs in at the start of the party, and weighs out at the end. The individual having the greatest differential between pre- and post-party weight will win the pool.

► An example of a matched participants t-test might be that of a researcher testing the effects of alcohol on a group of genii, measured through performance on a timed test.

The researcher matched 4 pairs of genii on variables of age, IQ and alcohol consumption. He then gave 1 genius in each pair 5 beers.

The researcher then administered the test, recording performance on the test, along with time required to complete it.

► A researcher wanted to study the effectiveness of a LSAT training program for undergraduates.

She sampled 25 students and had them complete the LSAT.

The students then participated in a 1-month LSAT training course.

At the end of this course, the participants took the LSAT again.

Results were as indicated below:

X_1 (LSAT score)	X_2 (LSAT score after course)	$D_{(x2-x1)}$	D^2
145	157	12	144
172	165	-7	49
135	142	7	49
157	163	6	36
164	155	-9	81
135	160	25	625
180	177	-3	9
177	179	2	4
162	165	3	9
157	153	-4	16
142	158	16	256
175	169	-6	36
132	160	28	784
133	157	24	576
154	149	-5	25
141	164	23	529
167	153	-14	196
173	165	-8	64
161	172	11	121
155	162	7	49
149	152	3	9
179	166	-13	169
153	153	0	0
144	152	8	64
117	140	23	529
$\bar{X}_1 = 154$	$\bar{X}_2 = 160$	$\Sigma D = 129$	$\Sigma D^2 = 4429$

► Step 1: State the hypotheses and select the alpha level:

H_0: $\mu_D = 0$ (there is no difference in LSAT scores)
H_1: $\mu_D \neq 0$ (there is a difference in LSAT scores)

The level of significance is set at $\alpha = .05$, for a two-tailed test.

► Step 2: Locate the critical region:

This is found by looking in a t-table. In this case, the critical region is ± 2.06.

If the t value obtained is higher than + 2.06, or lower than –2.06, we can conclude that our observation is significant.

► Step 3: Calculate the obtained t value:

The equation for a paired samples t-test is:

$$t = \frac{\overline{X}_D - \mu_D}{s_{XD}}$$

First, we have to calculate the Sum of Squares so that we can calculate the variance:

$$SS = \Sigma D^2 - \frac{(\Sigma D)^2}{n}$$

$$SS = 4429 - \frac{(129)^2}{25}$$

$$SS = 4429 - 666$$

$$SS = 3763$$

Next, we calculate the variance so that it can be used to calculate the standard error:

$$s^2 = \frac{SS}{n-1}$$

$$s^2 = \frac{3763}{25-1}$$

$$s^2 = 156.8$$

Now, we can calculate the standard error:

$$s_{XD} = \frac{s^2}{n}$$

$$s_{XD} = \frac{156.8}{25}$$

$$s_{XD} = 6.27$$

Finally we can calculate the obtained t-value

$$t = \frac{X_D - \mu_D}{s_{XD}}$$

$$t = \frac{(160\text{-}154) - 0}{6.27}$$

$$t = \frac{6}{6.27}$$

$$t = 0.96$$

Therefore, the obtained t-value is +0.96.

► Step 4: Make a decision:

The obtained t-value is not within the critical regions. Therefore, we can conclude that the LSAT training course did not significantly improve the LSAT scores of the participants.