

**University of Guelph  
Numeracy Project**

# **About Compound Interest**



# TABLE OF CONTENTS

About Compound Interest.....	1
What is COMPOUND INTEREST?.....	1
Interest.....	1
Simple Interest .....	1
Compound Interest.....	1
Calculations.....	3
Calculating How Much to Invest .....	3
Calculating Rate of Return.....	3
Compounding Periodically .....	4
Rules/Contributions .....	5
Rule of 72 - How to Estimate Compound Interest.....	5
Adding Contributions.....	5
Annuity .....	5
Annuities .....	5
Mortgage .....	6
Mortgages .....	6
Glossary .....	7
References.....	9

## **About Compound Interest**

### ***What is COMPOUND INTEREST?***

- Compound interest refers to interest applied to the principal, in addition to unpaid interest added to the principal. The growth of the balance is proportional to the frequency at which the interest is compounded.
- A distinguishing difference between compound interest and simple interest is that there is no compounding with simple interest. In other words, for simple interest, the interest is not added to the principal. As such, within finance and economics, compound interest is encountered much more commonly than simple interest.

## **Interest**

### ***Simple Interest***

- Before getting into compound interest, we need to look into what simple interest is:

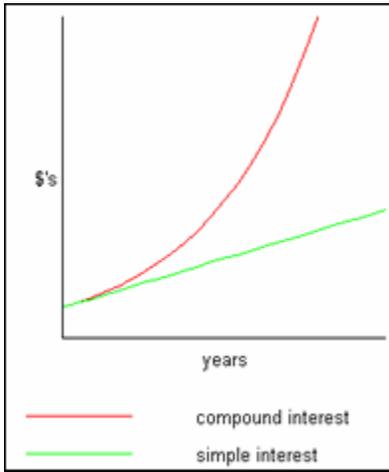
Interest is an amount of money which is paid to you when you make an investment or which you have to pay when you take out a loan.

- Interest is typically given as a percentage, at some fixed interval (every day, every month, every year). To calculate simple interest, just multiply this percentage by the amount invested or borrowed and then multiply by the number of intervals.

### ***Compound Interest***

- Albert Einstein said "The most powerful force in the universe is compound interest."
- Compound interest occurs when you receive or have to pay interest on the interest as well as the principal.

- Below is a graph that illustrates the difference between simple and compound interest.



The essential difference is clear: exponential vs. linear growth.

- There are three ways you can calculate compound interest:

- Use a financial calculator, such as the one found at:

[http://www.moneychimp.com/calculator/compound\\_interest\\_calculator.htm](http://www.moneychimp.com/calculator/compound_interest_calculator.htm)

- Use a Future Values Table, such as the one found at:

<http://members.shaw.ca/RetailInvestor/futurevaluetables.pdf>

- Use a formula.

- Let us begin to use some calculations to find compound interest:

To derive the formula, let us go through the calculations again using symbols instead of numbers. Let  $P$  be the amount of principal invested (or borrowed). Let  $r$  be the annual interest rate, and let  $n$  be the number of years.

Then, you get:

After year 1:  $P + Pr = P(1 + r)$

After year 2:  $[P(1 + r)] + r[P(1 + r)] = P(1 + r)(1 + r)$

After year 3:  $[P(1 + r)(1 + r)] + r[P(1 + r)(1 + r)]$   
 $= P(1 + r)(1 + r)(1 + r)$

After n years:  $P(1 + r)^n$

This gives us a formula we can use when money is compounded annually.

Future Value (FV) =  $P(1 + r)^n$

## Calculations

### ***Calculating How Much to Invest***

- Sometimes, instead of wanting to know the future value of your investment, you want to know how much you will need to invest to generate the amount of money you require: this is called "present value."
- If we take the equation  $FV = P(1+r)^n$ , solving for P, we get  $P = FV/(1+r)^n$ . It is traditional to change the P to PV to stand for "present value," instead of "principal." So, the formula to use is:

Present Value =  $PV = FV / (1 + r)^n$

### ***Calculating Rate of Return***

- You can also solve the equation  $FV = PV (1+r)^n$  for the interest rate: This is called the "rate of return," the interest rate you need to get the amount of money you require.
- Solving for r, the formula is:

$$r = \sqrt[n]{\frac{FV}{PV}} - 1$$

## ***Compounding Periodically***

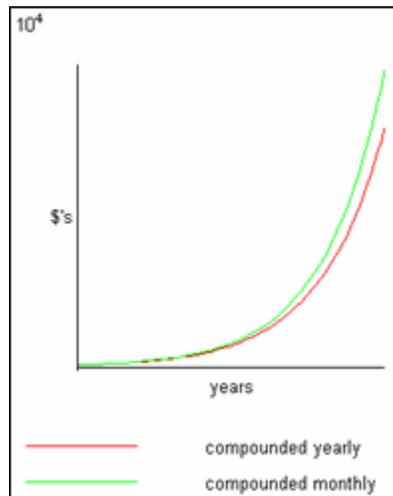
- Now, suppose the interest is compounded not once a year, but once a month: this is called "compounding periodically."
- The interest rate for one month is  $r/12$  and the number of times the money is compounded is  $12n$ . (Recall that  $r$  is the annual interest rate;  $n$  is the number of years.) So, the formula becomes:

$$P(1 + r/12)^{12n}$$

- More generally, if  $t$  is the number of times the money is compounded in a year, the formula is:

$$\text{Future Value} = P(1 + r/t)^{nt}$$

- Below is a graph showing the difference between compounding monthly and compounding yearly.



- Suppose you compounded continuously, every fraction of a fraction of a second. Then, the formula becomes:

$$\text{Future Value} = Pe^{nr}$$

where  $e$  is a special mathematical constant, named after Leonhard Euler (pronounced "oiler"), the 18th century Swiss mathematician. It is an irrational number with value  $\sim 2.7182818$ .

## Rules/Contributions

### ***Rule of 72 - How to Estimate Compound Interest***

- Imagine you are discussing making an investment, at a restaurant, with no calculator or computer handy. You can use the Rule of 72 to figure out how long it will take to double your money.

The number of years required is approximately  $72/r$ , where  $r$  is the interest rate expressed as a percentage, and the investment is compounded annually.

- If you know the number of years,  $n$ , you can calculate the interest rate you need to double your money using the formula  $72/n$ . This estimate works well, as long as the interest rate is not too high.

### ***Adding Contributions***

- People making an investment often want to make contributions to it as it grows.
- Generally, for  $P$  = principal,  $r$  = interest rate,  $c$ =annual contribution, and  $n$ =number of years, we have:

$$FV = P(1+r)^n + c \sum_{t=0}^{n-1} (1+r)^t = P(1+r)^n + c \left( \frac{(1+r)^n - 1}{r} \right)$$

## Annuity

### ***Annuities***

- Sometimes, instead of putting money into an investment, you want to take money out: this is called an annuity.
- Here is the general form for finding annuity:

$$w = \frac{Pr(1+r)^{n-1}}{(1+r)^n - 1}$$

where  $P$ =principal,  $r$ =rate, and  $n$ =number of years.

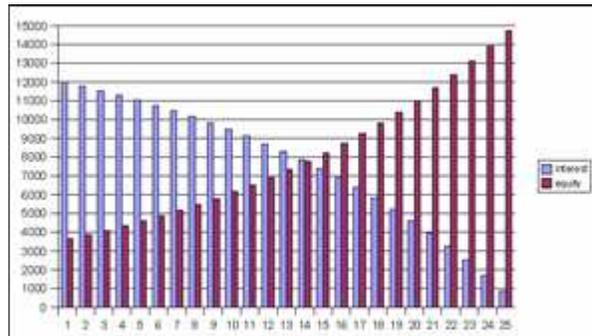
# Mortgage

## Mortgages

- Mortgages work somewhat like annuities, except that you make payments, as opposed to receiving them.
- Working with symbols, instead of numbers, the formula is:

$$a = \frac{P(1 + r)^n \cdot r}{(1 + r)^n - 1}$$

- Each time you make a payment on a mortgage, part of the payment is interest and part of the payment is equity. It is important to know which is which.
- The equity is the amount of money you would be able to keep if you sold the house; the interest might be tax deductible. The easiest way to calculate this is using a spreadsheet. Make three columns: balance, interest, and equity. The balance is last year's balance plus last year's interest. The interest is the balance times the interest rate. The equity is the annual payment minus the interest.



## **Glossary**

Annual Percentage Rate (APR):	effective interest rate paid annually on a loan taking into account fees.
Annual Percentage Yield (APY):	effective interest rate paid annually on an investment.
Annuity:	a type of investment which pays a fixed amount at some interval (month, year) for a set number of intervals.
Compound Interest:	interest calculated based on both the principal and the interest already accrued.
Effective Interest Rate:	interest rate stated as a rate which is compounded annually.
Equity:	the worth of a property beyond its mortgage.
Future Value (FV):	the value money will have in the future if invested.
Inflation rate:	rate at which prices rise relative to purchasing power.
Nominal Interest Rate:	interest rate which hasn't been adjusted to take into account the effect of compounding.
Present Value (PV):	the value of money you have now which you plan to invest.
Principal:	starting value of a debt or investment.
Rate of Return (ROR):	the amount of money accumulated or lost in light of the amount invested (also called return on investment, or rate of profit).

**Real Interest Rate:**

a subtraction of the inflation rate from the effective interest rate.

**Simple Interest:**

interest calculated based on the principal alone.

## References

[http://en.wikipedia.org/wiki/Compound\\_interest](http://en.wikipedia.org/wiki/Compound_interest)