About Compound Interest: Examples
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About Compound Interest: Examples

**Simple Interest**

$500 invested at 7% annually. After one year, the interest is $0.07 \times $500 = $35$, giving you a total of $535$.

$4500$ borrowed at 8% monthly. After three months, the interest is $0.08 \times 4500 \times 3 = $1080$, meaning you will have to pay $5580$.

$1,000,000$ borrowed at 6% annually. After 10 years, the interest is $0.06 \times 1,000,000 \times 10 = $600,000$, meaning you will have to pay $1,600,000$.

$2200$ invested at 2% monthly. After two years, the interest is $0.02 \times 2200 \times 24 = $1056$, giving you a total of $3256$.

**Compound Interest**

Say you invest $500 at 5% interest, compounded annually.

After the first year, you get $500 + 5\% \text{ of } 500 = $525$

After the second year, you get $525 + 5\% \text{ of } 525 = $551.25$

After the third year, you get $551.25 + 5\% \text{ of } 551.25 = $578.81$

With simple interest, after three years you would get:

$500 + 0.05 \times 500 \times 3 = $575$

Suppose you invest $200 at 1%, compounded annually for 25 years. Then, you will have $200(1 + 0.01)^{25} = $256.49$.

Suppose you borrow $15,000 at 4%, compounded annually for 30 years. Then, you will owe $15,000(1 + 0.04)^{30} = $48,650.96$. 

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Calculations

Calculating How Much to Invest

► Suppose you want to have $60,000 to pay for a newborn child's university expenses, and you can invest the money at 3.2%. How much do you need to invest?

For our example, FV = $60,000, n = 18, and r = 0.032. So, we get:

\[ PV = \frac{FV}{(1+r)^n} = \frac{60,000}{1.032^{18}} = 34,034.29 \]

Calculating Rate of Return

► You need $11,500 in 2 years, and you have $4,000 to invest. What interest rate do you need to find?

\[ r = \sqrt[2]{\frac{11,500}{4,000}} - 1 \approx 0.696 \]

So, you need an investment that earns 69.6% interest. Good luck!

Compounding Periodically

► Suppose you borrowed $52,000 at a 1.8% annual rate, compounded monthly for 12 years. Then, you would owe:

\[ $52,000 (1 + \frac{0.018}{12})^{12*12} = $52,000 (1.0015)^{144} = 64,526.88 \]

Instead, if you compounded yearly, you would owe:

\[ $52,000 (1+0.018)^{12} = 64,413.47 \]

Compounded weekly, it would be:

\[ $52,000 (1 + \frac{0.018}{52})^{52*12} = $52,000 (1.00035)^{624} = 64,689.93 \]

► $52,000 compounded continuously for 12 years at 1.8% is:

\[ $52,000 \times e^{12 \times 0.018} = 64,537.32 \]
Rules/Contributions

Rule of 72 - How to Estimate Compound Interest

Suppose you know you can get an interest rate of 2.8%. How long will it take to double your money? \(72/2.8 = 25.7\) years

Suppose you are investing for 3 years. What interest rate do you need to double your money? \(72/3 = 24\%\)

How long does it take to double your money with an interest rate of 4.3%? \(72/4.3 = 16.7\) years

How long does it take to double your money with an interest rate of 1.1%? \(72/1.1 = 65.5\) years

What interest rate do you need to double your money in 18 years? \(72/18 = 4\%\)

What interest rate do you need to double your money in 50 years? \(72/50 = 1.44\%\)

Suppose you invest $100 at 4.4% and add $20 every year for 25 years.

You would have:

after year 1: $100(1.044) + $20
after year 2: $100(1.044)^2 + $20(1.044) + $20
after year 3: $100(1.044)^3 + $20(1.044)^2 + $20(1.044) + $20
after year 25: $100(1.044)^{25} + $20(1.044)^{24} + $20(1.044)^{23} + \ldots + $20

This can be rewritten as:

\[
$100(1.044)^{25} + \sum_{i=0}^{24} 1.044^i \cdot 20 \]

The geometric series \(\sum_{i=0}^{24} 1.044^i\) is:

\[
1 + 1.044 + 1.044^2 + 1.044^3 + \ldots = \frac{1.044^{24+1} - 1}{1.044 - 1} = \frac{1.044^{25} - 1}{1.044 - 1}
\]
So, we can simplify the formula to get:

$$100 \cdot (1.044)^{25} + 20 \left( \frac{1.044^{25} - 1}{0.044} \right) = 1172.69$$

**Annuity**

**Annuities**

Suppose you have $50,000, which you want to use up over 25 years, earning interest at a rate of 4% annually. Let \( w \) represent the amount you withdraw each year. Then, you would have:

After year 1: $50,000 - w
After year 2: $50,000(1.04) - w(1.04) - w
After year 3: $50,000(1.04)^2 - w(1.04)^2 - w(1.04) - w
After year 25: $50,000(1.04)^{24} - w(1.04)^{24} - w(1.04)^{23} - \ldots - w

We can rewrite this as:

$$50,000(1.04)^{24} - w \sum_{i=0}^{24} (1.04)^i$$

The geometric series \( \sum_{i=0}^{24} 1.04^i = 1 + 1.04 + 1.04^2 + 1.04^3 + \ldots + 1.04^{24} \)

$$= \frac{1.04^{24+1} - 1}{1.04 - 1} = \frac{1.04^{25} - 1}{0.04}$$

So, we can simplify the formula to get:

$$50,000(1.04)^{24} - w((1.04)^{25} - 1) / 0.04$$

Now, set this equal to 0 (since you do not want to have any money left after 25 years), and solve for \( w \).

$$50,000(1.04)^{24} = w((1.04)^{25} - 1) / 0.04$$

$$w = \frac{0.04 \times 50,000 \times (1.04)^{24}}{(1.04)^{25} - 1} = 3077.50$$
Mortgages

Suppose you buy a $1,500,000 house and borrow the money at 3% for 20 years. Let $a$ represent your annual payment. Then, you will owe:

After year 1: $1,500,000(1.03) - a$

After year 2: $1,500,000(1.03)^2 - a(1.03) - a$

After year 3: $1,500,000(1.03)^3 - a(1.03)^2 - a(1.03) - a$

After year 20: $1,500,000(1.03)^{20} - a(1.03)^{19} - a(1.03)^{18} - \ldots - a$

Again, this is a geometric series. So, we have:

\[
1,500,000(1.03)^{20} - a(1+(1.03)+(1.03)^2+\ldots+(1.03)^{19})
= 1,500,000(1.03)^{20} - \frac{a((1.03)^{20} - 1)}{1.03-1}
\]

So, $a = 1,500,000(1.03)^{20}(0.03) = 100,823.56$ annual payment or $8,401.96$ per month.