University of Guelph Numeracy Project

About Compound Interest: Examples

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About Compound Interest: Examples

Interest

Simple Interest

► $500 invested at 7% annually. After one year, the interest is
0.07 * $500 = $35, giving you a total of $535.

$4500 borrowed at 8% monthly. After three months, the interest is
0.08 * $4500 * 3 = $1080, meaning you will have to pay $5580.

$1,000,000 borrowed at 6% annually. After 10 years, the interest is
0.06 * $1,000,000 * 10 = $600,000, meaning you will have to pay
$1,600,000.

$2200 invested at 2% monthly. After two years, the interest is
0.02 * $2200 * 24 = $1056, giving you a total of $3256.

Compound Interest

► Say you invest $500 at 5% interest, compounded annually.

After the first year, you get $500 + 5% of $500 = $525
After the second year, you get $525 + 5% of $525 = $551.25
After the third year, you get $551.25 + 5% of $551.25 = $578.81

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With simple interest, after three years you would get:

$500 + 0.05 * $500 * 3 = $575

► Suppose you invest $200 at 1%, compounded annually for 25 years.
Then, you will have $\$200(1 + 0.01)^{25} = \$256.49$.

Suppose you borrow $15,000 at 4%, compounded annually for 30
years. Then, you will owe $\$15{,}000(1 + 0.04)^{30} = \$48{,}650.96$.

Calculations

Calculating How Much to Invest

► Suppose you want to have $60,000 to pay for a newborn child's university expenses, and you can invest the money at 3.2%. How much do you need to invest?

For our example, FV = $60,000, n = 18, and r = 0.032. So, we get:

$PV = \$60{,}000/1.032^{18} = \$34{,}034.29$

Calculating Rate of Return

► You need $11,500 in 2 years, and you have $4,000 to invest. What interest rate do you need to find?

$$r = \sqrt{\frac{\$11{,}500}{\$4{,}000}} - 1 = 0.696$$

So, you need an investment that earns 69.6% interest. Good luck!

Compounding Periodically

► Suppose you borrowed $52,000 at a 1.8% annual rate, compounded monthly for 12 years. Then, you would owe:

$\$52{,}000(1 + 0.018/12)^{12*12} = \$52{,}000(1.0015)^{144} = \$64{,}526.88$

Instead, if you compounded yearly, you would owe:

$\$52{,}000(1+0.018)^{12} = \$64{,}413.47$

Compounded weekly, it would be:

$\$52{,}000(1 + 0.018/52)^{52*12} = \$52{,}000(1.00035)^{624} = \$64{,}689.93$

► $52,000 compounded continuously for 12 years at 1.8% is:

$\$52{,}000 * e^{12*0.018} = \$64{,}537.32$

Rules/Contributions

Rule of 72 - How to Estimate Compound Interest

► Suppose you know you can get an interest rate of 2.8%. How long will it take to double your money? 72/2.8 = 25.7 years

Suppose you are investing for 3 years. What interest rate do you need to double your money? 72/3 = 24%

How long does it take to double your money with an interest rate of 4.3%? 72/4.3 = 16.7 years

How long does it take to double your money with an interest rate of 1.1%? 72/1.1 = 65.5 years

What interest rate do you need to double your money in 18 years? 72/18 = 4%

What interest rate do you need to double your money in 50 years? 72/50 = 1.44%

► Suppose you invest $100 at 4.4% and add $20 every year for 25 years.

You would have:

after year 1: $\$100(1.044) + \$20$
after year 2: $\$100(1.044)^2 + \$20(1.044) + \$20$
after year 3: $\$100(1.044)^3 + \$20(1.044)^2 + \$20(1.044) + \$20$
after year 25: $\$100(1.044)^{25} + \$20(1.044)^{24} + \$20(1.044)^{23} + \ldots + \$20$

This can be rewritten as:

$\$100(1.044)^{25} + \$20(\sum_{i=0}^{24} 1.044^i)$

The geometric series $\sum_{i=0}^{24} 1.044^i = 1 + 1.044 + 1.044^2 + 1.044^3 + \ldots$

$$= \frac{1.044^{24+1} - 1}{1.044 - 1} = \frac{1.044^{25} - 1}{1.044 - 1}$$

So, we can simplify the formula to get:

$$\$100\ (1.044)^{25} + \$20\left(\frac{1.044^{25} - 1}{0.044}\right) = \$1172.69$$

Annuity

Annuities

► Suppose you have $50,000, which you want to use up over 25 years, earning interest at a rate of 4% annually. Let w represent the amount you withdraw each year. Then, you would have:

After year 1: $50,000 - w
After year 2: $50,000(1.04) - w(1.04) - w
After year 3: $50,000$(1.04)^2$ - w$(1.04)^2$ - w(1.04) - w
After year 25: $50,000$(1.04)^{24}$ - w$(1.04)^{24}$ - w$(1.04)^{23}$ - . . . – w

We can rewrite this as:

$$\$50{,}000(1.04)^{24} - w \sum_{i=0}^{24} (1.04)^i$$

$$\text{The geometric series } \sum_{i=0}^{24} 1.04^i = 1 + 1.04 + 1.04^2 + 1.04^3 + \ldots + 1.04^{24} = \frac{1.04^{24+1} - 1}{1.04 - 1} = \frac{1.04^{25} - 1}{0.04}$$

So, we can simplify the formula to get:

$50,000$(1.04)^{24}$ - w(((1.04)^{25} -1) / 0.04)

Now, set this equal to 0 (since you do not want to have any money left after 25 years), and solve for w.

$50,000$(1.04)^{24}$ = w(((1.04)^{25} -1) / 0.04)

$$w = \frac{0.04 * \$50{,}000 * (1.04)^{24}}{(1.04)^{25} - 1} = \$3077.50$$

Mortgage

Mortgages

► Suppose you buy a $1,500,000 house and borrow the money at 3% for 20 years. Let a represent your annual payment. Then, you will owe:

After year 1: $\$1{,}500{,}000(1.03) - a$

After year 2: $\$1{,}500{,}000(1.03)^2 - a(1.03) - a$

After year 3: $\$1{,}500{,}000(1.03)^3 - a(1.03)^2 - a(1.03) - a$

After year 20: $\$1{,}500{,}000(1.03)^{20} - a(1.03)^{19} - a(1.03)^{18} - \ldots - a$

Again, this is a geometric series. So, we have:

$$\$1{,}500{,}000(1.03)^{20} - a(1+(1.03)+(1.03)^2+\ldots+(1.03)^{19}) = \$1{,}500{,}000(1.03)^{20} - \frac{a((1.03)^{20}-1)}{1.03-1}$$

So, $a = \frac{\$1{,}500{,}000(1.03)^{20}(0.03)}{(1.03)^{20}-1} = \$100{,}823.56$ annual payment or $8,401.96 per month.