About t-tests:
Examples
TABLE OF CONTENTS

Single Sample t-test ........................................................................................................ 1
Single Sample t-test ........................................................................................................ 1
Independent Samples t-test ............................................................................................ 3
Independent Samples t-test ............................................................................................ 3
Paired Samples t-test ....................................................................................................... 6
Paired Samples t-test ....................................................................................................... 6
About t-tests: Examples

Single Sample t-test

Single Sample t-test

A biochemistry professor noted that last semester, of 321 registered students, 153 attended lecture regularly by the time the third week had passed. The class average was 54%, while the average of those students who attended lecture was 78% with SS = 525.

In this example, the average of students who attend class is being compared against the overall class average.

There is only one sample and it is being compared to a population.

Step 1: State the hypotheses and alpha level:

H₀: μ₀D = 0 (there is no difference in the averages)
H₁: μ₀D ≠ 0 (there is a difference in the averages)

The level of significance is set at α = .05, for a two-tailed test.

This means that if the obtained value of t falls in the bottom or top 2.5% of the distribution, we can conclude that our observations are significant.

Step 2: Locate the critical region:

This is found by looking in a t-table. To find the degrees of freedom:

df = n - 1
df = 153 - 1
df = 152
In this case, the critical region is ± 1.98.

If the t value obtained is higher than +1.98, or lower than −1.98, we can conclude that our observation is significant.

► Step 3: Calculate the obtained t value:

First, we calculate the variance so that it can be used to calculate the standard error:

\[
s^2 = \frac{SS}{n-1}
\]

\[
s^2 = \frac{525}{152}
\]

\[
s^2 = 3.45
\]

Now, we can calculate the standard error:

\[
s_x = \frac{s^2}{n}
\]

\[
s_x = \frac{3.45}{153}
\]

\[
s_x = 0.02
\]
Finally, we can calculate the obtained t-value:

$$t = \frac{\bar{X} - \mu}{s_x}$$

$$t = \frac{78 - 54}{0.02}$$

$$t = 1200$$

Therefore, the obtained t-value is +1200.

► Step 4: Make a decision:

The obtained t-value is greater than the critical t-value of +1.98.

Therefore, we can conclude that attending class significantly increased grades.

**Independent Samples t-test**

**Independent Samples t-test**

► A chaperone notes that some high school students at a theme park have a greater fear of roller coasters than others. She suspects this is due to the fact that some are significantly taller than their peers.

She notes the students’ indicated comfort levels, and their heights, as listed below:

<table>
<thead>
<tr>
<th>Height (cm) for Students Comfortable on Roller Coasters</th>
<th>Height (cm) for Students Uncomfortable on Roller Coasters</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td>12</td>
</tr>
<tr>
<td>$\bar{X}$</td>
<td>165</td>
</tr>
<tr>
<td>SS</td>
<td>1200</td>
</tr>
</tbody>
</table>
Step 1: State the hypotheses and select the alpha level:

H₀: μ₃ = 0 (there is no difference in the students’ scores)

H₁: μ₃ ≠ 0 (there is a difference in the students’ scores)

The level of significance is set at α = .05, for a two-tailed test.

This means that if the obtained value of t falls in the bottom or top 2.5% of the distribution, we can conclude that our observations are significant.

Step 2: Locate the critical region:

This is found by looking in a t-table. To find the degrees of freedom:

\[ df = df_1 + df_2 \]
\[ df = (n₁ - 1) + (n₂ - 1) \]
\[ df = (12 - 1) + (34 - 1) \]
\[ df = 44 \]

In this case, the critical region is ± 2.02.

If the t value obtained is higher than + 2.02, or lower than - 2.02, we can conclude that our observation is significant.
Step 3: Calculate the obtained t value:

The equation for an independent samples t-test is:

\[
t = \frac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{S_{(x_1 \cdot x_2)}}
\]

Next, we calculate the pooled variance. The pooled variance is used, rather than the sample variance, because there are unequal sample sizes:

\[
s^2_p = \frac{SS_1 + SS_2}{df_1 + df_2}
\]

\[
s^2_p = \frac{1200 + 1150}{44} = \frac{2350}{44} = 53.41
\]

Now, we can calculate the standard error:

\[
s_{(x_1 \cdot x_2)} = \frac{s^2_p}{n_1} + \frac{s^2_p}{n_2}
\]

\[
s_{(x_1 \cdot x_2)} = \frac{53.41}{12} + \frac{53.41}{34} = 6.02
\]

Finally, we can calculate the obtained t-value:

\[
t = \frac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{S_{(x_1 \cdot x_2)}}
\]

\[
t = \frac{(165 - 174) - (0 - 0)}{6.02} = \frac{-9}{6.02} = -1.50
\]

Therefore, the obtained t-value is -1.50.
Step 4: Make a decision:

The obtained t-value is not within the critical regions. Therefore, we can conclude that student height did not significantly affect roller coaster comfort levels.

**Paired Samples t-test**

**Paired Samples t-test**

- An example of a repeated measures t-test might occur at a Super Bowl party. A pool of money is put together, and each attendee \( (n = 37) \) weighs in at the start of the party, and weighs out at the end. The individual having the greatest differential between pre- and post-party weight will win the pool.

- An example of a matched participants t-test might be that of a researcher testing the effects of alcohol on a group of genii, measured through performance on a timed test.

  The researcher matched 4 pairs of genii on variables of age, IQ and alcohol consumption. He then gave 1 genius in each pair 5 beers.

  The researcher then administered the test, recording performance on the test, along with time required to complete it.

- A researcher wanted to study the effectiveness of a LSAT training program for undergraduates.

  She sampled 25 students and had them complete the LSAT.

  The students then participated in a 1-month LSAT training course.

  At the end of this course, the participants took the LSAT again.
Results were as indicated below:

<table>
<thead>
<tr>
<th>$X_1$ (LSAT score)</th>
<th>$X_2$ (LSAT score after course)</th>
<th>$D_{(x_2 - x_1)}$</th>
<th>$D^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>145</td>
<td>157</td>
<td>12</td>
<td>144</td>
</tr>
<tr>
<td>172</td>
<td>165</td>
<td>-7</td>
<td>49</td>
</tr>
<tr>
<td>135</td>
<td>142</td>
<td>7</td>
<td>49</td>
</tr>
<tr>
<td>157</td>
<td>163</td>
<td>6</td>
<td>36</td>
</tr>
<tr>
<td>164</td>
<td>155</td>
<td>-9</td>
<td>81</td>
</tr>
<tr>
<td>135</td>
<td>160</td>
<td>25</td>
<td>625</td>
</tr>
<tr>
<td>180</td>
<td>177</td>
<td>-3</td>
<td>9</td>
</tr>
<tr>
<td>177</td>
<td>179</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>162</td>
<td>165</td>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>157</td>
<td>153</td>
<td>-4</td>
<td>16</td>
</tr>
<tr>
<td>142</td>
<td>158</td>
<td>16</td>
<td>256</td>
</tr>
<tr>
<td>175</td>
<td>169</td>
<td>-6</td>
<td>36</td>
</tr>
<tr>
<td>132</td>
<td>160</td>
<td>28</td>
<td>784</td>
</tr>
<tr>
<td>133</td>
<td>157</td>
<td>24</td>
<td>576</td>
</tr>
<tr>
<td>154</td>
<td>149</td>
<td>-5</td>
<td>25</td>
</tr>
<tr>
<td>141</td>
<td>164</td>
<td>23</td>
<td>529</td>
</tr>
<tr>
<td>167</td>
<td>153</td>
<td>-14</td>
<td>196</td>
</tr>
<tr>
<td>173</td>
<td>165</td>
<td>-8</td>
<td>64</td>
</tr>
<tr>
<td>161</td>
<td>172</td>
<td>11</td>
<td>121</td>
</tr>
<tr>
<td>155</td>
<td>162</td>
<td>7</td>
<td>49</td>
</tr>
<tr>
<td>149</td>
<td>152</td>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>179</td>
<td>166</td>
<td>-13</td>
<td>169</td>
</tr>
<tr>
<td>153</td>
<td>153</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>144</td>
<td>152</td>
<td>8</td>
<td>64</td>
</tr>
<tr>
<td>117</td>
<td>140</td>
<td>23</td>
<td>529</td>
</tr>
<tr>
<td>$\bar{X}_1 = 154$</td>
<td>$\bar{X}_2 = 160$</td>
<td>$\Sigma D = 129$</td>
<td>$\Sigma D^2 = 4429$</td>
</tr>
</tbody>
</table>

► Step 1: State the hypotheses and select the alpha level:

$H_0$: $\mu_D = 0$ (there is no difference in LSAT scores)

$H_1$: $\mu_D \neq 0$ (there is a difference in LSAT scores)

The level of significance is set at $\alpha = .05$, for a two-tailed test.
Step 2: Locate the critical region:

This is found by looking in a t-table. In this case, the critical region is ± 2.06.

If the t value obtained is higher than + 2.06, or lower than –2.06, we can conclude that our observation is significant.

Step 3: Calculate the obtained t value:

The equation for a paired samples t-test is:

\[ t = \frac{\bar{D} - \mu_0}{s_{XD}} \]

First, we have to calculate the Sum of Squares so that we can calculate the variance:

\[ SS = \sum D^2 - \frac{(\sum D)^2}{n} \]
\[ SS = 4429 - \frac{(129)^2}{25} \]
\[ SS = 4429 - 666 \]
\[ SS = 3763 \]

Next, we calculate the variance so that it can be used to calculate the standard error:

\[ s^2 = \frac{SS}{n-1} \]
\[ s^2 = \frac{3763}{25-1} \]
\[ s^2 = 156.8 \]
Now, we can calculate the standard error:

\[ s_{XD} = \frac{s^2}{n} \]
\[ s_{XD} = \frac{156.8}{25} \]
\[ s_{XD} = 6.27 \]

Finally we can calculate the obtained t-value

\[ t = \frac{X_D - \mu_D}{s_{XD}} \]
\[ t = \frac{(160-154) - 0}{6.27} \]
\[ t = \frac{6}{6.27} \]
\[ t = 0.96 \]

Therefore, the obtained t-value is +0.96.

► Step 4: Make a decision:

The obtained t-value is not within the critical regions. Therefore, we can conclude that the LSAT training course did not significantly improve the LSAT scores of the participants.