University of Guelph Numeracy Project

About Factoring Quadratics

UNIVERSITY of GUELPH

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About Factoring Quadratics

What is FACTORING?

- Factoring refers to the decomposition of an object into a product of other objects, or factors, which yield the original when multiplied together.

- Factoring objects come in many forms: quadratic polynomials, perfect squares, etc.

Quad Polynomials

Quadratic Polynomials

- Quadratic polynomials are polynomials of order 2, with general form as follows: $ax^2 + bx + c$.

Perfect Squares

Perfect Squares

- Recall that:

  $(x + c)^2 = x^2 + 2cx + c^2$
  $(x - c)^2 = x^2 - 2cx + c^2$

  Reverse these to get:

  $x^2 + 2cx + c^2 = (x + c)^2$
  $x^2 - 2cx + c^2 = (x - c)^2$

- To check if a polynomial is a perfect square, look first at the constant term. This is the c term in the general form: $ax^2 + bx + c$. Is it positive? Is it a square? If not, this method of factoring will not work.

- Case 1: The coefficient of the x^2 term is 1 (i.e. a = 1, in $ax^2 + bx + c$).

  • Take the square root of c (If unable to do this, the polynomial is not a perfect square)
  • Calculate $2\sqrt{c}$.
  • If b = $2\sqrt{c}$, then the factored form is $(x + c)^2$.
  • If b = $-2\sqrt{c}$, then the factored form is $(x - c)^2$.
  • If $b \neq 2\sqrt{c}$ and $b \neq -2\sqrt{c}$, then you do not have a perfect square.

- Case 2: The coefficient of the x^2 term is not 1 (i.e. $a \neq 1$, in $ax^2 + bx + c$).

  • Is a a positive square? If not, you do not have a perfect square.
  • Find $\sqrt{a}$ and $\sqrt{c}$.
  • Calculate $2\sqrt{a}\sqrt{c}$.
  • If b = $2\sqrt{a}\sqrt{c}$, then the factored form is $(ax + c)^2$.
  • If b = $-2\sqrt{a}\sqrt{c}$, then the factored form is $(ax - c)^2$.
  • If $b \neq 2\sqrt{a}\sqrt{c}$ and $b \neq -2\sqrt{a}\sqrt{c}$, then you do not have a perfect square.

Difference of Squares

Difference of Squares

- Notice that $(ax - c)(ax + c) = (ax)^2 - acx + acx - c^2 = (ax)^2 - c^2$. Due to the cancelling of the middle terms, there is ease in factoring the difference of two squares.

- The problem gets a bit harder when a = 1, in ax2 + bx + c. In this case, you have to look at the factors of a and the factors of c. You multiply a factor of a by a factor c and the other factor of a by the other factor of c, summing the results.

Quadratic Formula

Quadratic Formula

- If unsure how to factor a polynomial, use the quadratic formula. It is worth being familiar with the formula, as given below, since it will allow you to know the factored form of a polynomial and positively confirm whether the polynomial is factorable.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

$$(x - \frac{-b + \sqrt{b^2 - 4ac}}{2a})(x - \frac{-b - \sqrt{b^2 - 4ac}}{2a}).$$

Completing the Square

Completing the Square

- Sometimes, instead of factoring an expression, you wish to express it in terms of a perfect square plus a constant. This is called completing the square. What do we need to add to this to make it a perfect square?

  Step 1: Remember the perfect square formulas:
  $(x + c)^2 = x^2 + 2cx + c^2$ and $(x - c)^2 = x^2 + 2cx + c^2$.

  Step 2: We wish to add c^2, to give a perfect square polynomial. However, we do not wish to change the value of the expression, so we must also subtract c^2.

  Step 3: Express as a perfect square.

- When working with quadratics that already have a c term, ignore this term when calculating the constant, but remember it when you complete Step 3.

- For a final complication, suppose the x^2 term has a coefficient $a \neq 1$. This necessitates the addition of a Step 0: Factor a from the x^2 and x terms.

Glossary

Binomial:	a polynomial equation with two terms.
Coefficient:	a constant value multiplying a variable.
Completing the Square:	is a technique in which an expression involving a quadratic polynomial is transformed to one involving a squared linear polynomial and a constant.
Constant term:	a term of fixed value.
Factors:	all the numbers that can be divided evenly into a given number.
Perfect Square:	a term or expression which can be re-expressed as the square of another term or expression.
Polynomial:	algebraic expression consisting of one or more summed terms.
Quadratic Formula:	a method for determining the roots of a quadratic polynomial.
Quadratic Polynomial:	polynomials of order 2.
Square:	a multiplication of a term by itself.

References

http://en.wikipedia.org/