Social Network Effects on Investment Behaviour in the Stock Market

by

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ABSTRACT

SOCIAL NETWORK EFFECTS ON INVESTMENT BEHAVIOUR IN THE STOCK MARKET

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We develop an agent-based model to examine the behaviour of brokers trading in a financial marketplace and determine whether social communication between brokers influence their overall trading decisions. The agents’ decision-making process is determined by their total utility, which is broken down into objective and subjective utility; an agent’s perceived value of the stock and the trading decision suggested from one’s peers. To accurately represent an agent’s trading decisions, sensitivity analysis is conducted to determine the required number of average trial runs within each simulation. Random and preferential attachment networks are implemented into simulations with 50, 100, 150 and 200 agents and the trading decisions over the course of 30 days are analyzed to examine if different social networks can influence a trader’s decision. Affected total utility (ATU) represents the agent’s decisions which are influenced by their peers. The frequency of affected total utility decreased significantly with the implementation of weak links in the social network. The occurrence of affected total utility remains stable for 50, 100 and 150 agents when the social network is a preferential attachment network.
Acknowledgements

I would like to thank Dr. Monica G. Cojocaru for supervising my research project during my fourth year of undergraduate and now as a master’s student. She has guided me through the development of the agent-based model and the writing of this thesis, and I look forward to working with her in the future.
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Chapter 1

Introduction

Financial instruments such as stocks are traded daily; individuals often rely on funds such as hedge funds or mutual funds to make investments for them. Brokers constantly use statistical algorithms and insight from peers to predict future trends. How does this social aspect of trading affect the brokers’ decisions? Through the use of agent-based models (ABM), social network models and momentum in market prices, this thesis attempts to determine if brokers’ social communication influences their trading decisions and if so, how does changing the brokers’ social relationships affect their peers’ influence.

Agent-based modeling is a method in complex dynamics that permits the creation, analysis, and experimentation of models composed of agents that interact within an environment [1] used to simulate a particular phenomenon [2]. Agents are initialized as distinct entities that are used to represent individual people, organizations or even nation-states [1]. They are able to interact with one another transferring information and properly react based on the new information acquired. Depending on the model at hand, the environment can
act as a neutral medium where it has little or no impact on the agent’s decisions, or the architect of the model can design the environment to play a significant role in the agent’s decision making at each iteration of time [1]. Alongside these two defined entities, the ABM must allow for decision-making heuristics, the learning of rules, and topological interactions.

ABMs are often thought to be extensions of the Ising model, a mathematical model of ferromagnetism initially established in 1925, or cellular automata (CA), a concept developed in the 1940s by John von Neumann [2]. One notable and enticing characteristic of CA and an ABM is the flexibility of implementing synchronous or asynchronous interactions. Asynchronous interactions within an ABM are desirable as the agent’s actions follow discrete-event cues rather than constant time-steps.

Fields such as biology, economics, and social sciences find ABMs very appealing because of the detail that can be attributed to the agents and the environment [2]. ABMs are used in numerous sectors of biology; common topics in which ABMs are used include the spread of epidemics, population dynamics, forced displacement/migration and cognitive modeling [2,6]. There has been a growing interest in the field of economics to use ABMs to aid in economic analysis [7,8]. These simulations use the bottom-up approach, which allows for simulations of extremely complex and volatile economies. With the agents’ ability to learn and adapt to their environment, economists find ABMs helpful in regards to simulating unstable systems where non-linear responses occur from proportionally small changes in the system [9]. Social sciences such as the study of politics highly benefit from ABMs, as they allow for agents with in-depth behaviours to join networks, seek coalitions, and continue learning in multidimensional environments [10].

Social networks stem from network theory; they are social structures composed of nodes (often representing individuals or organizations), dyadic links, and a corresponding social in-
interaction attribute \cite{11}. These social structures are often analyzed in an attempt to determine local or global patterns, locate influential entities, and examine network dynamics. Social scientists have broken down social networks into three networks: ego-centric, socio-centric and open-system \cite{11}. Ego-centric networks are defined as a network with one link, often used to represent the relationship with a good friend. Socio-centric networks are the most commonly studied of the three; these networks are closed networks where nodes represent discrete entities within a specified environment. Open-system networks are open networks, where the domain of individuals or organizations is not exclusively defined.

The current state of social network theory emerges from three lines of research: sociometric analysis, which implements graph theory to study social structures, interpersonal relations, which examines the formations of cliques among individuals within a group and an anthropologic view, which studies the structure of community relations in less developed societies \cite{12}. Centrality, cohesion and structural equivalence are three network attributes that have driven research on network effects \cite{13}. In \cite{14} Granovetter discusses the theory of weak ties stating the role in which weak social ties relays information and ideas. In a labour-market study, Granovetter concluded that individuals more frequently found jobs through their weak social ties rather than their strong social ties.

Within this paper, we implement the Edgar Gilbert model and the Barabasi-Albert model to form the random and preferential attachment social networks used in the ABM. The concept of the random network is to construct a network that is truly random; a challenge with this idea is deciding where to place the links between nodes which will replicate the complexity of a real system \cite{15}. In 1959 two formal definitions of random graphs were published, the Erdős-Rényi model and the Edgar Gilbert model \cite{16,17}. The Erdős-Rényi model states that $G(n, M)$ consists of all graphs with the vertex set $V = \{1, 2, ..., n\}$ having $M$ edges, where each graph has the same probability. With $N = \binom{n}{2}$ and $0 \leq M \leq N$,
then \( G(n, M) \) has \( \binom{N}{M} \) elements. If \( G^M \) is a random variable, where \( G^M \in G(n, M) \), then the probability of generating this graph variant is \( P(G^M) = \binom{N}{M}^{-1} \). The Edgar Gilbert model states that \( G(n, p) \) consists of all graphs with the vertex set \( V = \{1, 2, ..., n\} \), where each edge is independently chosen with a probability \( 0 \leq p \leq 1 \). Let \( N = \binom{n}{2} \), then the probability of selecting a graph with \( M \) edges from the set \( G(n, p) \) is \( \binom{N}{M} p^M (1 - p)^{N-M} \). As previously mentioned this paper uses the Edgar Gilbert model to develop random graphs.

In order to construct a random network consisting of \( N \) nodes where probability \( p \) represents the likelihood of any pair of nodes being connected, let \( x_{i,j} \) be a random variable representing the connection probability for distinct nodes \( i, j \) where \( x_{i,j} \in [0, 1] \) and \( i, j \in \{1, \ldots, N\}, i \neq j \). For each of the \( \frac{N(N-1)}{2} \) node pairs if \( x_{i,j} > p \) then \( i \) and \( j \) are connected.

The preferential attachment network emulates real networks where new nodes have a tendency to link to more established/connected nodes. The Barabasi-Albert model implements the idea that growth and preferential attachment coexist in real networks. At \( t = 0 \), the links connecting the \( m_0 \) nodes are assigned arbitrarily with the condition that each node has at least 1 link. At each time step a new node is added into the system with \( m \) links connected to established nodes, where \( m \leq m_0 \). Let \( k_i \) be the degree of node \( i \) and \( \pi(k_i) \) be the probability that node \( i \) connects to a new node such that \( \pi(k_i) = \frac{k_i}{\sum_j k_j} \). After \( t \) timesteps, the network will result in \( N = t + m_0 \) nodes and \( m_0 + mt \) links. Figure 5.4 from [15] illustrates that the degree distribution from the Barabasi-Albert model is a power law of the form \( P(k) \sim k^{-\gamma} \) where \( \gamma = 3 \).

Analysis of social networks is conducted in a vast range of disciplines and applications. Security organizations such as the National Security Agency (NSA) analyze the social networks of terrorist cells to determine the structure of the network and the leaders of these
organizations [18]. Textual corpora can be converted into networks and subsequently analyzed using a method of social network analysis. Within these textual corpora networks, the nodes represent social actors and the actions of the actors’ act as links. The analysis of these networks is used to determine key actors, communities or parties, and general properties of the network such as robustness, structural stability or centrality of certain nodes [19].

Momentum is one of many concepts that traders use to predict market performance. It stems from behavioural finance and is the rate of acceleration of a financial instrument’s price or volume [20]. Momentum traders often take a short or long position in a stock and hope that the stock follows the expected trend. The traders study the short-term movement of the stock’s market price as opposed to analyzing the stock’s intrinsic value. Rather than use the popularized strategy to buy low and sell high, these momentum traders tend to sell low and buy lower and buy high and sell higher [20]. In 1985 and 1987, DeBondt and Thaler reported that in the successive three to five years, stocks that were long-term losers would outperform long-term winners [21]. In 1990 Jegadeesh and Lehmann concluded that in the short-term the reversal effect holds true [21]. Three years later in 1993, Jegadeesh and Titman discovered over a period of three to twelve months that stocks deemed past winners on average continued to outperform stocks that were losers [22].

With the use of various analytics and the knowledge of competition in the market, it would be assumed that brokers would not be inclined to exchange information with one another. However, multiple studies suggest the contrary. For instance, a telephone survey, conducted in [23], among German mutual fund managers confirms that fund managers actively exchange information, which alludes to the idea that brokers value the opinion of their peers. Much behavioural economics literature has suggested that individuals have tendencies to follow influential people. Tversky and Kahneman conclude that individuals are inclined to overreact to superficial evidence without factual backing [24]. It can be argued that the evaluation of
stock prices is vulnerable to psychological biases due to the ambiguity of the true value of the stock [25]. Group psychology dynamics such as peer pressure are known to alter investors’ trade decisions and reinforce these psychological biases.

In this thesis we will proceed to define the main elements of the agent-based model, design and implement the required sensitivity analysis, and examine the trading decisions with random and preferential attachment networks. The motivation behind this thesis stemmed from [26], which uses different trust networks to examine the possible impact of social influence on investment behaviour.
Chapter 2

Model Setup

2.1 Model Description and Assumptions

There are two individual evaluations associated with a company: the intrinsic value and the market value. The intrinsic value, which is often difficult to determine, is an estimate of the true value of a company, whereas the market value is an evaluation of the company based on the company’s stock price. The market value of a company often does not reflect the actual value of a company; it is dependent on the supply and demand in the investing market. Since the market value varies from the true value, it can result in an overvaluation or undervaluation; because of the complexities of the market, people often rely on brokers to manage their investments for them [27].

This thesis attempts to model momentum traders within the stock market. The brokers communicate with one another in an attempt to gather information to predict future trends and better understand the behaviour of the stock market. Based on their intuition, knowl-
edge, influence from peers, and expectations of momentum in the market, the agents will then proceed to make a trading decision before the market closes. The risk-seeking variable represents the agent’s willingness to use their perception of the market price. Agents are also assigned a subgraph of a random or preferential attachment social network, known as a neighbourhood, to make a trading decision at time $t$.

Within our model, assumptions are made to simplify the interaction and thought process of humans. The first assumption made is that the market is deemed open, which provides an alternative buyer/seller for the brokers defined within the marketplace. Implementing an open market removes the necessity of one-to-one interactions and allows for the focus of the thesis to examine the effects of social influence rather than the outcome of non-zero-sum games. The alternative buyer/seller is always willing to trade regardless of the circumstance. To reduce complexity and ensure focus on the agent’s decision-making process, the agents do not negotiate prices when undergoing a transaction. Instead, a pricing mechanism defines the closing prices after all trade negotiations are executed. Throughout the simulations, it is assumed that there are not any exterior factors or events that could hinder the value of the stock.

2.2 Construction

An agent-based model is used to simulate the stock market, agents in the financial trading environment use the market price and their perceived stock values to trade financial instruments.
2.2.1 Stock Prices

<table>
<thead>
<tr>
<th>Variable</th>
<th>Symbol</th>
<th>Range</th>
<th>Value Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Price</td>
<td>$\bar{p}_m$</td>
<td>[10, 1000]</td>
<td>$\bar{p}_m \in \mathbb{N}$</td>
</tr>
<tr>
<td>Daily Market Price</td>
<td>$p_m$</td>
<td>-</td>
<td>$p_m \in \mathbb{N}$</td>
</tr>
<tr>
<td>Minimum Price</td>
<td>$p_m^{\text{min}}$</td>
<td>-</td>
<td>$p_m^{\text{min}} \in \mathbb{N}$</td>
</tr>
<tr>
<td>Maximum Price</td>
<td>$p_m^{\text{max}}$</td>
<td>-</td>
<td>$p_m^{\text{max}} \in \mathbb{N}$</td>
</tr>
<tr>
<td>Relative Price Difference</td>
<td>$p_m^r$</td>
<td>-</td>
<td>$p_m^r \in \mathbb{N}$</td>
</tr>
</tbody>
</table>

Table 2.1: The different price variables defined within the price class.

Within the price class of the ABM model all of the price variables are elements of the natural number set, and any calculation that results in a float/double decimal value is rounded to the nearest natural number. The system is also designed such that the market price $p_m(t) > 0$, $\forall t \in [1, T]$.

In general, each financial instrument has an associated price value; in order to replicate similar market price evaluations and fluctuations observed in the stock market, data collected from Advanced Micro Devices, Inc., Eli Lilly and Company, Xerox Corporation, Check Point Software Technologies Ltd., Enterprise Products Partners L.P., Telefonaktiebolaget LM Ericsson (publ), The Scotts Miracle-Gro Company, and VelocityShares 3x Inv Natural Gas ETN stocks were analyzed over the course of a year (February 21st 2017 - February 21st 2018) to determine relative price fluctuations. These 8 different companies were selected from a list of 30 companies found on the trending page [28] due to their stable price fluctuation in market over the span of a year. From the collected data the average closing price $\bar{p}_\nu$, where $\nu \in \{1, ..., 8\}$, and the extrema were determined. The relative difference in price between the stocks’ average price and the corresponding extrema were used to replicate the relative
price fluctuation boundaries observed in the stock market. Let \( \nu \) represent the indices of the eight trending stocks. Let \( p_{\nu}^{\text{min}} = \min\{p_1, p_2, \ldots, p_{365} \} \), \( p_{\nu}^{\text{max}} = \max\{p_1, p_2, \ldots, p_{365} \} \) and \( \bar{p}_{\nu} = \frac{1}{365} \sum_{\tau=1}^{365} p_{\tau} \), where \( p_{\nu}^{\text{min}}, p_{\nu}^{\text{max}}, \bar{p}_{\nu} \in \mathbb{R}^+ \), represent the minimum, maximum and average closing price, respectively, for the \( \nu^{th} \) indexed trending stock in Table 2.2. If \( p_{\nu}^{\text{r}}(p_{\nu}) = \frac{|p_{\nu} - \bar{p}_{\nu}|}{\bar{p}_{\nu}} \) is the general relative price difference for the \( \nu^{th} \) indexed trending stock, then the relative minimum price difference is defined as:

\[
p_{\nu}^{\text{r}}(p_{\nu}^{\text{min}}) = \frac{|p_{\nu}^{\text{min}} - \bar{p}_{\nu}|}{\bar{p}_{\nu}}
\]

and

\[
p_{\nu}^{\text{r}}(p_{\nu}^{\text{max}}) = \frac{|p_{\nu}^{\text{max}} - \bar{p}_{\nu}|}{\bar{p}_{\nu}}
\]

is the relative maximum price difference.

<table>
<thead>
<tr>
<th>Stock Name</th>
<th>( \nu ) Index</th>
<th>Stock ID</th>
<th>Average Closing Price ($)</th>
<th>52 Week Domain ($)</th>
<th>Relative Difference (Minimum)</th>
<th>Relative Difference (Maximum)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Advanced Micro Devices, Inc.</td>
<td>1</td>
<td>AMD</td>
<td>12.48</td>
<td>[9.70, 15.65]</td>
<td>0.22</td>
<td>0.25</td>
</tr>
<tr>
<td>Eli Lilly and Company</td>
<td>2</td>
<td>LLY</td>
<td>82.93</td>
<td>[73.69, 89.09]</td>
<td>0.11</td>
<td>0.07</td>
</tr>
<tr>
<td>Xerox Corporation</td>
<td>3</td>
<td>XRX</td>
<td>30.13</td>
<td>[26.64, 37.42]</td>
<td>0.12</td>
<td>0.24</td>
</tr>
<tr>
<td>Check Point Software Technologies Ltd.</td>
<td>4</td>
<td>CHKP</td>
<td>107.11</td>
<td>[95.03, 119.20]</td>
<td>0.11</td>
<td>0.11</td>
</tr>
<tr>
<td>Enterprise Products Partners L.P.</td>
<td>5</td>
<td>EPD</td>
<td>26.64</td>
<td>[23.59, 29.51]</td>
<td>0.11</td>
<td>0.11</td>
</tr>
<tr>
<td>Telefonaktiebolaget LM Ericsson (publ)</td>
<td>6</td>
<td>ERIC</td>
<td>6.47</td>
<td>[5.52, 7.47]</td>
<td>0.15</td>
<td>0.16</td>
</tr>
<tr>
<td>The Scotts Miracle-Gro Company</td>
<td>7</td>
<td>SMG</td>
<td>95.25</td>
<td>[81.48, 110.12]</td>
<td>0.14</td>
<td>0.16</td>
</tr>
<tr>
<td>VelocityShares 3x Inv Natural Gas ETN</td>
<td>8</td>
<td>DGAZ</td>
<td>25.76</td>
<td>[17.05, 42.92]</td>
<td>0.34</td>
<td>0.67</td>
</tr>
</tbody>
</table>

Table 2.2: Analytic results of the 8 trending stocks over the course of a year (February 21st 2017 - February 21st 2018). The values of the minimum relative difference are associated with the variable \( pr_{s_m}^{\text{min}} \) and the maximum relative difference are associated with the variable \( pr_{s_m}^{\text{max}} \).

With the initialization of the model of our trading environment, there are \( M \) financial instruments defined, each instrument is associated with an index \( m \), where \( m \in \{1, \ldots, M \} \).
and \( M \in \mathbb{N} \). Pseudo random number generated (prng) values are selected to represent the mean price \( \bar{p}_m \), where \( \bar{p}_m \in \{10, \ldots, 1000\} \), for the corresponding financial instrument. For the \( m^{th} \) financial instrument let \( s_m \in \{1, \ldots, 8\} \) equate to a \( \nu \) index found in Table (2.2). Using the relative price differences and the mean price value, the minimum and maximum price boundaries are determined.

\[
\begin{align*}
p_{m}^{\min} &= \bar{p}_m - (\bar{p}_m \cdot p_{sm}^{r}(p_{s_m}^{\min})) \\
p_{m}^{\max} &= \bar{p}_m - (\bar{p}_m \cdot p_{s_m}^{r}(p_{s_m}^{\max}))
\end{align*}
\]

The price boundaries allow for the standard deviation to be determined, which is required for the initialization of the normal distribution for the stock prices. The standard deviation for the \( m^{th} \) financial instrument is calculated with the equation:

\[
\sigma_m = \sqrt{\frac{\sum_{l=0}^{n}(x_l - \bar{p}_m)^2}{n - 1}}
\]

where

\[
n = p_{m}^{\max} - p_{m}^{\min} \quad \text{and} \quad x_l = p_{m}^{\min} + l
\]

With the mean and standard deviation determined for each financial instrument the normal variate function, from the random package in python, is used to determine the daily market prices such that \( p_m(t) \sim N(\bar{p}_m, \sigma_m^2) \).
2.2.2 Agents

Below are the parameters assigned to each agent initialized in the computational simulations.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk-Seeking</td>
<td>$\alpha_i$</td>
<td>[0, 1]</td>
<td>Willingness to deviate from momentum trading and rely on intuition</td>
</tr>
<tr>
<td>Potential Trading Gain</td>
<td>$ptg_i$</td>
<td>[0.05, 0.15] / 5 - 15%</td>
<td>Profit Margin Percentage</td>
</tr>
<tr>
<td>Social Coefficient</td>
<td>$c_{ij}$</td>
<td>[0, 1]</td>
<td>Strength in relationship with agent $j$</td>
</tr>
</tbody>
</table>

Table 2.3: The parameters assigned to agent $i$ within each simulation.

It is assumed that there is a fixed number of agents who trade over a given time interval. Let $N$ be the number of agents in the financial trading environment where each agent is assigned an identification number $i$, where $i \in \{1, \ldots, N\}$ and a predetermined amount of currency to build a diversified portfolio. The agents proceed to purchase stock and depending on the agent, the amount of stock purchased over the $M$ financial instruments will vary.

Stock price trends are constantly tracked and analyzed in an attempt to develop better methods of predicting future stock values. To predict future stock values, the agents assume there is momentum and select a potential future price from a normal distribution. During initialization of the trading environment, the agents are given the mean price $\bar{p}_m$ as well as the corresponding standard deviation $\sigma_m$ required to build the distribution.

The agent’s idea of momentum is that the trend in price will continue in the same direction as time progresses [29]. Associated with each agent is a risk-seeking coefficient $\alpha(t)$, where $\alpha \in [0, 1]$, which acts as a weighting factor for the agent. The risk-seeking coefficient determines the magnitude in which an agent is willing to use their intuition over their prediction determined by momentum. With trading financial instruments comes the
question of when to buy, sell or hold. Each agent is assigned a potential trading gain variable $ptg$, where $ptg_i \in [0.05, 0.15]$. This variable determines at what deviation from the agent’s expected price point the agent is willing to buy or sell the financial instrument. Let $p_{im}^e(t)$ represents the expected price, and $k_{im}$ represent the deviation from the expected price where

$$k_{im}(t) = p_{im}^e(t) \cdot ptg_i$$

Each agent has a defined neighbourhood, a subgraph of the social network, where each linking node represents the relationship to the other $N - 1$ agents. Along with each link is a corresponding social coefficient $c_{ij}$ where $i \neq j$, and $i, j \in \{1, ..., N\}$. The social coefficient $c_{ij} \in [0, 1]$, represents agent $i$’s ability to obtain agent $j$’s previous stock trading decision.

$$c_{ij} = \begin{cases} 
0 & \text{No communication} \\
(0, 1) & \text{Communication} \\
1 & \text{Perfect communication}
\end{cases}$$

The perceived relationship between $i$ and $j$ may differ from that of $j$ and $i$ and could result in a varying likelihood of obtaining previous trading decisions. Due to the possible variation in relationships, it creates three different potential social coefficient comparisons.

1. If $c_{ij} > c_{ji}$ then amount of information gathered by $i$ is greater than that of $j$.
2. If $c_{ij} = c_{ji}$ then amount of information gathered by each agent is the same.
3. If $c_{ij} < c_{ji}$ then amount of information gathered by $i$ is less than that of $j$. 

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2.3 Utility and an Agent’s Daily Trading Decision

Each day, before the market closes, the agents trade their financial instruments based on their expectations for the closing price. With the unknown behaviour of the market price and the potential price fluctuation, the agents require a mechanism to aid in their decision whether to buy, hold or sell. The decision-making process for each agent is determined using the concept of total utility, defined in equation (2.1).

\[ U_{im}^{\text{total}}(t) = (1 - \lambda_{im}(t))U_{im}^{\text{obj}}(t) + \lambda_{im}(t)U_{im}^{\text{sub}}(t) \]  

(2.1)

where \( m \in \{1, \ldots, M\} \), and \( i \in \{1, \ldots, N\} \). The function \( \lambda_{im}(t) \) is the average social coefficient based on the most commonly chosen trading decision at time \( t - 1 \).

The formulation of the total utility found in (2.1) is broken into two components, objective and subjective. The objective utility function \( U_{im}^{\text{obj}}(t) \) attempts to predict the future stock price. Depending on how risk-seeking the agent is, the expected price for iteration \( t \) will be dependent on the stock’s price history, the concept of momentum, and a price selected from a normal distribution. The subjective utility \( U_{im}^{\text{sub}}(t) \) uses the social coefficients/relations with \( i \)’s peers and their past trade decision to suggest a trade decision.

Once both the objective and subjective utility values are computed, and the total utility is then determined, the agents will then make their trading decision. Below demonstrates
how the agents will make their decision based on the total utility value.

\[
D_{im}^{\text{overall}}(t) = \begin{cases} 
\text{Buy} & \text{if } U_{im}^{\text{total}}(t) < 0 \\
\text{Hold} & \text{if } U_{im}^{\text{total}}(t) = 0 \\
\text{Sell} & \text{if } U_{im}^{\text{total}}(t) > 0 
\end{cases}
\]

### 2.3.1 Objective Utility

The objective utility at each iteration is defined by two components and is represented as the equation (2.2):

\[
U_{im}^{\text{obj}}(t) = acc_{im}(t-1)D_{im}^{\text{obj}}(t)
\]  

(2.2)

where \(acc_{im}(t-1)\) represents the accuracy of the agent’s previous price estimation and \(D_{im}^{\text{obj}}(t)\) represents the objective trade decision. In order for \(acc_{im}(t-1) \in [0,1]\), the conditions in (2.3) must be satisfied.

\[
acc_{im}(t-1) = \begin{cases} 
0 & \text{if } \frac{|p_{im}^{e}(t-1) - p_{m}(t-1)|}{p_{m}(t-1)} > 1 \\
1 - \frac{|p_{im}^{e}(t-1) - p_{m}(t-1)|}{p_{m}(t-1)} & \text{if } 0 \leq \frac{|p_{im}^{e}(t-1) - p_{m}(t-1)|}{p_{m}(t-1)} \leq 1 
\end{cases}
\]

(2.3)

The objective trade decision \(D_{im}^{\text{obj}}(t)\) is dependent on the expected price \(p_{im}^{e}(t)\), the price trend \(p_{im}^{\text{trend}}(t-1)\) and the trading deviation \(k_{im}\). Each agent has a variable \(\alpha_i(t)\) which represents how risk-seeking the agent is, the value of alpha is determine by a truncated normal distribution \(\alpha_i(t) \sim N(0.50, 0.36)\), such that \(0 \leq \alpha_i(t) \leq 1\). The expected price
defined in (2.4) represents the agent’s evaluation of the financial instrument at time $t$

$$p_{im}^e(t) = (1 - \alpha_i(t)) \cdot p_{im}^{trend}(t) + \alpha_i(t) \cdot \zeta_{im}(t)$$

(2.4)

where $\zeta_{im}(t) \sim N(\bar{p}_m, \sigma^2_m)$. The variable $p_{im}^{trend}(t)$ is used to implement momentum into the agent’s evaluation of the future stock price, (2.5) assumes that the future price will follow the trend of the previous two days.

$$p_{im}^{trend}(t) = p_{im}(t - 1) + [p_{im}(t - 1) - p_{im}(t - 2)]$$

(2.5)

As previously defined $k_{im}$ represents the deviation from the expected price at which the agent is willing to trade.

$$k_{im}(t) = p_{im}^e(t) \cdot ptg_i$$

(2.6)

Once the values of (2.4), (2.5), and (2.6) are determined, the agent compares the expected price to the prices it’s willing to trading at to determine the objective trade decision at time $t$.

$$D_{im}^{obj}(t) = \begin{cases} 
1 & \text{If } p_{im}^e(t) \geq p_{im}^{trend}(t - 1) + k_{im}(t) \\
0 & \text{If } p_{im}^{trend}(t - 1) - k_{im}(t) < p_{im}^e(t) < p_{im}^{trend}(t - 1) + k_{im}(t) \\
-1 & \text{If } p_{im}^e(t) \leq p_{im}^{trend}(t - 1) - k_{im}(t) 
\end{cases}$$

In the absence of subjective factors and utility, then the agent’s decision making is given by
the value of $D_{im}^{obj}(t)$.

$$
\text{If } D_{im}^{obj}(t) = \begin{cases} 
1 & \text{Then Sell} \\
0 & \text{Then Hold} \\
-1 & \text{Then Buy}
\end{cases}
$$

2.3.2 Subjective Utility

The idea behind the subjective utility is that brokers actively communicate with one another to gauge their perception of the stock value. As described in [2.2.2], the agent’s are connected through a social network, where each link represents the agents’ relationship and has a corresponding social coefficient. The corresponding coefficients represent the likelihood to obtain the previous trading decision made by the agent’s peer. The subjective utility is represented by the equation below:

$$U_{im}^{sub}(t) = D_{im}^{sub}(t) \quad (2.7)$$

To determine the subjective utility of $i$, the agents in $i$’s neighbourhood are grouped based on their previous trade decisions. Let $B_{im}, H_{im}, S_{im}$ represent the set of agents in agent $i$’s neighbourhood who bought, held and sold stock the previous day, respectively and $SN_i$ represent the set containing all of the agents in agent $i$’s network. Then agent $i$’s subjective
Let \( V, W, Z \in \mathbb{N} \), where \( 0 \leq \{V, W, Z\} \leq N \) and \( b = \{b_1, ..., b_V\} \), \( h = \{h_1, ..., h_W\} \), \( s = \{s_1, ..., s_Z\} \) be distinct sets containing the agent indices of agents who had chosen to buy, hold and sell, with respect to the financial instrument \( m \), where \(|b|+|h|+|s| = N\). Depending on agent \( i \)'s subjective trade decision, the corresponding average social coefficient will either be

\[
\begin{align*}
    c_{ibm} &= \frac{\sum_{b=1}^{b_V} c_{ib}}{|B_{im}|} \\
    c_{ism} &= \frac{\sum_{s=1}^{s_W} c_{is}}{|S_{im}|} \\
    c_{ihm} &= \frac{\sum_{h=1}^{h_Z} c_{ih}}{|H_{im}|}
\end{align*}
\]
and the associated average social coefficient will be

\[
\lambda_{im}(t) = \begin{cases} 
\frac{c_{ihm}}{c_{ilm}} & \text{if } D_{im}^{sub} = -1 \\
\frac{c_{ilm}}{c_{ilm}} & \text{if } D_{im}^{sub} = 0 \\
\frac{c_{ism}}{c_{ilm}} & \text{if } D_{im}^{sub} = 1
\end{cases}
\]

where \(\lambda_{im}(t) \in \mathbb{R}_{\geq 0}\) and \(\lambda_{im}(t) \in [0, 1]\).

### 2.4 Model Objectives and Investigative Methods

Below are the parameters and indices used within the thesis or initialized in the computational simulations.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Index Symbol</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Agents</td>
<td>i</td>
<td>N</td>
<td>50, 100, 150, 200</td>
<td>-</td>
</tr>
<tr>
<td>Number of Financial Instruments</td>
<td>m</td>
<td>M</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>Number of Iterations</td>
<td>t</td>
<td>T</td>
<td>30</td>
<td>Days</td>
</tr>
<tr>
<td>Number of Trials per Simulation</td>
<td>(\omega)</td>
<td>(\Omega)</td>
<td>6, 58, 100</td>
<td>-</td>
</tr>
<tr>
<td>Tolerance</td>
<td>-</td>
<td>tol</td>
<td>0.006</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 2.4: The varying parameters used to determine the effects of social influence.

Python version 3.5.3 was used to write the agent-based model discussed in this paper; the program uses the following libraries/packages: python standard library, matplotlib version 2.0.2, NumPy version 1.12.1, SciPy version 1.1.0 and XlsWriter version 0.9.6. Matplotlib is a free 2D plotting library designed to produce MATLAB-like interface [30]. NumPy is
a free library used for scientific computing in Python; the package contains a practical  
N-dimensional array object and useful linear algebra, Fourier transformation and random  
number capabilities [31]. SciPy free library commonly used for mathematics, science, and  
engineering; this library provides a collection of numerical algorithms and domain-specific  
toolboxes [32]. XlsWriter is a free library used to write files in the Excel+ XLSX file format;  
this module can be used to write text, numbers, and formulas to multiple worksheets [33].  
Throughout the simulation runs data is stored in Comma-Separated Values (CSV) files and  
Microsoft Excel XLSX files. The plots observed throughout the paper are saved as Portable  
Network Graphic (PNG) files. The construction and execution of the agent-based model  
discussed in this paper was run on a laptop running Windows 10, with a Intel(R) Core(TM)  
i7-4510U CPU @ 2.00GHz 2.60 GHz processor and 12.0 GB of installed memory (RAM).  

With 1 financial instrument, 30 iterations/days, 6 trials and 50 agents the simulation  
run time was 27.40 seconds. Keeping the same parameters as listed above but with 100 and  
150 agents, the simulation run times were 48.01 and 82.03 seconds, respectively. With 1  
financial instrument, 30 iterations/days, 58 trials and 200 agents the simulation run time  
was 19 minutes and 07 seconds.  

Through the use of the agent-based model, we attempt to determine whether social  
influence affects brokers trade decisions. We examine the trading decisions of the agents  
with and without a social network over a period of 30 days and then examine how the  
agents’ total utility was affected.
Chapter 3

Model in the Absence of Human Networks

3.1 Sensitivity Analysis

Simulations without social interaction will act as a baseline test to demonstrate the averaged agent’s normal behaviour. We require $\lambda_{im}(t) = 0$ for the agents total utility at time $t$ to be strictly dependent on the objective utility. Before conducting the simulations it was determined that the number of days the agents would interact within the market would be 30 days, and the number of agents initialized in the system would be one of 50, 100, 150 and 200. The selection of agents initialized was designed to emulate the behaviour of brokers within a company, as opposed to examining the interaction between brokers and any other personnel that trade within the stock market. Within each simulation there are $x$ trials initialized, each trial is a duplication of the initial set of agents, where all trials in parallel...
proceed to trade within the market over the course of 30 days. After the completion of the simulation the decisions of each agent over each day are averaged with the other \(x - 1\) trials to create an averaged decision for each agent. Sensitivity analysis is required to determine the number of trials sufficient enough to cause the system dynamics to settle into a form of equilibrium. Equilibrium represents the scenario in which the agent’s decisions will not sufficiently change as the number of trials increases.

The approach to the sensitivity analysis is to use the sum of square errors and variance to determine the number of trials required for the agents trading decisions to be accurately represented. Let \(\omega, \beta \in \mathbb{N}, \omega \in \{1, ..., \Omega\}, \beta \in \{2, ..., \Omega\}\) and \(X^\omega\) be a \(N \times T\) matrix, given the equations

\[
X_{i,t}^{\omega} = |U_{im}^{\omega}(t)|
\]

\[
\bar{X}_{i,t}^{\beta} = \frac{1}{\beta} \sum_{\omega=1}^{\beta} X_{i,t}^{\omega}
\]

\[
\sigma_{i,t}^{2,\beta} = \frac{1}{\beta - 1} \sum_{\omega=1}^{\beta} (X_{i,t}^{\omega} - \bar{X}_{i,t}^{\beta})^2.
\]

\(\forall i \in \{1, ..., N\}, t \in \{1, ..., T\}\) and \(\beta \in \{2, ..., \Omega\}\). If

\[
\max |\sigma_{i,t}^{2,\beta}| > \max |\sigma_{i,t}^{2,\beta-1}|
\]

and

\[
\max |\sigma_{i,t}^{2,\beta}| - \max |\sigma_{i,t}^{2,\beta-1}| < tol
\]

\(\forall i \in \{1, ..., N\}\) and \(t \in \{1, ..., T\}\), then \(\beta\) represents the number of trials required such that the difference in variance, in the agent’s decisions, is strictly less than the defined tolerance.

Let \(U\) be defined as the set that contains all \(\sigma_{i,t}^{2,\beta}\), such that \(U = \{\sigma_{i,t}^{2,1}, \sigma_{i,t}^{2,2}, ..., \sigma_{i,t}^{2,\Omega}\} = \)
\{u_1,\ldots,u_{\Omega-1}\}$, and let $V$ be the set where $V = \{\max|\sigma_{i,t_m}^2| - \max|\sigma_{i,t_m}^2|, \ldots, \max|\sigma_{i,t_m}^2| - \max|\sigma_{i,t_m}^2|\} = \{v_1,\ldots,v_{\Omega-2}\}$. Below are figures which illustrate the difference between the elements within both sets $U$ and $V$.

Figure 3.1: Four different perspectives of the determined variance for multiple trials within simulation 1, when initialized with 50 agents and 100 trials.
Figure 3.2: Four different perspectives of the determined variance for multiple trials within simulation 1, when initialized with 100 agents and 100 trials.
Figure 3.3: Four different perspectives of the determined variance for multiple trials within simulation 1, when initialized with 150 agents and 100 trials.
Figure 3.4: Four different perspectives of the determined variance for multiple trials within simulation 1, when initialized with 200 agents and 100 trials.
Figure 3.5: The difference in maximal variance between trials in four separate simulations, each initiated with 50 agents and 100 trials.
Figure 3.6: The difference in maximal variance between trials in four separate simulations, each initiated with 100 agents and 100 trials.
Figure 3.7: The difference in maximal variance between trials in four separate simulations, each initiated with 150 agents and 100 trials.
Figure 3.8: The difference in maximal variance between trials in four separate simulations, each initiated with 200 agents and 100 trials.

Table 3.1: The averaged $\beta$ values determined through the simulation of 4 different agent values.

<table>
<thead>
<tr>
<th>Number of Agents</th>
<th>Average $\beta$ Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 Agents</td>
<td>6.0</td>
</tr>
<tr>
<td>100 Agents</td>
<td>$5.8 \approx 6.0$</td>
</tr>
<tr>
<td>150 Agents</td>
<td>6.0</td>
</tr>
<tr>
<td>200 Agents</td>
<td>$57.6 \approx 58.0$</td>
</tr>
</tbody>
</table>

Comparing the results of the average maximal variances plots, it is apparent that in the domain $\beta \in [0, 20]$ as the number of agents increases within the system the variation in the difference of maximal variance decreases. From Table 3.1 and observed in the plots above, when 50, 100 or 150 agents are initialized within the system the difference in maximal
variance approaches 0 quickly and reaches the tolerance value when $\beta = 6$. With the increase of agents in the system, the variation in the difference of maximal variance decreases. The behaviour observed with 200 agents is vastly different from the cases of 50, 100 and 150 agents. The variation in the difference of maximal variance is minimal, the rate in which the difference in maximal variance approaches 0 is much smaller and the required averaged trials to reach the tolerance value is $\beta = 58$. Because of these differences, the examination of 200 agents with a social network will be conducted separately.
Chapter 4

Model with Networks

4.1 Implementing Social Networks

The agents require a social network to interact with one another. The two types of social networks examined are random and preferential attachment networks. With the implementation of the social network and influence of the agent’s peers, the subjective utility can either solidify the agent’s opinion on the future closing price or sway the agent to undergo a different method of trading. The situations in which the agents are swayed by their peers will be referred to as affected total utility (ATU) cases and are indexed accordingly.

1. If $U_{obj} = 0$ and $U_{sub} > 0$ then $U_{obj} + U_{sub} = U^{Tot} > 0$
2. If $U_{obj} = 0$ and $U_{sub} < 0$ then $U_{obj} + U_{sub} = U^{Tot} < 0$
3. If $U_{obj} > 0$ and $U_{sub} < 0$ and $|U_{obj}| < |U_{sub}|$ then $U_{obj} + U_{sub} = U^{Tot} < 0$
4. If $U_{obj} > 0$ and $U_{sub} < 0$ and $|U_{obj}| = |U_{sub}|$ then $U_{obj} + U_{sub} = U^{Tot} = 0$
5. If $U_{obj} < 0$ and $U_{sub} > 0$ and $|U_{obj}| < |U_{sub}|$ then $U_{obj} + U_{sub} = U^{Tot} > 0$
6. If $U^{obj} < 0$ and $U^{sub} > 0$ and $|U^{obj}| = |U^{sub}|$ then $U^{obj} + U^{sub} = U^{Tot} = 0$

The presence of these 6 ATU cases will be examined in the Agents with ATU figures. Throughout the various simulations, the average total utility with a social network initialized will be directly compared to the agent’s average total utility without a network.

## 4.2 Random Networks

With the implementation of a random network, $i$ is randomly assigned links to other agents within the system and a corresponding social coefficient. The social coefficient determines the strength of communication between agents. There are three different variations of the random network which will be examined.

1. $c_{ij} \sim unif(0, 1), \forall i, j \in \{1, ..., N\}$ where $i \neq j$.

2. If $c_{ij} \sim unif(0, 1) < 0.50$ then $c_{ij} = 0, \forall i, j \in \{1, ..., N\}$ where $i \neq j$.

3. $c_{ij} \sim unif(0.5, 1), \forall i, j \in \{1, ..., N\}$ where $i \neq j$.

### 4.2.1 First Variation

The first random network is designed to implement weak, strong and perfect relationships; this scenario attempts to examine the influence of the varying relationship. The social coefficients are determined through a uniform distribution; if $c_{ij} > 0$ then a link is formed.
Figure 4.1: Comparing the averaged total utility of 50 agents over the course of 30 days. Six independent trials are initialized to determine the average total utility. Figures (a) and (b) demonstrate the differences of $U_{total}$ for the case with and without $U_{sub}$. Figure (c) and (d) examine the cases where the subjective utility influence the agents trading decisions and illustrates the differences in $U_{total}$ when the subjective utility is and is not present.

Figure (4.1a) illustrates the behaviour of the agents without the influence of their peers. Throughout the course of the 30 days the total utility of the agents appears to be mixed, there are agents with various levels of confidence in their trading decisions whether it be to buy or sell. The distinction between the total utility and the objective utility in Figure (4.1b) is that the majority of the total utility appears to be within the domain of [-0.75, 0.75]. Figures (4.1c) and (4.1d) examine the cases where the social utility influences the agents trading decisions. From the two plots we observe that the ATU has a rather small magnitude compared to the total utility displayed in Figure (4.1c). This suggests that the
agents were not confident in their swayed trading decision.

Figure 4.2: Comparing the averaged total utility of 100 agents over the course of 30 days. Six independent trials are initialized to determine the average total utility. Figures (a) and (b) demonstrate the differences of $U_{total}$ for the case with and without $U_{sub}$. Figure (c) and (d) examine the cases where the subjective utility influence the agents trading decisions and illustrates the differences in $U_{total}$ when the subjective utility is and is not present.

Similar to the case with 50 agents, Figure 4.2a showed that there were various opinions on the behaviour of the financial instrument as some agents were inclined to buy when others wanted to sell. In Figure 4.2b, it appears that the total utility is still prominently found in the domain of [-0.75, 0.75]. The magnitude of ATU observed in Figure 4.2d remains relatively consistent of that found with 50 agents, but it should be noted that the ATU case 2 is now observed. This suggests that the agents’ social influence swayed them to purchase stock instead of holding.
Figure 4.3: Comparing the averaged total utility of 150 agents over the course of 30 days. Six independent trials are initialized to determine the average total utility. Figures (a) and (b) demonstrate the differences of $U_{total}$ for the case with and without $U_{sub}$. Figure (c) and (d) examine the cases where the subjective utility influence the agents trading decisions and illustrates the differences in $U_{total}$ when the subjective utility is and is not present.

From Figure (4.3), it appears that the implementation of 150 agents does not result in different conclusions observed with 50 or 100 agents. Once again Figure (4.3b) shows that the total utility is confined to the domain of $[-0.75, 0.75]$. The ATU seen in Figure (4.3d) follows the same behaviour found in Figure (4.2d).
Table 4.1: The breakdown of the average affected total utility observed in the four simulations and the percentage in which the agents’ decisions are changed because of its peers.

Table 4.1 highlights two interesting observations. Firstly, as we increase the number of agents within the system the occurrence of ATU decreases and secondly the percentage of decisions which are affected is significantly low. With the occurrence of ATU ranging from 4.62 – 11.80% it suggests that the impact of a random network with weak links on the agents’ trading decisions are minimal. It appears that as more agents enter the system, the effect of the social influence lessens.

4.2.2 Second Variation

Different to the first variation of the random network, the second variation attempts to eliminate poor relationships between agents, where it is assumed that acquaintances are unlikely to gather any information about the brokers past trading decisions. From the uniform distribution if $c_{ij} < 0.5$ then $c_{ij} = 0$, resulting in no link.
Figure 4.4: Comparing the averaged total utility of 50 agents over the course of 30 days. Six independent trials are initialized to determine the average total utility. Figures (a) and (b) demonstrate the differences of $U_{total}$ for the case with and without $U_{sub}$. Figure (c) and (d) examine the cases where the subjective utility influence the agents trading decisions and illustrates the differences in $U_{total}$ when the subjective utility is and is not present.

Figure (4.4a) reveals the trading decisions of 50 agents over the course of 30 days, without any social influence. The total utility over the 30 days appears to be varied as agents are inclined to buy and sell. In Figure (4.4b) the total utility with social influence appears to form a surface, where the agents seem to be eager to sell their stock. Figures (4.4c) and (4.4d) reveal a rather interesting situation: on day 1 up until day 20 the agents in Figure (4.4c) have a strong desire to purchase stock. Examining the same time frame the agents’ total utility in Figure (4.4d) appears to be highly influenced, as the agents now desire to sell their stock. From day 20 to 30, it appears that agents who had a strong desire to sell their
stock are now influenced as well to purchase stock. We can speculate that as time progressed the social influences were great enough to sway agents’ trading decisions.

\[
\lambda_i = 0, \quad \forall i \in \{1, \ldots, N\}. \\
\lambda_i \neq 0, \quad \forall i \in \{1, \ldots, N\}.
\]

Figure 4.5: Comparing the averaged total utility of 100 agents over the course of 30 days. Six independent trials are initialized to determine the average total utility. Figures (a) and (b) demonstrate the differences of \(U_{total}\) for the case with and without \(U_{sub}\). Figure (c) and (d) examine the cases where the subjective utility influence the agents trading decisions and illustrates the differences in \(U_{total}\) when the subjective utility is and is not present.

In Figure (4.5b) it appears that the surface produced by the total utility is flatter compared to that observed in (4.4b). This suggests that the social influence experienced on the last 10 days is more prominent when 100 agents are initialized within the system. It appears that Figures (4.5c) and (4.5d) illustrate the same behaviour seen with 50 agents. It is worth noting that the frequency of ATU case 3 over the span of the last 10 days is less frequent compared to the environment with 50 agents.
Figure 4.6: Comparing the averaged total utility of 150 agents over the course of 30 days. Six independent trials are initialized to determine the average total utility. Figures (a) and (b) demonstrate the differences of $U_{\text{total}}$ for the case with and without $U_{\text{sub}}$. Figure (c) and (d) examine the cases where the subjective utility influence the agents trading decisions and illustrates the differences in $U_{\text{total}}$ when the subjective utility is and is not present.

Keeping with the progression observed with 100 agents it appears that the surface in Figure (4.6b) is flatter and even more pronounced. This observation highlights the previous claim that the increase in agents in the system results in an increased social influence over the last 10 days. Figures (4.6c) and (4.6d) appear to follow the trend observed with 100 agents, where the frequency of ATU case 3 continues to decreases.
<table>
<thead>
<tr>
<th>Number of Agents</th>
<th>Average Affected Total Utility</th>
<th>Average Affected Total Utility / Decisions (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Case 1 (%) Case 2 (%) Case 3 (%) Case 4 (%) Case 5 (%) Case 6 (%)</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>0.00 0.00 18.27 0.00 81.73 0.00</td>
<td>47.80</td>
</tr>
<tr>
<td>100</td>
<td>0.00 0.00 8.96 0.00 91.04 0.00</td>
<td>46.87</td>
</tr>
<tr>
<td>150</td>
<td>0.00 0.00 3.53 0.00 96.47 0.00</td>
<td>43.49</td>
</tr>
</tbody>
</table>

Table 4.2: The breakdown of the average affected total utility observed in the four simulations and the percentage in which the agents decisions are changed because of its peers.

Table 4.2 shows that the frequency of ATU decreases as the number of agents increase, this suggests that as the number of agents providing feedback increases, the value of the information decreases. It should be noted that the frequency in the agents decisions is rather large due to the removal of weak links.

4.2.3 Third Variation

The third variation is a complete network, where all agents have strong relationships with one another; this scenario will demonstrate how multiple opinions can affect the overall decision of an agent.
Figure 4.7: Comparing the averaged total utility of 50 agents over the course of 30 days. Six independent trials are initialized to determine the average total utility. Figures (a) and (b) demonstrate the differences of $U_{total}$ for the case with and without $U_{sub}$. Figure (c) and (d) examine the cases where the subjective utility influence the agents trading decisions and illustrates the differences in $U_{total}$ when the subjective utility is and is not present.

Figure (4.7a) examines the trading decisions of 50 agents over the course of 30 days, without any social influence. The total utility over the 30 days appears to be varied as agents are inclined to buy and sell. In Figure (4.7b) the total utility with social influence appears to form two surfaces, from day 1 to 20 it appears that the agents seem to be eager to purchase their stock but after day 20 the agents have a desire to sell of their stock. In Figure (4.7d), examining the behaviour from day 1 to 20 we notice that the agents are influenced to purchase stock. Over this time span we see that ATU case 2 and 3 are present which states that agents who were inclined to hold and sell their stock are now influenced to buy. In the
last 10 days we observed that majority of the agents are now inclined to sell their stock as a result of ATU case 1 and 5, but it should be noted that there is a minority of agents who are still influenced to purchase stock.

\[ \lambda_i = 0, \forall i \in \{1,..,N\}. \]

\[ \lambda_i \neq 0, \forall i \in \{1,..,N\}. \]

Figure 4.8: Comparing the averaged total utility of 100 agents over the course of 30 days. Six independent trials are initialized to determine the average total utility. Figures (a) and (b) demonstrate the differences of \( U_{\text{total}} \) for the case with and without \( U_{\text{sub}} \). Figure (c) and (d) examine the cases where the subjective utility influence the agents trading decisions and illustrates the differences in \( U_{\text{total}} \) when the subjective utility is and is not present.

In Figure (4.8b) the total utility appear to make two defined surfaces split at day 20, the agents in the last 10 days are have a desire to sell their stock while the first 20 days suggest that the agents were influenced to buy stock. In Figures (4.8c) and (4.8d) the social influence on the agents is apparent, over the course of the first 20 days the agents that were initially inclined to sell or hold are now being influenced by their peers to buy. Over the last
10 days we see that the agents are influenced to sell their stock. During this time span we no longer see evidence of any agents influenced to buy stock.

Figure 4.9: Comparing the averaged total utility of 150 agents over the course of 30 days. Six independent trials are initialized to determine the average total utility. Figures (a) and (b) demonstrate the differences of $U_{total}$ for the case with and without $U_{sub}$. Figure (c) and (d) examine the cases where the subjective utility influence the agents trading decisions and illustrates the differences in $U_{total}$ when the subjective utility is and is not present.

The case of 150 agents appears to be identical to the case of 100 agents this suggests that the social influence of a complete random network with either 100 or 150 agents does not vary enough to observe a different outcome.
Table 4.3: The breakdown of the average affected total utility observed in the four simulations and the percentage in which the agents' decisions are changed because of its peers.

The frequency of ATU in Table 4.3 suggests that the agents are the most socially influenced with 50 agents. The difference in ATU frequency between 100 and 150 agents confirms the suggested claims above, that the increase in agents does not have a significant impact on the agents' social influence.

4.3 Preferential Attachment Network

Implementing preferential attachment (PA) networks results in a smaller social network compared to the random network. When assigning the social coefficients to the agent's links, two variations will be examined.

1. \( c_{ij} \in (0, 1], \forall i, j \in \{1, ..., N\} \) where \( i \neq j \).

2. \( c_{ij} \in [0.5, 1], \forall i, j \in \{1, ..., N\} \) where \( i \neq j \).

4.3.1 First Variation

With this first variation, it allows for various types of relationships to be formed, demonstrating strong or weak communication between agents.
Figure 4.10: Comparing the averaged total utility of 50 agents over the course of 30 days. Six independent trials are initialized to determine the average total utility. Figures (a) and (b) demonstrate the differences of $U_{total}$ for the case with and without $U_{sub}$. Figure (c) and (d) examine the cases where the subjective utility influence the agents trading decisions and illustrates the differences in $U_{total}$ when the subjective utility is and is not present.

Figures (4.10a) and (4.10b) demonstrate the trading decisions of 50 agents with and without a preferential attachment network. In both Figures we observe that the trading decisions are rather mixed where agents are inclined to buy and sell throughout the 30 days. In Figures (4.10c) and (4.10d), over the simulation period we see that agents with the desire to buy and sell are continuously being influenced by their peers to partake in the opposing trading decision. In Figure (4.10d) it should be noted that there are various magnitudes of affected total utility.
Figure 4.11: Comparing the averaged total utility of 100 agents over the course of 30 days. Six independent trials are initialized to determine the average total utility. Figures (a) and (b) demonstrate the differences of $U_{total}$ for the case with and without $U_{sub}$. Figure (c) and (d) examine the cases where the subjective utility influence the agents trading decisions and illustrates the differences in $U_{total}$ when the subjective utility is and is not present.

The behaviour of the trading decisions observed in Figure (4.11) does not vary significantly from the case of 50 agents. One noticeable difference is found in Figure (4.11d) is that with the increase in agents ATU case 1 is now present.
Figure 4.12: Comparing the averaged total utility of 150 agents over the course of 30 days. Six independent trials are initialized to determine the average total utility. Figures (a) and (b) demonstrate the differences of $U_{\text{total}}$ for the case with and without $U_{\text{sub}}$. Figure (c) and (d) examine the cases where the subjective utility influence the agents trading decisions and illustrates the differences in $U_{\text{total}}$ when the subjective utility is and is not present.

Comparing the figures of 150 agents to those of 100 or 50 agents it appears that there isn’t a significant difference in results. Once again it is worth noting that in Figure 4.12d both ATU case 1 and 2 are now present.

<table>
<thead>
<tr>
<th>Number of Agents</th>
<th>Average Affected Total Utility</th>
<th>Average Affected Total Utility / Decisions (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Case 1 (%)  Case 2 (%)  Case 3 (%)  Case 4 (%)  Case 5 (%)  Case 6 (%)</td>
<td>Case 1 (%)</td>
</tr>
<tr>
<td>50</td>
<td>0.00  0.00  51.02  0.00  48.98  0.00</td>
<td>26.13</td>
</tr>
<tr>
<td>100</td>
<td>0.26  0.00  50.92  0.00  48.81  0.00</td>
<td>25.27</td>
</tr>
<tr>
<td>150</td>
<td>0.26  0.17  52.14  0.00  47.43  0.00</td>
<td>25.91</td>
</tr>
</tbody>
</table>

Table 4.4: The breakdown of the average affected total utility observed in the four simulations and the percentage in which the agents decisions are changed because of its peers.
The results from Table (4.4) illustrates two different findings. The first finding which is apparent from the graphs is that the social influence in a system of 50, 100 or 150 agents remains rather similar as seen by the difference in the frequency of ATU. Secondly the frequency of the ATU is quite low, an average of $\approx 26\%$, this suggests a slight social influence as a result of a preferential attachment network with both weak and strong links.

4.3.2 Second Variation

In this case, each link between agents is defined with a social coefficient $c_{ij} \geq 0.5$, demonstrating strong relationships only.
Figure 4.13: Comparing the averaged total utility of 50 agents over the course of 30 days. Six independent trials are initialized to determine the average total utility. Figures (a) and (b) demonstrate the differences of $U_{total}$ for the case with and without $U_{sub}$. Figure (c) and (d) examine the cases where the subjective utility influence the agents trading decisions and illustrates the differences in $U_{total}$ when the subjective utility is and is not present.

Figures (4.13a) and (4.13b) demonstrate the trading decisions of 50 agents with and without a preferential attachment network. In both figures we observe that the trading decisions are rather mixed where agents are inclined to buy and sell throughout the 30 days. In Figures (4.13c) and (4.13d), over the simulation period we see that agents with the desire to buy, hold and sell are continuously being influenced by their peers to partake in an alternative trading decision. The magnitude of the ATP observed in Figure (4.13d) appears to be within the domain of $[-0.75, 0.75]$. 

50
Figure 4.14: Comparing the averaged total utility of 100 agents over the course of 30 days. Six independent trials are initialized to determine the average total utility. Figures (a) and (b) demonstrate the differences of $U_{total}$ for the case with and without $U_{sub}$. Figure (c) and (d) examine the cases where the subjective utility influence the agents trading decisions and illustrates the differences in $U_{total}$ when the subjective utility is and is not present.

The results of 100 agents appears to be significantly different compared to the results observed with 50 agents. In Figure 4.14(d) there are two noticeable differences from the simulation with 50 agents. Firstly, ATU case 1 is now present, secondly the magnitude in case 1 and 2 are significant, indicating that agents that were inclined to hold are now eager to buy/sell as a result of their peers.
Figure 4.15: Comparing the averaged total utility of 150 agents over the course of 30 days. Six independent trials are initialized to determine the average total utility. Figures (a) and (b) demonstrate the differences of $U_{total}$ for the case with and without $U_{sub}$. Figure (c) and (d) examine the cases where the subjective utility influence the agents trading decisions and illustrates the differences in $U_{total}$ when the subjective utility is and is not present.

The social influence observed with 150 agents appears to be similar to that observed with 50 agents. Although ATU case 1 and 2 remains in Figure (4.15d) the frequency of the two cases is less apparent compared to the case with 100 agents.

<table>
<thead>
<tr>
<th>Number of Agents</th>
<th>Average Affected Total Utility</th>
<th>Average Affected Total Utility / Decisions (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Case 1 (%)</td>
<td>Case 2 (%)</td>
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<tr>
<td>150</td>
<td>0.18</td>
<td>0.27</td>
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</table>

Table 4.5: The breakdown of the average affected total utility observed in the four simulations and the percentage in which the agents decisions are changed because of its peers.
From Table (4.5) it appears that the frequency of ATU is rather significant, which suggests that the agents are socially influenced by a preferential attachment network with only strong links. The table also confirms the observations previously suggested, claiming that ATU case 1 and 2 is more apparent with 100 agents compared to the results of 50 and 150 agents.

4.4 Social Networks with 200 Agents

The initialization of 58 trials each with 200 agents in the financial trading environment resulted in vastly different results compared to the systems with 50, 100 or 150 agents. Figures (4.16), (4.17), (4.18) illustrates the effect of the random network on 200 agents in the financial trading environment. Figures (4.19), (4.20) illustrates the effect of the preferential attachment network on 200 agents.
Figure 4.16: Comparing the averaged total utility (With the first variant of the random network) of 200 agents over the course of 30 days. Within the simulation there were 58 trials initialized to determine the average total utility. Figures (a) and (b) demonstrate the differences of $U_{total}$ for the case with and without $U_{sub}$. Figure (c) and (d) examine the cases where the subjective utility influence the agents trading decisions and illustrates the differences in $U_{total}$ when the subjective utility is and is not present.
Figure 4.17: Comparing the averaged total utility (With the second variant of the random network) of 200 agents over the course of 30 days. Within the simulation there were 58 trials initialized to determine the average total utility. Figures (a) and (b) demonstrate the differences of $U_{\text{total}}$ for the case with and without $U_{\text{sub}}$. Figure (c) and (d) examine the cases where the subjective utility influence the agents trading decisions and illustrates the differences in $U_{\text{total}}$ when the subjective utility is and is not present.
Figure 4.18: Comparing the averaged total utility (With the third variant of the random network) of 200 agents over the course of 30 days. Within the simulation there were 58 trials initialized to determine the average total utility. Figures (a) and (b) demonstrate the differences of $U_{total}$ for the case with and without $U_{sub}$. Figure (c) and (d) examine the cases where the subjective utility influence the agents trading decisions and illustrates the differences in $U_{total}$ when the subjective utility is and is not present.
Figure 4.19: Comparing the averaged total utility (with the first variant of the preferential attachment network) of 200 agents over the course of 30 days. Within the simulation there were 58 trials initialized to determine the average total utility. Figures (a) and (b) demonstrate the differences of $U^{t_{\text{otal}}}$ for the case with and without $U^{sub}$. Figure (c) and (d) examine the cases where the subjective utility influence the agents trading decisions and illustrates the differences in $U^{t_{\text{otal}}}$ when the subjective utility is and is not present.
Figure 4.20: Comparing the averaged total utility (With the second variant of the preferential attachment network) of 200 agents over the course of 30 days. Within the simulation there were 58 trials initialized to determine the average total utility. Figures (a) and (b) demonstrate the differences of $U_{\text{total}}$ for the case with and without $U_{\text{sub}}$. Figure (c) and (d) examine the cases where the subjective utility influence the agents trading decisions and illustrates the differences in $U_{\text{total}}$ when the subjective utility is and is not present.

<table>
<thead>
<tr>
<th>Network Type</th>
<th>Average AFFECTED Total Utility</th>
<th>Average AFFECTED Total Utility / Decision (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Case 1 (%)</td>
<td>Case 2 (%)</td>
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<tr>
<td>RN 1</td>
<td>6.83</td>
<td>3.78</td>
</tr>
<tr>
<td>RN 2</td>
<td>1.73</td>
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<td>RN 3</td>
<td>12.79</td>
<td>20.33</td>
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<tr>
<td>PA 1</td>
<td>1.99</td>
<td>1.37</td>
</tr>
<tr>
<td>PA 2</td>
<td>2.18</td>
<td>1.59</td>
</tr>
</tbody>
</table>

Table 4.6: The breakdown of the average affected total utility observed when the three the random networks (RN) and two preferential attachment networks (PA) are initialized.

At first glance the behaviour of 200 agents appears to be similar to the behaviour of
150 agents but looking at Table 4.5 it appears that the frequency of ATU is drastically different. With the implementation of 200 agents we move from small number statistics to large number statistics. Cases with weak links, observed in Figure 4.16 and 4.19, now have significantly higher ATU frequencies this suggests that there are enough agents within the system that the summation of weak influence is now enough to influence the agents trading decisions. In the other three cases which only have strong links, the ATU frequency is significantly greater than expected because the number of agents with strong links is large enough that the agents have a higher likelihood of using their peers opinion over the suggested decision from momentum trading.
Chapter 5

Conclusion and Future Work

5.1 Conclusion

In this thesis, we developed a financial trading environment through an agent-based model to investigate whether social interactions between agents affect their trading decisions.

Before we were able to investigate the problem at hand, we had to set rules and determine parameters for our agents and environment in the ABM. Data from 8 stable stocks were collected, analyzed and used as a reference for the price mechanism. The total utility equation which is composed of the objective and subjective utility will determine the agent’s trading decision at time $t$. The objective utility uses momentum and the agent’s intuition to predict the future price which results in a trading decision. The subjective utility acquires the previous favoured trading decision from the agent’s neighbourhood, a subgraph of the social network, to influence the objective trading decision. Once the system design is complete, a sensitivity analysis is conducted to determine the number of averaged trials required to
obtain an accurate representation of the agent’s behaviour without a social network. With the implementation of the random and preferential attachment networks, we examined the impact that social influence play on the agents trading decisions.

We found that the weak connections in random networks resulted in the occurrence of ATU to be rather low and as the number of agents initialized in the system increases, the percentage of ATU / Decision decrease significantly. In both variations of the preferential attachment networks, the systems demonstrated stability where the increase in agents did not have a significant impact on the percentage of ATU / Decision. When 200 agents are implemented into the system, the results appear to be vastly different compared to those of 50, 100 or 150 agents. Regardless of the variation or type of network, the percentage of ATU / Decision was always significantly greater compared to the systems with fewer agents.

5.2 Future Work

Using agent-based models to simulate the behaviour of brokers in a financial trading market has allowed for future studies through the adjustment of numerous parameters. This thesis explores the behaviour of 50, 100, 150 and 200 agents connected by a random or preferential attachment social network throughout a period of 30 days.

Further studies should be conducted to better understand the social influence of 200 or more agents. Furthermore, adjusting the number of days that the agents trade should also be investigated to determine how longer periods of time influence the agents’ trade decisions.

The implementation of different social networks could be another avenue to explore. Using a small-world network could not only allow for the potential analysis of agents influenced
by this type of social network but could also create potential comparisons of the effect of large and small social networks.
Bibliography


