About t-tests

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What is a T-TEST?

- A t-test is used to test hypotheses about a population when the standard deviation of that population is unknown.

t-tests

t-tests

- Generally, the standard deviation of a population is not known.

  This makes the z-score useful only in very limited situations, when the mean and standard deviation of the population are known.

- A t-test is used when the population standard deviation is unknown, with up to two samples of interest.

- The formula for calculating a t-score is:

$$t = \frac{\overline{X} - \mu}{s_x}$$

  where $\overline{X}$ is the sample mean, μ is the population mean, and s_x is the standard error of the mean.

- The population mean is hypothesized from the null hypothesis.

- The standard error is calculated using the sample variance:

$$\text{Estimated standard error} = s_x = \frac{s^2}{n}$$

  where s^2 is the sample variance and n is the sample size.

$$s^2 = \frac{SS}{df}$$

  where SS is the Sum of Squares and df is the degrees of freedom(n-1).

t-tests and hypothesis testing

- In hypothesis testing, researchers use sample data to draw logical conclusions on the results of a research study and to make inferences on a population of interest.

- There are 4 main steps involved in hypothesis testing using t-tests:

  1) State the hypotheses and alpha level:

     There are 2 hypotheses that need to be stated:

  a) Null hypothesis (H_0): claims the procedure has no effect.

  b) Alternate hypothesis (H_1): claims the procedure has an effect.

  Both hypotheses are stated in terms of the population parameter (μ).

  The alpha level is a probability value used to define the critical regions.

  Usually the alpha level is set at $\alpha = .05$.

  This means that there is a 5% chance of finding a statistically significant result.

  If the alpha level is $\alpha = .01$, this means that there is a 1% chance of finding a statistically significant result.

  Therefore, the smaller the alpha level, the smaller the chance of finding a significant result.

  2) Locate the critical region in the t-distribution:

  The t-distribution is a normal curve, with the majority of the scores in the middle of the distibution and fewer scores at the extremes.

  The location of the critical regions depends on the alpha level and the degrees of freedom.

The critical region is found by referring to a t-distribution table.
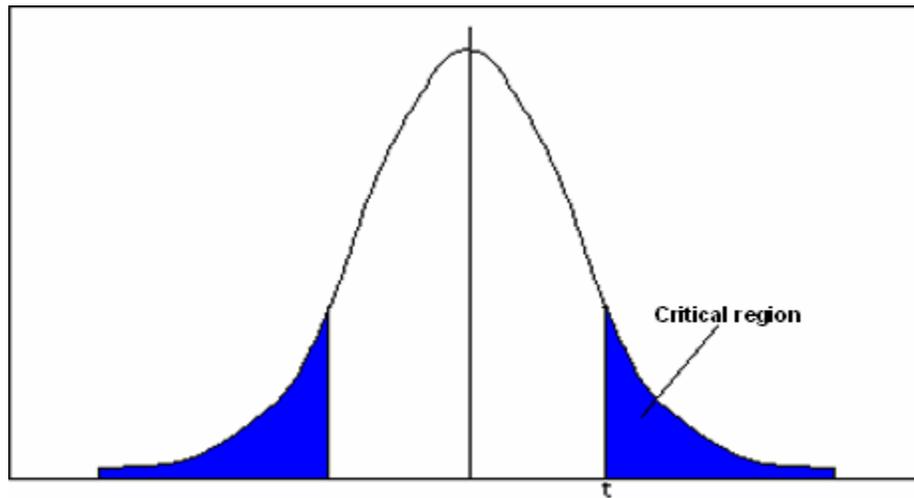
Results that fall in the critical regions are statistically significant because they are unlikely to have occurred by chance.

3) Collect the data and calculate the t-test.

4) Make a decision:

For t-statistics within the critical region, H_0 is rejected, allowing us to conclude that the procedure has an effect.

If the t-statistic is not within the critical region, we fail to reject H_0, allowing us to conclude that the procedure has no effect.

- There are 3 types of t-tests:

  1) Single sample

  2) Independent samples

  3) Paired samples
  > a) Repeated Samples t-test

  > b) Matched Samples t-test

How do you know which test to use?
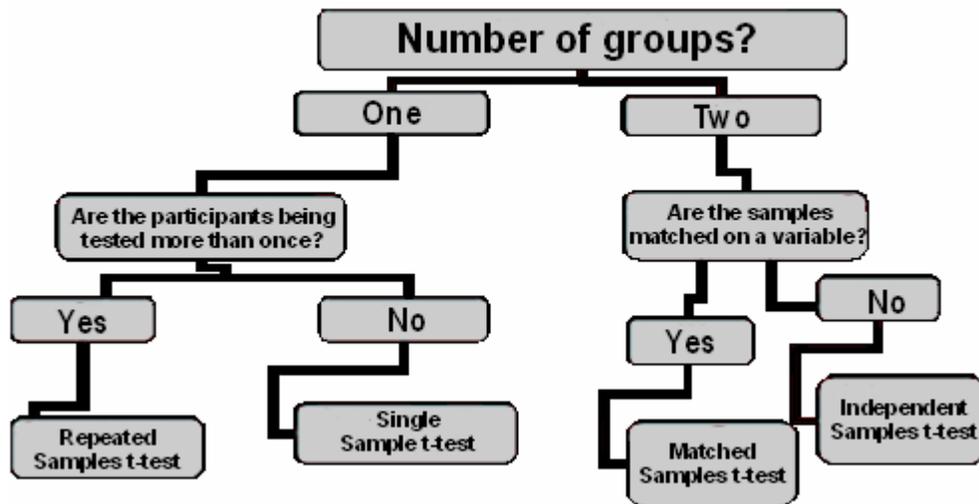
Single Sample t-test

Single Sample t-tests

- A single sample t-test is used when there is only one independent sample that is being compared to a population of interest.

- The formula for a single sample t-test is:

$$t = \frac{\overline{X} - \mu}{s_x}$$

where $\overline{X}$ is the sample mean, μ is the population mean, and s_x is the standard error of the mean.

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- The population mean can be hypothesized from the null hypothesis.

Independent Samples t-test

Independent Samples t-test

- An independent samples t-test is used to determine differences between the means of two distinct samples within a population.

  It is also referred to as a between-subjects design.

- The formula for the independent samples t-test is:

$$t = \frac{(\overline{X}_1 - \overline{X}_2) - (\mu_1 - \mu_2)}{s_{(x1 - x2)}}$$

  where $s_{(x1 - x2)}$ is the standard error that is calculated using the pooled variance.

  Standard error of the estimate measures how accurately the sample represents the population.

- In an independent samples t-test, there are 2 sources of error:

  $\overline{X}_1$ and $\overline{X}_2$.

- Using the standard error formula, $s_X = s^2 / n$, restricts us to using groups whose sample sizes are the same.

  This formula is biased, as it treats both samples equally.

  The solution to this bias is using pooled variance.

  Pooled variance corrects this bias by combining the two sample variances into a single value.

- The pooled variance equation is:

$$s^2p = \frac{SS_1 + SS_2}{df_1 + df_2}$$

where SS is the sum of squares and df are the degrees of freedom.

Using the pooled variance, the equation for calculating the standard error becomes:

$$s_{(x1-x2)} = \sqrt{\frac{s^2p}{n_1} + \frac{s^2p}{n_2}}$$

- There are 3 assumptions that must be met in order to conduct an independent samples t-test.

    1) Independent observations: this means that the results of one sample are not due to the other sample.

    2) Normal distribution: the population from which the sample is drawn is normally distributed. This assumption can be violated if there is a large enough sample size in each group.

    3) Homogeneity of variance: the two populations from which the samples are drawn have equivalent variance. This assumption is very important. If violated, a meaningful interpretation of the results can be negated.

Paired Samples t-test

Paired Samples t-test

- A paired samples t-test is a type of t-test where a single sample of participants is used more than once on the same dependent variable.

- The main advantage of a paired samples t-test is that it uses the same individuals in all the treatment conditions.

  Therefore, there is no risk that the participants in one treatment are substantially different than those in another.

  There is the risk that the participants are significantly different in an independent samples design.

- A paired samples t-test is calculated using the formula:

$$t = \frac{\overline{X}_D - \mu_D}{s_{XD}}$$

$$\text{where } s_{XD} = \frac{s^2}{n}$$

- A paired samples t-test uses a difference score, which is obtained by subtracting the pre-treatment score from the post-treatment score.

  The difference score shows the direction of change.

- There are 2 types of paired samples t-test:

  1) Repeated measures t-test

  2) Matched- participants t-test

- Repeated meaures t-tests are used when there is only one sample and the participants are tested twice.

  This type of t-test is often used in pre-test/post-test situations, where the researchers want to see the effects of some variable compared to the baseline.

- In a matched participants research design, each individual in one sample is matched with an individual in the other sample.

  The individuals used in these studies are often siblings or are matched on a specific variable of interest, such as IQ.

  The participants can be matched on more than one variable of interest (i.e. IQ, sex, race, level of education, etc.); however, as the number of variables that have to be matched increases, the more difficult it becomes to find participants.

- The goal of a matched participants design is to simulate a repeated samples design as closely as possible.

Glossary

Carryover effects: occur when a participant's response in the second treatment is affected by lingering after effects from the first treatment.

Difference score: in a paired samples t-test, obtained by subtracting the pre-treatment score from the post-treatment score. It shows the direction of change.

Independent samples t-test: used when there are two samples that are being compared to the population.

Individual differences: the inherent differences between the participants in a study that could confound the results.

Paired samples t-test: used when there is one sample, or two samples, that are matched on a variable of interest. This variable is tested twice.

Pooled variance: in independent samples t-tests, when the sample sizes are uneven, this is used to correct the bias in the standard error statistic by combining the two sample variances into a single value.

Progressive error: occurs when the subject's performance/response changes consistently over time (e.g. a decline over time due to fatigue, an increase over time due to practice).

Single sample t-test: used when there is one sample, which is to be compared to the population.

Standard deviation: a measure, in relation to the mean, which
                        indicates the spread of values within the
                        distribution. If the values are close to the
                        mean, the standard deviation is small. If the
                        values are spread out, the standard deviation
                        is large.

Standard error: measures how accurately the sample
                        represents the population.

Variance: a measure, in relation to the mean, which
                        indicates the distribution of scores.

References

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