University of Guelph Numeracy Project

About Factoring Quadratics: Examples

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About Factoring Quadratics: Examples

Quad Polynomials

Quadratic Polynomials

► $2x^2 - 3x + 4$

► $8x^2 - 2$

► $7x^2$

Perfect Squares

Perfect Squares

► Examples of numbers which are positive squares:

1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 1/4, 9/16

► Examples of numbers which are not positive squares:

3, 10, -72, 13, 84, -66, -16, -4, 37, 18, 1/2, ¾

► $x^2 - 4x + 4 = (x - 2)^2$

$x^2 + 10x + 25 = (x + 5)^2$

► $x^2 + x + 1$ is not a perfect square

$x^2 + 5x + 3$ is not a perfect square

► $16x^2 + 40x + 25 = (4x + 5)^2$

$4x^2 - 28x + 49 = (2x - 7)^2$

► $3x^2 - 7x + 3$ is not a perfect square

$9x^2 + 7x + 7$ is not a perfect square

Difference of Squares

Difference of Squares

► $x^2 - 49 = (x + 7)(x - 7)$

$x^2 - 25 = (x + 5)(x - 5)$

► $2x^2 - 81$ is not a difference of squares

$x^2 + 25$ is not a difference of squares

► To factor $x^2 + 5x + 6$, notice that 5*1 = 5 and 5 + 1 = 6, so you have (x + 3)(x + 2). To factor $x^2 + 9x + 8$, notice that 8 * 1 = 8 and 8 + 1 = 9, so you have (x + 1)(x + 8). Under the same property, you cannot factor $x^2 + 11x + 2$.

In $x^2 + bx + c$, if c is negative, you will also add its factors to yield b; however, one of the two factors will be negative.

b = 9, c = -10 : b = 10 + (- 1) = 10 - 1
b = -3, c = -10 : b = 2 + (- 5) = 2 - 5
b = 7, c = -10 There is no way to add the factors of c to yield b.

► $x^2 + 4x - 21 = (x + 7)(x - 3)$ b = 4, c = -21; b = 7 + (-3) = 7 - 3

$x^2 - 5x - 36 = (x - 9)(x + 4)$ b = -5, c = -36; b = 4 + (-9) = 4 - 9

► $x^2 - 7x - 7$ cannot be factored

$x^2 - 12x - 8$ cannot be factored

► Factor $7x^2 + 10x - 8$

The factors of 7: 7 and 1. The factors of -8: -8 and 1, -4 and 2, 8 and -1, 4 and -2

Possibilities: 7 * -4 = -28 and 1 * 2 = 2; 2 + (- 28) = -26 ≠ 10
1 * -4 = -4 and 7 * 2 = 14; 14 + (- 4) = 10, meaning these are the factors needed. So, the answer is (7x - 4)(x + 2).

► In a similar fashion to the above, $15x^2 - 11x - 12 = (5x + 3)(3x - 4)$.

Quadratic Formula

Quadratic Formula

► $3x^2 + 27x - 99$; a = 3, b = 27, c = -99

The quadratic formula tells us that:

$$x = \frac{-27 \pm \sqrt{729 - 4*3*-99}}{6} = \frac{-27 \pm \sqrt{729+1188}}{6}$$

729 + 1188 > 0, so the polynomial is factorable.

The factorization is $\left(\frac{x + 27 + \sqrt{1917}}{6}\right)\left(\frac{x + 27 - \sqrt{1917}}{6}\right)$

► $7x^2 - 3x - 2$; a = 7, b = -3, c = -2

The quadratic formula tells us that:

$$x = \frac{3 \pm \sqrt{9 - 56}}{14}$$

9 – 56 = -47 < 0, meaning this polynomial is not factorable.

Completing the Square

Completing the Square

► Complete the square for $x^2 + 18x$.

Step 1: Find c: c = 18/2 = 9.
Step 2: Add and subtract c^2: $(x^2 + 18x + 81) - 81$.
Step 3: Express as a perfect square: $(x + 9)^2 - 81$.

► Complete the square for $x^2 – 11x$.

Step 1: Find c: c = -11/2
Step 2: Add and subtract c^2: $(x^2 - 11x + 121/4) – 121/4$.
Step 3: Express as a perfect square: $(x – 11/2)^2 – 121/4$.

► Complete the square for $x^2 + 5x - 3$.

Step 1: Find "c": c = 5/2
Step 2: Add and subtract "c^2": $(x^2 + 5x + 25/4) – 25/4 - 3$
Step 3: Express as a perfect square: $(x + 5/2)^2 – 37/4$

► Complete the square for $x^2 - 72x + 25$.

Step 1: Find "c": c = -72/2 = -36
Step 2: Add and subtract "c^2": $(x^2 - 72x + 1296) - 1296 + 25$
Step 3: Express as a perfect square: $(x - 36)^2 – 1271$

► Complete the square for $7x^2 + 49x + 11$.

Step 0: $7(x^2 + 7x) + 11$
Step 1: c = 7/2
Step 2: $7(x^2 + 7x + 49/4) – 49/4 * 7 + 89$
Step 3: $7(x + 7/2)^2 + 13/4$