Theoretical and Experimental Characterisations of Optofluidic Lenses with Subunit Horizontal-to-Vertical Aspect Ratios

by

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ABSTRACT

THEORETICAL AND EXPERIMENTAL CHARACTERISATIONS OF OPTOFLUIDIC LENSES WITH SUBUNIT HORIZONTAL-TO-VERTICAL ASPECT RATIOS

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Optofluidic systems are a subset of microfluidics and are used in a variety of applications that require reconfigurable optics, such as detection systems, biochemical applications, spectrometry, and in-plane light focusing. Elliptical optofluidic lenses can provide tunable optical parameters in different optical planes. This tunability is achieved through modifications to the aspect ratio (AR). In this thesis, we present an optofluidic lens with a subunit AR, starting with the theory behind the working of this system, to simulation and finally analysis of the experimental results. In the theoretical analysis, improved tunability of focal length, longitudinal spherical aberration, and beam cone angle is observed in the subunit AR regime compared to the superunit AR regime. In the experimental analysis, the shape of the microdroplet is altered, and a ten percent reduction in AR on applying a high voltage across the microdroplet is observed. We ultimately test and characterise the optofluidic lens in an imaging application.
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# TABLE OF CONTENTS

Abstract .................................................................................................................................................. ii

Acknowledgements .......................................................................................................................... iii

Table of Contents .................................................................................................................................. iv

List of Figures ........................................................................................................................................ vi

list of tables ........................................................................................................................................... ix

1 Introduction ......................................................................................................................................... 1

1.1 Microfluidic Systems ..................................................................................................................... 1

1.1.1 Continuous Flow Microfluidic Systems ..................................................................................... 1

1.1.2 Microdroplet-Based Microfluidic Systems ................................................................................. 3

1.1.3 Digital Microfluidic Systems .................................................................................................... 4

1.2 Optofluidic Lenses ......................................................................................................................... 4

1.3 Thesis Scope and Outline ............................................................................................................... 9

2 Photonic Characterisation .............................................................................................................. 10

2.1 Electromagnetic (EM) Wave Approach ......................................................................................... 10

2.1.1 Electro-Static Equation ............................................................................................................. 11

2.1.2 Magneto-Static Equation ......................................................................................................... 11

2.1.3 Electro-Dynamic Equation ...................................................................................................... 12

2.1.4 Magneto-Dynamic Equation .................................................................................................. 12

2.1.5 Helmholtz Wave Equation ..................................................................................................... 13

2.2 Optics Approach ............................................................................................................................. 16

2.2.1 General Ray Matrix .................................................................................................................. 16

2.2.2 Ray Matrix for Propagation of Light in a Medium .................................................................... 17
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.2.3</td>
<td>Ray Matrix for Refraction at a Flat Interface</td>
<td>18</td>
</tr>
<tr>
<td>2.2.4</td>
<td>Ray Matrix for Refraction at a Curved Interface (Thick Lens)</td>
<td>19</td>
</tr>
<tr>
<td>2.2.5</td>
<td>Ray Matrix for Refraction at a Curved Interface (Thin Lens)</td>
<td>22</td>
</tr>
<tr>
<td>2.3</td>
<td>Geometric Model of the Optofluidic Lens Setup</td>
<td>23</td>
</tr>
<tr>
<td>3</td>
<td>Optofluidic Lens: Theoretical Results</td>
<td>31</td>
</tr>
<tr>
<td>3.1</td>
<td>Ray Tracing Simulation In MATLAB</td>
<td>31</td>
</tr>
<tr>
<td>3.2</td>
<td>Ray Tracing Results</td>
<td>34</td>
</tr>
<tr>
<td>4</td>
<td>Microfluidic Characterisation: Dielectrophoresis (DEP)</td>
<td>40</td>
</tr>
<tr>
<td>5</td>
<td>Optofluidic Lens: Experimental Results</td>
<td>44</td>
</tr>
<tr>
<td>5.1</td>
<td>Optofluidic Tuning</td>
<td>44</td>
</tr>
<tr>
<td>5.2</td>
<td>Optofluidic Implementation</td>
<td>47</td>
</tr>
<tr>
<td>5.3</td>
<td>Limitations</td>
<td>49</td>
</tr>
<tr>
<td>6</td>
<td>Conclusion</td>
<td>51</td>
</tr>
<tr>
<td>6.1</td>
<td>Summary and Conclusions</td>
<td>51</td>
</tr>
<tr>
<td>6.2</td>
<td>Future Work</td>
<td>52</td>
</tr>
<tr>
<td></td>
<td>References</td>
<td>53</td>
</tr>
<tr>
<td></td>
<td>Appendices</td>
<td>60</td>
</tr>
<tr>
<td>APPENDIX A</td>
<td>Main MATLAB Code</td>
<td>60</td>
</tr>
<tr>
<td>APPENDIX B</td>
<td>MATLAB Function used in Code</td>
<td>66</td>
</tr>
<tr>
<td>APPENDIX C</td>
<td>Considerations for Operation with Light Rays Originating Orthogonal to the Optical Axis</td>
<td>69</td>
</tr>
</tbody>
</table>
LIST OF FIGURES

Figure 1. Ellipsoid microdroplet in a 3D Cartesian coordinate system, with the coronal, sagittal, and transverse plane views labelled as CPV, SPV, and TPV, respectively, and hydrophobic base plate labelled as HBP................................................................. 6

Figure 2. Propagation of a ray vector in a medium......................................................... 17

Figure 3. Refraction of a ray vector at a flat interface....................................................... 18

Figure 4. Refraction of a ray vector at a curved interface.................................................. 19

Figure 5. Refraction of a ray vector through a thick lens. .................................................. 20

Figure 6. Refraction of a ray vector through a thin lens.................................................... 22

Figure 7. Modelled optofluidic lens setup in the (a) coronal plane view (positive z-direction), (b) sagittal plane view (negative y-direction), and (c) transverse plane view (negative x-direction). The horizontal line below the centre of the droplet indicates the hydrophobic base plate, below which lies the imagined volume (to complete the ellipsoid) represented by a dotted line. The inset shows Figure 1 again for convenience. .......... 26

Figure 8. Refraction of a ray at a curved surface. ............................................................. 29

Figure 9. Ray tracing simulations comparing a subunit AR = 0.75 optofluidic lens and a superunit AR = 1.25 optofluidic lens in the sagittal plane view, (a) and (c), respectively, and the transverse plane view, (b) and (d), respectively. This is for a fixed volume, fixed input beam width, and fixed image plane and object plane distances. The dark grey volume in these figures represents the portion of the microdroplet ellipsoid that is cut off due to pinning, and the volume of the light grey area is fixed for all the simulations. The focal distance of the AR = 1.00 case is included to compare its relative position with the focal distances of the other ARs................................................................. 33
Figure 10. Focal length from the vertex of the microdroplet ellipsoid, in the five planes, as a function of AR in (a) coarse increments of $\Delta AR = 0.1$ and (b) fine increments of $\Delta AR = 0.01$ in the linear region...

Figure 11. The LSA of the microdroplet ellipsoid as a function of F and AR in (a) coarse increments of $\Delta AR = 0.1$ and (b) fine increments of $\Delta AR = 0.01$ in the linear region. ...

Figure 12. Beam cone angle $\Theta$ for a fixed input beam width, measured at the focal distance of the microdroplet ellipsoid as a function of AR in (a) coarse increments of $\Delta AR = 0.1$ and (b) fine increments of $\Delta AR = 0.01$ in the linear region.

Figure 13. Schematic of the optofluidic lens system for an imaging application, with light from an image passing through the optofluidic lens (i.e., microdroplet) onto a CMOS image sensor. The shape of the optofluidic lens is controlled through an applied voltage onto the indium tin oxide (ITO). The substrate (i.e., glass) adds rigidity onto which the hydrophobic coating is applied.

Figure 14. Ellipsoid microdroplet AR at six different voltages (a) to (f), along with images captured by the CMOS image sensor at each applied voltage. The blue circle is the best-fit circle for the first image, and is overlapped with the other images to facilitate comparison between microdroplet shapes obtained. The results are plotted in the lower image. There is a negligible change in microdroplet shape below applied voltage of 0 V$_{\text{rms}}$.

Figure 15. Images recorded by the CMOS image sensor with the microdroplet ellipsoid (a) at rest, and (b) with the high voltage (800 V$_{\text{rms}}$) applied; along with zoomed-in versions of the resulting image. The outline of the ‘E’ produced with the initial microdroplet shape has been overlapped with the ‘E’ produced after the application of voltage to show the resulting change in dimensions.
Figure 16. Ray tracing simulations comparing (a) a sub-unit AR = 0.75 optofluidic lens, (b) a unit AR = 1.00 optofluidic lens, and (c) a super-unit AR = 1.25 optofluidic lens in the sagittal plane view, with light propagating perpendicular to the substrate.

Figure 17. Focal length from the vertex of the microdroplet ellipsoid, in the five planes, as a function of AR in coarse increments of ΔAR = 0.1, with light rays originating orthogonal to the substrate.
LIST OF TABLES

Table 1: Mean microdroplet and image dimensions and calculations based on the optofluidic implementation. ........................................................................................................................................... 50
1 Introduction
This thesis presents a novel optofluidic lens. In this introduction chapter, microfluidic systems are initially discussed, with details provided on continuous flow, microdroplet-based, and digital microfluidic systems, as these are the technological developments necessary to enable optofluidic lenses. This chapter provides background information and details on the scope of the work.

1.1 Microfluidic Systems

Microfluidic systems—manifesting themselves as lab-on-a-chip devices that can control and manipulate fluids—have a plethora of applications in various fields. These fields include photonics (point-of-care spectroscopy [1,2], micro-opto-electro-mechanical systems (MOEMS) [3,4]), biomedicine (cell enrichment, extraction of blood plasma, solution exchange [5,6]), point-of-care diagnostics (pathogen detection [7], enzyme kinetics analysis [8]), biotechnology (cell culturing and identification [9]), and electronic engineering (radio frequency devices, antennas [10]). These microfluidic systems can be broadly categorised into continuous flow and microdroplet-based microfluidic systems.

1.1.1 Continuous Flow Microfluidic Systems

Continuous flow microfluidic systems involve the manipulation of continuously flowing liquid through microfluidic channels. The motion of liquid within microchannels is enabled through micro-pumps that drive the fluid and valves that control the direction of fluid flow. These continuous flow microfluidic systems can be divided further into subcategorizes
relating to the actuation mechanisms they employ. In piezoelectric actuation, the volume of fluid is determined by the conversion of electric voltage converted to mechanical translation [11]. Thermopneumatic actuation uses temperature control to control the fluid volume, as a fluid is warmed using a thin heater and in turn expands a membrane, that exerts force on a fluid reservoir [12, 13]. Electrostatic actuation uses the force produced between two conducting electrodes when a voltage is applied between these electrodes [14]. Electrochemical actuation uses redox reactions to modify the volume of dopant ions and maintain electrical balance in a conductive polymer membrane, thus causing change in fluid volume [15].

These continuous flow microfluidic systems offer many advantages. One such advantage over larger conventional fluid systems is improved control and manipulation of fluids and fluid interfaces. To facilitate this comparison, Jahn et al. [16] studied the preparation of nanoparticles by continuous flow microfluidics and found that this method provides a tighter particle distribution and homogeneous particle sizes, for applications such as lipid self-assembly. Another major advantage of using continuous flow microfluidic systems is the continuous injection and separation of a fluid or sample for real-time monitoring, rather than using a fixed volume at a time (batch procedure) which is labour intensive and time consuming [17]. Batch procedures also have disadvantages such as a limited loading capacity and accumulation of collected particles, a slow rate of stirring and cooling, and a tendency towards fluctuations in concentration. These disadvantages can be overcome using continuous flow systems as they provide steady state reactions for better control and reproducibility [18, 19]. Continuous flow microfluidic systems are also useful for other
applications such as environment testing and bio warfare agent detection [20], measurement of enzyme kinetics [21], and polymerase chain reactions (PCR) for forensics, cloning, and DNA diagnostics [22].

1.1.2 Microdroplet-Based Microfluidic Systems

In contrast to continuous flow microfluidic systems, microdroplet-based microfluidic systems involve the manipulation of discrete microdroplets that are positioned in microchannels [23, 24]. This reduces reactant volume from microlitres to nano- or picolitres. These microdroplet-based microfluidic systems do not require micro-pumps and -valves, which enables miniaturization and offers higher flexibility, reduced sample consumption, and lower fabrication costs. Microdroplets, within microdroplet-based microfluidic systems, can also be used to partition experiments into individual compartments, which prevents interactions and cross-contamination [25] for applications such as PCR [26]. Although the use of microdroplets has limitations such as scalability and controllability (as individual droplets are manipulated), such compartments can help mimic conditions similar to a single cell in terms of volume, pH, and salt concentration, for studying biological and chemical reactions [27]. Other benefits of microdroplet-based microfluidic systems include increased surface area-to-volume ratio. This enhances mixing and heat transfer and reaction rates. These systems are particularly suitable when solid particle reagents are used, as channel clogging is avoided [28]. These microdroplet-based microfluidic systems have evolved into digital microfluidic systems, wherein the filled fluid is removed, and microdroplets are moved and positioned through electrocapillary forces [24, 29-30].
1.1.3 Digital Microfluidic Systems

In recent years, digital microfluidic systems have gained popularity in MOEMS research, due to their automation, scalability, high throughput, and reduced reagent volumes. They are a subset of microdroplet approaches, wherein several microdroplets can be controlled individually and independently. In digital microfluidic systems, microdroplets are manipulated on open planar surfaces rather than in channels, and each microdroplet can be individually controlled by programmable software [31]. This provides benefits such as reconfigurability of the system and software-based sensing and actuation for fluid control applications [24, 32]. The software can also be programmed for user-defined analytical tests such as fault detection, path planning, and process scheduling [29, 33] and is therefore application-specific unlike continuous flow microfluidic systems. Digital microfluidics can also be used in applications such as clinical diagnostics on human physiological fluids (blood, urine, sweat, etc.) [34], airborne chemical detection, DNA sequencing by synthesis, and tissue engineering [35].

In fact, these digital microfluidic systems have been widely used in photonics, as volume can be carefully controlled for geometric and refractive properties, and this led to the development of optofluidic lenses [36-40], which is the broad topic of this thesis.

1.2 Optofluidic Lenses

In optofluidic lenses, the liquid interfaces of a microdroplet can be tuned via electric fields [41], thereby offering significant advantages over their glass counterparts including in-situ
variability of optical parameters (e.g., focal length and radius of curvature) and scalability of optical dimensions. The electric fields for shaping liquid interfaces can be created through the application of voltage across the microdroplet, and can be used to alter the width-to-height aspect ratio (AR) of the microdroplet and achieve desired optical parameters. This alteration transitions a spherical optofluidic lens into an elliptical optofluidic lens.

Elliptical optofluidic lenses, in contrast with spherical optofluidic lenses, can provide lower longitudinal spherical aberration (LSA) and differing (and tunable) focal lengths in the sagittal (side view) and transverse (top view) planes, i.e., the planes perpendicular and parallel, respectively, to the base plate. These planes are shown in Figure 1, represented in a computer-aided drafting image of the optical lens setup, and discussed in more detail in Section 2.3 (Figure 7). Tunability of the focal length in these different planes (i.e., astigmatism) is particularly useful in laser beam shaping. Specifically, Huang et al. [51] mentioned that a tightly focused circular beam is required for direct laser writing, to fabricate high-quality waveguides. Jabbour et al. [54] studied the use of a diffractive optical element to reshape a beam for applications such as optical lithography and laser-based materials processing. Watts et al. [42] controlled the width of a shaped beam by changing their lens design for flow cytometric analysis. All of these applications can benefit from an elliptical optofluidic lens.
Elliptical optofluidic lenses have been explored in the literature. Zhao et al. [36] utilised laser-induced surface bubbles on a metal film to develop a plasmonic lens that provides advantages such as tunability, reconfigurability, and lower fabrication cost. The optical properties were manipulated by changing flow rates and liquid compositions. Such an integration of microfluidics with plasmonics allows the development of plasmonic elements with multiple functionalities, high sensitivity, and high throughput, for applications such as biomedical detection systems and on-chip optical information processing techniques. Chen et al. [38] used a laser-induced thermal gradient applied to chromium strips at the bottom of a microfluidic chamber to create an optofluidic lens with advantages such as fast tuning, aberration-free focusing, and remote control.
separate study, Chen et al. [39] designed an optofluidic lens for in-plane light focusing (manipulating light in the plane of the substrate) by shaping a silicone oil-air interface in an open microfluidic channel with voltage. Such an optofluidic lens setup is useful for light manipulation in microfluidic networks due to low aberration, low power consumption, high tunability, and simple fabrication, but it suffers from a long switching time from concave to convex, and the requirement of placing the lens at the end of the channel. Chen et al. [40] also studied the applications of in-plane optofluidic lenses for lab-on-a-chip applications, and found that the integration of in-plane beam shaping and microfluidics techniques enables manipulation and analysis of nano-scale particle or cells for biological research and provides a portable system for biochemical applications. For instance, optical tweezers can trap particles with the help of a tightly focused beam for non-destructive applications. Liu et al. [43] amplified an emitted fluorescent signal to focus light on a fluorescent fluidic sample (to enhance lab-on-a-chip performance), by developing an array of optofluidic lenses that were tuned by pneumatic manipulation. Shi et al. [44] demonstrated light enhancement for applications such as microscope imaging, cell sorting, and optical trapping, by tuning an in-plane lens formed by a divergent air-liquid interface with a static PDMS lens using the fluid flow rate. Rosenauer et al. [45] and Li et al. [46] both presented an optofluidic lens which amplified the signal from a dye laser to improve optical sensor systems. This is useful for applications that are limited by the sensitivity of the optical sensor, such as fluorescence spectrometry. The lens is tuned by laminar flow, which forms an interface between carefully chosen core and cladding fluids.
A similar core-cladding setup has also been used by Nguyen et al. [47] for sensing and feedback control of hydrodynamic focusing in lab-on-a-chip devices.

These above examples of optofluidic lenses all have in common a desire for tunability and control in the shape of the microdroplet in each plane. With this in mind, a noteworthy study is the optofluidic lens work of Born et al. [41], where superunit (width-to-height) ARs were used to achieve focusing and retroreflection. (The work of Born et al. defines the AR as height-to-width rather than width-to-height, and is therefore subunit according to their convention, and superunit according to the convention in this work. Here, a width-to-height definition is used as this is the norm in image processing.) This manifestation causes a reduction in the AR of the microdroplet due to an increase in the (electro-) wetting of the hydrophobic base plate. The resulting subunit AR geometry enables a greater degree of focusing in the sagittal plane than in the transverse plane and is thus suitable for applications such as on-chip interconnects and photonic cross-connects [48]. However, the tunability of the elliptical optofluidic lens in this setup is limited by a maximum AR of 1.3, beyond which the focal distance shifts inside the microdroplet due to the oblate shape of the microdroplet.

This internal focusing is detrimental in applications such as laser beam profile tailoring [42]; instead, significantly greater tunability of the system (i.e., higher sensitivity of optical parameters to a change in AR) is needed. Given that it is undesirable to rotate the optofluidic lenses to reverse the sagittal and transverse planes, as gravitational effects and hydrophobicity become significant factors, an elliptical optofluidic lens of subunit AR is required. In response, this thesis develops an optofluidic lens with subunit AR.
1.3 Thesis Scope and Outline

As stated above, this work presents an elliptical optofluidic lens with subunit AR. Subunit AR is achieved by tuning the shape of the microdroplet using dielectrophoresis (DEP), rather than using electrowetting, as in traditional optofluidic lenses [41]. This elliptical optofluidic lens with subunit AR is investigated through the chapters of this thesis.

Photonic characterisation is presented in Chapter 2, wherein the required optical concepts are summarized and implemented. Theoretical results of the optofluidic lens are presented in Chapter 3, wherein investigations are carried out on focal length, LSA, and beam cone angle in sub- and superunit AR regimes. Microfluidic characterisation is presented in Chapter 4, wherein the tunability in the subunit AR regime is shown through dielectrophoresis. Experimental results of the optofluidic lens are presented in Chapter 5, wherein the microfluidic characterisation and optical analysis is connected in an experimental realisation of the elliptical optofluidic lenses. Finally, concluding remarks and recommendations for future work are presented in Chapter 6.
2 Photonic Characterisation

This chapter investigates the Photonic Characterisations required for the analyses. The Photonic characterisation considers both electromagnetic (EM) and optics approaches, as well as the ultimate geometric model of the optofluidic system that is implemented. Although the EM approach is not directly used in this thesis, it is added here for completeness, as it forms the basis for the optics approach.

2.1 Electromagnetic (EM) Wave Approach

In order to understand optical analysis, the propagation of light, i.e. an electro-magnetic (EM) wave, should be considered first. Light is an EM wave that can be characterised by Maxwell’s equations. These equations relate the electric and magnetic vector fields ($E$ and $H$, respectively), as well as the displacement field and magnetic flux density vectors ($D$ and $B$, respectively), all of which vary with position $r$ and time $t$.

Maxwell’s equations characterise an EM wave by describing the divergence of the displacement field $D$ and magnetic flux $B$, and the curl of the electric field $E$ and magnetic field $H$, respectively. The first two equations are the electro- and magneto-static equations. The latter two equations are the electro- and magneto-dynamic equations. Although the latter two equations are of particular interest for the propagation of EM waves, the former two equations will be considered first as they are required for understanding of other elements of the optofluidic analyses.
2.1.1 Electro-Static Equation

Maxwell's first equation states that the divergence of $D$ equals the free charge density $\rho_f$:

$$\nabla \cdot D = \rho_f. \quad (1)$$

A re-arrangement of these terms leads to Gauss' electrostatic law that states that electric flux lines move from a positive to a negative charge in an electrostatic field, and the electric charge enclosed is equal to the surface integral of the electric flux density. This also leads to Coulomb’s law which states that the electric flux density equals the charge divided by the surface area of the sphere, implying a diminishing electric flux density with increased distance from a point charge. In this work, an electric field is applied across an (initially) spherical microdroplet, producing electric flux lines that tune the shape of the microdroplet. This is discussed in more detail in Chapter 4.

2.1.2 Magneto-Static Equation

Maxwell's second equation states that the divergence of $B$ equals zero; i.e. the magnetic field entering a point equals the magnetic field leaving the point:

$$\nabla \cdot B = 0. \quad (2)$$

Similar to Maxwell's first equation, rearranging the terms gives Gauss' magnetic law, which states that the net magnetic flux out of any closed surface is zero. In this work, electric fields are used instead of magnetic fields, so Maxwell's second law is not of interest as much as Maxwell's first law.
2.1.3 Electro-Dynamic Equation

Maxwell's third equation states that the curl of $E$ equals the negative time-rate-of-change of $B$:

$$\nabla \times E = -\frac{\partial B}{\partial t}. \tag{3}$$

This equation leads to Faraday's law, which states that changing the magnetic flux in a closed wire loop produces a voltage within the circuit. This voltage is termed the electro-motive force. We will later see that Maxwell's third equation, in combination with Maxwell's fourth equation, can be used to directly show the propagation of EM waves, as the electric and magnetic fields are connected and spur each other forward.

2.1.4 Magneto-Dynamic Equation

The fourth equation states that the curl of $B$ equals the time-rate-of-change of $D$ and the free current density $J_f$, i.e.

$$\nabla \times H = J_f + \frac{\partial D}{\partial t}. \tag{4}$$

From this equation, we can derive Ampere's law for zero displacement current ($\partial D/\partial t = 0$), which states that the enclosed current in a wire loop equals the line integral of the magnetic field wrapping around it.

In contrast to Ampere's law, time-varying cases are of interest for the propagation of electromagnetic radiation. In fact, the electro- and magneto-dynamic equations can be
combined to show how light and EM waves propagate. This is necessary for our understanding of light propagation through an optofluidic lens.

2.1.5 Helmholtz Wave Equation

The last two of Maxwell's equations deal with the time and spatial connection of electric and magnetic fields and can be written, respectively, as

\[ \nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} = \varepsilon_r \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \]  
(5)

and

\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} = -\mu_r \mu_0 \frac{\partial \mathbf{H}}{\partial t}, \]  
(6)

where \( \varepsilon_r \) is the relative permittivity, \( \varepsilon_0 \) is the free space permittivity, \( \mu_r \) is the relative permeability, and \( \mu_0 \) is the free space permeability. The equations can be combined to get the Helmholtz wave equation. This derivation begins by taking the partial derivative of (5) with respect to time to get

\[ \nabla \times \frac{\partial}{\partial t} \mathbf{H} = \varepsilon_r \varepsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2}. \]  
(7)

Substituting (6) into (7) and simplifying, we progress from

\[ \nabla \times \frac{-(\nabla \times \mathbf{E})}{\mu_r \mu_0} = \varepsilon_r \varepsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} \]  
(8)

to

\[ -\nabla \times \nabla \times \mathbf{E} = \varepsilon_r \varepsilon_0 \mu_r \mu_0 \frac{\partial^2 \mathbf{E}}{\partial t^2}. \]  
(9)
Through the mathematical relation, $∇ × ∇ × E = ∇(∇ · E) − ∇^2 E$, and noting that the divergence term is zero non-conductive medium (from Maxwell’s first equation), the Helmholtz wave equation can be expressed as

$$∇^2 E = εμ \frac{∂^2 E}{∂t^2}$$

(10)

or

$$∇^2 E − εμ \frac{∂^2 E}{∂t^2} = 0.$$  

(11)

A one-dimensional (1D) time-varying electric field propagating in the $z$ direction, with speed $ν$ and amplitude $A$ and electric field in the $x$ direction and magnetic field in the $y$ direction, can be characterised by the equation $E_x(z, t) = A \sin(\omega(t - z/ν))$ for $x$ direction electric field. This $x$ direction electric field equation and can be shown to satisfy the wave equation, as follows:

$$∇^2 E = \frac{∂^2 E_x}{∂x^2} = \left(\frac{ω}{ν}\right)^2 \sin\left(\omega \left(t - \frac{z}{ν}\right)\right) = \left(\frac{ω}{ν}\right)^2 \sin \left[\omega \left(t - \frac{z}{ν}\right)\right]$$

(12)

and

$$εμ \frac{∂^2 E}{∂t^2} = (εμ) \left(−ω^2 \sin \left[ω \left(t - \frac{z}{ν}\right)\right]\right).$$

(13)

Equating (12) and (13) reveals that

$$−\left(\frac{ω}{ν}\right)^2 \sin \left[ω \left(t - \frac{z}{ν}\right)\right] = (εμ) \left(−ω^2 \sin \left[ω \left(t - \frac{z}{ν}\right)\right]\right).$$

(14)
Since $\omega$ and $\sin$ terms in (14) cancel, and we are finally left with

$$\frac{1}{v^2} = \varepsilon \mu \Rightarrow v = \frac{1}{\sqrt{\varepsilon \mu}}.$$  \hspace{1cm} (15)

This is the speed of the EM wave. As such, (14) represents a travelling wave for which the position $z$ must change as time $t$ changes. For this equation, the crests are located at 

$$\omega \left(t - \frac{z}{v}\right) = (4n + 1)\frac{\pi}{2},$$

where $n = 0, 1, 2, \ldots$. Therefore, the position $z$ is given by $z = v t - \frac{(4n+1)\pi v}{2\omega}$. As the second term is a constant, and the first term is velocity multiplied by time, this means that as the time changes, the position (peak location) also changes. Therefore, light must move as its distance changes with time and this movement is at the speed of light.

This EM wave approach is the most thorough analysis possible and will show a complete picture of the movement of light through an optical system. However, simulations of EM waves (typically finite-difference time-domain analyses) are computationally expensive. Given that the feature sizes of our optofluidic lens ($\gg$ microns) are much larger than the wavelengths of light ($<\text{one micron}$), this EM wave approach is unnecessary. As such, an optical approach is considered next, which is typically used when feature sizes of optical systems are much larger than the wavelength.
2.2 Optics Approach

For feature sizes much larger than the wavelength ($\gg \lambda$), a ray approximation (approximate solution to Maxwell’s equations) can be used. Here, the propagation of light is represented by a straight line (ray vector) in the direction of propagation, rather than a wave. This simplifies optical and geometrical analyses of the system. Ray matrices relevant to this work are discussed below.

2.2.1 General Ray Matrix

The input and output ray vectors are represented by column vectors with two rows, the first row being the distances $r_1$ and $r_2$ of the starting points of the rays from the optical axis (OA), and the second row being the angles $r_1'$ and $r_2'$ between the ray vectors and the OA. The output ray from an optical element is obtained by multiplying the input ray vector with a unique 2×2 ray matrix describing the optical interaction. This ray matrix $M$ is characteristic to the optical element, and a general form is given by

\[
\begin{bmatrix}
    r_2 \\
    r_2'
\end{bmatrix}
= M \begin{bmatrix}
    r_1 \\
    r_1'
\end{bmatrix},
\]

(16)

where

\[
M = \begin{bmatrix}
    A & B \\
    C & D
\end{bmatrix}.
\]

(17)

Here, the variables $A$, $B$, $C$, and $D$ describe the optical interaction and modification of the distance and angle of the ray vector with respect to the OA.
2.2.2 Ray Matrix for Propagation of Light in a Medium

For light propagating in a medium, there will be no change in the angle between the ray vector and the OA, so $r_2' = r_1'$, as shown Figure 2 below.

If the refractive index of the medium is $n_1$ and the distance propagated is $d$, the height $r_2$ is given by $r_2 = r_1 + (\tan r_1')d$. Also, the angles and heights are considered to be small compared to the dimensions of the optical system (paraxial rays), so we can make use of the small angle approximation $\tan \theta \approx \theta$, so $r_2 = r_1 + r_1'd$. The ray matrix for propagation of light in a medium can therefore be calculated to be

\[
\begin{bmatrix}
    r_2' \\
    r_2
\end{bmatrix} =
\begin{bmatrix}
    1 & d \\
    0 & 1
\end{bmatrix}
\begin{bmatrix}
    r_1' \\
    r_1
\end{bmatrix}.
\]

(18)

It is worth noting that this ray matrix does not depend on the refractive index of the medium.

Figure 2. Propagation of a ray vector in a medium.
2.2.3 Ray Matrix for Refraction at a Flat Interface

A ray travelling through an interface between two media with refractive indices $n_1$ and $n_2$ respectively is shown in Figure 3 below.

As the input ray reaches a point on the interface, the output ray is refracted from the same point, so $r_2 = r_1$, as shown in the figure. The refracted ray can be calculated using Snell’s law $n_1 \sin r_1' = n_2 \sin r_2'$. For small angles, $\sin \theta \approx \theta$, so $n_1 r_1' = n_2 r_2'$. A re-arrangement yields $r_2' = \left( \frac{n_1}{n_2} \right) r_1'$, which leads to the following ray matrix:

$$
\begin{bmatrix}
    r_2 \\
    r_2'
\end{bmatrix} =
\begin{bmatrix}
    1 & 0 \\
    0 & \frac{n_1}{n_2}
\end{bmatrix}
\begin{bmatrix}
    r_1 \\
    r_1'
\end{bmatrix}.
$$

(20)

Figure 3. Refraction of a ray vector at a flat interface.
2.2.4 Ray Matrix for Refraction at a Curved Interface (Thick Lens)

As we are later considering optofluidic lenses, the ray matrix for a curved interface should be considered. When a ray travels through a curved interface, the normal vector $\mathbf{N}$ to the curved interface at that point must be considered, as well as the radius of curvature $R$. This introduces three additional angles that need to be considered: the incidence angle $\theta_i$ between the input ray and the normal, the transmittance angle $\theta_t$ between the output ray and the normal, and the angle $\phi$ between the normal and the optical axis. These angles are labelled in Figure 4 below.

From this figure, we have $r_2 = r_1$, $\theta_t = r_2' + \phi$, and $\theta_i = r_1' + \phi$. Snell’s law therefore becomes $n_1 (r_1' + \phi) = n_2 (r_2' + \phi)$. Using the small angle approximation, $\phi \approx \frac{r_1}{R}$.

Figure 4. Refraction of a ray vector at a curved interface.
Rearranging the terms, we get $r_2' = -\frac{n_2-n_1}{n_2R} r_1 + \frac{n_1}{n_2} r_1'$, which gives us the ray matrix for a curved interface:

$$\begin{bmatrix} r_2' \\ r_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{n_1-n_2} & 0 \\ \frac{n_1}{n_2R} & \frac{n_1}{n_2} \end{bmatrix} \begin{bmatrix} r_1' \\ r_1 \end{bmatrix}. \quad (22)$$

Using this ray matrix for a curved interface, we can derive the ray matrix for a thick lens. A thick lens consists of two curved interfaces separated by a distance, such that the total vertex-to-vertex thickness of the lens is $t$. To simplify calculation, a collimated input ray is considered ($r_1' = 0$), and an output ray reaching the focal point $f$ on the optical axis is considered ($r_2 = 0$). This is shown in Figure 5 below.

![Figure 5. Refraction of a ray vector through a thick lens.](image)
Here, the input ray is refracted at the first curved interface with radius of curvature $R_1$, and the ray matrix for this refraction is

$$M_1 = \begin{bmatrix} 1 & -n_2 & 0 \\ n_2R_1 & n_1 \\ n_2 \\ n_1 \\ n_1 \\ n_2 \\ n_2 \\ n_1 \\ n_2 \\ n_1 \end{bmatrix}.$$  \hfill (24)

This ray then propagates through the lens for a distance $d$, and the ray matrix for this propagation is

$$M_2 = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}.$$  \hfill (25)

Finally, the ray is refracted at the second curved interface with radius of curvature $R_2$, and the ray matrix for this refraction is

$$M_3 = \begin{bmatrix} 1 & -n_1 & 0 \\ n_2R_2 & n_2 \\ n_2 \\ n_1 \\ n_2 \\ n_1 \\ n_2 \\ n_1 \end{bmatrix}.$$  \hfill (26)

The overall ray matrix for this system is therefore a multiplication of the three constituent ray matrices, i.e., the ray matrix for this refraction is $M = M_3M_2M_1$. Thus, we have

$$M = \begin{bmatrix} n_1 & -n_2 & 0 \\ n_2 & 0 & 0 \\ n_2 & 0 & 0 \\ n_2R & n_1R_1 & n_1R_2 \\ n_2 & n_1 & n_1 \\ n_2 & n_1 & n_1 \\ n_2 & n_1 & n_1 \end{bmatrix}.$$  \hfill (27)

If the incident medium is air, we can assume $n_1 = 1$, and $n_2 = n$. The multiplication therefore yields:
\[ M = \begin{bmatrix} 1 + d \left( \frac{1 - n}{nR_1} \right) & \frac{d}{n} \\ - \left( \frac{1 - n}{R_2} \right) \left( 1 + d \left( \frac{1 - n}{nR_1} \right) \right) + \frac{1 - n}{R_1} & 1 - \frac{d}{n} \left( \frac{1 - n}{nR_2} \right) \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}. \quad (28) \]

Relating this equation to a thin lens equation (described in the next subsection) reveals the focal length of the thick lens, which is needed for the coming optofluidic analyses.

### 2.2.5 Ray Matrix for Refraction at a Curved Interface (Thin Lens)

The ray matrix derived for the thick lens can also be used for a thin lens, as the setup remains the same but with \( d \to 0 \) as shown in Figure 6.

![Figure 6. Refraction of a ray vector through a thin lens.](image)
As \( d \to 0 \), the matrix coefficients (of the thick lens ray matrix) are as follows: \( A \to 1 \), \( B \to 0 \), \( D \to 1 \). The thin lens ray matrix therefore simplifies to \[
\begin{bmatrix}
0 \\
r_2'
\end{bmatrix}
= \begin{bmatrix}
1 & 0 \\
C & 0
\end{bmatrix}
\begin{bmatrix}
r_1 \\
0
\end{bmatrix};
\]
i.e., \( r_2' = Cr_1 \).

From Figure 6, \( \tan(-r_2') \approx -r_2' = \frac{r_1}{f} = -Cr_1 \); therefore \( C = -\frac{1}{f} \). Thus, the ray matrix for a thin lens is

\[
M = \begin{bmatrix}
1 & 0 \\
-\frac{1}{f} & 1
\end{bmatrix}.
\] (29)

Similarly, applying \( C = -\frac{1}{f} \) to the thick lens ray matrix, we get

\[
C = -\left(\frac{1-n}{R_2}\right)
\left(1 + d\left(\frac{1-n}{nR_1}\right)\right)
+ \left(\frac{1-n}{R_1}\right) = -\frac{1}{f}.
\] (30)

\[
\therefore f = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{d(n-1)}{nR_1R_2}\right).
\] (31)

Equation (30) is the Lensmaker’s equation and it calculates the focal length for a thick lens. This equation will be used in this thesis to validate simulation and experimental results.

### 2.3 Geometric Model of the Optofluidic Lens Setup

The material in this section is heavily taken from work previously published in Applied Optics [49] and as a presentation at the Photonics North 2018 conference.
The above Lensmaker's equation for a thick lens is required for the analysis of this thesis; however, a geometric model of the optofluidic lens setup is also required. This optofluidic lens setup is a microdroplet at rest on a hydrophobic base plate. The microdroplet is modelled as an ellipsoid, and uniform vertical stretching is assumed. This makes the microdroplet elliptical in the sagittal and coronal planes, and circular in the transverse plane, as shown in Figure 1 and the inset of Figure 7 below. It is assumed that the surfaces of the microdroplet can be approximated to be spherical, so a radius of curvature for each surface can be obtained and used for refractive calculations.

Experimental characterisations of the optofluidic lenses will be considered later in the thesis; however, at this point it is important to note that the positioning of the microdroplet on the hydrophobic base plate results in a pinning effect, i.e., the perimeter of the microfluid contact is ‘pinned’ to the base plate. (It is believed that the pinning along the contact line is the result of friction forces whereby the horizontal motion of the microdroplet is inhibited, restricting the microdroplet to remain positioned along the same contact line [50].) This results in the microdroplet having a flat circular base, the radius of which will stay constant as the AR of the microdroplet changes. As such, in the microdroplet modelling, the ellipsoid microdroplet is divided into the actual volume, being the true volume of the microdroplet, and a pseudo-volume, being the imagined volume of the remaining ellipsoid (i.e., the volume of the portion of the ellipsoid that is below the base plate). The true volume remains constant during experimentation and simulation.

The modelled optofluidic lens setup is shown in Figure 7. Here, we consider five different optical planes with respect to the center of the microdroplet. The 0° plane (transverse
plane) is defined as being parallel to the base plate. The 90° plane (sagittal plane) is defined as being perpendicular to the base plate. The 30°, 45°, and 60° planes form respective angles of 30°, 45°, and 60° with respect to the hydrophobic base plate.

The coronal plane view (i.e., the front view) is shown in Figure 7(a) and is from the perspective of the origin of the light rays (red crosses indicating propagation into the page). Here, the vertical and horizontal axis radii of the ellipsoid are \(a\) and \(b\), respectively. The AR is defined as the ratio of the horizontal axis radius to the vertical axis radius, i.e., \(AR = b/a\). From Figures 6(b) and (c), it is clear that the focal length \(F_{90}\) in the sagittal plane view is longer than the focal length \(F_0\) in the transverse plane view, due to the subunit AR of the ellipsoid microdroplet; therefore, the focal length of each axis can be modified and tuned for specific applications.

In Figure 7, two extreme rays are shown to begin at the object plane and terminate at the image plane, as would be the case in a focusing application. In contrast, an imaging application would have light traveling from image plane to object plane.
Figure 7. Modelled optofluidic lens setup in the (a) coronal plane view (positive z-direction), (b) sagittal plane view (negative y-direction), and (c) transverse plane view (negative x-direction). The horizontal line below the centre of the droplet indicates the hydrophobic base plate, below which lies the imagined volume (to complete the ellipsoid) represented by a dotted line. The inset shows Figure 1 again for convenience.
Given the ability to shape the microdroplet in a subunit AR regime, we analyse the optical behaviour of the ellipsoid microdroplet. This optical analysis involves the propagation of light rays through the ellipsoid microdroplet, in the form of ray tracing and simulation results.

The first step in the ray tracing simulation process is to describe a microdroplet ellipsoid that will have an elliptical cross-section in the side view and a circular cross-section in the top view, due to the microdroplet ellipsoid being stretched vertically (positive $x$-direction).

For ray tracing, we consider a fixed fluid volume of 2 μL, which is a common size for micro-optofluidic systems. However, given that this is in a scalable regime (bond number less than 0.3), the results can be scaled as needed to represent another optofluidic lens. This scalability is dictated by the cubic relationship between volume and distance. Specifically, there is a relationship of volume $\propto$ distance$^{1/3}$. Based on experimental observations of microfluidic behaviour, the width of the base of the microdroplet ellipsoid has been set to 65% of the initial diameter. The microdroplet ellipsoid is centered at the origin and is defined by the ellipsoid equation,

$$
\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2 = 1.
$$

Here, $c$ denotes the axis radius of the ellipsoid in the $z$-direction. Due to the symmetry of the microdroplet in the transverse plane, the relation $c = b$ holds true. (In contrast, a spherical microdroplet has $a = b = 1$, and AR = 1.00.)

With the microdroplet ellipsoid generated, we begin ray tracing. A light beam of a fixed width (15% of the height of the initial spherical microdroplet, i.e., AR = 1.00 case) is...
centered about the optical axis, which is in the vertical centre of the microdroplet. Each light ray in the light beam is modelled as a vector. We choose a starting point and direction at a fixed distance from the microdroplet, and an initial propagation direction $<0, 0, 1>$ to represent propagation in the positive $z$-direction. As this ray vector reaches the first interface, we need to compute the resulting ray vector that will propagate through the microdroplet ellipsoid. The ray matrices described in Chapter 2 work based on the assumption that small angles and heights from the OA are used (i.e., paraxial rays). However, this may not produce accurate results for rays further away from the OA, and the small angle approximation no longer holds – so in this case, Snell’s law in vector form is used. Figure 8 below shows the variables required for this calculation: an input ray $s_1$, an output ray $s_2$, the normal $N$ to the surface at the point of intersection, and the two refractive indices $n_1$ and $n_2$. 
The standard form of Snell’s law is \( n_1 \sin r_1' = n_2 \sin r_2' \), as mentioned earlier in this chapter. This can be re-arranged to get \( \sin(r_2') = \frac{n_1}{n_2} \sin(r_1') \). The output ray can be computed by the vector addition of the component along the normal and the component perpendicular to the normal. For simplicity, unit vectors are considered (\(|N| = 1, |s_1| = 1\)). The component along the normal (\(-N\), due to the orientation of the vector) is given by

\[
-N \cdot s_2 = -|N||s_2| \cos(r_2') = -N \sqrt{1 - \sin^2(r_2')} = -N \sqrt{1 - \left(\frac{n_1}{n_2}\right)^2 \sin^2(r_1')}. \tag{33}
\]

As we can note that \( N \times s_1 = |N||s_1| \sin(r_1') \eta_1 \), we can rearrange to obtain
\[
\sin(r_1') = \frac{N \times s_1}{\eta_1},
\]

and the component along the normal becomes \[-N \sqrt{1 - \left(\frac{n_1}{n_2}\right)^2 (N \times s_1) \cdot (N \times s_1)}.\]

Similarly, the component perpendicular to the normal is given by

\[
N \times s_2 = \sin(r_2') (N \eta) = \frac{n_1}{n_2} \sin(r_1') (N \eta) = \frac{n_1}{n_2} - \frac{N \times s_1}{\eta} (N \eta) = \frac{n_1}{n_2} [N \times (-N \times s_1)].
\]

Finally, we add the two components to obtain the output ray:

\[
s_2 = \frac{n_1}{n_2} [N \times (-N \times s_1)] - N \sqrt{1 - \left(\frac{n_1}{n_2}\right)^2 (N \times s_1) \cdot (N \times s_1)}.
\]

This resulting vector \(s_2\) models an optical beam that propagates until it intersects the second interface, and this equation is used again to compute the final output ray. Finally, the intersection of this output ray with the optical axis determines the focal length of the microdroplet ellipsoid in the plane of interest. For validation, our ray tracing results are compared against open source ray tracing software (MATLAB, RayOptik package), with nearly identical results.
3 Optofluidic Lens: Theoretical Results

Given the photonic characterisation of an optical system, we analyse the optical behaviour of an ellipsoid microdroplet. This optical analysis involves the propagation of light rays through the ellipsoid microdroplet, in the form of ray tracing and simulation results.

The material in this section is heavily taken from work previously published in Applied Optics [49] and as a presentation at the Photonics North 2018 conference.

3.1 Ray Tracing Simulation In MATLAB

From the ray tracing simulations for subunit and superunit AR geometries in different views (Figure 9), we observe that, for a subunit AR lens, the focal distance in the sagittal plane is significantly further from the lens compared to the AR = 1.00 case, while in the transverse plane it differs only by a small amount and is shorter than the AR = 1.00 case. Conversely, the beam cone angle in the sagittal plane is much smaller in the transverse plane, resulting in a smaller spread of the output light rays. These focal length and beam cone angle effects are as expected given the reduced radius of curvature of the microdroplet in the transverse plane, which causes an increased refraction of the light rays. For this same reason, an optofluidic lens with superunit AR follows the opposite trend in terms of the focal length and beam cone angle, as the lens shape in the sagittal plane has a smaller radius of curvature compared with that of the transverse plane.
In both the subunit and superunit AR cases, the change in the focal length is greater in the sagittal plane—this is because the optofluidic lens is elliptical in the sagittal plane, so the radius of curvature varies non-linearly with AR compared to the variation in the transverse plane where the lens is circular. These effects imply that, for a collimated beam as an input, the shape of the beam would have greater change in the transverse plane than in the sagittal plane for a subunit AR lens and the reverse for a superunit AR lens. This is a useful result for applications such as laser beam tailoring [51-54] which require distinct focal lengths in different planes.

It should be noted that these simulations are carried out for a fixed collimated input beam, as would be the case in focusing applications. For imaging applications, the light rays from the image will pass through the focal distance of the lens to produce a (roughly) collimated output beam. This in turn would cause the opposite effect to that observed in Figure 9, which implies that, for a fixed object size, the subunit AR lens would produce a larger image compared to the unit and superunit lenses in the sagittal plane, and a smaller image in the transverse plane. The numerical results of the simulations are summarized in the following subsection.
Figure 9. Ray tracing simulations comparing a subunit $AR = 0.75$ optofluidic lens and a superunit $AR = 1.25$ optofluidic lens in the sagittal plane view, (a) and (c), respectively, and the transverse plane view, (b) and (d), respectively. This is for a fixed volume, fixed input beam width, and fixed image plane and object plane distances. The dark grey volume in these figures represents the portion of the microdroplet ellipsoid that is cut off due to pinning, and the volume of the light grey area is fixed for all the simulations. The focal distance of the $AR = 1.00$ case is included to compare its relative position with the focal distances of the other ARs.
3.2 Ray Tracing Results

We use ray tracing and other optical analyses (described in Chapter 2) to perform simulations to extract focal length, longitudinal spherical aberration (LSA, which accounts for the difference in focal lengths for light rays at different heights from the optical axis), and beam cone angle, and, ultimately, to quantify performance of various optofluidic lenses at several optical planes. The computer code for these simulations can be found in Appendices A and B. It should be noted that only operation with light rays originating parallel to the optical axis is considered here. Appendix C includes considerations for operation with light rays originating orthogonal to the optical axis.

The first parameter that we consider through simulations and optical analyses is focal length. The focal length is obtained by subtracting the horizontal axis radius $b$ from the point of intersection of a paraxial output ray and the optical axis in the plane of interest (the microdroplet ellipsoid is situated at the origin). As such, the theoretical paraxial focal length, $F$, is calculated using the thick lens formula to be

$$\frac{1}{F} = (n_2 - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} + \frac{(n_2 - 1)d}{n_2 R_1 R_2} \right)$$

for a total vertex-to-vertex lens thickness of $d$ and radii of curvature of the two interfaces of $R_1$ and $R_2$, respectively. Due to the symmetry of the microdroplet ellipsoid, $R_1 = -R_2$.

We then study focal lengths at different ARs in different planes (Figure 10) from a lower limit of $AR = 0.50$, due to the increasingly high voltages required to reduce the AR as
observed during the experimental realisation (Section 5.1), to an upper limit of AR = 1.30, beyond which internal focusing takes place with negative focal lengths.

Figure 10. Focal length from the vertex of the microdroplet ellipsoid, in the five planes, as a function of AR in (a) coarse increments of $\Delta AR = 0.1$ and (b) fine increments of $\Delta AR = 0.01$ in the linear region.
From the figure, we observe that as the optical plane transitions from the 0° plane (transverse plane) to the 90° plane (sagittal plane), the curves transition from a gradual monotonically-increasing curve to a rapid monotonically-decreasing curve. All curves converge at a unit AR as the microdroplet ellipsoid becomes a perfect sphere in this case and is therefore symmetrical about the z-axis. The extreme change in the curve of the 90° plane shows a sensitivity to change in AR (i.e., changes in the height of the ellipsoid microdroplet). The smaller change in the curve of the 0° plane shows a relative insensitivity to change in AR and the intermediate optical planes, being 30°, 45°, and 60° planes, form intermediate trends between these extremes. For the superunit AR regime, there are only minor differences between focal length of the 0° and 90° planes; for the subunit AR regime, there are extreme differences between focal length of these optical planes. As such, we can conclude that there is tremendous potential for tunability in a subunit AR regime. For the linear range of AR in Figure 10(b), there is an almost-linear relationship between \( F \) an AR. As such, this range of operation would be ideal for applications with integrated classical control systems.

The second parameter that we consider through simulations and optical analyses is LSA. The LSA is defined as the difference between the paraxial focal length and the focal length of the marginal ray from the optical axis used in the simulation, being 15% of the vertical axis radius \( a \) for the AR = 1.00 case. Therefore, the LSA quantifies how much the extreme light ray will disagree with the paraxial focal length. For a better understanding of this quantification, we consider the relative parameter LSA/\( F \) for analysis, rather than the absolute LSA parameter alone. Due to the (roughly) reciprocal relationship that has been
established between LSA and $F$, it is expected that the results (Figure 11) show a reciprocal pattern compared to the focal length in Figure 10 and this is seen to be true. The results in Figure 11(b) also show a similar linear range, implying that in this range it is easier to predict the amount of aberration and therefore tune the lens accordingly.

Figure 11. The LSA of the microdroplet ellipsoid as a function of $F$ and AR in (a) coarse increments of $\Delta AR = 0.1$ and (b) fine increments of $\Delta AR = 0.01$ in the linear region.
The third parameter that we consider through simulations and optical analyses is the beam cone angle, $\theta$. This beam cone angle defines the maximum spread of output rays from the focal distance (as seen in Figure 7(b) and (c)) based on an input beam of fixed width and therefore determines the degree to which an input beam can be reshaped. On graphing the beam cone angle at different ARs (Figure 12), we observe a trend similar to the LSA/F trend in Figure 11 in that the sagittal plane curve increases rapidly while the transverse plane decreases comparatively gradually. This trend agrees with the beam fan-out observed during ray tracing in Figure 9. The most significant outcome of this result, as in the case of the focal length, is that in the superunit AR regime the beam cone angle variations between the sagittal and transverse planes are much smaller compared to these variations in the subunit AR regime, indicating improved tunability in the subunit AR regime.

Based on the simulation results for the three parameters, we can conclude that subunit AR lenses offer a significantly greater tunability, through heightened sensitivity to changes in AR, over superunit AR lenses. This tunability, combined with the linearity observed in the AR ± 0.1 region and the ability to focus or reshape the beam differently in the transverse and sagittal planes, indicates that an optofluidic lens in the subunit AR regime is well-suited to applications such as laser beam profile tailoring [51-54].
Figure 12. Beam cone angle $\Theta$ for a fixed input beam width, measured at the focal distance of the microdroplet ellipsoid as a function of AR in (a) coarse increments of $\Delta AR = 0.1$ and (b) fine increments of $\Delta AR = 0.01$ in the linear region.
4 Microfluidic Characterisation: Dielectrophoresis (DEP)

This chapter investigates the Microfluidic Characterisation required for the analyses. The microfluidic characterisation considers dielectrophoresis considerations. The material in this section is heavily taken from work previously published in Applied Optics [49] and as a presentation at the Photonics North 2018 conference.

In order to achieve focal length tunability discussed in Chapter 3, the shape of the fluid micropdroplet must be controlled. Two methods for tuning an optofluidic lens are electrowetting and dielectrophoresis (DEP). Electrowetting is a phenomenon by which the conducting liquid-solid interface (contact line) is altered by an electrostatic force [55]. DEP is a bulk phenomenon that occurs due to polarization induced in the dielectric liquid, by a non-uniform electric field [34]. Unlike electrowetting, DEP does not require a conducting liquid to function. Additionally, as electrowetting increases the wettability of the liquid-solid interface, it would cause a spherical microdroplet to obtain a superunit AR (as in Born et al. [41]), while DEP can help produce subunit ARs and is therefore the chosen method for the optofluidic lens setup presented here.

In this work, the chosen liquid for experimentation is deionized (DI) water. In terms of optical properties, DI water was chosen as the refractive index of DI water, $n = 1.33$, is notably different to the refractive index of surrounding air, $n = 1.00$. Additionally, its absorption coefficient of $3 \times 10^{-4}$ cm$^{-1}$ is sufficiently low that there is negligible loss of optical power through the optofluidic lenses. In terms of electrical properties, DI water is suitable as it has a large dielectric constant of $\varepsilon_r = 80$. Such a dielectric constant is
sufficient for manipulation of the shape of a microdroplet using the electric field from an applied voltage. In terms of wetting properties, DI water was chosen as it remains unencumbered by gravitational effects up to relatively large sizes, which help facilitate the experimental characterisation and (manual) microdroplet dispensing. The bond number, $Bo$, can be estimated through $Bo = g L^2 (\rho_L - \rho_G) / \sigma$, where $g = 9.8 \text{ m/s}^2$ is gravitational acceleration, $R = 1,400 \mu\text{m}$ is the radius of the microdroplet, $\rho_L = 1,000 \text{ kg/m}^3$ is the liquid density, $\rho_G = 1 \text{ kg/m}^3$ is the air density, and $\sigma = 70 \text{ mN/m}$ is the surface tension. In this case the estimated bond number is below 0.3 for our experiments, indicating minimal gravitational effects [56]. As the operation is occurring within a regime with minimal gravitational effects, the results of our study can be scaled down to much smaller scales. However, as the scale reduces closer to the wavelength scale, apart from practicality issues (size of the syringe required, quality of the camera to take such an image, etc.), the ray approximation would no longer hold true and EM analysis would need to be carried out, as discussed in Chapter 2.

As shown in the schematic of Figure 13, the microdroplet is placed on a base plate, which is coated with NeverWet™ hydrophobic coating as it is commercially (and readily) available and can be used in low-cost optofluidic devices [57]. This hydrophobic coating comes in two parts, being the NeverWet™ base coat and NeverWet™ top coat, and is applied to the glass substrate. To apply the hydrophobic coating, the glass substrate is cleaned of any debris, and the NeverWet™ base coat is evenly sprayed onto its surface. The glass substrate is left for 30 minutes, after which the NeverWet™ top coat is then evenly sprayed and is left for 24 hours.
Figure 13. Schematic of the optofluidic lens system for an imaging application, with light from an image passing through the optofluidic lens (i.e., microdroplet) onto a CMOS image sensor. The shape of the optofluidic lens is controlled through an applied voltage onto the indium tin oxide (ITO). The substrate (i.e., glass) adds rigidity onto which the hydrophobic coating is applied.

As stated previously, a DEP effect is utilised in this work to achieve a subunit AR. This DEP effect differs from an electrowetting effect in that DEP causes a change in the shape of the microdroplet due to the forces on dielectric interfaces based on the gradient of the (magnitude squared) electric field [58]. With an electrowetting effect, AR will monotonically increase with increased applied voltages (e.g., Figure 5 in Born et al. [41]). With DEP, we see the opposite trend.

The force on the microdroplet can be estimated by following the theoretical description outlined in Ren et al. [59] and Techaumnat et al. [60]. Here, it is suggested that for our

\[ V_{AC} \text{ at a frequency of } 1.7 \text{ kHz.} \]

As the force on the microdroplet is proportional to the gradient of the square of the electric field magnitude, it is assumed that a DC voltage should produce similar effects in the optofluidic lens.

\[ \text{(magnitude squared)} \]

\[ E \]

---

\[ 1 \] The AC voltage is at a frequency of 1.7 kHz. As the force on the microdroplet is proportional to the gradient of the square of the electric field magnitude, it is assumed that a DC voltage should produce similar effects in the optofluidic lens.
experimental case of a microdroplet positioned on one of two electrode plates, the force can be approximated by the point-dipole model to be $F = 2\pi a^3 C \varepsilon_m \nabla |E|^2$, where $a$ is the radius of the microdroplet, $C = \text{Re}\left\{ \left( \varepsilon_d - \varepsilon_m \right) / \left( \varepsilon_d + 2 \varepsilon_m \right) \right\}$ is the Claussius-Mossotti factor, $\varepsilon_m$ is the permittivity of the surrounding medium, $\varepsilon_d$ is the permittivity of the microdroplet, and $E$ is the electric vector field. This force is non-zero along the surface of the microdroplet, for which the position of the interface changes continuously resulting in the electric field changing continuously from (approximately) $(V_{rms}/d)(\varepsilon_m/\varepsilon_d)$ at the border of the microdroplet to $V_{rms}/d$ at the apex of the microdroplet. Placing approximate values into this force equation reveals that the force is on the order of tens of micro-Newton, which is consistent with forces reported in other digital microfluidic systems in the literature [61].
5 Optofluidic Lens: Experimental Results

The material in this section is heavily taken from work previously published in Applied Optics [49] and as a presentation at the Photonics North 2018 conference.

5.1 Optofluidic Tuning

Based on the theory discussed in Chapter 3 and with consideration to the microfluidic characterisation of Chapter 4, we perform experimental analysis of the ellipsoid microdroplet. Figure 14 shows a series of subunit AR geometries of an ellipsoid microdroplet of DI water (refractive index 1.33), surrounded by air (refractive index 1.00).

The experiment is carried out by progressively applying an alternating current (AC) voltage across a microdroplet placed between two electrode plates. As the AC voltage is increased, the AR of the microdroplet decreases, from its value of approximately unity (eccentricity of 0.12), to about 0.9.

To facilitate successful application of the AC voltage, glass plates are coated with indium-tin-oxide (ITO) electrodes on one side and hydrophobic coating on the other side. Glass provides stability and insulation, ITO maintains the voltage value, and the hydrophobic coating minimises the contact between the microdroplet and the plate.

A microdroplet with volume of 10 μL is placed on this hydrophobic side of the bottom glass plate, and the ITO electrode is grounded. The top glass plate is positioned directly above the microdroplet, with 3,200 μm separation from the bottom glass plate, and
connected to a high AC voltage source, which is the sinusoidal output of a function generator (BK Precision 4011A) amplified through a transformer (Hammond 117E4). The voltage is increased in coarser increments where AR sensitivity is low, and in finer increments where AR sensitivity is higher.

Finally, a complementary metal-oxide-semiconductor (CMOS) image sensor records images of the microdroplet.

From the images obtained at six different voltages (Figure 14), we measure the AR by manually finding the best-fit ellipse for each image, and dividing its width by its height. This manual method is validated through comparison to a curve-fitting algorithm in MATLAB, ‘cftool’², with values in close agreement (within 5%). We then graphically represent this data by plotting AR against the root-mean-squared (RMS) AC voltage, \( V \).

The results show a 10% change in the AR at the highest applied AC voltage of \( V = 800 \) V\(_{\text{rms}} \) compared with the lowest applied AC voltage of \( V = 80 \) V\(_{\text{rms}} \). Aspect ratio decreases monotonically with increased AC voltages, as the microdroplet stretches vertically towards the top plate and the relationship between AR and applied voltage showed minimal hysteresis. Additionally, the hydrophobicity of the hydrophobic coating enables the microdroplet to have an initial contact angle of 130°, which allows headroom for

² In this curve-fitting tool, an image can be imported as an array after image processing that detects the edge of the droplet. A curve can then be fit over this edge of the droplet, and the equation for the curve is displayed, so the \( a \) and \( b \) of the ellipse can be derived.
expansion with a subunit AR. When an external voltage is applied, the contact angle decreases approximately linearly, to a contact angle of 115° for 800 V_{rms}.

![Ellipsoid microdroplet AR at six different voltages (a) to (f), along with images captured by the CMOS image sensor at each applied voltage. The blue circle is the best-fit circle for the first image, and is overlapped with the other images to facilitate comparison between microdroplet shapes obtained. The results are plotted in the lower image. There is a negligible change in microdroplet shape below applied voltage of 0 V_{rms}.](image)

Figure 14.
The findings of the microfluidic characterisation lead to the creation of subunit AR optofluidic lenses.

5.2 Optofluidic Implementation

Given the ability to control the microfluidic and optofluidic behaviour of the elliptical lens, we now illustrate the behaviour of a subunit AR lens by viewing an image through the lens. The same lens setup described in Chapter 4 (Figure 13) is used, by dispensing a microdroplet on a grounded hydrophobic base plate and connecting the top plate to the voltage source. An image is then chosen such that its dimensions can easily be measured. For this reason, a simple rectangular-shaped letter ‘E’ (8-point Arial font) is printed on white paper and placed on the other side of the microdroplet such that the image through the microdroplet can be viewed through a CMOS image sensor. A voltage of 0 V\text{rms} is applied and an image is captured, after which a voltage of 800 V\text{rms} is applied and an image is captured again, and this process is repeated several times.
Figure 15. Images recorded by the CMOS image sensor with the microdroplet ellipsoid (a) at rest, and (b) with the high voltage (800 V$_{rms}$) applied; along with zoomed-in versions of the resulting image. The outline of the ‘E’ produced with the initial microdroplet shape has been overlapped with the ‘E’ produced after the application of voltage to show the resulting change in dimensions.

Dimension labels are added to the images obtained (Figure 15) to highlight the changes that have taken place due to the application of voltage. As the variables $a$ and $b$ defined in this thesis represent the vertical and horizontal radii of the ellipsoid microdroplet, the height and width of the microdroplet are labelled as $2a$ and $2b$, respectively. The height and width of the final image are $h$ and $w$, respectively. It is observed that the image dimensions increase or decrease along with microdroplet dimensions, which is consistent with the results in Chapter 3.
Finally, we quantitatively compare the recorded images in terms of microdroplet and image size. Table 1 summarizes this data, obtained from 10 trials. The results show that for an 8.136% decrease in the AR, the image width decreases by 3.537% and the height increases by 6.039%, implying that the subunit AR lens changes the shape of the input object in the sagittal plane more than it does in the transverse plane. The relatively low error (assessed via the standard deviation $\sigma$) compared to the mean indicates the reproducibility of the experiment and shows the robust nature of our optofluidic lens.

### 5.3 Limitations

Limitations of the optofluidic lens should also be discussed. There is a fundamental minimum imposed on the AR due to the presence of the top plate. As the microdroplet stretches vertically towards this top plate it should not make contact. (This maintains the minimal energy consumption from the open circuit that is imposed by the open space between the top plate and the microdroplet.) Specifically, in our experimental system, an AR of 0.73 will result in contact of the microdroplet with the top plate. This represents the limitation in terms of AR. To overcome this limitation, the setup can be modified to accommodate larger plate separations. This will require the application of voltages higher than what the current setup can output (in the order of kilovolts [60]). Based on the AR vs voltage trend shown in Figure 14, such a setup may perhaps enable to AR to drop close to a value of 0.5.

Additionally, like all optofluidic systems, limitations due to evaporation must be considered. To characterise this limitation, we experimentally measure the evaporation
of the microdroplet and observed that 99.3% of the volume remains over a period of one minute. This was sufficient for the experimental characterisation; however, for protection against evaporation for a robust optofluidic lens system, it is envisioned that a thin film of oil can be coated over the optofluidic lens for longer term operation [62].

Table 1: Mean microdroplet and image dimensions and calculations based on the optofluidic implementation.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Variable</th>
<th>Mean</th>
<th>Error (σ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Microdroplet height [μm]</td>
<td>2a₁</td>
<td>2762</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>2a₂</td>
<td>2926</td>
<td>15</td>
</tr>
<tr>
<td>Microdroplet width [μm]</td>
<td>2b₁</td>
<td>2810</td>
<td>17</td>
</tr>
<tr>
<td></td>
<td>2b₂</td>
<td>2735</td>
<td>15</td>
</tr>
<tr>
<td>Aspect ratio</td>
<td>AR₁</td>
<td>1.018</td>
<td>0.007</td>
</tr>
<tr>
<td></td>
<td>AR₂</td>
<td>0.935</td>
<td>0.005</td>
</tr>
<tr>
<td>Percentage change in aspect ratio [%]</td>
<td>\frac{AR₂ - AR₁}{AR₁}</td>
<td>-8.136</td>
<td>0.626</td>
</tr>
<tr>
<td>Image height [μm]</td>
<td>h₁</td>
<td>859</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>h₂</td>
<td>911</td>
<td>6</td>
</tr>
<tr>
<td>Percentage change in image height [%]</td>
<td>\frac{h₂ - h₁}{h₁}</td>
<td>6.039</td>
<td>0.580</td>
</tr>
<tr>
<td>Image width [μm]</td>
<td>w₁</td>
<td>750</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>w₂</td>
<td>723</td>
<td>6</td>
</tr>
<tr>
<td>Percentage change in image width [%]</td>
<td>\frac{w₂ - w₁}{w₁}</td>
<td>-3.537</td>
<td>0.750</td>
</tr>
</tbody>
</table>
6 Conclusion

6.1 Summary and Conclusions

We investigated an elliptical optofluidic lens in the subunit AR regime through modelling and experiment. In Chapter 2, electromagnetic wave analysis was considered, but due to features sizes being much greater than wavelength, a ray vector approach was selected. In Chapter 3, the subunit AR lens was shown to avoid the internal focusing associated with superunit AR lenses (such as in the work of Born et al. [41]) and improved overall lens tunability, through heightened sensitivity of optical parameters to changes in AR. Here, we carried out ray tracing simulations for superunit and subunit ARs and found a linear trend in focal length, LSA, and beam cone angle, for small changes in the AR. In Chapter 4, we considered microfluidic characterisation and selected a dielectrophoresis approach to achieve the desired subunit AR. In Chapter 5, we performed experimental analyses to observe the tunability of AR for applied voltages and optofluidic implementation and found that sensitivity to the AR increases at higher voltages. In the optofluidic implementation, the effects of the change in AR on the dimensions of an image were studied. Based on the theoretical and experimental results, elliptical optofluidic lenses with subunit ARs have been found to hold much potential for applications requiring increased sensitivity and tunability of optical parameters, as opposed to the relatively limited tunability provided by lenses with superunit ARs [41].
6.2 Future Work

For future development of a full optofluidic lens system, with a carefully-chosen CMOS image sensor and testing image, analyses through the modulation transfer function (MTF) could be performed for quantification of performance. The MTF is able to quantify the contrast of the image by providing a measurement of the spatial frequencies. However, a detailed MTF analysis should not need to be performed at this proof-of-concept stage.

Another potential improvement to this work could be an in-plane 2D lens experiment, wherein the focusing properties of a microdroplet can be directly observed. This can be achieved by placing the microdroplet in a closed channel filled with dye, such that the two liquids are immiscible. As it is a closed channel, the droplet is cylindrical rather than elliptical, allowing for 2D analysis to be carried out. The dye should help to trace the actual light rays, and these results would help to verify the theoretical results obtained. Such a lens would also be helpful in applications such as in-plane light manipulation.

In addition, the optofluidic lens presented here has a subunit AR. By modifying the experiment to include electrowetting effects, a setup could be developed where the lens can be tuned in both the subunit and superunit regimes, which would combine the benefits of these regimes.
REFERENCES


Appendices

APPENDIX A: Main MATLAB Code

This is the code used to produce ray tracing simulations, as described in Chapter 3. It makes use of the function calc_abc.m, which is included in Appendix B.

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% This MATLAB code produces a ray tracing simulation for a given Aspect
% Ratio and Volume of the microdroplet.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

clear all;  % clear any previously stored variables
close all;  % close any open figures
clc;        % clear command window

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Inputs to the optofluidic system
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

AR = 0.75;   % width-to-height aspect ratio of microdroplet
V = 50;      % volume of microdroplet in microlitres

n_air = 1;   % refractive index of air
n_lens = 1.33; % refractive index of microdroplet fluid (water)

[ARla, ARlb, ~] = calc_abc(V,1);    % calculate ellipse height and width
                                   % for AR = 1 case (this calls another function named 'calc_abc.m')

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Startpoint coordinates: these coordinates are based off the AR = 1 case
% so that they stay consistent even as microdroplet shape changes
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
\begin{verbatim}
start_x = -2*AR1b;               % z-axis starting point
start_y = linspace(-0.15*AR1b, 0.15*AR1b, 8); % 8 linearly spaced points
% along y-axis as starting points
start_z = start_y;               % x-axis starting points

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Modelling the ellipsoid and Optical Axis (OA)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

[ellip_b, ellip_a, C2] = calc_abc (V, AR); % calculate ellipse height and
% width for given AR

[x_ellipse, y_ellipse, z_ellipse] = ellipsoid (0, 0, 0, ellip_b, ...
    ellip_b, ellip_a, 100); % solutions of ellipsoid equation

surf(x_ellipse,y_ellipse,z_ellipse, C2, 'EdgeColor', 'none')
% plot ellipsoid with colour map
hold on
alpha 0.2 % transparency
axis equal
view(0,0) % sagittal plane view

for i = 1:numel(start_y)
    for j = 1:numel(start_z)
        start_point = [start_x,start_y(i),start_z(j)];

% Plotting the Optical Axis
OA = [start_x,0,0,((7*AR1b)-start_x),0,0];
q = quiver3(OA(1),OA(2),OA(3),OA(4),OA(5),OA(6),0,'--',...  
    'ShowArrowHead','off','Tag','OA');

% Modelling the input vector
normray_in = [1,0,0]; % input unit vector
syms fact1 real % scaling factor variable
% vector form of a line: (x,y,z) = (starting point) + fact*(direction)
\end{verbatim}
\[
x_{\text{dist1}} = \text{start}_\text{point}(1) + \text{fact1} \times \text{normray}_\text{in}(1);
\]
\[
y_{\text{dist1}} = \text{start}_\text{point}(2) + \text{fact1} \times \text{normray}_\text{in}(2);
\]
\[
z_{\text{dist1}} = \text{start}_\text{point}(3) + \text{fact1} \times \text{normray}_\text{in}(3);
\]

% Solve for scaling factor such that vector intersects ellipse

\[
\text{mag1} = \text{solve}( (\frac{x_{\text{dist1}}}{\text{ellip}_b})^2 + (\frac{y_{\text{dist1}}}{\text{ellip}_b})^2 + (\frac{z_{\text{dist1}}}{\text{ellip}_a})^2 = 1, \text{fact1}, '\text{Real}', \text{true});
\]

% Calculate first intersection point and plot input vector

\[
\text{intPoint1} = \text{start}_\text{point} + (\text{double}(\text{mag1}(1))) \times \text{normray}_\text{in};
\]
\[
\text{ray}_\text{in} = \text{intPoint1} - \text{start}_\text{point};
\]
\[
\text{quiver3}(\text{start}_\text{point}(1), \text{start}_\text{point}(2), \text{start}_\text{point}(3), \text{ray}_\text{in}(1), \ldots \text{ray}_\text{in}(2), \text{ray}_\text{in}(3), 0, 'k', '\text{ShowArrowHead}', '\text{off}')
\]

% 3D Snell's Law to find transmitted vector through lens

\[
\text{norm}_\text{intPoint1} = [(2 \times \text{intPoint1}(1))/(\text{ellip}_b^2), \ldots
\]
\[
(2 \times \text{intPoint1}(2))/(\text{ellip}_b^2), (2 \times \text{intPoint1}(3))/(\text{ellip}_a^2)];
\]
\[
\text{norm}_\text{intPoint1} = \text{norm}_\text{intPoint1} ./ \text{norm}(\text{norm}_\text{intPoint1});
\]

% normal unit vector at first intersection point

\[
\text{crosspp} = \text{cross}(\text{norm}_\text{intPoint1}, \text{normray}_\text{in}); \ % \text{positive} \ \text{cross product}
\]
\[
\text{crosspn} = \text{cross}(-\text{norm}_\text{intPoint1}, \text{normray}_\text{in}); \ % \text{negative} \ \text{cross product}
\]
\[
\text{ratio} = \text{n}_\text{air}/\text{n}_\text{lens}; \ % \text{refractive} \ \text{index ratio}
\]
\[
\text{term1} = (\text{ratio}) \times (\text{cross}(\text{norm}_\text{intPoint1}, \text{crosspn}));
\]
\[
\text{term2} = (\text{norm}_\text{intPoint1}) \times (\sqrt{1 - ((\text{ratio}^2) \times (\text{dot}(\text{crosspp}, \text{crosspp}))))});
\]

\[
\text{normray}_\text{lens} = \text{term1} - \text{term2}; \ % \text{unit} \ \text{vector} \ \text{of} \ \text{ray} \ \text{through} \ \text{lens}
\]

% Modelling the transmitted vector

\[
\text{syms} \ \text{fact} \ \text{real}
\]
\[
x_{\text{dist}} = \text{intPoint1}(1) + \text{fact} \times \text{normray}_\text{lens}(1);
\]
\[
y_{\text{dist}} = \text{intPoint1}(2) + \text{fact} \times \text{normray}_\text{lens}(2);
\]
\[
z_{\text{dist}} = \text{intPoint1}(3) + \text{fact} \times \text{normray}_\text{lens}(3);
\]

\[
\text{mag} = \text{solve}( ((x_{\text{dist}}/\text{ellip}_b)^2)+((y_{\text{dist}}/\text{ellip}_b)^2)+\ldots
\]
\((z_{\text{dist}}/\text{ellip}_a)^2) == 1, \text{ fact, 'Real', true)};

% this solution is the magnitude of the ray vector through lens

\text{intPoint2} = \text{intPoint1} + (\text{double(mag(2))}.*\text{normray_lens});
\text{ray_lens} = \text{intPoint2} - \text{intPoint1};

\text{quiver3(intPoint1(1),intPoint1(2),intPoint1(3),ray_lens(1),...}
\text{ray_lens(2),ray_lens(3),0,'k','ShowArrowHead','off')}
\text{% trasmitted vector}

% 3D Snell's Law to find output vector
\text{norm_intPoint2} = [ (2*intPoint2(1))/(\text{ellip}_b^2) , ... \]
\text{2*intPoint2(2))/(\text{ellip}_b^2) , (2*intPoint2(3))/(\text{ellip}_a^2)];
\text{norm_intPoint2} = \text{norm_intPoint2} ./ \text{norm(norm_intPoint2)};

\text{crosspp_t} = \text{cross(-norm_intPoint2, normray_lens)};
\text{crosspn_t} = \text{cross(norm_intPoint2, normray_lens)};
\text{ratio2} = 1/\text{ratio};
\text{term1t} = (\text{ratio2}).*(\text{cross(-norm_intPoint2,crosspn_t)});
\text{term2t} = (-\text{norm_intPoint2}).*(\text{sqrt(1-((ratio2)^2)*...}
\text{(dot(crosspp_t,crosspp_t))))});

\text{normray_out} = \text{term1t} - \text{term2t};

% To draw screen for rays to terminate
\text{x_screen} = 7*\text{AR1b};
\text{y_screen} = 1.5 * \text{AR1b};
\text{z_screen} = 1.5 * \text{AR1a};
\text{Xs} = [\text{x_screen} \text{x_screen} \text{x_screen} \text{x_screen} \text{x_screen}];
\text{Ys} = [\text{y_screen} -\text{y_screen} -\text{y_screen} \text{y_screen} \text{y_screen}];
\text{Zs} = [\text{z_screen} \text{z_screen} -\text{z_screen} -\text{z_screen} \text{z_screen}];
\text{patch(Xs,Ys,Zs,'white');}
\text{axis([\text{start}_x \text{x_screen} -\text{y_screen} \text{y_screen} -\text{z_screen} \text{z_screen}])}

\text{xspc} = [-1000 -500 0 500 1000];
set(gca,'YTick',xspc)

% Modelling the output vector
fact4 = (x_screen-intPoint2(1))/normray_out(1);
raypoint_final(1) = x_screen;
raypoint_final(2) = intPoint2(2) + (fact4*normray_out(2));
raypoint_final(3) = intPoint2(3) + (fact4*normray_out(3));
ray_final = raypoint_final - intPoint2;
quiver3(intPoint2(1),intPoint2(2),intPoint2(3),ray_final(1),...
ray_final(2),ray_final(3),0,'black','ShowArrowHead','off')

% To calculate beam cone angle, we need the final intersection point
dist_screen = x_screen - ellip_b;
syms t real % scaling factor
yplane = intPoint2(3) + (t * normray_out(3));
t1 = double(solve(yplane == 0, t));
point = intPoint2 + (t1 .* normray_out);
scatter3(point(1), 0, 0,25,'k','filled')

focLength(i) = point(1) - ellip_b; % actual focal length for this ray
length(i) = abs(raypoint_final(2));
theta(i) = atand(length(i)/(dist_screen-focLength(i)));

end
end

% Calculating theoretical focal length
Radcurv = (ellip_a^2)/ellip_b; % radius of curvature
d = 2 * ellip_b; % thickness of lens
Z = (n_lens - 1)*((2/Radcurv)-((d*(n_lens - 1))/...
    (n_lens * Radcurv * Radcurv))); % optical power
Parax_foc = 1/Z; % paraxial focal length
BFL = Parax_foc * (1 - (((n_lens - 1)*d)/(n_lens * Radcurv)));

% Plot settings
title(['Aspect Ratio = ',num2str(AR)])
grid off
set(gcf,'color','white')
set(gca, 'FontName', 'Times New Roman', 'FontSize', 14)
xlabel(['{\textit{z}}', ' [', char(181), 'm']])
zlabel(['{\textit{x}}', ' [', char(181), 'm']])

% Calculating LSA and maximum beam cone angle
LSA = abs((abs(max(max(focLength)))) - (abs(min(min(focLength)))))
LSA_P = (LSA / abs(BFL)) * 100; % LSA as a percentage of focal length
theta_max = max(theta); % maximum beam cone angle
APPENDIX B: MATLAB Function used in Code

This is the function 'calc_abc.m', which is referred to in the main code.

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% This MATLAB function calculates the height and width (a and b) of the
% ellipsoid microdroplet for a given volume and Aspect Ratio. It also
% produces a colour map that shades the 'imagined' volume of the ellipsoid.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

function [a_ellipse, b_ellipse, c_map] = calc_abc (V,AR)
% function [outputs] = function_name (inputs)
% Outputs: height, width, colour map
% Inputs: volume, Aspect Ratio
% Function_name: unit_AR, the filename should be the same

syms rad h a_ellipse real
% symbolic variables: radius of base of microdroplet, height of portion
% below base plate, height 'a' of ellipse

base_rad_ratio = 0.65;       % ratio of base radius to microdroplet radius

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Calculating radius of microdroplet base as this will be fixed while
% tuning microdroplet shape
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

base    =  base_rad_ratio * rad;       % radius of droplet base
h       =  sqrt(rad^2 - base^2);     % height of portion below base plate
vol_sph =  (4/3) * (pi) * (rad^3);   % volume of total sphere
vol_seg =  (1/6) * (pi*h) * ((3*(base^2))+(h^2));  % imagined volume
vol    =  vol_sph - vol_seg;         % actual droplet volume
rad     =  double(solve(vol == V, rad));
% solving symbolic equation for microdroplet volume to find the radius
base    =  double(base_rad_ratio * rad);   % calculating base radius
% Calculating height of portion of ellipsoid microdroplet below base plate

\[ b_{ellipse} = \frac{a_{ellipse}}{AR}; \]

% width b of ellipsoid

\[ V_{ell\_total} = \frac{4}{3} \pi (a_{ellipse} a_{ellipse} b_{ellipse}); \]

% total volume of ellipsoid

\[ h_{ell} = \sqrt{\frac{((a_{ellipse} b_{ellipse})^2) - (b_{ellipse} base)^2)}{(a_{ellipse})^2}}; \]

% height of imagined volume segment

\[ V_{ell\_cap} = \frac{(\pi a_{ellipse} a_{ellipse} h_{ell}^2)((3*b_{ellipse}) - h_{ell})}{3*(b_{ellipse})^2}; \]

% imagined volume

\[ V_{ell\_actual} = V_{ell\_total} - V_{ell\_cap}; \]

% actual volume of microdroplet

\[ a_{ellipse} = \text{double(solve}(V_{ell\_actual} == V, a_{ellipse})); \]

% solving symbolic equation for actual 'a'

\[ b_{ellipse} = \frac{a_{ellipse}}{AR}; \]

% calculating 'b' based on 'a'

\[ h_{ell} = \sqrt{\frac{((a_{ellipse} b_{ellipse})^2) - (b_{ellipse} base)^2)}{(a_{ellipse})^2}}; \]

% calculating height of imagined volume segment

% Developing a colour map to shade portion of ellipsoid below base plate

[~, ~, z_ellipse] = ellipsoid (0, 0, 0, a_{ellipse}, a_{ellipse},...
\[ b_{ellipse}, 100); \]

% solutions of ellipsoid equation

for i = 1:size(z_ellipse)

for j = 1:size(z_ellipse)

if (z_ellipse(i,j) < -h_{ell})

% this 'if' statement sets the colour of the imagined
% volume (all z-values below base plate) to 0, i.e.
% black, and all values above 0.3, i.e. grey

\[ c\_map(i,j,1) = 0; \quad c\_map(i,j,2) = 0; \quad c\_map(i,j,3) = 0; \]

end

end

end
else c_map(i,j,1) = 0.3; c_map(i,j,2) = 0.3; c_map(i,j,3) = 0.3;
end
end
end
end
APPENDIX C: Considerations for Operation with Light Rays Originating Orthogonal to the Optical Axis

For some cases, an optical axis that is orthogonal to the substrate is desirable in optofluidic lenses. However, these optofluidic lenses tend to have superunit ARs. This superunit aspect ratio geometry encourages the focal distance to be outside of the microdroplet. On the other hand, our optofluidic lenses with subunit ARs, when operated with an orthogonal optical axis, can potentially suffer from a focal distance that is within the microdroplet due to the exaggerated vertical dimension. As such, we have focused our analyses on parallel orientation of the optical axis. For comparison, an analysis for the case of orthogonal orientation of the optical axis is shown in this Appendix.

Figure 16. Ray tracing simulations comparing (a) a sub-unit AR = 0.75 optofluidic lens, (b) a unit AR = 1.00 optofluidic lens, and (c) a super-unit AR = 1.25 optofluidic lens in the sagittal plane view, with light propagating perpendicular to the substrate.
Figure 16 shows ray tracing simulations for the optofluidic lens in orthogonal orientation. It can be seen that as the AR reduces from 1.25 to 0.75, the focal length reduces as well. This trend continues as we move further into the sub-unit AR region.

Figure 17 shows this trend of focal length versus AR. It should be noted that all of the optical planes (0° through 90°) show the same response, indicating that this configuration would not be acceptable for applications requiring different focal lengths in the different optical planes. Additionally, the focal length becomes negative (i.e., the optofluidic lens focuses within the microdroplet) for AR less than 0.65 for this orthogonal orientation. As such, the ideal orientation of the optical axis is parallel to the substrate.

Figure 17. Focal length from the vertex of the microdroplet ellipsoid, in the five planes, as a function of AR in coarse increments of $\Delta$AR = 0.1, with light rays originating orthogonal to the substrate.