About Limits and Continuity
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About Limits and Continuity

What is a LIMIT?

- Limits are essentially the first step into the wonderful world of Calculus. Limits describe the behaviour of a function as $x$ approaches some number.

- Limits can also be used to determine a function's end behaviour, that is, when $x$ approaches positive infinity ($+\infty$) and negative infinity ($-\infty$).

What is CONTINUITY?

- We say a curve is continuous if you can draw it without having to lift up your pencil.

- If the opposite is true, then we say the curve is discontinuous.

- The point at which you need to lift your pencil, or where the graph is no longer continuous, is called a point of discontinuity.

Introduction to Limits

Introduction to Limits

- We know from past work with functions that if you plug in a certain $x$-value, you will get a $y$ value back. Sometimes, though, we need to know what happens to the $y$-value as a function approaches some $x$-value.

- It is important to know how to read and write limits. We write limits the following way:

\[
\lim_{x \to a} f(x)
\]

This reads as: "the limit of $f(x)$ as $x$ approaches 'a'."
Limits

**Limits**

- Now that we know what limits are, we must introduce some definitions.

Every limit has a right-hand, and a left-hand, limit. Both are needed to find actual limits.

**Right-hand Limit**: The right-hand limit of \( f(x) \) is the value \( f(x) \) approaches as \( x \) is approached from the right (i.e. plugging in values greater than \( 'x' \)). We write this as:

\[
\lim_{x \to a^+} f(x)
\]

**Left-hand Limit**: The left-hand limit of \( f(x) \) is the value \( f(x) \) approaches as \( x \) is approached from the left (i.e. plugging in values less than \( 'x' \)). We write this as:

\[
\lim_{x \to a^-} f(x)
\]

- In order for a limit to exist, the left-hand and right-hand limits must be equivalent.

**Definition of a Limit**

- Having examined right-hand and left-hand limits, we are prepared to give a formal definition of a limit.

1) \( \lim_{x \to a} f(x) \) exists if and only if \( \lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x) \).

2) If \( \lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x) = L \), then \( \lim_{x \to a} f(x) = L \).

3) If \( \lim_{x \to a^+} f(x) = L \) and \( \lim_{x \to a^-} f(x) = M \), where \( L \neq M \), then \( \lim_{x \to a} f(x) \) d.n.e.
Properties of Limits

Before we dive deeper into the topic of limits, we will first discuss some of their properties.

Given the functions \( f(x) \) and \( g(x) \), such that \( \lim_{x \to a} f(x) \) and \( \lim_{x \to a} g(x) \) both exist:

1. \( \lim_{x \to a} c = c \)
2. \( \lim_{x \to a} [f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x) \)
3. \( \lim_{x \to a} [f(x) - g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x) \)
4. \( \lim_{x \to a} [f(x) \cdot g(x)] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x) \)
5. \( \lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)} \) if \( g(x) \neq 0 \) and \( \lim_{x \to a} g(x) \neq 0 \)

- Property 1 tells us that the limit of a constant is that constant.
- Property 2 tells us that the limit of the sum of two or more functions is equivalent to the sum of the limits of those functions.
- Property 3 tells us that the limit of the difference of two or more functions is equivalent to the difference of the limits of those functions.
- Property 4 tells us that the limit of the product of two functions is equivalent to the product of the limits of those functions.
- Property 5 says the limit of the quotient of two functions is equivalent to the quotient of the limits of those functions.
Continuity

Continuity

- Earlier, we said a curve is continuous if we can draw it without lifting our pencil, but now our knowledge of limits can permit a more mathematical definition.

f is continuous at the point a if:

1) \( f(a) \) is defined

2) \( \lim_{x \to a} f(x) \) exists

3) \( f(a) = \lim_{x \to a} f(x) \)

- Since there are three rules for continuity, this implies that there are three types of discontinuity.

The first type of discontinuity is called a "removable discontinuity" (because it looks like someone has "removed" the point), which violates the first condition that \( f(a) \) must be defined.

The second type of discontinuity is called a "jump discontinuity," as it appears the point has "jumped" to a new location, from which the graph continues. This violates the second condition of continuity: that \( \lim f(x) \) must exist.

The final type of discontinuity occurs when both \( f(x) \) and \( \lim f(x) \) exist, but are unequal. This creates another type of "jump," in which the curve maintains its original shape, but has a displaced point.
Glossary

Continuity: When you can draw a curve without lifting your pencil, we say the curve is continuous.

Discontinuity: When you cannot draw a curve without lifting your pencil, we say the curve is discontinuous.

Left-hand Limit: The left-hand limit of \( f(x) \) is the value \( f(x) \) approaches as we approach \( x \) from the left (i.e. plugging in values less than 'x').

Limits: When we approach a certain \( x \) value from the left and right, plugging the values into \( f(x) \), we denote this as "the limit of \( f(x) \) as \( x \) approaches 'a'."

Rational Function: A function in which both its numerator and denominator are functions themselves.

Right-hand Limit: The right-hand limit of \( f(x) \) is the value \( f(x) \) approaches as we approach \( x \) from the right (i.e. plugging in values greater than 'x').
References

