High-Precision Half-Life Measurements for the Superallowed Fermi $\beta^+$

Emitters $^{10}\text{C}$ and $^{22}\text{Mg}$

by

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ABSTRACT

HIGH-PRECISION HALF-LIFE MEASUREMENTS FOR THE SUPERALLOWED FERMI $\beta^+$ EMITTERS $^{10}$C AND $^{22}$MG

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High precision measurements of the $ft$ values for superallowed Fermi $\beta$ transitions between $J^\pi = 0^+, T = 1$ isobaric analogue states allow for stringent tests of the electroweak interaction described by the Standard Model. These transitions provide an experimental probe of the Conserved-Vector-Current hypothesis, the most precise determination of $V_{ud}$, the up-down element of the Cabibbo-Kobayashi-Maskawa quark-mixing matrix, and set stringent limits on the existence of scalar currents in the weak interaction.

This thesis focuses on high-precision half-life measurements of two superallowed Fermi $\beta^+$ emitters, $^{10}$C and $^{22}$Mg. These half-life measurements were performed at TRIUMF’s Isotope Separator and Accelerator facility. For both cases, measurements of their half-lives were motivated by the discrepancies between previous measurements found in literature that resulted in inflations in the uncertainties to account for the inconsistent measurements.

Two independent measurements of the $^{10}$C half-life were performed. The first measurement was performed via $\gamma$-ray photopeak counting using the $8\pi \gamma$-ray spectrometer, an array of 20 Compton-suppressed high-purity germanium detectors, by measuring the time profile of the characteristic 718-keV $\gamma$-ray. This analysis yielded $T_{1/2} = 19.2969 \pm 0.0072$ s. A second measurement was performed via direct $\beta$ counting, using a $4\pi$ continuous-flow gas
proportional $\beta$ counter. The results from this analysis yields $T_{1/2} = 19.3009 \pm 0.0017$ s, and is the most precise superallowed half-life measurement reported to date and the first to ever achieve a relative precision below $10^{-4}$.

A half-life measurement of the superallowed $\beta^+$ emitter $^{22}\text{Mg}$ was performed via direct $\beta$ counting using the $4\pi$ continuous-flow gas proportional counter. This analysis yielded a half-life of $T_{1/2} = 3.87400 \pm 0.0079$ s. This measurement resolved a discrepancy between the previous world-average dataset, which was composed of only two measurements. These two half-life measurements showed a disagreement characterized by a $\chi^2/\nu = 4.0$, however, upon the inclusion of the new measurement presented in this thesis, now has a $\chi^2/\nu = 1.1$.

The high-precision half-life measurements reported in this thesis, once included with the other high-precision superallowed data, can be used to test fundamental properties of the Standard Model. In particular, the improved uncertainty of the $^{10}\text{C}$ half-life can be used to provide a more stringent limit on the contribution of scalar currents in the Standard Model. The improvement in the uncertainty of the $^{22}\text{Mg}$ half-life provides further tests of the isospin symmetry breaking corrections in superallowed Fermi $\beta$ decays.
Pour ma famille.

Pour ma famille.
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Chapter 1

Introduction

1.1 Nuclear $\beta$ Decay

Nuclear $\beta^\pm$ decay is a process in which an unstable radioactive nucleus with atomic number $Z$ and neutron number $N$ is converted into a more energetically stable nucleus with atomic number $Z \mp 1$ and neutron number $N \pm 1$ together with the emission of a positron/electron and electron neutrino/anti-neutrino. This is a weak interaction process which, at a fundamental level, involves an up quark changing into a down quark (or vice versa) mediated by the charged $W^\pm$ intermediate vector boson. There are three types of $\beta$ decay transitions that exist in nature. These are:

\begin{align*}
A_z X_N &\rightarrow A_{z+1} Y_{N-1} + \beta^- + \bar{\nu}_e \quad (\beta^-\text{decay}) \\
A_z X_N &\rightarrow A_{z-1} W_{N+1} + \beta^+ + \nu_e \quad (\beta^+\text{decay}) \\
A_z X_N + e^- &\rightarrow A_{z-1} W_{N+1} + \nu_e \quad (\text{EC decay})
\end{align*}
where $X$ represents the parent nucleus, $(Y, W)$ represent the daughter nuclei, $(\beta^\pm)$ represents the emitted positron or electron, $e^-$ represents a captured atomic electron in EC decay, and $(\nu_e, \bar{\nu}_e)$ are the electron neutrino and antineutrino. The electron capture (EC) process given in Equation 1.3 is similar to the $\beta^+$ decay given in Equation 1.2 in that the proton number $Z$ is decreased by one and the neutron number $N$ is increased by one. However, instead of the emission of a positron, this process involves the capture of an atomic electron which leaves the daughter atom in an excited atomic state. Since each of the three processes involves the conversion of a proton into a neutron, or vice versa, the total mass number $A = Z + N$ of the nucleus remains unchanged.

The energies released in these decays are given by:

$$Q_{\beta^-} = m(A_Z X_N)c^2 - m(A_{Z+1} Y_{N-1})c^2 \quad (\beta^- \text{decay})$$
$$Q_{\beta^+} = m(A_Z X_N)c^2 - m(A_{Z-1} W_{N+1})c^2 - 2m_e c^2 \quad (\beta^+ \text{decay})$$
$$Q_{EC} = m(A_Z X_N)c^2 - m(A_{Z-1} W_{N+1})c^2 - B_n \quad (\text{EC decay}),$$

where $m$ is the mass of the neutral atom with nucleus $A_Z X_N$, $m_e$ is the mass of the electron, and $B_n$ is the atomic binding energy of the electron in the $n$th electron shell. In Equations 1.4 and 1.5, the small differences in the atomic binding energy between the parent and daughter atoms are ignored since these differences are typically of $O(eV)$ and are usually negligible relative to the $Q$-values, which are typically of $O(MeV)$. The mass of the electron neutrino/anti-neutrino, experimentally known to be $m_{\nu_e} < 2 \ eV$ [1], has also been neglected. For $Q_{EC} \geq 2m_e c^2 = 1022 \ \text{keV}$, $\beta^+$ and EC decays are in direct competition since the initial and final state nuclei are the same for both processes. For $0 < Q_{EC} < 1022 \ \text{keV}$, the corresponding $Q_{\beta^+}$ is negative and is therefore energetically forbidden.
The energy released in these decays is converted into kinetic energy of the decay particles and possibly to the excitation energy of the daughter nucleus or atom. For the decay of a nucleus at rest, the $Q$-value is thus equal to

$$ Q = \sum_i T_i + E^*_D + E^*_{De}, $$ (1.7)

where $T_i$ is the kinetic energy of the $i$th decay product, $E^*_D$ is the excitation energy of the daughter nucleus, and $E^*_{De}$ is the excitation energy of the daughter atom. It is common, however, to define an effective $Q$ value as the sum of the kinetic energies of the final state particles:

$$ Q_{\text{eff}} = \sum_i T_i = Q - E^*_D - E^*_{De}. $$ (1.8)

For the 2-body final state of EC decay, conservation of momentum and energy provide the following constraints:

$$ \vec{p}_\nu + \vec{p}_D = 0 $$ (1.9)

$$ Q_{\text{eff}} = T_\nu + T_D. $$ (1.10)

From these equations, it can be seen that the corresponding neutrino energy spectrum results in a discrete $T_\nu$ solution for a given $Q_{\text{eff}}$. For $\beta^{\pm}$ decays, the 3-body final state satisfies the constraints:

$$ \vec{p}_\nu + \vec{p}_\beta + \vec{p}_D = 0 $$ (1.11)

$$ Q_{\text{eff}} = T_e + T_\nu + T_D. $$ (1.12)

Since the mass of the daughter nucleus is significantly larger than that of the electron and neutrino, the recoil kinetic energy is small and the electron and neutrino effectively
share the energy released in the decay. The electron and neutrino energy spectra are thus continuous from \(T_e \approx 0\), corresponding to the neutrino carrying away all of the kinetic energy, to \(T_e^{\text{max}} \approx Q_{\text{eff}}\), corresponding to the kinetic energy of the neutrino being zero.

### 1.2 Fermi Theory of \(\beta\) Decay

The transition rate between initial and final states for a time-dependent quantum-mechanical system is given by Fermi’s golden rule:

\[
\lambda = \frac{2\pi}{\hbar} |M_{fi}|^2 \frac{dn}{dE_f},
\]

(1.13)

where \(|M_{fi}|\) is the matrix element connecting the initial and final states, and \(\frac{dn}{dE_f}\) is the density of final states. The number of quantum states for a single particle confined within an arbitrary box of volume \(V\) is given by:

\[
n = \frac{V}{(2\pi\hbar)^3} \int d^3p \; d^3x.
\]

(1.14)

For \(\beta^\pm\) decays, conservation of momentum of the 3-body final state given in Equation 1.12 results in only two independent momenta. The number of independent quantum states for the \(\beta\) decay system is therefore the product of these independent quantum states. Choosing the independent states to be those of the electron and neutrino, the number of quantum states is thus:

\[
n = n_en_\nu = \frac{V^2}{(2\pi\hbar)^6} \left( \int_0^{p_{e,\text{max}}} p_e^2 dp_e \int d\Omega_e \int_0^{p_{\nu,\text{max}}} p_{\nu}^2 dp_{\nu} \int d\Omega_{\nu} \right),
\]

(1.15)

where \(\int d\Omega_e = \int d\Omega_{\nu} = 4\pi\) since both particles are emitted over all angles. The corresponding density of final states is:

\[
\frac{dn}{dE_f} = \frac{V^2}{4\pi^4\hbar^6} \frac{d}{dE_f} \left( \int_0^{p_{e,\text{max}}} p_e^2 dp_e \int_0^{p_{\nu,\text{max}}} p_{\nu}^2 dp_{\nu} \right).
\]

(1.16)
For β decay, a relativistic treatment of the motion is necessary and the variation in the momentum with respect to the total energy, \( E_f = E_\nu + E_e \) (where the small recoil energy of the heavy nucleus is ignored), must be calculated using the relativistic energy, \( E^2 = p^2c^2 + m^2c^4 \). Evaluating the density of states for electrons emitted with momenta between \( p_e \) and \( p_e + dp_e \) gives the result:

\[
\frac{dn}{dE_f} = \frac{V^2}{4\pi^4\hbar^6c^3}p_e^2(E - E_e)^2 \left( 1 - \frac{m_\nu^2c^4}{(E - E_e)^2} \right)^{1/2} dp_e. \tag{1.17}
\]

Under the assumption that \( m_\nu = 0 \), this expression simplifies to:

\[
\frac{dn}{dE_f} = \frac{V^2}{4\pi^4\hbar^6c^3}p_e^2(Q - T_e)^2 dp_e, \tag{1.18}
\]

where the substitution of \( (E - E_e) = (Q - T_e) \), which holds for \( m_\nu = 0 \), was made. This expression for the density of final states plays a dominant role in determining the β particle energy spectrum. As previously discussed in Section 1.1, the energy spectrum of the electron is continuous from \( T_e = 0 \to T_e^{max} = Q \). An example of the energy distribution corresponding to Equation 1.18 is given in Figure 1.1.

In order to calculate the transition rate, the matrix element must be calculated. The β decay matrix element is given by the integral of the weak interaction transition operator, \( \hat{H}_{int} \), which contains all of the dynamical information of the interaction, and connects the initial and final state wavefunctions:

\[
M_{fi} = \langle \psi_f | \hat{H}_{int} | \psi_i \rangle = \int [\psi_D^* \psi_e^* \hat{\psi}_n] \hat{H}_{int} \psi_f d\vec{r}. \tag{1.19}
\]

The wavefunction of the neutrino can be described by a plane wave which is taken to be normalized in the same box of volume \( V \) used to calculate the density of final states, and is given by:

\[
\psi_\nu(\vec{r}_\nu) = \frac{1}{\sqrt{V}} e^{-i\vec{p}_\nu \cdot \vec{r}_\nu / \hbar}. \tag{1.20}
\]
Figure 1.1: Calculated kinetic energy spectrum for the positron from the superallowed $\beta^+$ decay of $^{10}\text{C}$. The distribution goes to zero at the endpoint energy of $Q_{\beta^+} = 886$ keV.

The wavefunction of the electron on the other hand, cannot be solely described by a plane wave, but must also account for the distortion of the wavefunction due to the interaction between the charged electron and the electrostatic field of the nucleus. The wavefunction of the electron is therefore given by:

$$\psi_e(r_e) = \frac{1}{\sqrt{V}} e^{-ip_e r_e/\hbar} \sqrt{F(Z, p_e)},$$  \hspace{1cm} (1.21)

where $F(Z, p_e)$ is the Fermi function, which distorts the shape of the $\beta$ spectrum. The
relativistic description of the Fermi function is given as [2]:

\[
F(Z, p_e) = 2(1 + \gamma) \left( \frac{2p R}{\hbar} \right)^{2 \gamma - 2} e^{\pi \eta} \frac{\left| \Gamma(\gamma + i\eta) \right|^2}{\left| \Gamma(2\gamma + 1) \right|^2},
\]  
(1.22)

where \( \gamma = \sqrt{1 - (\alpha Z)^2} \), \( \eta = \mp Z\alpha E/pc \) for \( \beta^\pm \) decays, \( \alpha \) is the fine structure constant, and \( Z \) and \( R \) are the charge and radius of the daughter nucleus, respectively. This simplified description of the Fermi function, however, was derived assuming a uniformly charged sphere of radius \( R \). In order to obtain a more accurate description of the Fermi function additional higher order corrections must be applied to Equation 1.22. These corrections include: a revision in the nuclear charge distribution from a spherical and uniformly charged nucleus to a more realistic description, a term that accounts for the screening of the outgoing positron from the electric charge of the daughter nucleus as it moves outward from the atom, as well as a nuclear recoil term which is introduced by the re-definition of the system from a 2-body to a 3-body system [2]. Although these corrections are small, between 0.1% to 1%, they are nonetheless important, in particular, for high-precision studies.

The \( A \)-body wavefunctions of the parent and daughter nucleus can both be expressed in terms of the product of single particle wavefunctions. Thus, for \( \beta^\pm \) decays, the wavefunctions are:

\[
\Psi_P = \prod_{i=1}^{Z} \phi_{P_i}(r_i) \prod_{j=1}^{N} \phi_{P_j}(r_j) + \text{antisymmetric terms.} 
\]  
(1.23)

\[
\Psi_D = \prod_{i=1}^{Z} \phi_{P_i}(r_i) \prod_{j=1}^{N+1} \phi_{P_j}(r_j) + \text{antisymmetric terms.} 
\]  
(1.24)

Since the \( \beta \) decay transition involves the conversion of a proton into a neutron (or vice versa) the contributions from the other \( A - 1 \) nucleons that are not involved in the interaction
integrate to one. The resulting matrix element then becomes

\[ |M'_{fi}| = \int \phi_D^*(\vec{r})e^{i(\vec{p}_e - \vec{p}_\nu + \vec{r}_n - \vec{r}_e)/\hbar} \hat{H}_{\text{int}} \phi_P(\vec{r}) d\vec{r}, \]  

(1.25)

where the matrix element in the above expression was redefined with the volume normalization and the Fermi function removed:

\[ |M_{fi}| = \sqrt{\frac{F(Z, p_e)}{V}} |M'_{fi}|. \]  

(1.26)

Since the mediating $W^\pm$ boson is heavy ($M_{W^\pm} = 80.379 \pm 0.012$ GeV/$c^2$ [1]), the low energy $\beta$ decay operator is very short range and can be described, to a good approximation, by a contact operator which takes the form

\[ \hat{H}_{\text{int}} = g\delta(\vec{r}_p - \vec{r}_n)\delta(\vec{r}_e - \vec{r}_\nu)\delta(\vec{r}_\nu - \vec{r}_n) \hat{O}(n \leftrightarrow p), \]  

(1.27)

where $\hat{O}(n \leftrightarrow p)$ describes the operator which converts a proton into a neutron (or vice versa) and $g$ is the weak coupling strength. Substituting the above Hamiltonian into Equation 1.25 yields:

\[ |M'_{fi}| = \int \phi_D^*(\vec{r})e^{i(\vec{p}_e + \vec{p}_\nu - \vec{r}_n)/\hbar} \hat{O}(n \leftrightarrow p)\phi_P(\vec{r}) d\vec{r}. \]  

(1.28)

The electron and neutrino plane waves can be expanded in terms of the spherical harmonics:

\[ e^{i(\vec{k}_e + \vec{k}_\nu) \cdot \vec{r}} = \sum_{L_\beta M} 4\pi i^L j_{L_\beta}(kr)Y_{L_\beta}^M(\hat{k})Y_{L_\beta}^{M*}(\hat{r}), \]  

(1.29)

where $\vec{k} = \vec{p}/\hbar$, $k = |\vec{k}_e + \vec{k}_\nu|$, $L_\beta = L_e + L_\nu$, and $j_{L_\beta}(kr)$ are the spherical Bessel functions. Furthermore, for a central potential, the single particle wavefunctions of the proton and neutron involved in the interaction can be separated into their radial and angular components:

\[ \phi_P(\vec{r}) = R(\vec{r})Y_{L_\beta}^{M_P}(\theta, \phi) \]

\[ \phi_D(\vec{r}) = R(\vec{r})Y_{L_\beta}^{M_D}(\theta, \phi) \]  

(1.30)
Substituting Equations 1.29 and 1.30 into the expression for the matrix element given in Equation 1.25 gives:

\[
M_{f_i}^{L_{\beta'}} = g 4\pi i^L \beta \sum_{M=-L_{\beta}}^{L_{\beta}} Y_{L_{\beta}}^M(\hat{k}) \int Y_{L_D}^{M*}(\hat{r})Y_{L_{\beta}}^M(\hat{r})Y_{L_P}^{M*}(\hat{r})d\Omega \\
\times \int R_D^{*}(r)j_{L_{\beta}}(kr)\hat{O}(n \leftrightarrow p)R_P(r)r^2dr,
\]

(1.31)

where the first integral vanishes unless \(L_P = L_D + L_{\beta}\), giving rise to the \(\beta\) decay angular momentum selection rule. Moreover, since the spherical harmonics, \(Y^M_L\), have even or odd parity depending on the quantum number \(L\), of the form \(\pi = (-1)^L\), the parity selection rule \(\pi^D = \pi^P (-1)^L\) emerges.

In general, the total matrix element is a sum from all possible contributions, satisfying the angular momentum \(|L_P - L_D| < L_{\beta} < |L_P + L_D|\) and parity \(\pi^D = \pi^P (-1)^L_{\beta}\) selection rules

\[
|M_{f_i}'|^2 = \sum_{L_{\beta}} |M_{f_i}^{L_{\beta'}}|^2.
\]

(1.32)

However, in practice the sum is almost always completely dominated by the lowest allowed value of \(L_{\beta}\).

Finally, it is typical to remove the weak coupling strength, \(g\), and define the reduced matrix element, \(|\bar{M}_{f_i}'|\), as:

\[
|M_{f_i}'| = g|\bar{M}_{f_i}'|.
\]

(1.33)

The total \(\beta\) decay transition rate, \(\lambda\), is obtained by integrating over all possible electron momenta, and is given by:

\[
\lambda = \int_0^{p_{max}} d\lambda(p_e).
\]

(1.34)
Substituting the density of states from Equation 1.18 and the matrix element from Equation 1.33 yields:

\[
\lambda = \frac{g^2}{2\pi^3\hbar^2c^3} |\bar{M}'_{fi}|^2 \int_0^{p_{max}} F(Z, p_e) p_e^2 (e - E_e)^2 dp_e.
\]  (1.35)

Conventionally, the integral in Equation 1.35 is expressed in terms of dimensionless quantities by defining the reduced variables \( \rho = \frac{p_e}{m_ec} \) and \( W = \frac{E_e}{m_ec^2} \) where \( m_e \) is the mass of the electron. The relativistic energy-momentum relation yields \( W^2 = \rho^2 + 1 \). Substituting these variables into Equation 1.35, the transition rate can be expressed as:

\[
\lambda = \frac{g^2 m_e^5 e^4}{2\pi^3 \hbar^2} |\bar{M}'_{fi}|^2 \int_1^{W_0} F(Z, W) W(W^2 - 1)^{1/2}(W_0 - W)^2 dW
\]  (1.36)

where \( W_0 = \frac{E_{max}}{m_ec^2} \). The dimensionless integral in the above expression is known as the statistical rate function, \( f \).

The decay constant, \( \lambda \), can be re-written in terms of the partial half-life, \( t \), for decay between the initial and final states. For \( \beta^+ \) decays, the fraction of events which occur via the competing \( EC \) decay mode, \( P_{EC} = \frac{\lambda_{EC}}{\lambda_{EC} + \lambda_3} \), must also be included. The resulting expression for the partial half-life is thus:

\[
t = \frac{\ln 2}{\lambda_{fi}} = \frac{T_{1/2}}{BR} (1 + P_{EC}),
\]  (1.37)

where \( T_{1/2} \) is the half-life of the decaying nucleus and \( BR \) is the branching ratio to the particular final state of interest. Substituting Equation 1.37 into Equation 1.36 gives the expression for the \( \beta \) decay \( ft \)-values:

\[
ft = \frac{2\pi^3 \hbar^7 \ln 2}{m_e^5 e^4 g^2 |\bar{M}'_{fi}|^2} = \frac{K}{g^2 |\bar{M}'_{fi}|^2},
\]  (1.38)

where the constants have been grouped together into \( K/(hc)^6 = 8120.2776(9) \times 10^{-10} \text{ GeV}^{-4} \text{ s}^{-1} \).
1.3 Classification of $\beta$ Decays

As discussed in Section 1.2, it is the lowest allowed value of $L_\beta$ that dominates the sum in the calculation of the matrix element. Since the $ft$ values depend inversely on the square of the reduced matrix element, transitions of lower $L_\beta$ are seen to occur more quickly than the higher-$L_\beta$ decays. The lowest order ($L_\beta = 0$) are referred to as “allowed” transitions and are the fastest of the $\beta$ decays. Higher order values of $L_\beta$ are referred to as “forbidden” transitions and occur more slowly as $L_\beta$ increases. The classifications for the different values of $L_\beta$ and corresponding typical log($ft$) values are given in Table 1.1.

Table 1.1: Typical log($ft$) values for each $L_\beta$ are taken from Ref. [4].

<table>
<thead>
<tr>
<th>Transition</th>
<th>$L_\beta$</th>
<th>log($ft$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Allowed</td>
<td>0</td>
<td>2.9-6.0</td>
</tr>
<tr>
<td>1st Forbidden</td>
<td>1</td>
<td>6-10</td>
</tr>
<tr>
<td>2nd Forbidden</td>
<td>2</td>
<td>10-13</td>
</tr>
<tr>
<td>3rd Forbidden</td>
<td>3</td>
<td>$&gt; 15$</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

In the derivation of the $\beta$ decay formalism thus far, the intrinsic spins of the particles have been ignored. Each $\beta$ decay can be further characterized by the combined spins of the outgoing electron and neutrino, $S_\beta = S_e + S_\nu$. Since both particles have an intrinsic spin of $\frac{1}{2}$ in units of $\hbar$, they can couple to either $S_\beta = 0$ or $S_\beta = 1$. Transitions in which the electron and neutrino couple to $S_\beta = 0$ are referred to as vector or Fermi $\beta$ decays. On the other hand, transitions in which the electron and neutrino couple to $S_\beta = 1$ are referred to as axial-vector or Gamow-Teller $\beta$ decays.

The total angular momentum is the vector sum of the orbital and spin angular momentum, $\vec{J} = \vec{L} + \vec{S}$, with the corresponding conservation of angular momentum selection rule
\vec{J}_P = \vec{J}_D + \vec{J}_\beta. \text{ Thus, the selection rule for the total angular momentum for allowed Fermi decays is } \Delta J = 0 = |\vec{J}_P - \vec{J}_D| \text{ since both } L_\beta = 0 \text{ and } S_\beta = 0. \text{ On the other hand, since } S_\beta = 1 \text{ for allowed Gamow-Teller decays, the selection rule for the change in the total angular momentum of the nucleus is } \Delta J = 0, \pm 1, \text{ but no } J = 0 \to J = 0 \text{ transitions.}

The strength of the weak interaction is different for Fermi and Gamow-Teller decays and as such the matrix element must be separated into the vector and axial-vector components:

$$g^2|M_{fi}|^2 = g^2_V|M_{fi}(F)|^2 + g^2_A|M_{fi}(GT)|^2,$$

(1.39)

where \( g_V \) and \( g_A \) are the weak interaction vector and axial-vector coupling strengths, respectively. For a given \( \beta \) decay, contributions from both Fermi and Gamow-Teller matrix elements are possible provided that the \( \beta \) decay selection rules are satisfied.

Decays which occur between \( J^\pi = 0^+ \) states are, however, to leading order pure Fermi \((S_\beta = 0)\) allowed \((L_\beta = 0)\) transitions because the \( J = 0 \to J = 0 \) case is excluded for allowed Gamow-Teller decays with \( J_\beta = S_\beta = 1 \) by the angular momentum selection rules. Since these transitions are pure Fermi, the Gamow-Teller matrix element is zero. The \( ft \) values for such pure Fermi \( \beta \) decay transitions are thus given by:

$$ft = \frac{K}{g^2_V|M_{fi}|^2}.$$  

(1.40)

\section*{1.4 Superallowed \( \beta \) Decays}

\subsection*{1.4.1 Isospin}

In 1932, the concept of isospin was introduced \cite{5} in recognition of the very similar properties of protons and neutrons beyond their different electric charges. The proton and neutron were collectively referred to as nucleons and were considered to be different quantum
states of the same particle \[5\]. Mirroring the formalism used for spin, the wavefunction of a nucleon with isospin \( t = 1/2 \) and projection \( t_Z = \pm 1/2 \) are of the form \(|t, t_Z\rangle\). The isospin states of a single neutron and proton are thus defined as:

\[
|n\rangle = |\frac{1}{2}, +\frac{1}{2}\rangle \\
|p\rangle = |\frac{1}{2}, -\frac{1}{2}\rangle.
\]  

(1.41) 

(1.42)

Since a nucleus is composed of many protons and neutrons, the total isospin \( T \) and projection \( T_z \) of a nucleus with \( Z \) protons and \( N \) neutrons is obtained from the vector sum of the individual nucleon isospins \( t_i \) with the results:

\[
\frac{1}{2}|N - Z| \leq T \leq \frac{1}{2}|N + Z| \\

T_z = \frac{1}{2}(N - Z).
\]  

(1.43) 

(1.44)

Although many values of \( T \) are possible for different states within a given nucleus, the nuclear symmetry energy favours the smallest possible value of \( T \) \[6\]. Almost all nuclei thus have ground states with \( T = |T_z| = \frac{1}{2}|N - Z| \). There are, however, a few exceptions, specifically for \( A \geq 34 \), odd-odd \( N = Z \) nuclei in which the pairing energy (which favours \( T = 1 \)) overcomes the symmetry energy (which favours \( T = 0 \)) and the ground state becomes \( T = 1 \) instead of \( T = 0 \) \[7\].

In the isospin formalism, the operator for converting a proton into a neutron (or vice versa) is the isospin ladder operator:

\[
\hat{T}^\pm|T, T_z\rangle = \sqrt{(T \mp T_z)(T \pm T_z + 1)}|T, T_z \pm 1\rangle.
\]  

(1.45)
The Fermi $\beta^\pm$ operator is, in fact, the isospin ladder operator $\hat{T}^\pm$, whereas the Gamow-Teller $\beta^\pm$ decay operator is $\hat{Y}^\pm = \sigma \hat{T}^\pm$, where $\sigma$ represents the Pauli spin matrices. Similar to the selection rules for total angular momentum, isospin selection rules for Fermi and Gamow-Teller $\beta$ decays are $\Delta T = 0$ and $\Delta T = 0, \pm 1$, respectively [6].

As can be seen from Equation 1.45, the Fermi isospin ladder operator connects members of an isospin multiplet that have the same total isospin $T$, but different $T_Z$. States within this multiplet are known as isobaric analogue states. Besides the changing of a proton into a neutron, or vice versa, isobaric analogue states have identical nuclear wavefunctions. The difference between their wavefunctions results from the fact that members of the isobaric multiplet have different number of protons and neutrons. In particular, the differing number of protons leads to differences in the energies and, hence masses, between members of the isobaric multiplet due to the different Coulomb energies.

In the isospin formalism, pure Fermi allowed $\beta$ decays between isobaric analogue states are referred to as superallowed $\beta$ decays and are among the fastest $\beta$ decay transitions. The reduced matrix element for such superallowed Fermi transitions is given by:

$$|\bar{M}_{fi}(F)|^2 = |\langle T, T_Z \pm 1|\hat{T}^\pm|T, T_Z\rangle|^2$$
$$= (T \mp T_Z)(T \pm T_Z + 1)$$

(1.46)

If the $\beta^+$ transition occurs between members of a $T = 1$ multiplet, the matrix element is:

$$|\bar{M}_{fi}(F)|^2 = (1 + 1)(1 - 1 + 1) = 2 \quad T_Z = -1 \rightarrow T_Z = 0$$

$$|\bar{M}_{fi}(F)|^2 = (1 - 0)(1 + 0 + 1) = 2 \quad T_Z = 0 \rightarrow T_Z = 1$$

(1.47)
Thus, for the specific subset of pure superallowed Fermi $\beta$ decays within $T = 1$ multiplets, the $ft$-value is greatly simplified to:

$$ft = \frac{K}{2g_V^2},$$

(1.48)

and is independent of the complexities of the parent and daughter nuclear wavefunctions.

The sole dependence of the superallowed $ft$ values on $g_V$ is of particular interest. While some interactions of particles within many-body bound systems differ from their free particle counterparts other interactions are completely independent of their environment. In 1958, the conserved-vector-current (CVC) hypothesis \[8\] was first postulated, stating that, unlike its axial-vector counterpart, $g_A$, the vector coupling constant, $g_V$, is not renormalized in the nuclear medium and is therefore a fundamental constant. We thus make the notational replacement, $g_V \rightarrow G_V$, and note that the $T = 1$ superallowed $ft$ values given in Equation (1.48) should be constant, $ft = \frac{K}{2G_V^2}$, and independent of the actual nuclei involved.

Experimentally, the $ft$ values are determined via the measurement of three quantities: the total transition energy, $Q_{EC}$, which is used to calculate $f$, as well as the half-life of the parent nucleus, $T_{1/2}$, and the branching ratio, $BR$, to the $0^+$ isobaric analogue state in the daughter nucleus, which are combined to calculate $t$ as given in Equation (1.37).

To date, $ft$ values have been experimentally determined to a precision better than $\pm 0.3\%$ for 14 different superallowed decays in $N \approx Z$ nuclei from $^{10}\text{C}$ ($Z = 6$) to $^{74}\text{Rb}$ ($Z = 37$) \[3\]. The experimental data for these 14 transitions are given in Table (1.2) and a plot of the resulting $ft$ values is given in Figure (1.2).
Table 1.2: The $ft$ data for the 14 most precisely determined $T = 1 \,$ superallowed emitters. The data in table were taken from the most recent superallowed survey [3].

<table>
<thead>
<tr>
<th>Parent Nucleus</th>
<th>$f$</th>
<th>$P_{EC}$ (%)</th>
<th>$T_{1/2}$ (ms)</th>
<th>Branching Ratio (%)</th>
<th>$ft$ (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{10}\text{C}$</td>
<td>$2.30169 \pm 0.00070$</td>
<td>0.299</td>
<td>19305.2 ± 7.1</td>
<td>1.4646 ± 0.0019</td>
<td>3043.0 ± 4.3</td>
</tr>
<tr>
<td>$^{14}\text{O}$</td>
<td>$42.771 \pm 0.023$</td>
<td>0.088</td>
<td>70619 ± 11</td>
<td>99.374 ± 0.068</td>
<td>3042.2 ± 2.7</td>
</tr>
<tr>
<td>$^{22}\text{Mg}$</td>
<td>$418.37 \pm 0.17$</td>
<td>0.069</td>
<td>3875.2 ± 2.4</td>
<td>53.16 ± 0.12</td>
<td>3051.9 ± 7.2</td>
</tr>
<tr>
<td>$^{26m}\text{Al}$</td>
<td>$478.232 \pm 0.081$</td>
<td>0.083</td>
<td>6346.02 ± 0.54</td>
<td>100.0000$^{+0}_{-0.0015}$</td>
<td>3037.38 ± 0.058</td>
</tr>
<tr>
<td>$^{34}\text{Cl}$</td>
<td>$1996.003 \pm 0.096$</td>
<td>0.080</td>
<td>1526.55 ± 0.44</td>
<td>100.0000$^{+0}_{-0.0012}$</td>
<td>3049.43$^{+0.08}_{-0.95}$</td>
</tr>
<tr>
<td>$^{34}\text{Ar}$</td>
<td>$3410.97= \pm 0.61$</td>
<td>0.069</td>
<td>843.81 ± 0.40</td>
<td>94.45 ± 0.25</td>
<td>3049.6 ± 8.1</td>
</tr>
<tr>
<td>$^{38m}\text{K}$</td>
<td>$3297.39 \pm 0.15$</td>
<td>0.085</td>
<td>924.33 ± 0.27</td>
<td>99.9670$^{+0.0043}_{-0.0044}$</td>
<td>3051.45 ± 0.92</td>
</tr>
<tr>
<td>$^{38}\text{Ca}$</td>
<td>$5238.88 \pm 0.30$</td>
<td>0.075</td>
<td>443.77 ± 0.35</td>
<td>77.28 ± 0.16</td>
<td>3062.3 ± 6.8</td>
</tr>
<tr>
<td>$^{42}\text{Sc}$</td>
<td>$4472.23 \pm 1.15$</td>
<td>0.099</td>
<td>680.72 ± 0.26</td>
<td>99.9941 ± 0.0014</td>
<td>3047.5 ± 1.4</td>
</tr>
<tr>
<td>$^{46}\text{V}$</td>
<td>$7209.25 \pm 0.54$</td>
<td>0.101</td>
<td>422.622 ± 0.053</td>
<td>99.9848$^{+0.0013}_{-0.0042}$</td>
<td>3050.32$^{+0.44}_{-0.46}$</td>
</tr>
<tr>
<td>$^{50}\text{Mn}$</td>
<td>$10745.97 \pm 0.50$</td>
<td>0.107</td>
<td>282.21 ± 0.11</td>
<td>99.9423 ± 0.0030</td>
<td>3048.4 ± 1.2</td>
</tr>
<tr>
<td>$^{54}\text{Co}$</td>
<td>$15766.7 \pm 2.9$</td>
<td>0.111</td>
<td>193.271 ± 0.063</td>
<td>99.9955$^{+0.0006}_{-0.0306}$</td>
<td>3050.7$^{+1.1}_{-1.5}$</td>
</tr>
<tr>
<td>$^{62}\text{Ga}$</td>
<td>$26400.3 \pm 8.3$</td>
<td>0.135</td>
<td>116.121 ± 0.040</td>
<td>99.862 ± 0.011</td>
<td>3074.0 ± 1.5</td>
</tr>
<tr>
<td>$^{74}\text{Rb}$</td>
<td>$47281 \pm 93$</td>
<td>0.191</td>
<td>64.776 ± 0.043</td>
<td>99.541 ± 0.030</td>
<td>3082.7 ± 6.5</td>
</tr>
</tbody>
</table>

Although the $ft$ values are relatively constant in comparison to the large range of general $\beta$ decay $ft$ values, as characterized in Table 1.1, it is clear that the $ft$ values plotted in Figure 1.2 are not exactly constant but vary by approximately 1.5% between the low-$Z$ and high-$Z$ superallowed transitions. The small difference between the $ft$-values is a result of several approximations that were made in deriving Equation 1.48. In order to obtain a more precise test of the CVC hypothesis, several refinements must be considered.

### 1.4.2 Corrected-$ft$ Values

The derivation of the $ft$ values thus far has neglected some higher ordered processes which must be considered in order to test the constancy of the superallowed $ft$ values implied by the CVC hypothesis. These theoretical corrections account for isospin symmetry breaking as well as radiative effects. These small theoretical corrections, which are on the order of
one percent, must be applied to the expression for the \( ft \) value given in equation 1.48. Upon the addition of these corrections, the “corrected”-\( ft \), \( \mathcal{F}t \), value is defined as[3]:

\[
\mathcal{F}t = ft(1 + \delta_R)(1 - \delta_C) = \frac{K}{2G_F^2(1 + \Delta_R^V)} = \text{constant},
\]

(1.49)

where \( \delta_C \) is the isospin symmetry breaking correction, \( \delta_R \) is a transition-dependent radiative correction, and \( \Delta_R^V \) is a transition-independent radiative correction. Although the various theoretical corrections are small, improvements in the accuracy of the theoretical approaches is important for many of the weak interaction tests that will be discussed at the end of this

Figure 1.2: Plot of the \( ft \) values for the 14 most precisely determined \( T = 1 \) superallowed emitters. Data are taken from Ref. [3].
Figure 1.3: The $\gamma W$-box and $ZW$-box diagrams used to calculate the radiative corrections. Diagrams taken from Ref. [9].

chapter. For many of the superallowed transitions, the experimentally determined superallowed $ft$ values have been measured to such high precision that it is now the uncertainties in the theoretical corrections that dominate the uncertainty in their $Ft$-values.

1.4.3 Radiative Corrections

Experimental measurements of the decay rate encompass the rate of both the "bare" $\beta$ decay process as well as other indistinguishable radiative decay processes. The processes which are evaluated as part of the radiative corrections are the one-photon bremsstrahlung, the $\gamma W$-box, and the $ZW$-box diagrams [3]. Feynmann diagrams of the $\gamma W$-box and $ZW$-box diagrams are shown in Figure 1.3.

The first radiative correction, $\Delta^V_R$ is the transition-independent correction and is a short-distance, or high-energy, correction. It includes contributions from the $ZW$-box diagram as well as the high-energy part of the $\gamma W$-box and is evaluated by ignoring the hadronic structure of the nucleus. Since this correction is independent of the specific transition, and is therefore the same for all transitions, it is placed on the right-hand side of Equation 1.49. The currently adopted value is $\Delta^V_R = (2.361 \pm 0.038\%)$ [10], and improvements in the precision of this universal correction remains an active area of theoretical research [9].
The second radiative correction, $\delta_R$, is a long distance, or low energy, correction which includes contributions from one photon bremsstrahlung and the low-energy part of the $\gamma W$-box diagrams. This correction is unique for each transition and requires a calculation of the specific hadronic structure of each nucleus. Conventionally, this term is further separated into two components:

$$
\delta_R = \delta'_R + \delta_{NS},
$$

(1.50)

where $\delta'_R$ is independent of the nuclear structure and depends only on the endpoint energy of the $\beta$ transition and the $Z$ of the daughter nucleus, and $\delta_{NS}$, which depends on the nuclear structure.

Currently, the model independent radiative correction, $\delta'_R$, has been calculated with contributions of order $\alpha$, $\alpha^2$ and $Z\alpha^2$, while the leading-log term of order $Z^2\alpha^3$ has been estimated \[11-14\]. The uncertainty in $\delta'_R$ is currently estimated as one third of the $Z^2\alpha^3$ term and is taken to be a systematic uncertainty as the effect of neglected higher-order terms will likely have the same sign for all of the superallowed decays \[3\].

The final radiative term, $\delta_{NS}$ depends on the specifics of the nuclear structure. Although superallowed transitions are purely vector, axial-vectors interactions may contribute to the measured $\beta$ rate at higher-order, via, for example, an axial-vector interaction that flips the spin of the nucleon followed by an electromagnetic interaction which flips a nucleon spin back with the net result of no nuclear spin change. Correspondingly, this interaction involves the evaluation of the $\gamma W$-box diagram including axial-vector couplings.
Table 1.3: Radiative corrections for the 14 most precisely determined $T = 1$ superallowed emitters from Ref. [15]. The uncertainties in the $\delta_{NS}$ values are treated as statistical and added in quadrature with other uncertainties in calculating the individual $\mathcal{F}t$ values. Following Ref. [3], the uncertainties in the $\delta'_R$ values are treated as systematic and shifted to their $\pm 1\sigma$ values simultaneously for all cases to determine the overall contribution of $\delta'_R$ to the uncertainty in $\mathcal{F}t$.

<table>
<thead>
<tr>
<th>Parent Nucleus</th>
<th>$\delta'_R$ (%)</th>
<th>$\delta_{NS}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{10}$C</td>
<td>1.679(1)</td>
<td>-0.345(35)</td>
</tr>
<tr>
<td>$^{14}$O</td>
<td>1.543(3)</td>
<td>-0.245(50)</td>
</tr>
<tr>
<td>$^{22}$Mg</td>
<td>1.466(6)</td>
<td>-0.290(20)</td>
</tr>
<tr>
<td>$^{26}$mAl</td>
<td>1.478(7)</td>
<td>0.005(20)</td>
</tr>
<tr>
<td>$^{34}$Cl</td>
<td>1.443(11)</td>
<td>-0.085(15)</td>
</tr>
<tr>
<td>$^{34}$Ar</td>
<td>1.412(12)</td>
<td>-0.180(15)</td>
</tr>
<tr>
<td>$^{38}$mK</td>
<td>1.440(13)</td>
<td>-0.100(15)</td>
</tr>
<tr>
<td>$^{38}$Ca</td>
<td>1.414(14)</td>
<td>-0.175(15)</td>
</tr>
<tr>
<td>$^{42}$Sc</td>
<td>1.453(16)</td>
<td>0.035(20)</td>
</tr>
<tr>
<td>$^{46}$V</td>
<td>1.445(18)</td>
<td>-0.035(10)</td>
</tr>
<tr>
<td>$^{50}$Mn</td>
<td>1.444(21)</td>
<td>-0.040(10)</td>
</tr>
<tr>
<td>$^{54}$Co</td>
<td>1.443(24)</td>
<td>-0.035(10)</td>
</tr>
<tr>
<td>$^{62}$Ga</td>
<td>1.459(29)</td>
<td>-0.045(20)</td>
</tr>
<tr>
<td>$^{74}$Rb</td>
<td>1.499(40)</td>
<td>-0.075(30)</td>
</tr>
</tbody>
</table>

1.4.4 Isospin Symmetry Breaking Correction

The $\delta_C$ correction must be included to account for the fact that isospin is not a perfect symmetry in the nucleus. It is broken by the Coulomb interaction as well as charge-dependent forces of nuclear origin which affects the charged proton but not the neutral neutron. Two consequences arise from the fact that isospin is not an exact symmetry: the overlap of the radial wavefunction for the proton in the parent nucleus and the neutron in the daughter nucleus is no longer exact, and the decaying $0^+$ state in the parent nucleus becomes a mixture of configurations, dominated by the isobaric analogue state but also including small components of other non-analogue $0^+$ states in the daughter nucleus. This results in a slight
reduction in the matrix element previously defined in Equation 1.47 which is redefined as:

\[ |\bar{M}_{fi}'(F)|^2 = 2 \rightarrow |\bar{M}_{fi}'(F)|^2 = 2(1 - \delta_C). \quad (1.51) \]

Many theoretical approaches have been used to calculate \( \delta_C \) \([15\text{–}27]\). A subset of some of the \( \delta_C \) corrections for the high-precision superallowed transitions is given in Figure 1.4 which demonstrates the significant model dependence between the different theoretical approaches. Most of these theoretical calculations are, however, not currently used in the evaluation of the superallowed data. This is primarily due to the fact that they have not been constrained to the same degree by independent experimental data such as nuclear charge radii and coefficients of the isobaric multiplet mass equation (IMME). These calculations can nonetheless be used to compare and contrast with the phenomenologically constrained models that are currently used.

Historically, two approaches have been used in the evaluation of the \( \delta_C \) corrections in surveys of the world superallowed data \([3,28,30]\). In both of these approaches, the \( \delta_C \) correction is separated into two components:

\[ \delta_C = \delta_{C1} + \delta_{C2}, \quad (1.52) \]

where \( \delta_{C1} \) accounts for the difference in configuration mixing between the parent and daughter \( 0^+ \) states while assuming perfect radial overlap between the parent and daughter wavefunctions in its evaluation, and \( \delta_{C2} \) accounts for the imperfect radial overlap of the parent and daughter nucleus due to small differences in the single-particle neutron and proton radial wavefunctions, while neglecting differences in configuration mixing. Since both of these corrections are small, the \( \delta_C \) correction can be further factorized, and the correction to the matrix element given in Equation 1.51 of the form \( (1 - \delta_C) \) becomes:

\[ 1 - \delta_C = 1 - \delta_{C1} - \delta_{C2} \approx (1 - \delta_{C1})(1 - \delta_{C2}). \quad (1.53) \]
In both approaches used in the evaluation of the superallowed data, the $\delta_{C1}$ term is obtained from large-basis shell model calculations. A two-body Coulomb interaction between valence protons, as well as a charge-dependent interaction of nuclear origin, are included and their strengths are adjusted for each isobaric multiplet in order to reproduce the measured $b$ and $c$ coefficients of the IMME \[3,31\]. As the differences in configuration mixing depend sensitively on a faithful reproduction of the spectrum of excited $0^+$ states in the daughter nucleus, the calculated $\delta_{C1}$ values are also adjusted by the ratio of excitation energies for the first excited $0^+$ state between theory and experiment \[15\].
Since isospin is not a perfect symmetry, the differences in configuration mixing between the parent and daughter nuclei result in $\beta$ feeding not only through the superallowed branch to the isobaric analogue state in the daughter but also to other non-analogue $0^+$ states. The total $\delta_{C1}$ correction can then be expressed as the sum of the contributions to each of the non-analogue $0^+$ states:

$$\delta_{C1} = \sum_n \delta^n_{C1},$$

(1.54)

where each $\delta^n_{C1}$ accounts for the mixing to the $n$th non-analogue $0^+$ state in the daughter nucleus into the decaying $0^+$ state of the parent nucleus.

The magnitude of the $\delta_{C1}$ correction can be experimentally constrained by measuring the $\beta$ decay branching ratios to these non-analogue $0^+$ states. For $0^+$ states within the $\beta$ decay $Q$ value window, the $\delta^n_{C1}$ corrections can thus be measured experimentally. For the simple example where the daughter has only a single non-analogue $0^+$ state, the matrix element squared for the non-analogue state is $2\delta^1_{C1}(1 - \delta_{C2})$ while the matrix element squared for the isobaric analogue state is $2(1 - \delta^1_{C1})(1 - \delta_{C2})$. The $\beta$ decay branching ratios, as given in Equation 1.40, can then be expressed as:

$$B_1 = B_0 \frac{t_0}{t_1} = B_0 \frac{f_1}{f_0} \frac{\delta^1_{C1}}{(1 - \delta^1_{C1})} \approx B_0 \frac{f_1}{f_0} \delta^1_{C1}$$

(1.55)

$$\delta^1_{C1} = \frac{B_1 f_0}{B_0 f_1},$$

(1.56)

where $B_1$ ($B_0$) represents the branching ratios to the non-analogue (analogue) state, and $f_1$ ($f_0$) represents the statistical rate function for the transitions to the non-analogue (analogue) states. The assumption that $\delta_{C1} << 1$ is also made and therefore $(1 - \delta_{C1}) \approx 1$. It should be noted that this result can be generalized for the case in which the daughter nucleus has
multiple non-analogue $0^+$ states:

$$\delta_{C1}^n = \frac{B_n f_0}{B_0 f_n}. \quad (1.57)$$

Experimental measurements of the $Q$-values and branching ratios for the decays to the $0^+$ non-analogue states can therefore be used to directly test the theoretical $\delta_{C1}^n$ values. In practice, however, many of the non-analogue $0^+$ states will lie outside of the $\beta$ decay $Q$-value window, rendering the measurement of such $\delta_{C1}^n$ components experimentally inaccessible. Measurements of the experimentally accessible $\delta_{C1}^n$, however, can be used to constrain different theoretical models, while the entire $\delta_{C1}$ correction must be obtained from theoretical calculations. Moreover, Equation 1.54 is only strictly correct if all of the $0^+$ state are $T = 1$ states. Mixing to other non-analogue $0^+$ states with $T \neq 1$ is generally very small and the expression in Equation 1.54 is thus approximately true.

The second component of the isospin breaking correction, $\delta_{C2}$, corrects for the imperfect radial overlap between the wavefunction of the proton in the parent nucleus and that of the neutron in the daughter nucleus resulting from both their differences in separation energies as well as the fact that the proton feels the Coulomb potential while the neutron does not. In both calculations of the radial wavefunctions considered, the proton and neutron separation energies from the shell model calculations are constrained to match their experimentally determined values.

The radial wavefunctions in the first $\delta_{C2}$ calculation are derived from a phenomenological Woods-Saxon (WS) potential [15]. In the second method, calculations originally pioneered by Ormand and Brown [16,32], and updated in Ref. [28], use radial wavefunctions from a self-consistent Hartree-Fock (HF) mean-field potential.
1.4.5 Tests of the Isospin Symmetry Breaking Corrections

As mentioned in the previous section, although many different theoretical approaches to the isospin symmetry breaking corrections have been taken, they depend sensitively on nuclear structure and as such have been seen to exhibit significant model dependence. Accurate determinations of the $\delta_C$ corrections are, however, crucial as the superallowed $\mathcal{F}t$ data are used to test fundamental properties of the Standard Model and set limits on possible extensions to it, as will be discussed in the subsequent section of this chapter.

Table 1.4: Isospin symmetry corrections for the 14 most precisely determined superallowed emitters for both the Wood-Saxon and Hartree-Fock calculations. The corresponding $\mathcal{F}t$ values for both theoretical approaches are also given. The WS $\mathcal{F}t$ values are taken from Ref. [3]. The HF $\delta_{C2}$ values are from Ref. [28] while the remaining data ($\mathcal{F}t$, $\Delta_R^V$, $\delta_R'$, $\delta_{NS}$, $\delta_{C1}$, and $\delta_{C2}(\text{WS})$) are taken from Ref. [3].

<table>
<thead>
<tr>
<th>Parent Nucleus</th>
<th>$\delta_{C1}$ (%)</th>
<th>$\delta_{C2}$ (WS) (%)</th>
<th>$\delta_{C2}$ (HF) (%)</th>
<th>$\mathcal{F}t$ (WS) (s)</th>
<th>$\mathcal{F}t$ (HF) (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{10}$C</td>
<td>0.010(10)</td>
<td>0.165(15)</td>
<td>0.215(35)</td>
<td>3078.0 ± 4.5</td>
<td>3076.5 ± 4.5</td>
</tr>
<tr>
<td>$^{14}$O</td>
<td>0.055(20)</td>
<td>0.275(15)</td>
<td>0.255(30)</td>
<td>3071.4 ± 3.2</td>
<td>3071.9 ± 3.3</td>
</tr>
<tr>
<td>$^{22}$Mg</td>
<td>0.010(10)</td>
<td>0.370(20)</td>
<td>0.250(55)</td>
<td>3077.9 ± 7.3</td>
<td>3081.6 ± 7.5</td>
</tr>
<tr>
<td>$^{26}$Al</td>
<td>0.030(10)</td>
<td>0.280(15)</td>
<td>0.410(50)</td>
<td>3072.9 ± 1.0</td>
<td>3068.9 ± 1.8</td>
</tr>
<tr>
<td>$^{34}$Cl</td>
<td>0.100(10)</td>
<td>0.550(45)</td>
<td>0.595(55)</td>
<td>3070.7 ± 1.8</td>
<td>3069.3 ± 2.0</td>
</tr>
<tr>
<td>$^{34}$Ar</td>
<td>0.030(10)</td>
<td>0.665(55)</td>
<td>0.510(60)</td>
<td>3065.6 ± 8.4</td>
<td>3070.4 ± 8.4</td>
</tr>
<tr>
<td>$^{38m}$K</td>
<td>0.105(20)</td>
<td>0.565(50)</td>
<td>0.640(60)</td>
<td>3071.6 ± 2.0</td>
<td>3069.2 ± 2.2</td>
</tr>
<tr>
<td>$^{38}$Ca</td>
<td>0.020(10)</td>
<td>0.757(70)</td>
<td>0.600(60)</td>
<td>3076.4 ± 7.2</td>
<td>3081.0 ± 7.1</td>
</tr>
<tr>
<td>$^{42}$Sc</td>
<td>0.020(10)</td>
<td>0.645(55)</td>
<td>0.620(55)</td>
<td>3072.4 ± 2.3</td>
<td>3073.1 ± 2.3</td>
</tr>
<tr>
<td>$^{46}$V</td>
<td>0.075(30)</td>
<td>0.545(55)</td>
<td>0.525(55)</td>
<td>3074.1 ± 2.0</td>
<td>3074.8 ± 2.0</td>
</tr>
<tr>
<td>$^{50}$Mn</td>
<td>0.035(20)</td>
<td>0.610(50)</td>
<td>0.575(55)</td>
<td>3071.2 ± 2.1</td>
<td>3072.3 ± 2.2</td>
</tr>
<tr>
<td>$^{54}$Co</td>
<td>0.050(30)</td>
<td>0.720(60)</td>
<td>0.635(55)</td>
<td>3069.8 ± 2.6</td>
<td>3072.5 ± 2.5</td>
</tr>
<tr>
<td>$^{62}$Ga</td>
<td>0.275(55)</td>
<td>1.20(20)</td>
<td>0.93(16)</td>
<td>3071.5 ± 6.7</td>
<td>3079.9 ± 5.5</td>
</tr>
<tr>
<td>$^{74}$Rb</td>
<td>0.115(60)</td>
<td>1.50(26)</td>
<td>1.29(16)</td>
<td>3076 ± 11</td>
<td>3082.6 ± 8.4</td>
</tr>
</tbody>
</table>

The $\delta_{C1}$ and $\delta_{C2}$ corrections which have been discussed above are given in Table 1.4. Both the SM-WS and SM-HF $\delta_{C2}$ corrections are given. The corresponding $\mathcal{F}t$-values using
the different $\delta_{C2}$ corrections are also given in Table 1.4. The corresponding $\delta_C = \delta_{C1} + \delta_{C2}$ corrections for the two methods are shown in Figure 1.5 and the difference between the SM-WS and SM-HF $\delta_{C2}$ corrections are plotted separately for the decays of the $T_Z = -1$ and $T_Z = 0$ parent nuclei in Figure 1.6.

As can be seen from the data in Table 1.4 as well as in Figures 1.5 and 1.6, the HF $\delta_{C2}$ corrections are systematically smaller than the WS $\delta_{C2}$ corrections for the majority of the superallowed decays. Thus, in past evaluations of the superallowed data [28, 30], the
Figure 1.6: Plot of the difference between the $\delta_{C2}$ corrections calculated using the WS calculations (data from Ref. [3]) and the HF calculations (data from Ref. [28]). The blue and red data points correspond to the decays in which the parent nucleus has $T_Z = 0$ and $T_Z = -1$, respectively. The systematic $Z$-dependent difference between the two theoretical calculations is demonstrated by the non-zero slopes of the data points.

The difference between these two models was treated as a systematic uncertainty. The central value of $\mathcal{F}t$ was taken as the average of the $\mathcal{F}t$(HF) and $\mathcal{F}t$(WS) values, and a systematic uncertainty corresponding to half of the difference between the two $\mathcal{F}t$ values was assigned to account for the model dependence of the theoretical calculations.

In the most recent evaluation of the superallowed data [3], however, no systematic uncertainty relating to the $\delta_C$ correction was assigned. Rather, the HF $\delta_{C2}$ corrections were
omitted from the evaluation altogether and the WS $\delta_{C2}$ values were adopted with no additional model dependent systematic uncertainty assigned. A key motivation for this decision was the better agreement with the CVC hypothesis obtained when using the WS $\delta_{C2}$ corrections \cite{3}. The weighted averages of the 14 most precisely determined $Ft$ values given in Table 1.4 for the two different models, yield:

$$\overline{Ft}(WS) = 3072.30(62)_{stat.(36)\delta_{R}^{s}} s \quad \chi^{2}/\nu = 0.52$$  \hspace{1cm} (1.58)

$$\overline{Ft}(HF) = 3071.88(74)_{stat.(42)\delta_{R}^{s}} s \quad \chi^{2}/\nu = 1.26.$$  \hspace{1cm} (1.59)

Although there is a 23\% chance that statistically independent data with 13 degrees of freedom would yield $\chi^{2}/\nu = 1.26$, the worse $\chi^{2}/\nu$ obtained with the HF $\delta_{C2}$ corrections was a key motivating factor for removing this method in the evaluation of the world superallowed data since the larger $\chi^{2}/\nu$ implies a poorer agreement with the CVC hypothesis. As the constancy of the $Ft$ values is used to test the CVC hypothesis, it is, of course, questionable to use this as a reason to discriminate between the different theoretical models. Moreover, upon further inspection of Figure 1.7 it is noted that the larger $\chi^{2}/\nu$ for the HF corrections originates primarily from a small subset of the decays whose $ft$ values are currently measured with the least precision, namely $^{22}$Mg, $^{38}$Ca, $^{62}$Ga, and $^{74}$Rb. If only the 9 transitions with $Ft$ values determined to $\pm 0.15\%$ or better are retained, one obtains:

$$\overline{Ft}(WS) = 3072.20(63)_{stat.(36)\delta_{R}^{t}} s \quad \chi^{2}\nu = 0.67$$ \hspace{1cm} (1.60)

$$\overline{Ft}(HF) = 3071.43(76)_{stat.(42)\delta_{R}^{t}} s \quad \chi^{2}\nu = 1.00.$$ \hspace{1cm} (1.61)

Both of these $\overline{Ft}$ are consistent with the CVC hypothesis but with central values differing
Figure 1.7: $\mathcal{F}t$ values for the 14 most precisely measured $T = 1$ superallowed transitions. The upper (lower) plot use the WS (HF) $\delta C_2$ calculations used in the evaluation of the $\mathcal{F}t$ data.

by 0.77 s, which is more than the statistical uncertainty associated with the entire world dataset of experimental measurements combined in quadrature with the quoted theoretical uncertainties within either set of $\delta C_2$ calculations. The corresponding plot of $\mathcal{F}t$ values is given in Figure 1.8. Upon the removal of the 5 least precisely measured $\mathcal{F}t$ values, the $\chi^2/\nu$ corresponding to $\mathcal{F}t$(WS) increases while the $\chi^2/\nu$ corresponding to $\mathcal{F}t$(HF) is improved.

The uncertainty in the $\mathcal{F}t$ values for the two high-$Z$ superallowed emitters, $^{62}$Ga and $^{74}$Rb, is dominated by theoretical corrections, namely uncertainties within the calculations
Figure 1.8: $F_t$ values for the 9 most precisely measured $T = 1$ superallowed transitions. The upper (lower) plot use the WS (HF) $\delta_{C2}$ calculations in the evaluation of the $F_t$ data.

of the $\delta_{C2}$ corrections themselves. However, uncertainty in the $F_t$ value for the lower-$Z$ $T_Z = -1$ emitters, $^{22}$Mg, $^{34}$Ar, and $^{38}$Ca, is currently dominated by the experimentally determined $ft$ values. Improvements in the $ft$ values for these cases can thus provide critical tests of the different theoretical models used to calculate the $\delta_C$ corrections.

Additional tests of the $\delta_C$ corrections can also be performed by studying mirror superallowed transitions within an isospin triplet. Again, under the assumption of the CVC
hypothesis, the ratio of the $ft$ values for a pair of superallowed transitions is given by [3]:

$$
\frac{ft^a}{ft^b} = 1 + (\delta_R^b - \delta_R^a) + (\delta_{NS}^b - \delta_{NS}^a) - (\delta_C^b - \delta_C^a),
$$

(1.62)

where $a$ denotes the decay of the $T_Z = -1$ parent and $b$ denotes the decay of the $T_Z = 0$ parent. These transitions are particularly sensitive to the $\delta_C$ corrections because the theoretical uncertainty on the difference of the $\delta_C$ corrections within an isospin triplet at constant mass number $A$ is significantly smaller than the uncertainties on the absolute value of the corrections themselves [3]. This is a result of how the theoretical uncertainties are calculated. For example, the $\delta_C$ corrections are taken to be the average of many calculations obtained using different parameter sets in the Shell Model. The uncertainty in the absolute value of the correction is estimated by the scatter between the different results. When these calculations are done for the difference, $(\delta_C^b - \delta_C^a)$ within an isospin multiplet, the scatter in the results is significantly smaller than the scatter in the individual $\delta_C^a$ and $\delta_C^b$ corrections.

For the $T = 1$ superallowed transitions, there are 4 candidates to study such mirror superallowed decays: $A = 26$, $A = 34$, $A = 38$, and $A = 42$. Of these 4 candidates, only the $A = 38$ and $A = 34$ mirror transitions have been measured experimentally with sufficient precision and are thus currently the only available data to compare to the different $\delta_C$ calculations. The $ft^a/ft^b$ ratio calculated using the HF and WS $\delta_C$ calculations [33], as well as the experimentally determined ratios, are given in Table 1.5 for both sets of mirror transitions and are plotted in Figure 1.9. Comparison of the $A = 38$ transitions favours the WS $\delta_C$ calculations, while disagreeing with the HF calculations at the $1.7\sigma$ level and this observation was used in Ref. [3] as an additional motivation for rejection the HF $\delta_{C2}$ corrections in favour of the WS ones. We note, however, that the case of $A = 34$, currently measured with comparable precision to $A = 38$, was not considered in Ref. [3]. As shown in
Table 1.5: Ratio of the $ft$ values for mirror superallowed transitions. The transitions labelled $a$ and $b$ refer to the transitions to the left and right of the semicolon given in the first column, respectively. The ratios of $ft$ values using the HF and WS $\delta_C$ corrections are taken from Ref. [33] and compared to the experimentally determined values.

<table>
<thead>
<tr>
<th>Mirror Transition</th>
<th>$ft^a/ft^b$ (WS)</th>
<th>$ft^a/ft^b$ (HF)</th>
<th>$ft^a/ft^b$ (Expt.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{26}\text{Si} \rightarrow ^{26}\text{mAl}; ^{26}\text{mAl} \rightarrow ^{26}\text{Mg}$</td>
<td>1.00389(26)</td>
<td>1.00189(26)</td>
<td></td>
</tr>
<tr>
<td>$^{34}\text{Ar} \rightarrow ^{34}\text{Cl}; ^{34}\text{Cl} \rightarrow ^{34}\text{S}$</td>
<td>1.00171(26)</td>
<td>0.99971(43)</td>
<td>1.00005(268)</td>
</tr>
<tr>
<td>$^{38}\text{Ca} \rightarrow ^{38}\text{mK}; ^{38}\text{mK} \rightarrow ^{38}\text{Ar}$</td>
<td>1.00196(39)</td>
<td>0.99976(43)</td>
<td>1.00356(225)</td>
</tr>
<tr>
<td>$^{42}\text{Ti} \rightarrow ^{42}\text{Sc}; ^{42}\text{Sc} \rightarrow ^{42}\text{Ca}$</td>
<td>1.00566(65)</td>
<td>1.00296(42)</td>
<td></td>
</tr>
</tbody>
</table>

Figure 1.9, the experimental ratio of the $ft$ values for the $A = 34$ mirror transitions, although in agreement with the WS $\delta_C$ calculations at the $1\sigma$ level, could be argued to actually favour the HF $\delta_C$ calculations.

Based on the above observations, additional tests of the $\delta_C$ corrections clearly remain crucial. In addition to the $T = 1$ superallowed mirror transitions mentioned above, the $\delta_C$ corrections from superallowed mixed mirror transitions can also be used to test the different $\delta_C$ corrections [36]. At this point, it would appear premature to reject the SM-HF $\delta_{C2}$ corrections in favour of the SM-WS $\delta_{C2}$ corrections, therefore, we continue to adopt the procedure of the previous superallowed surveys and average $\bar{F}t$(WS) and $\bar{F}t$(HF) given in Equation 1.59 to obtain the final result:

$$
\bar{F}t = 3072.09(62)_{\text{stat}}(36)_{\delta_R}(21)_{\delta_C} \\
= 3072.09(75) \text{ s.}
$$  

(1.63)
Figure 1.9: A plot of the ratio of the superallowed $ft$ values given in Table 1.5. The red and green bands represent the WS and HF theoretical corrections, respectively. The data point for the $A = 38$ mirror transitions show better agreement with the WS $\delta_C$ corrections while the $A = 34$ mirror transitions favour the HF $\delta_C$ theoretical corrections. The uncertainties of the published $ft$ values of $^{26}$Si and $^{42}$Ti are currently too large to provide a sensitive test of the $\delta_C$ corrections. Although a new measurement of the $^{26}$Si superallowed branching ratio, preliminary results of which have been presented [34,35], is expected to add an additional point at the $A = 26$ in the near future.

1.5 Weak Interaction Tests

Although the superallowed decays between $J^\pi = 0^+$, $T = 1$ isobaric analogue states represent only a tiny fraction of all nuclear $\beta$ decays, they provide crucial tests of fundamental properties of the Standard Model. Including all of the theoretical corrections discussed in
Section 1.4, the $\mathcal{F}_t$ values are observed to be constant over the range of superallowed emitters which have been measured to date. A plot of the $\mathcal{F}_t$ values for the 14 most precisely determined superallowed decays is shown in Figure 1.7 and confirms the CVC hypothesis at the level of $1.2 \times 10^{-4}$ \cite{3}. Once the CVC hypothesis is confirmed, $G_V$ can be extracted from the superallowed data. Re-arranging Equation 1.49 for $G_V$ yields:

$$G_V^2 = \frac{K}{2(1 + \Delta VR)\mathcal{F}_t}$$

$$\frac{G_V}{(\hbar c)^3} = 1.13628(14)\mathcal{F}_t(21)\Delta VR \times 10^{-5} \text{ GeV}^{-2}. \quad (1.64)$$

$$\frac{G_V}{(\hbar c)^3} = 1.13628(25) \times 10^{-5} \text{ GeV}^{-2}. \quad (1.65)$$

### 1.5.1 The CKM Quark Mixing Matrix

Another property of the Standard Model that the superallowed data allow us to probe is the unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing matrix. It is the flavour-changing nature of the weak interaction that allows the $\beta$ decay process to occur. The quark couplings between the different generations is described by the CKM quark-mixing matrix, which represents the unitarity transformation between the weak interaction eigenstates and the mass eigenstates of the quarks. The CKM matrix is defined as:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}, \quad (1.66)$$

where $V_{ij}$ represents the non-zero coupling between quarks $i$ and $j$, $\{d', s', b'\}$ represents the weak interaction eigenstates of the quarks, and $\{d, s, b\}$ represents their mass (physical) eigenstates.
The determination of $G_V$ using the high-precision superallowed data given in Equation 1.65, in combination with the Fermi coupling constant, $G_F/(hc)^3 = 1.1663787(6) \times 10^{-5} \text{ GeV}^{-2}$ [1], as deduced from purely leptonic muon decays, provide the most precise determination of $|V_{ud}|$, the up-down element of the CKM matrix. It is given as:

$$|V_{ud}| = \frac{G_V}{G_F}$$

$$= 0.97420(13)\frac{\Delta V}{\Delta \nu}$$

$$= 0.97420(22). \quad (1.67)$$

The CKM quark mixing matrix is a unitary matrix within the Standard Model. The sum of the squares of the elements of each row and each column in the CKM matrix should thus sum to unity. It is the top row elements of the CKM matrix that provide the most precise test of the CKM unitarity. Thus, taking $|V_{us}| = 0.2243(5)$ and $|V_{ub}| = 0.00394(36)$, obtained from the Particle Data Group [1], the sum of the squares of the top row elements yields:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.99939 \pm 0.00048 \quad (1.68)$$

which satisfies the CKM unitarity test at the level of $\pm 0.05\%$.

### 1.5.2 Limits on Additional Interactions

The description of the weak interaction in the Standard Model is an equal mixture of vector and axial-vector interactions which maximizes parity violation. Before the establishment of the “$V - A$” nature of the weak interaction, several choices for the form of the interaction were considered, namely; scalar, pseudoscalar, vector, axial vector, and tensor. The general
form of the Hamiltonian including all of these potential interactions is of the form \[37,38\]:

\[
H_\beta = (\bar{p}n)[\bar{e}(C_s + C'_S\gamma_5)\nu] \\
+ (\bar{p}\gamma_\nu(C_V + C'_V\gamma_5)\nu] \\
+ \frac{1}{2}(\bar{p}\sigma_{\lambda\mu}n)[\bar{e}\sigma_{\lambda\mu}(C_T + C'_T\gamma_5)\nu] \\
- (\bar{p}\gamma_\nu\gamma_5n)[\bar{e}\gamma_\nu\gamma_5(C_A + C'_A\gamma_5)\nu] \\
+ (\bar{p}\gamma_5)[(C_P + C'_P\gamma_5)\nu] + \text{H. C.} \tag{1.69}
\]

where the tensor operator, \(\sigma_{\lambda\mu}\), is given by:

\[
\sigma_{\lambda\mu} = -\frac{i}{2}(\gamma_\lambda\gamma_\mu - \gamma_\mu\gamma_\lambda) \tag{1.70}
\]

where \(\gamma_i\) \((i = 0, 1, 2, 3)\) are the Dirac gamma matrices, and \(C_j\) and \(C'_j\) \((j = S, P, V, A, \text{and } T)\) represent the coupling constants for each of the parity conserving and non-conserving interactions listed above, respectively.

Although no statistically significant discrepancies from the Standard Models’ “\(V - A\)” form have been observed, there remains interest in searching for contributions from any of the other potential interactions. In the non-relativistic limit, the contributions from pseudoscalar interactions vanish and provide no contributions in nuclear beta decay \[39\]. In 1937 \[40\], Fierz showed that if an admixture of either vector and scalar, or axial-vector and tensor, interactions were present, then the interference between the two interactions would be of the form \((1 + a/W)\), where \(a\) is a constant and \(W\) is the energy of the \(\beta\) particle in electron rest mass units. Thus, searches for additional interactions to the “\(V - A\)” form of the Standard Model in the low energy regime often focus on evidence that either the weak interaction is not maximally parity violating or that there are additional small contributions from scalar interactions.
or tensor interactions.

Since $0^+ \rightarrow 0^+$ superallowed decays are purely vector, the $\mathcal{F}t$ data are uniquely sensitive to potential additional contributions from scalar interactions. The relevant general form of the weak interaction Hamiltonian with both scalar and vector couplings is given by [37]:

$$\mathcal{H}_{S+V} = (\bar{\psi}_p \psi_n)(C_S \bar{\phi}_e \phi_{\bar{\nu}_e} + C'_S \bar{\phi}_e \gamma_5 \phi_{\bar{\nu}_e}) + (\bar{\psi}_p \gamma_\nu \psi_n)(C_V \bar{\phi}_e \gamma_\nu \phi_{\bar{\nu}_e} + C'_V \bar{\phi}_e \gamma_5 \gamma_\nu \phi_{\bar{\nu}_e}), \quad (1.71)$$

where $C_S$ and $C_V$ represent the scalar and vector coupling constants for parity conserving interactions, respectively, and the corresponding couplings $C'_S$ and $C'_V$ represent the analogous parity non-conserving interactions.

The redefined Hamiltonian given in Equation (1.71) directly changes the matrix element. Thus, in the presence of a scalar interaction, the experimentally measured $\mathcal{F}t$ values would no longer be constant, but instead:

$$\mathcal{F}t \rightarrow \mathcal{F}t(1 + b_F \gamma \langle W^{-1} \rangle) = \text{constant}, \quad (1.72)$$

where $\gamma = \sqrt{1 - (\alpha Z)^2}$, $\langle W^{-1} \rangle$ is the mean value of $W^{-1}$ averaged over the $\beta$ spectrum, and $b_F$ is the Fierz interference term [30]. Naturally, if $b_F = 0$, the Standard Model description with only contributions from vector interactions is restored. The plot shown in Figure 1.10 highlights the sensitivity of the superallowed $\mathcal{F}t$ data to the contribution from scalar interactions. In particular, the slope of the graph of $1/\mathcal{F}t$ as a function of $\gamma \langle W^{-1} \rangle$ can be used to directly extract $b_F$. Because of the $\langle W^{-1} \rangle$ dependence which arises in the Equation (1.72) it is the lightest superallowed decays, which have the lowest $Q$-values and thus the largest values of $\langle W^{-1} \rangle$, that are most sensitive to possible contributions from scalar currents in the weak interaction. Therefore, precise determinations of the $\mathcal{F}t$ values for $^{26}\text{m}Al$, $^{22}\text{Mg}$, $^{18}\text{Ne}$, $^{14}\text{O}$, and $^{10}\text{C}$ are of particular interest to set stringent limits on $b_F$. 

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Figure 1.10: Plot of $1/Ft$ as a function of $\gamma <W^{-1}>$ for the 14 most precisely measured $0^+ \rightarrow 0^+$ superallowed emitters. The data were taken from the most recent evaluation of the superallowed data and use the WS $\delta_C$ corrections [3]. The slope yields a value of $b_F = -0.0028(25)_{stat}(6)\delta_{R'}$ for the Fierz interference term. The best limit on $b_F$ currently comes from the superallowed $Ft$ data, with the most recent superallowed evaluation yielding $b_F = -0.0028(25)_{stat}(6)\delta_{R'}$ [3], which is consistent with zero at the 1.1$\sigma$ level. Since this result is taken from Ref. [3], the limit on $b_F$ is obtained using the $Ft$ values which were evaluated using the WS $\delta_C$ corrections. If the HF $\delta_C$ corrections are used to calculate the limit on $b_F$, one obtains $b_F = 0.0012(26)_{stat}(6)\delta_{R'}$. The limits on $b_F$ using the WS and HF $\delta_C$ corrections are shown in Figures 1.10 and 1.11 respectively. Although both are nearly consistent with zero at the 1$\sigma$ level, the central values obtained
from the two sets of $\delta_{C2}$ corrections have opposite signs and differ by 1.5\(\sigma\).

Again, adopting the procedure of averaging the results obtained with the WS and HF $\delta_{C2}$ corrections and assigning a systematic uncertainty equal to half their difference, we obtain $b_F = -0.0008(26)_{stat}(6)_{\delta R}(20)_{\delta C} = -0.0008(33)$. This result illustrates that the model dependence of the theoretical isospin symmetry breaking corrections contributes an uncertainty to the Fierz interference term approximately equal to that of the statistical uncertainty of the world superallowed dataset.
Limits on the scalar couplings, $C_S$ and $C'_S$, can be deduced from $b_F$. First, if the assumption that the Hamiltonian is invariant under time reversal is made, then the couplings $C_j$ and $C'_j$ are real. If the further assumption that parity is maximally violated is also made, then $C'_j = C_j$. In this limit, $b_F$ can be used directly to extract limits on the ratio of the scalar coupling to the vector coupling [3]:

$$\frac{C'_S}{C_V} = -\frac{b_F}{2},$$

with the above value of $b_F$ from the superallowed data yielding $C_S/C_V = 0.0004 \pm 0.0017$, consistent with no weak scalar current interactions.

If, however, the assumption of maximal parity violation is removed, then $C_j \neq C'_j$ and the relationship between the scalar and vector couplings and $b_F$ is given by [3]:

$$b_F = \frac{-2C_V(C_S + C'_S)}{2|C_V|^2 + |C_S|^2 + |C'_S|^2} \approx -\left(\frac{C_S}{C_V} + \frac{C'_S}{C_V}\right).$$

In this scenario, the superallowed $Ft$ data alone are insufficient to set limits on the scalar current couplings. The superallowed $Ft$ data, however, can be used in combination with $\beta$-$\nu$ angular correlation coefficient data. The $\beta$-$\nu$ angular correlation coefficient $a$, for superallowed $0^+ \to 0^+$ transitions, is also sensitive to the scalar couplings, with the relation [3]:

$$a = \frac{2|C_V|^2 - |C_S|^2 - |C'_S|^2}{2|C_V|^2 + |C_S|^2 + |C'_S|^2} \approx 1 - \left(\frac{|C'_S|^2}{|C_V|^2} + \frac{|C'_S|^2}{|C'_V|^2}\right).$$

The most stringent limit on the scalar couplings in this scenario uses the superallowed $Ft$ data along with the $\beta$-$\nu$ angular correlation measurements from $^{38\text{m}}$K decay [41], yielding the result $\left|\frac{C_S}{C_V}\right| \leq 0.065$ and $\left|\frac{C'_S}{C_V}\right| \leq 0.065$. 40
1.6 Summary

The study of superallowed $\beta$ decays is used to test fundamental properties of the weak interaction as described by the Standard Model. Currently, the constancy of the $\mathcal{F}t$ values from these decays provide the most stringent tests of the CVC hypothesis, which is confirmed at the level of $10^{-4}$. The constancy of these $\mathcal{F}t$ values also provides the most stringent limit on the Fierz interference, which is used to characterize the strength of a possible scalar current contribution to the weak interaction. Currently, the limit on the Fierz interference term remains consistent with the absence of a scalar current in agreement with the prediction of the Standard Model.

In Chapter 2, an overview of the experimental facilities, including the accelerator complex and the detector systems used in the experiments leading to the data presented in this thesis will be described. A brief description of the relevant experimental techniques necessary for high-precision half-life measurements will be discussed in Chapter 3. In Chapters 4 and 5, the motivation prompting the measurements of the half-lives of $^{10}\text{C}$ and $^{22}\text{Mg}$ will be discussed and the full details of the half-life analyses will be presented. The impact of these new results will also be discussed. Finally, conclusions and future measurements relevant to high-precision superallowed Fermi $\beta$ decays will be examined in Chapter 6.
Chapter 2

Experimental Facilities

2.1 TRIUMF

The experiments leading to the results presented in this thesis were performed at TRIUMF’s Isotope Separator and Accelerator (ISAC) facility which is located in Vancouver, BC. Founded in 1968, TRIUMF is Canada’s national laboratory for nuclear and particle physics and is one of the world’s leading subatomic physics laboratories. The broad scientific research performed at TRIUMF include: nuclear physics using radioactive ion beams, nuclear theory, molecular and material science, particle physics, and nuclear medicine. The Advanced Rare Isotope Laboratory (ARIEL) which is scheduled to be operational in the near future will further expand TRIUMF’s scientific program.

At the core of TRIUMF is a sector-focused H⁻ cyclotron which produces proton beams with energies up to 520 MeV with a total current of up to 300 µA. A photograph of the main cyclotron during its construction is shown in Figure 2.1. The H⁻ ions are obtained from an ion source and are accelerated in the cyclotron with a high frequency alternating
electric field. Once the H\(^-\) ions are accelerated, they are stripped of two electrons via passage through thin (11 \(\mu m\)) carbon foils yielding positively charged protons that bend in the opposite direction in the applied magnetic field. The extracted protons can subsequently be sent to up to four different experimental halls simultaneously, where they can be used directly or can be collided onto stationary targets to produce secondary beams of muons, pions, or radioactive ions.
2.2 The Isotope Separator and Accelerator Facility

Radioactive ion beam (RIB) studies are performed at TRIUMF’s Isotope Separator and Accelerator (ISAC) facility using the Isotope Separator On-Line (ISOL) technique. The production of radioactive nuclei occurs via the bombardment of the high intensity proton beams onto stationary targets where they induce spallation and fragmentation reactions. The targets, located two floors below the experimental halls, are operated at high temperatures (up to $\sim 2300^\circ$C) to promote the diffusion of the reaction products out of the target. The target is coupled to an ion source in order to ionize the diffused particles [45]. In order to provide a large variety of rare isotope beams, several different ion sources are available at TRIUMF. These include: surface ionization sources, the TRIUMF Resonant Laser Ion Source (TRILIS) [46], the Ion Guide Laser Ion Source (IGLIS) [47] and plasma ion sources [48,49].

For the $^{10}$C experiments which will be presented in this thesis, a forced electron beam induced arc discharge (FEBIAD) plasma ion source was used. The FEBIAD ionization source is primarily used for elements or molecules that have an ionization energy of more than 9 eV because they cannot be efficiently ionized by either surface or laser ionization techniques [48–51]. Electrons are extracted from a heated cathode and are accelerated into a plasma. Neutral atoms and molecules are then ionized via collisions with these electrons.

For the $^{22}$Mg experiment, the target was coupled to IGLIS. This ionization technique was developed specifically to reduce the background contribution from surface-ionized species, which are not suppressed by the other laser-ionization techniques [47]. In this method, surface-ionized ions diffusing from the target are repelled using electrostatic electrodes. The neutral atoms that diffuse from the target pass through the region of electrostatic repulsion and are then laser-ionized. In particular for the $^{22}$Mg experiment, IGLIS was used to produce
high-intensity beams of laser-ionized $^{22}$Mg while suppressing the surface-ionized contaminant of $^{22}$Na by a factor of up to $10^6$ [47].

Following the ionization of the neutral particles, the ions are extracted from the ion source which is held at an electric potential of 10 kV–60 kV above ground potential. The charged ions are subsequently transported to a mass separator where they are deflected along a trajectory of radius $r$ by their mass-to-charge ratio according to:

$$r = \frac{1}{B} \sqrt{\frac{2m\Delta V}{q}},$$

(2.1)

where $B$ is the applied magnetic field, $\Delta V$ is the change in voltage between the mass separator and the ion source, and $m$ and $q$ are the mass and charge of the particular nuclei, respectively. TRIUMF’s mass separator is typically operated in a mode with mass resolution of $\delta m/m \approx 1/1000$ which provides clean separation between neighbouring mass isotopes, but is, however, often unable to separate molecular and isobaric contaminants with the same $A/q$.

Following mass separation, the resultant low energy beams, typically with energies between 10 keV–60 keV, are delivered along the beam line to the different experimental stations in the ISAC-I hall. In the ISAC-I hall, the RIBs can be further accelerated using a radio frequency quadrupole (RFQ) and a room temperature drift-tube linear (DTL) accelerator to energies up to 0.15 – 1.7 MeV/nucleon. The RIBs can also be accelerated using the ISAC-II superconducting linear accelerator [52], to energies between 6 – 15 MeV/nucleon, and sent to the experimental stations located in ISAC-II experimental hall. A schematic figure of the ISAC experimental halls is shown in Figure [27,2].
2.3 The $8\pi$ Spectrometer

The $8\pi$ spectrometer is a $\gamma$-ray spectrometer which is used to determine the properties of radioactive nuclei, such as the spins, transition rates, transition multipolarities, and half-lives of nuclear states. It is a spherical array composed of 20 Compton-suppressed High-Purity Germanium (HPGe) detectors that provide approximately 13% coverage of the $4\pi$ solid angle \cite{53,54}. The geometry of the array is a truncated icosahedron where the 20 HPGe detectors are located in the positions of the 20 hexagons. A photograph of one hemisphere
of the array is shown in Figure 2.3. Each HPGe crystal has a diameter of 5.3 cm and an efficiency of \( \sim 23\% \) relative to a standard 7.62 x 7.62 cm cylindrical NaI(Tl) crystal, while the absolute \( \gamma \)-ray photopeak efficiency of the entire array is approximately 1% at 1.3 MeV [54].

One disadvantage of HPGe detectors with crystals of relatively modest size such as the ones used in the \( 8\pi \) spectrometer is the large Compton continuum background that results from the fact that the majority of \( \gamma \)-rays will undergo Compton scattering and exit the crystal without depositing their full energy [55]. When the scattered \( \gamma \)-ray escapes from a crystal, the full energy photopeak will not be recorded, and the partial energy deposition results in a continuous background. In order to combat this effect, Compton suppression shields are often used with HPGe detectors. In the \( 8\pi \) spectrometer setup, the Compton suppression shields are made of bismuth germanate (BGO) scintillators which surround the HPGe crystals. There are two particular advantages to using BGO scintillators for Compton suppression: it is a high-density (7.13 g/cm\(^3\)) and high-\( Z \) (\( Z = 83 \) for the Bi component) inorganic crystal. Both of these characteristics ensure a large probability per unit volume for the Compton scattering and photoelectric absorption of \( \gamma \)-rays [55]. Each BGO crystal is connected to a pair of photomultiplier tubes (PMTs) in order to collect the light emitted from each scintillator and convert it into an electrical signal.

In order to reduce the probability that the first interaction of a \( \gamma \)-ray occurs in the BGO scintillator, potentially causing a “false veto” of another \( \gamma \)-ray photopeak recorded in the neighbouring HPGE detector, a 2.54 cm thick Heavimet collimator is placed in front of each BGO scintillator. In order to reduce the production of bremsstrahlung radiation resulting from the interaction of high-energy \( \beta \) particles with the Heavimet collimator, each of the
collimators are covered with a 2 cm layer of Delrin. Since Delrin is a high-density but low-$Z$ plastic, it has a low $\gamma$-ray scattering cross section and the $\beta$ particles will lose energy primarily through collision processes rather than via bremsstrahlung. A schematic of the
Figure 2.4: Single $8\pi$ HPGe detector surrounded by its BGO Compton suppression shield. PMTs are attached to each BGO scintillator in order to collect the emitted light. The front of the HPGe and BGO detectors are surrounded by Heavimet (Tungsten alloy) collimators which are, in turn, covered by Delrin absorbers.

configuration of each HPGe detector with all of the aforementioned components is shown in Figure 2.4. It should be noted that, although the active use of BGO scintillators greatly reduces the Compton background, their use also results in rate-dependent losses of good photopeak events due to false vetoes and, as such, were only used as passive shields.
during the high-precision half-life measurements reported in this thesis.

The radioactive ion beams that are delivered to the $8\pi$ spectrometer are implanted into a tape transport system, which is under vacuum and located at the center of the $8\pi$ array. The moveable tape transport system, shown in Figure 2.5, is used to remove long lived isobaric contamination out of view of the detector. The system is composed of a $\sim$120 m long continuous tape. The aluminized Mylar tape used in these experiments had a width of 1.27 cm with a 9.4 $\mu$m thick aluminum coating into which the radioactive beams are implanted. Once the radioactive sample is built up on the tape, the beam is deflected after the mass separator which is located two floors below the experimental hall. Once the decay has been measured, the tape is moved into a tape disposal box which is shielded by a 5 cm thick lead wall to ensure that the activity is not measured in the detector array.

### 2.3.1 The $8\pi$ Data Acquisition System (DAQ)

Both the beam implantation and tape movement are controlled through a Jorway timing and sequencing controller in CAMAC which is programmed to automatically move the tape. The Jorway is used to control timing parameters of the cycle such as tape movement, background counting, beam on/off times, etc., which can be easily programmed by the user to optimize the cycle for each experiment.

Following the interaction of a $\gamma$-ray in an HPGe detector, the signal from the HPGe is pre-amplified and fanned to two different streams in order to extract and record both the energy and timing information of the event. The energy signal is sent to an Ortec 572 spectroscopy amplifier with an adjustable shaping time. This amplified signal is sent to an Ortec AD114 14-bit analog-to-digital converter (ADC) in order to extract the $\gamma$-ray energy from the collected charge signal. The time signal is amplified using a timing-filter amplifier.
and discriminated with a Ortec 583b constant fraction discriminator (CFD). The signal from the CFD is then sent to a Lecroy 4516 logic module. The first output of the logic module is sent to a Lecroy 3377 32-bit multi-hit time-to-digital converter (TDC) which provides intra-event nanosecond timing of the $\gamma$-rays, and, along with the master trigger, provide coincidence timing information between $\gamma$-rays from different crystals. A second set of these TDCs are used to time pile-up signals from the spectroscopy amplifiers in order to record events that are flagged as “piled-up” (which will be discussed in further detail in Chapter 3). When the BGO scintillators are being used as active suppression shields, a third set of TDCs also provides timing information for Compton coincidence rejection. Finally, a Lecroy 2367
universal logic module (ULM), which consists of two 32-bit latching scalars and one 16-bit bit register, is used to: (i) count pulses from a Stanford Research Systems high-precision 10 MHz ± 0.1 Hz oscillator to provide a global time stamp for each event relative to the start of the cycle, (ii) record the event-by-event system wide dead-time by counting the pulses from the 10 MHz oscillator that remain after a veto by an acquisition dead-time, and (iii) record the total number of master triggers.

2.3.2 The GRIFFIN Spectrometer

The $8\pi$ $\gamma$-ray spectrometer was decommissioned in December 2013 after being operated at TRIUMF for approximately 10 years. The $8\pi$ spectrometer was replaced by the new high-efficiency Gamma-Ray Infrastructure For Fundamental Investigation of Nuclei (GRIFFIN) spectrometer, an array of 16 clover-type HPGe detectors [57, 58]. A photograph of the GRIFFIN array is shown in Figure 2.6. Similar to the $8\pi$ spectrometer, the beam is implanted into a tape transport system located at the center of the GRIFFIN array. A Figure of the Delrin sphere in which the tape is located is shown in Figure 2.7.

Each GRIFFIN clover consists of four HPGe crystals that are housed together within a single cryostat. A GEANT4 simulated model of a single clover is given in Figure 2.8. The individual crystals used in the GRIFFIN spectrometer are 60 mm in diameter and 90 mm in length, each with a $\sim 40\%$ relative efficiency [59]. The edges of the crystals are tapered at 22.5 degrees over the first 30 mm of their length in order to allow a close-packed configuration between neighbouring clovers [59].

The clover detector configuration of the GRIFFIN spectrometer was designed to have a much higher photopeak efficiency than the $8\pi$ spectrometer. Since Compton scattering is the dominating interaction process in HPGe for $\gamma$-rays with energies between $\sim 200$ keV and
7 MeV, there is a possibility that a $\gamma$-ray scatters between neighbouring HPGe crystals. The full energy of the $\gamma$-ray can be recovered by summing individual events of the neighbouring crystals within a clover. This procedure, known as addback, increases the detection efficiency of GRIFFIN relative to the $8\pi$ since these Compton scattering events in the $8\pi$ spectrometer would have been lost.

The improved photopeak efficiency of the GRIFFIN spectrometer in comparison to that of the $8\pi$ spectrometer is shown in Figure 2.9. This increased photopeak efficiency improves
Figure 2.7: A photograph of the implantation site of the GRIFFIN array. The RIB is implanted onto a tape transport which is located within the Delrin sphere at the center of the GRIFFIN array.

the $\gamma$-$\gamma$ coincidence detection efficiency by a factor of 100-400 depending on the $\gamma$-ray energies, greatly improving the ability to measure very weak branches. Moreover, the improved
sensitivity of the GRIFFIN spectrometer provides the ability to perform low-rate $\gamma$-ray spectroscopy experiments with radioactive beam intensities as low as 0.01 ions/s [57].

Many of the auxiliary systems of the $8\pi$ spectrometer system have been carried over to the GRIFFIN array, including the RIB beam implantation into the tape transport system and the full suite of $8\pi$ auxiliary detection systems [57,58]. However, unlike the $8\pi$ spectrometer,
Figure 2.9: GEANT4 simulations of the GRIFFIN and 8π γ-ray spectrometers. a) Absolute photopeak efficiency, b) photopeak-to-total ratio, c) addback factor, and d) ratio of photopeak efficiencies for GRIFFIN compared to 8π. This figure illustrates the greatly improved performance of the GRIFFIN spectrometer. Figure adapted from Ref. [57].

which utilized an analog DAQ system, the GRIFFIN spectrometer is operated with a custom-designed digital DAQ system. The GRIFFIN DAQ system was designed to accommodate high data rates for the nuclear structure program and to allow for precision measurements relevant for the study of superallowed Fermi β decays [57, 58]. In particular, these design goals included the ability to collect data at high rates up to 50 kHz per HPGe crystal, while maintaining careful control of system deadtime in order to perform branching ratio and half-life measurements to a precision of better than ±0.05%.
2.3.3 The Zero Degree Scintillator

Both the $8\pi$ and GRIFFIN spectrometers are designed to operate with several auxiliary detectors in order to obtain complimentary information relevant to the decays of interest. In the $^{10}$C experiment discussed in this thesis, the Zero Degree Scintillator (ZDS) was used to measure the $\beta$ particles which are emitted in the $\beta$ decay of the parent nucleus. A photograph of the ZDS is given in Figure 2.10. It is comprised of a 1 mm thick, 2.5 cm diameter, BC-422Q ultrafast timing plastic scintillator, manufactured by Saint-Gobain, and is coupled to a H6522 Hamamatsu PMT. The distance of the ZDS detector relative to the implantation tape at the center of the $8\pi$ spectrometer can be adjusted between $\sim 1$ and 5 cm, and can provide up to $\approx 25\%$ coverage of the solid angle. The use of an organic scintillator is ideal because, since it is a low-$Z$ material, it is able to measure the $\beta$ particles while being largely transparent to the $\gamma$ radiation. For the experiments performed in this thesis, the ZDS detector was used to tag the $\beta$ particles in order to perform $\beta$-$\gamma$ coincidence measurements, and the energy information was thus inconsequential.

2.3.4 The Scintillating Electron-Positron Tagging Array

Another auxiliary detector that can be used in conjunction with the $8\pi$ and GRIFFIN spectrometers is the Scintillating Electron-Positron Tagging Array (SCEPTAR). It is comprised of twenty 1.6 mm thin BC-404 plastic scintillators, arranged in four pentagonal rings, that surround the implantation site on the tape transport system and is also used to measure $\beta$ particles. SCEPTAR provides significantly greater coverage around the tape, in comparison to the ZDS, with a solid angle coverage of $\sim 80\%$ of the $4\pi$ solid angle. The SCEPTAR array is also located within the spherical Delrin vacuum chamber of the $8\pi$ spectrometer,
and is divided into two hemispheres. Each of the twenty scintillators are located in front of one of the 8π HPGe detectors. With this configuration, the resulting contamination of the γ-ray spectrum by bremsstrahlung radiation due to the stopping of the β particle can be greatly reduced by vetoing the HPGe detector located directly behind the scintillator in which the β particle was detected. A photograph of a single hemisphere of the SCEPTAR array is shown in Figure 2.11.
2.4 \(4\pi\) Continuous-Flow Proportional Gas Counter

High precision half-life measurements, via direct \(\beta\) counting, were performed using a \(4\pi\) continuous-flow proportional gas counter. A schematic diagram of the gas counter is shown in Figure 2.12. The active cylindrical volume consists of two identical hemispheres constructed from low background copper \([60]\), each with a length of \(~2.5\) cm and diameter
of \(\sim 3.7\) cm. Within each hemisphere is a gold plated tungsten anode wire with a nominal diameter of 0.013 mm. The gas counter is operated at atmospheric pressure with methane gas (CH\(_4\)) which is continuously flushed through at a rate of 0.5 cc/min. Thin Ni foils are used to protect the active volume from contamination from air while still allowing the \(\beta\) particles to easily penetrate the active volume. A photograph of the gas counter is shown in Figures 2.13.

For this setup, the beam is implanted under vacuum into the thick (17.2 \(\mu\)m) Al layer of an aluminized mylar tape which, unlike the \(\gamma\)-ray spectrometer setup, is not located at the center of the gas counter. Typically after implantation, and before the sample is moved into the gas counter, the sample is allowed to "cool" for some time. This is done to either allow any short-lived contaminants to decay before measuring the activity in the gas counter or to
Figure 2.13: Photograph of the $4\pi$ proportional gas counter. The beam is delivered on the left, and implanted into the tape under vacuum out of the gas counter. The methane gas is continuously flushed through the gas counter via the green and clear tubes pictured in this photograph.

Limit the maximum count rate in the gas counter. Limiting the count rate in proportional gas counters is important in order to minimize space charge effects, which can alter the operational stability of the detection system [55]. If the number of ion pairs created in the gas counter is too large, the drifting charged ions will reduce the electric field near the anode wire resulting in a loss of proportionality in the signal from the gas counter. In order to ensure that such space charge effects do not affect the data, the maximum count rate in the gas counter is typically limited to approximately 10 kHz. After the sample is sufficiently “cooled”, it is moved into the center of the $4\pi$ proportional gas counter via 2 stages of differential pumping from the vacuum to atmospheric pressure. Following the decay, the tape is moved out of the gas counter into a tape disposal box. A schematic of the setup for
Figure 2.14: Schematic of the $4\pi$ gas counter and its tape transport system. The beam is implanted into the tape under vacuum via a differential pumping system and moved into the gas counter which is located 28 cm from the implantation site. Following the decay of the nuclei of interest, the tape is moved into the tape disposal box in order to remove any long lived contaminants from the view of the detector.

The signals from the two hemispheres of the gas counter were summed and processed by a Phillips 300 MHz bipolar preamplifier. The preamplified pulse was then sent to an Ortec 579 fast timing filter amplifier operated at high differentiation for maximum high-rate pulse processing. Following this, the signal was sent to an Ortec 436 100 MHz constant fraction discriminator (CFD) where the threshold is adjusted in order to reduce background contributions from low energy noise. The amplified and discriminated pulses from the gas
counter were then fanned to two LeCroy 222N gate-and-delay generators with fixed applied deadtimes of \( \sim 3 \mu s \) and \( \sim 4 \mu s \), respectively. The dead-time-affected pulses from the two gate-and-delay generators are simultaneously multiscaled in two multichannel scalars (MCS). The first ("Old MCS") is a Data Design Corporation IS1s0A integrating MCS and the second ("New MCS") is a LeCroy 3521A MCS. The Old MCS is limited to a maximum of 999 channels, each limited to a 14-bit count. The New MCS is limited to 500 channels of constant time width, which is adjustable in software, with each bin limited to a 24-bit maximum count. The total cycle time, which includes beam implantation, "cooling", tape movement, and total decay time within the aforementioned limits is controlled through a CAMAC standardized Jorway controller. An external time standard for the system was provided by a Stanford Research Systems model DS335 1 MHz \( \pm 1 \) Hz temperature stabilized precision clock scaled to 10 kHz.

In order to perform high-precision half-life measurements with the proportional gas counter, it is essential to operate within the plateau region of the gas counter. In the plateau region, the measured activity in the detector is essentially independent of the applied voltage \[55\]. The data in this thesis were taken with two nearly identical \( 4\pi \) gas counters. The only practical difference between the two gas counters was their operating plateau region which is due to a slight difference in the thicknesses of their anode wires. The plateau region for each of the gas counters was determined by measuring the activity of a \( ^{90}Sr \) source for various applied bias voltages. Sample plateau measurements are shown in \[2.15\] and \[2.16\]. Since the plateau region is also dependent on the applied CFD threshold, a set of plateau measurements were performed for several CFD threshold settings in order to determine the mutually inclusive plateau region for the different bias voltage and threshold settings.
Figure 2.15: Plateau measurements taken for the “old” gas counter. The measurements were taken in October 2013 during the $^{10}$C half-life experiment. The 70 mV data was taken with a 15 kBq $^{90}$Sr source, whereas the 95 mV and 120 mV data were taken with a 10 kBq $^{90}$Sr source. The 15 kBq data have been normalized to the (2700 V, 70 mV, 10 kBq) data point in order to directly compare the plateau regions for the different thresholds.
Figure 2.16: Plateau measurements taken for the “new” gas counter. The measurements were taken in October 2016 during the $^{22}\text{Mg}$ half-life experiment. All of the data was taken with a 10 kBq $^{90}\text{Sr}$ source. The data have been normalized to the 2300 V, 100 mV threshold data point.
Chapter 3

Experimental Corrections and Methods

In order to use the superallowed data to test fundamental properties of the Standard Model, as described in Chapter 1, the \( ft \) values for the superallowed emitters must be determined to a precision of \( \pm 0.3\% \) or better. To accomplish this, the \( \beta \) decay half-life, superallowed branching ratio, and \( f \)-value must all typically be measured to a precision of \( \pm 0.05\% \) or better. In this chapter, the rate dependent losses associated with measurements of radioactive decay processes, in particular dead-time and detector pulse pile-up, will be introduced. General methodologies required for high-precision half-life measurements through both \( \beta \) counting and \( \gamma \)-ray photopeak counting, as well as methods used to correct for these rate-dependent losses, will be discussed.

3.1 Dead Time

In all detector systems, there will exist a finite time following the detection of an event during which the detection system will be unable to record any subsequent events. This time is referred to as the dead time of the counting system. The dead time can result from
both the time required to collect the charge in the detector and the amount of time required to process the signal in the associated electronic system. Due to the random nature of radioactive decays, there is a probability that two events will occur too closely in time to one another for both events to be separately registered in the detector. This probability leads to dead time losses. In order to perform high-precision half-life measurements, it is essential to perform these experiments at high rates in order to obtain the statistical precision required. Running at high rates, however, can ultimately lead to significant dead time losses. Thus, in order to perform both accurate and precision measurements, it is essential to correct for these inherent dead time losses.

There are two modes in which the dead-time of a detection system can occur: extendible and non-extendible [55]. These two operational modes are illustrated in Figure 3.1. In this figure, the true distribution of events is given in the first row. In the non-extendible scenario, which is depicted in the third row, if a true event occurs while the system is live, then the system will be dead for some fixed time interval $\tau_D$ following the detection of this event. If a second true event occurs within the time interval $\tau_D$ relative to the first event, this second event will be undetected. Thus, given six true events distributed according to the first row in Figure 3.1, only four events would be recorded by the detector. The situation is slightly more complex for the extendible scenario, which is depicted in the second row of Figure 3.1. If an event is detected when the system is live, the system will go dead. Unlike the non-extendible regime, however, if a second event occurs a time $t < \tau_D$ after the initial event, the event will not be detected but the dead time of the system will be extended by another period $\tau_D$ from time $t$. Thus, given six true events distributed according to Figure 3.1, only three events will be recorded by the detector.

At low rates, the observed rate for the extendable case and the non-extendable case
Figure 3.1: A schematic illustration of the detector response for dead-time and pile-up. The first row depicts the true distribution of events, where $\tau_D$ represents the fixed time interval following each signal and is depicted by the dotted line. In the second and third rows, the two dead-time scenarios are depicted: extendible and non-extendible, respectively. The fourth and fifth rows depict the events that are flagged as not piled-up and piled-up, respectively. A description of each scenario is discussed in the text.

are nearly identical. As the rate is increased, this symmetry is broken. In particular, at extremely high rates, the extendible scenario would only record the first event because the interval between each event would fall within the time interval $\tau_D$. On the other hand, in the non-extendible scenario, the observed count rate in the system approaches an asymptotic value of $1/\tau_D$.

Since the dead time of the system in the extendable case is not fixed, recovering the true
count rate based on the measured rate is less straightforward than in the non-extendable case [61]. As extendable dead-times were not used in any of the experiments presented in this thesis, they will not be discussed further here. In the non-extendable case, given an observed count rate in the detector of \( R' \), the fraction of time in which the detector is dead is \( R' \tau_D \). Thus, for a true count rate, \( R \), the event rate loss will, statistically, be equal to \( RR' \tau_D \). The event rate loss is the difference between the true and observed counting rates and equating the two yields:

\[
R - R' = RR' \tau_D. \tag{3.1}
\]

Re-arranging Equation 3.1 to solve for the true count rate leads to:

\[
R = \frac{R'}{1 - R' \tau_D}. \tag{3.2}
\]

Equation 3.2 allows the true count rate \( R \) to be determined based on the observed count rate \( R' \) in the detector provided the fixed non-extendable of the dead-time \( \tau_D \) per event is known.

### 3.1.1 Measuring the Dead Time

For the \( \beta \) counting experiment using the 4\( \pi \) proportional gas counter, the events are counted in time bins using multi-channel scalers. If the system is live and an interaction occurs in the detector, a fixed non-extendable dead-time that is intentionally much longer than any other dead-times in the system is applied in the electronic processing of the signal. As described above, in order to determine the true count rate, it is essential that this dead-time is determined both accurately and precisely. The traditional method used to measure the dead-time is a Paired Source Method [55]. In this method, the count rates from two sources are measured both individually and in combination. Since the losses in the observed counting
rate are non-linear, the measured rate with the combined sources will be less than the sum of
the count rates of the individual sources. Let \( R_1 \), \( R_2 \), and \( R_{12} \) be the combined background
and source rate of source 1, source 2, and the combined source system, respectively, and let
\( R_b \) be the background rate when all of the samples are removed. In the absence of dead-time,
the following equations can be used to describe the system:

\[
R_{12} - R_b = (R_1 - R_b) + (R_2 - R_b)
\]
\[
R_{12} + R_b = R_1 + R_2.
\]

(3.3)

For a non-extendible dead-time, Equation 3.2 can be substituted into Equation 3.3 leading
to the result:

\[
R'_{12} - R'_b = (R'_1 - R'_b) + (R'_2 - R'_b)
\]
\[
R'_{12} + R'_b = R'_1 + R'_2.
\]

(3.4)

Typically the background rate is significantly less than the sources rates, and the approxi-
mations \( R'_b \tau_D = 0 \) and \( R'_b (1 - R'_1 \tau_D)(1 - R'_2 \tau_D)(1 - R'_{12} \tau_D) = R'_b \) can be made. This reduces
Equation 3.4 to a quadratic equation which can be solved for \( \tau_D \) given measurements of \( R'_1 \),
\( R'_2 \), \( R'_{12} \) and \( R'_b \).

One of the difficulties encountered using the Paired Source Method, however, is ensuring
that the geometry for each of the sources is exactly reproducible. In 1965, A. P. Baerg
proposed a variation of the Paired Source Method to overcome this problem [62]. In this
method, a pulser is used instead of a secondary source. Since the pulser is not randomly
distributed, the pulser does not induce dead-time losses on itself provided that the pulser
rate \( R_p \) is less than \( 1/\tau_D \). The probability of measuring a signal from the pulser is thus
\( (1 - R_s \tau_D) \), where \( R_s \) is the observed source rate when measured alone. The observed pulser
rate, \( R'_p \), is therefore:

\[
R'_p = R_p (1 - R_s \tau_D),
\]  

(3.5)

where \( R_p \) is the true pulser rate. If the periodic pulser induces the same dead-time in the system as a signal from the source, the count rate from the source will be further reduced when the pulser is present. Assuming that the two contributions to the dead-time are independent, which is valid when the total fractional dead-time is small, the probability of recording a randomly distributed source event becomes:

\[
R'_s \approx R_s (1 - R'_p \tau_D)
\]

\[
R'_s \approx R_s [1 - R_p (1 - R_s \tau_D) \tau_D]
\]  

(3.6)

Thus, the observed counting rate when the two systems are combined is given by:

\[
R_{sp} = R'_s + R'_p
\]

\[
R_{sp} \approx R_p + R_s - 2R_pR_s \tau_D + R_s^2 R_p \tau_D^2
\]  

(3.7)

Re-arranging Equation 3.7 for the dead-time yields:

\[
\tau_D \approx \frac{1}{R_s} \left[ 1 - \left( \frac{R_{s+p} - R_s}{R_p} \right)^2 \right]
\]  

(3.8)

Since this method involves the use of a pulser, rather than a second source, the position of the source can be kept constant. However, in order to effectively use this method, it is essential to have a stable pulse generator.

It should be noted, however, that Equations 3.6 and all equations derived thereafter, are only approximate. In particular, the derived expressions ignore higher order effects in which
more than 2 events occur within the dead-time interval. An extensive investigation of the
effect of this approximation was documented in Ref. [63]. From this analysis, the results from
Equation 3.8 are determined to be valid provided $\tau_D < 1/3R_P$. For dead-times longer than
this, significant breakdowns due to the approximations in the derivation begin to appear.
As detailed in Ref. [63], by ensuring that a proper pulser rate is selected for a given applied
dead-time, precise measurements of the dead-time via this method can be obtained.

Since an accurate and precise determination of the dead-time is essential to obtain a
high-precision half-life measurement, dead-time measurements are typically performed before
and after each experiment. Thus, the dead-time analyses for the individual experiments
performed in this thesis will be given in the subsequent analysis chapters.

For the $\gamma$-ray counting experiment, the time and energy information from each event
was recorded in “list mode” rather than being binned in a multi-channel scalar. The total
system dead-time was also measured on an event-by-event basis. In this case, the dead-time
is determined by counting the pulses from a 10 MHz master clock as well as separately
counting the pulses from this same clock that are not vetoed by the dead-time signal. The
difference of the differences between these two counters for consecutive events gives an event-
by-event measurement of the dead-time. Since the $\gamma$-ray experiments are also performed in
the non-extendible dead-time regime, the true count rate can be determined by substituting
the measured count rate into Equation 3.2 with $R'\tau_D$ replaced by the sum of the event-by-
event dead-times measured over any desired time bin divided by the length of that time
bin.
3.2 Detector Pulse Pile-up

In γ-ray spectroscopy, the fact that two events can occur within a short time interval introduces an additional effect. When two or more γ-rays strike the detector too closely in time, the correct energy information from both of the individual γ-rays is lost. This effect is referred to as detector pulse pile-up. Since the energy of these γ-rays is not properly determined, γ-ray pile-up results in a decrease in the number of counts within a given photopeak. When a γ-ray interaction occurs in the HPGe detector, there is a minimum time required to process the signal. For analog electronics systems, this signal processing time, and therefore the probability of pile-up, is related to the shaping time on the spectroscopy amplifiers. Experimentally, the shaping times on the spectroscopy amplifiers are fixed and define the pile-up time, $\tau_P$. Although decreasing the shaping time would decrease the effects resulting from pile-up, the gain from the reduction in piled-up events is counteracted by a decrease in energy resolution.

A simplified schematic representations of the detector output for events that are piled-up and not piled-up are given in the last two rows of Figure 3.1. In this example, the events in the first row that are separated by a time interval larger than $\tau_P$ will not be piled-up. The corresponding detector output signal is given in the fourth row. The energy for these events is correct and the amplitude of the charge signal shown in the fourth row is proportional to the initial charge depicted in the first row. On the other hand, both the third and fourth events, as well as the sixth and seventh events given in the first row occur within a time interval $\tau_P$ of one another, resulting in a piled-up signal in the detector. The corresponding detector output signal for the piled-up events is given in the last row. Since these piled-up events result from the overlap of two events, the amplitude of the charge signal is larger.
than the true charge collected from the individual events depicted in the first row. For these events, the true energy signal from the individual piled-up $\gamma$-rays cannot be deconvoluted in an analog electronics system. It should be noted that, for typical running conditions, the system dead-time, $\tau_D$, is much longer than the pile-up time, $\tau_P$. The subsequent methodology derived to correct for the effects of pile-up are only valid if this condition is met.

Since high-precision half-life measurements via $\gamma$-ray counting are performed by measuring the activity within the characteristic photopeak of the decay of interest, a rate-dependent loss of these events would prohibit the ability to accurately determine the half-life. In order to perform high-precision half-life measurements using this method, it is thus critical that the effects of pile up be understood and corrected for. A thorough investigation of the effects arising from pile-up, and a detailed description of pile-up corrections was performed in Refs. [56,64]. A brief overview of the relevant methods will be discussed.

The pile-up correction methodology relies on the ability to flag detected events as either piled-up or not piled-up. Since the true energy information of piled-up events is lost, these events are rejected. However, analogous to the dead-time correction, a pile-up correction can be applied to statistically correct for this effect. Experimentally, the pile-up detection circuitry is built into the HPGe spectroscopy amplifiers. Events in which one or more $\gamma$-rays are detected within the pile-up time, $\tau_P$, following a trigger event are flagged as a piled-up event. Events that are piled-up can be categorized into two groups, post-piled-up or pre-piled-up, each of which is depicted in Figure 3.2. Post-pile-up, which is shown in the Figure 3.2a, refers to events in which the first $\gamma$-ray is the trigger event and is followed by one or more $\gamma$-rays within a time $\tau_P$. The probability of observing $n$ events in a time interval $\Delta t$ when the true event rate is $R$ is given by the Poisson distribution:

$$P(n, \Delta t) = \frac{(R\Delta t)^ne^{-R\Delta t}}{n!}. \quad (3.9)$$
The probability of observing no events within a time interval $\tau_p$ is thus:

$$P_{\text{post}} = P(0, \tau_p) = e^{-R\tau_p} \quad (3.10)$$

and thus the probability of observing at least one event in the same interval is:

$$P_{\text{post}} = P(> 0, \tau_p) = 1 - e^{-R\tau_p} \quad (3.11)$$

In order for pre-pile-up to occur, there must be at least three $\gamma$-rays involved. A schematic representation of pre-pile-up is given in Figure 3.2b. We consider the first event to be a trigger event if a second event occurs within a time $\tau_p < t < \tau_D$ relative to the first event, the energy of the first event will be unaffected. However, the second $\gamma$-ray will not trigger the system
since the event occurs within the dead-time window induced by the first \( \gamma \)-ray. If a third \( \gamma \)-ray is detected at a time \( \Delta t > \tau_D \) relative to the first event, the third event will trigger the detector system. If this third event also occurs within \( \tau_P \) of the second \( \gamma \)-ray event, the charge integration of the third event will be a combination of the charge of both the second and third \( \gamma \)-rays. In this scenario, the third \( \gamma \)-ray is pre-piled-up and, despite triggering the system, is recorded with an incorrect energy. It is thus flagged as piled-up and rejected in the analysis.

The probability that an event is pre-piled-up by a preceding \( \gamma \)-ray interacting in the detector is:

\[
P_{\text{pre}} = \int_{0}^{\tau_P} P(>0; \Delta t = \tau_P - t) dP
= \int_{0}^{\tau_P} (1 - e^{-R(\tau_P-t)}) Re^{Rt} dt
= 1 - e^{-R\tau_P(1 + R\tau_P)}.
\]  

(3.12)

Correspondingly, the probability that the \( \gamma \)-ray is not pre-piled-up is therefore:

\[
P_{\text{pre}} = 1 - P_{\text{pre}}
= e^{-R\tau_P(1 + R\tau_P)}.
\]  

(3.13)

(3.14)

A single \( \gamma \)-ray can be both pre- and post-piled-up, however, the information can only be lost once. Therefore, the probability that the \( \gamma \)-ray is piled-up is given by:

\[
P = P_{\text{pre}}P_{\text{post}} + P_{\text{pre}}P_{\text{post}} + P_{\text{pre}}P_{\text{post}}.
\]  

(3.15)
and the probability that the $\gamma$-ray is not piled up is:

$$N = P_{\text{pre}} P_{\text{post}}. \quad (3.16)$$

There are, however, several refinements to the above description that must be implemented in order to provide an accurate description of the detector system. Although it is essential that these refinements be implemented, they will only be discussed briefly here. A more thorough description and detailed derivation of each of the following refinements is given in Refs. [56, 64].

The first refinement involves replacing the constant dimensionless rate term, $x = R\tau_P$ used in the above description, for a time-dependent rate term in which the sample undergoes radioactive decay. The dimensionless detector rate for a radioactive source, with a decay constant $\lambda$, and a constant background rate, for example, would be written as $x(t) = A\tau_P e^{-\lambda t} + B\tau_P$, where $A$ and $B$ represent the rates at $t = 0$. Thus, the substitution $x = R\tau_P \rightarrow x(t) = A\tau_P e^{-\lambda t} + B\tau_P$ in the above equations is required. The second modification accounts for a finite time interval during which two interactions cannot be distinguished and thus will not be properly flagged in the pile-up circuitry but will nonetheless be piled-up. This is referred to as the pile-up time resolution, $\tau_r$, and for typical running conditions satisfies $\tau_r << \tau_P$. Another effect that must be considered is the existence of a non-zero trigger energy threshold. Therefore, $\gamma$-rays whose energy are below the CFD threshold may not trigger the system, but can still be piled-up with another $\gamma$-ray. Similarly, the energy of a $\gamma$-ray may exceed the CFD threshold, but may be piled-up by an event whose energy does not exceed the pile-up energy detection threshold, and will thus not be flagged as pile-up. Finally, the existence of high-energy cosmic rays that interact in the detector can saturate the pile-up circuitry and induce self pile-up. Cosmic ray pile-up is independent of the true detector pulse pile-up and can thus be included as an additional term with a constant rate.
as a function of time. Implementation of each of these refinements is discussed in detail in Ref. [64].

Figure 3.3: Sample pile-up probability curve from a single run. The dashed aqua line represents the cosmic-ray self pile-up and the blue dot-dashed line represents the pile-up from $\gamma$-ray multiplicity. Only the $\gamma$-ray multiplicity pile-up correction is applied to the $\gamma$-ray gated data.

Experimentally, the pile-up circuitry flags every event as piled-up or not piled-up. Therefore, the time distribution of events that trigger the pile-up circuitry, $P(t)$, and those that
do not trigger the pile-up circuitry, $N(t)$, are defined as:

\begin{align}
P(t) &= \frac{p_i}{n_i + p_i} \\
N(t) &= \frac{n_i}{n_i + p_i}
\end{align}

(3.17)

(3.18)

where $p_i$ is the number of events flagged as piled-up within the $i$th time bin and $n_i$ is the number of events within the time bin that were flagged as not piled-up. Accounting for all of the refinements discussed above, the expression of the pile-up probability becomes:

\begin{align}
x(t) &= a_1 e^{-(\ln 2/a_2)t} + a_3 \\
P(t) &= a_6 (1 - e^{-(2-a_4)x})[e^{a_4x} + a_5 (1 - a_4)x] + \frac{a_7}{x},
\end{align}

(3.19)

(3.20)

where the $a_i$ variables are free parameters in the fit to the data flagged as “piled-up” and correspond to the refinements mentioned above and discussed in great detail in Ref [64]. A sample fit to the pile-up data using the expression given in Equation 3.20 is shown in Figure 3.3.

Once the best-fit parameters are obtained, a correction to the data flagged as non-piled-up can be applied to statistically correct for the fraction of events in each time bin that were detected with incorrect energies due to the pile up. It should be noted, however, that since cosmic-ray self pile-up does not affect the $\gamma$-ray gated data, a subtraction of this term, given by the second term in Equation 3.20, must be subtracted before the correction is applied to the gated $\gamma$-ray data.
3.3 Half-Life Fitting Methodologies

In this thesis, three high-precision half-life measurements will be presented. Each of these measurements showed a number of common procedures that were used to analyze the data. An overview of these standard methodologies will be introduced below while further details corresponding to the individual analyses will be presented in subsequent chapters.

3.3.1 Cycle Selection

As described in Chapter 2, the experimental data are collected in cycles mode, where a cycle consists of implanting the beam onto a tape, measuring the decay activity in the detector, and then removing the sample into a tape disposal box in order to remove any long lived contaminants out of view of the detector. Typically, many cycles are taken, which is referred to as a “run”. Following the completion of a run, different experimental parameters are changed in order to study possible systematic effects resulting from the choice of the selected running conditions.

The quality of the data from each individual cycle is assessed before including the cycle in the final analysis. Cycles are rejected if the observed bin-by-bin counts in the decay curve exhibit a large statistical variation in any individual time bin, which could originate from a noise burst in the detector or a transient source of noise in the experimental hall. Generally, this effect is flagged by large residuals for individual time bins, which will be discussed at the end of this section. Moreover, in order to increase the signal-to-noise ratio, cycles in which the total number of observed counts is drastically reduced, \( \leq 50\% \) of the average number of counts within a cycle, are rejected. A sample run in which a single cycle is removed due to low counts is shown in Figure 3.4. A significant decrease in events typically occurs when
Figure 3.4: Total number of counts within a cycle for the individual cycles within a single run. The cycle with counts below the red dashed line was rejected due to low statistics.

there is an interruption in the delivery of the primary proton beam from the main cyclotron, resulting in a decrease in, or total loss of, the RIB intensity to the experimental station.

### 3.3.2 Correcting for Dead-Time and Pile-Up Losses

After assessing the quality of the data in each cycle, the aforementioned dead-time and pile-up corrections must be applied in order to extract an accurate half-life measurement. For the $\beta$ counting analyses, there are no loses due to pile-up since the energies of the detected $\beta$ particles are not recorded and the data only requires a dead-time correction. For an observed
number of counts in the $i$th time bin, $n_i \pm \sigma_i \ (\sigma = \sqrt{n})$, the result from Equation 3.2 is used to determine the true number of counts in the $i$th time bin $n_{0i} \pm \sigma_{0i}$ as:

$$n_{0i} = \frac{n_i}{1 - D_i},$$  \hspace{1cm} (3.21)

$$\sigma_{0i}^2 = \frac{\sigma_i^2}{(1 - D_i)^2}$$  \hspace{1cm} (3.22)

where the fractional dead-time within the $i$th time bin is $D_i = n_i \tau_D/t_{bin}$, where $t_{bin}$ is the duration of the time bin, also referred to as the dwell time.

For $\gamma$-ray counting experiments, both dead-time and pile-up corrections must be applied to the experimental data. The correction, however, is only applied to a subset of the non-piled-up data that lies within a specific $\gamma$-ray energy gate, $g_i$. Therefore, the dead-time and piled-up corrected number of counts within the $i$th time bin gated on the $\gamma$-ray photopeak of interest, $g_{0i} \pm \sigma_{0i}$, is defined as:

$$g_{0i} = \frac{g_i}{(1 - D_i)(1 - P(t))}$$  \hspace{1cm} (3.23)

$$\sigma_{0i}^2 = \frac{\sigma_i^2}{[(1 - D_i)(1 - P(t))]^2},$$  \hspace{1cm} (3.24)

where $D_i = (n_i + p_i)\tau_D/t_{bin}$. Unlike the dead-time fraction which is defined exactly on a bin-by-bin basis, the pile-up probability as a function of time, $P(t)$, is taken from a fit of the piled-up flagged data to the function given in Equation 3.20 with a subtraction of the cosmic ray self pile-up term. This procedure corrects the $\gamma$-ray gated data from pile-up losses while avoiding statistical fluctuations in the bin-by-bin probability of pile-up data given by Equation 3.17.
### 3.3.3 Maximum Likelihood Fitting

Following dead-time and pile-up corrections, the activity from each run is fit using a Poisson log-likelihood function in a Levenberg-Marquardt $\chi^2$-minimization procedure \[64\]. Initial values for the half-lives and intensities are provided for each nucleus, each of which can either be defined as a fixed parameter or as a free parameter in the fit. The method is iterative, with the parameters varied until convergence that minimizes the $\chi^2$. The definition of the $\chi^2$ that was used in the fitting procedure is appropriate for Poisson distributed data \[65\] and is given by:

$$\chi^2 = 2 \sum_{i=1}^{N} W_i \left( y_{fit} - y_i + y_i \ln \frac{y_i}{y_{fit}} \right), \quad (3.25)$$

where $y_i$ is the number of counts in the $i$th time bin, as defined from either Equation $3.21$ for $\beta$ counting analyses or Equation $3.23$ for $\gamma$-ray counting analyses, and $y_{fit}$ is the value of the fit function in the $i$th bin. The bin-by-bin weighting factor, $W_i$, is introduced to account for the increased variance that results from performing the dead-time and pile-up corrections. This weighting factor is a ratio of the variance for pure Poisson counting statistics, $y_i$ to the true variance $\sigma^2_{0i}$ in the dead-time and pile-up corrected data and is defined as:

$$W_i = \frac{y_i}{\sigma^2_{0i}}, \quad (3.26)$$

where the $\sigma^2_{0i}$ values are obtained from Equation 3.20 or 3.22 for $\beta$ and $\gamma$-ray counting experiments, respectively.

The quality of the fit to the data can be assessed from the $\chi^2$ per degree of freedom, $\nu$, of the fit \[66\]. Although it should be noted that for low-statistics Poisson distributed data, each time bin does not contribute exactly 1 degree of freedom, as described in detail in Ref \[64\]. The reduced-$\chi^2$, denoted $\chi^2/\nu$, nonetheless remains useful to quantify the dispersion of the
data relative to the fit. For an ideal fit, the $\chi^2/\nu$ would be equal to one, while a large $\chi^2/\nu$ would indicate that the function used to fit the data does not accurately represent the data points.

Another way to assess the quality of the data is to look at the residuals. The residuals are defined bin-by-bin as:

$$R_i = \frac{y_i - y_{\text{fit}}}{\sigma_i},$$  \hspace{1cm} (3.27)

where $y_{\text{fit}}$ is the integral of the fit function over the $i$th time bin, and $y_i$ and $\sigma_i$ are the dead-time and pile-up (if applicable) corrected counts and variance, respectively. Statistically, the residuals should be scattered about zero with approximately a Gaussian normal distribution in the limit of large numbers of counts per bin. An example of the residuals from a “good” cycle of data is shown in Figure 3.5. On the other hand, a cycle in which a spurious noise signal was detected has a large residual for that specific bin.

An example of a cycle with a large residual corresponding to a noise spike in the decay curve is shown in Figure 3.6, where the corresponding noisy data point is highlighted by the red circle in both the residuals plot and the decay curve. Calculating residuals for every time bin of every individual cycle and flagging events with large residuals ($\geq 6$) provides an effective tool to flag the small number of cycles with such noise bursts for further investigation and possible rejection from the final data sample.
Figure 3.5: Typical residual distribution for “good” cycle. The bin-by-bin residuals are scattered about $R = 0$. The pattern of “striping” at late times represent the different integer numbers of detected counts per time bin in the low count rate background region. A histogram of the residuals is also shown.
Figure 3.6: Example residual distribution for a cycle in which a spurious noise burst was detected in one of the time bins. The corresponding decay curve is shown on the lower panel. The data point in the residuals plot which prompted the “bad residuals” flag is highlighted in red and the corresponding data point in the decay spectrum is also highlighted in red. This cycle was rejected from the final analysis.
3.3.4 Summing Data

Previous investigations have shown that a bias is introduced in the half-life determination when a weighted average of many low statistic runs is used to calculate the half-life [67]. This bias originates from the fact that the individual runs have low statistics, and hence are Poisson distributed, but the weighted average is done assuming Gaussian statistics. This effect is highly dependent on the counting statistics per run; the larger the statistics of individual runs, the smaller the effect. To perform high-precision half-life measurements, the detector rates are typically high, and the bias introduced by averaging the individual runs is usually negligible. Nonetheless, it is both good practice and more accurate to sum the dead-time corrected data from the individual runs and perform a single fit to the summed dataset whenever possible.

3.3.5 Chop Plot Analysis

When performing high-precision half-life measurements, a thorough investigation of potential rate dependent effects should be investigated. One method to search for such rate dependent effects is referred to as a “chop plot” analysis. Chop plots are made by systematically removing the data from the leading, high-rate, channels and re-fitting the decay curve following the removal of these data. The half-life as a function of channels removed is plotted to investigate potential rate-dependent effects on the deduced half-life.

The presence of a rate dependence, that has not been properly account for, will lead to a systematic change in the deduced half-life as a function of channels removed. This effect is demonstrated in Figure 3.7. In this plot, the blue data points correspond to the observed data while the black points correspond to data where a dead-time correction has
been applied. Since dead-time is a rate dependent effect, it is the higher rate data points that are most affected. As more leading channels are removed, the effects of dead-time decrease, and the two results converge.

The absence of any rate dependence, on the other hand, leads to a constant distribution in the deduced half-life as a function of channels removed as shown by the distribution of the black data points in Figure 3.7. However, it should be noted that since each data point contains all of the data to the right of it, the data points are highly correlated. Therefore, these data are not statistically scattered about the mean half-life value, but tend to follow a slowly varying oscillatory trend.
Figure 3.7: Chop plots corresponding to data with (black) and without (blue) dead-time corrections applied. The deduced half-life from both datasets converges at later times since the effect of dead-time are much less significant at low rates. This example highlights the sensitivity of the chop plot analysis to rate dependent effects as well as the importance of applying dead-time corrections to the observed data.
Chapter 4

$^{10}\text{C}$ Half-Life Measurements

The history of $^{10}\text{C}$ half-life measurements is depicted in Figure 4.1. Although several of the earlier, and less precise, measurements are no longer included in modern evaluations [3], the half-life used in the current evaluated dataset, nonetheless, suffers from inconsistencies between the independent measurements. Most notably, a comparison between the two most recent and most precise measurements, $T_{1/2} = 19.310 \pm 0.004$ s [68] and $T_{1/2} = 19.282 \pm 0.011$ s [69], show very poor agreement, with a $\chi^2/\nu = 5.7$. In the most recent survey of the superallowed data [3], the four most precise measurement are used since it is common practice to remove measurements from the weighted average if they are more than a factor of 10 less precise than the most precise measurement [30]. The inconsistency between the four most precise $^{10}\text{C}$ half-life measurements leads to the adoption of a weighted average value of $T_{1/2}(^{10}\text{C}) = 19.3052 \pm 0.0071$ s [3], with a significantly inflated uncertainty, as shown by the solid bar in Figure 4.1. This inconsistency is further illustrated by the ideograph for the four most precise $^{10}\text{C}$ half-life measurements shown in Figure 4.1, which reveals two distinct peaks.
Figure 4.1: Previous $^{10}\text{C}$ half-life measurements as reported in Ba09 [69], Ia08 [68], Ba90 [70], Az74 [71], Ro74 [72], Ba63 [73], and Ea62 [74]. The solid curve is an ideograph representing the sum of normal distributions from the four most precise measurements [68–71], yielding the adopted world average [3] shown by the grey band.

Because of the low $Q$-value for the $^{10}\text{C}$ superallowed decay, and hence its high sensitivity to a potential scalar current contribution, the $\mathcal{F}t$ value of $^{10}\text{C}$ has a significant impact on the determination of Fierz interference term, $b_F$, from the superallowed data. The importance of obtaining an accurate measurement of the $^{10}\text{C}$ half-life is highlighted in Figures 4.2 and 4.3. In Figure 4.2, the Fierz interference term is calculated using $T_{1/2}(^{10}\text{C}) = 19.310 \pm 0.004$ s from Ref. [68], whereas in Figure 4.3, the Fierz interference term is calculated using $T_{1/2}(^{10}\text{C}) = 19.282 \pm 0.011$ s from Ref. [69], i.e. the two most recent, and most precise, measurements of the
$b_F = -0.0031 \pm 0.0025$

$\chi^2 / v = 0.43$

Figure 4.2: Plot of $1/Ft$ as a function of $\gamma \langle W^{-1} \rangle$ for the 14 most precisely measured $0^+ \rightarrow 0^+$ superallowed emitters. The Fierz interference term, $b_F$, is calculated using $T_{1/2}(^{10}C)=19.310 \pm 0.004$ s from Ref. [68]. All of the remaining world superallowed data is the same in the two cases. A shift in $b_F$ by $0.5\sigma$ is obtained when the two different $^{10}C$ half-life measurements are used to extract $b_F$.

$^{10}C$ half-life which correspond, approximately, to the two distinct peaks in the $^{10}C$ half-life ideograph of Figure 4.1. The limit on $b_F$ is otherwise calculated using the data compiled in the most recent survey of the world superallowed data [3], which included 222 measurements of comparable precision. The difference between these two $^{10}C$ half-life measurements would, by itself, shift the central value of the Fierz interference term determined from the entire world superallowed data set by more than half of its quoted uncertainty. An accurate, and
Figure 4.3: Plot of $1/Ft$ as a function of $\gamma \langle W^{-1} \rangle$ for the 14 most precisely measured $0^+ \rightarrow 0^+$ superallowed emitters. The Fierz interference term, $b_F$, is calculated using $T_{1/2}(^{10}\text{C})=19.282 \pm 0.011$ s from Ref. [69]. All of the remaining world superallowed data is the same in the two cases. A shift in $b_F$ by 0.5$\sigma$ is obtained when the two different $^{10}\text{C}$ half-life measurements are used to extract $b_F$.

precise, determination of the $^{10}\text{C}$ half-life is thus clearly critical to setting limits on $b_F$ from the superallowed data.

In this chapter, two new $^{10}\text{C}$ high-precision, and independent, half-life measurements are presented [75]. The first measurement was performed using the $\gamma$-ray photopeak counting technique. During the same beam time, a second, independent, half-life measurement was
performed via direct $\beta$ counting at a second experimental facility. These half-life measurements were performed in October 2013 at the TRIUMF-ISAC facility. A NiO target was used to produce the radioactive ions beams. A high-resolution mass separator was then used to select a singly ionized radioactive beam of either molecular $^{10}\text{C}^{16}\text{O}$ ($A = 26$) with an average yield of $1.5 \times 10^5$ s$^{-1}$ or atomic $^{10}\text{C}$ ($A = 10$) with an average yield of $2.5 \times 10^4$ s$^{-1}$, which were delivered to the experimental facilities at an energy of 20.4 keV. During this experimental running time, both the $\gamma$-counting and $\beta$ counting measurements were performed simultaneously. This was accomplished by first sending the beam to one experimental station. Following the implantation, the decay was measured for approximately 500 s ($\approx 25$ half-lives) in order to ensure a measurement of the background. During the decay period at one of the detector stations, the beam was sent to the second experimental station and implanted onto the tape at that experimental station. Switching the radioactive ion beam between the two experimental stations maximized the counting statistics collected during the allotted beam time. For the $\gamma$ counting experiment, only the $^{10}\text{C}^{16}\text{O}$ beam was delivered to the $8\pi$ spectrometer due to the higher beam intensity, whereas both the $^{10}\text{C}$ and $^{10}\text{C}^{16}\text{O}$ beams were delivered to the $4\pi$ proportional gas counter for the $\beta$-counting experiment.

4.1 The $^{10}\text{C}$ Half-Life via $\gamma$-ray Photopeak Counting

In this section, the $\gamma$-ray photopeak analysis will be presented. In order to perform the $^{10}\text{C}$ half-life measurement via $\gamma$-ray photopeak counting, the activity within the characteristic photopeak at 718 keV was measured. Although this $\gamma$-ray transition does not correspond to the direct feeding from the superallowed branch, as shown in Figure 4.4, nearly 100% of $^{10}\text{C}$ $\beta$ decays feed through the 718-keV energy level and this $\gamma$-ray therefore corresponds to the
Figure 4.4: Decay scheme of the $\beta$ decay of $^{10}$C to $^{10}$B.

highest statistics photopeak.

4.1.1 Run Settings

The data was collected in cycles mode, as described in Chapter 2. At the $8\pi$ spectrometer, cycles were comprised of 9 s of background counting, a “beam-on” period of 60 s during which a sample was accumulated in the tape, followed by a decay period of approximately 500 s ($\approx 25$ $^{10}$C half-lives) during which the decay activity was measured, with the radioactive beam deflected after the mass separator. Following the decay, the tape was moved from
Figure 4.5: A sample $^{10}$C half-life run at the $8\pi$ spectrometer which is used to depict the different components of the cycle. To begin, a background measurement of 9 s was taken. Following the background measurement, the beam was implanted on the tape for 60 s. The decay activity was then measured for 500 s. In order to obtain a good background measurement, the decay is measured for approximately 25 half-lives. This particular run is composed of 12 cycles and the spectrum shown here is the sum of the data from the 12 cycles.

the centre of the array into a box shielded by a lead wall in order to remove any potential long-lived contaminants. The cycle was then repeated. On average, 12 cycles with identical experimental running conditions were taken before the run was stopped. A sample run is shown in Figure 4.5. Following the end of a run, experimental parameters were varied and another run was started. In particular, the shaping times used in the spectroscopy amplifiers
were varied between 0.5, 1.0, and 2.0 $\mu$s, while the dead-time lengths per event were varied between 27.0 $\mu$s, 40 $\mu$s and variable (measured event by event). A summary of the settings for each run is given in Table 4.1.

Table 4.1: Summary of the run settings used at the $8\pi$ spectrometer.

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4.1.2 Half-life Analysis

Cycle Selection

In Table 4.1, the total number of cycles collected, as well as the number of “good” cycles within each run is shown. As discussed in Section 3.3, the cycles were individually assessed to ensure that the quality of the data was good.

Only fifteen out of a total of 577 cycles were removed from the analysis. Of these fifteen cycles, twelve were removed due to low statistics, which typically resulted from a loss of

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protons from TRIUMF’s main cyclotron. A single cycle was removed due to a large noise spike in the decay curve that resulted in large individual channel residuals. An additional cycle was removed due to an apparent data write issue since there was missing data across multiple bins in the time spectrum. Finally, a single cycle was removed due to a poor fit that resulted in an inexplicably short $^{10}\text{C}$ half-life, $T_{1/2} = 18.35(11)$ s, that differed by more than $8\sigma$ from the remainder of the statistically consistent data set. After these fifteen cycles were removed, a total of 562 good cycles remained in the $\gamma$-counting dataset.

The $\gamma$-ray energy spectra obtained from the sum of good runs with shaping times of 0.5, 1.0, and 2.0 $\mu$s are shown in Figures 4.6, 4.7, and 4.8, respectively. The corresponding $\gamma$-ray gate used for each shaping time is shown in the inset of each spectra. The reduced energy resolution resulting from shorter shaping times is evident from the photopeak width shown in the inset of Figure 4.6 in comparison to, for example, Figure 4.8, and results in an increase in the $\gamma$-ray photopeak gate width used in the analysis. Upon investigation of the $\gamma$-ray spectrum, no additional peaks other than those from the $^{10}\text{C}$ decay or known room background lines were observed. The lack of characteristic peaks from other sources provides a good indication that there was no significant $\gamma$-ray emitting contamination in the $^{10}\text{C}^{16}\text{O}$ molecular beam, and therefore, within the 718-keV photopeak, as will be discussed in more detail in Section 4.2.3.
Figure 4.6: Sum of the $\gamma$-ray energy spectra for all cycles with shaping times of 0.5 $\mu$s. The energy gate used in the analysis is shown in the inset.
Figure 4.7: Sum of the γ-ray energy spectra for all cycles with shaping times of 1.0 µs. The energy gate used in the analysis is shown in the inset.
Figure 4.8: Sum of the $\gamma$-ray energy spectra for all cycles with shaping times of 2.0 $\mu$s. The energy gate used in the analysis is shown in the inset.

The activity curves gated on the 718-keV photopeak were corrected for both dead-time and pile-up losses. Pile-up probability curves, corresponding to single runs, for each of the shaping times are shown in Figure 4.9, 4.10, and 4.11. An average pile-up correction, $P(t)$, was determined for each run by fitting these data to the function given in Equation 3.20 and then removing the cosmic-ray self pile-up component, as described in Section 3.2.
Figure 4.9: Pile-up probability curves from a single run with a shaping time of 0.5 $\mu$s.
Figure 4.10: Pile-up probability curves from a single run with a shaping time of 1.0 μs.
Figure 4.11: Pile-up probability curves from a single run with a shaping time of 2.0 µs.

The $^{10}$C half-life measurements determined from the fits to the dead-time and pile-up corrected data for each cycle, along with the corresponding $\chi^2/\nu$ values for each fit are shown in Figure 4.12. Cycle-by-cycle fits to the data were primarily performed as a method to assess the quality of the data within the individual cycles, and Figure 4.12 demonstrates that the entire data set is statistically consistent with a $\chi^2/\nu$ of 0.97 for the 562 half-life measurements from the individual cycles.
Figure 4.12: Cycle-by-cycle $^{10}$C half-life measurements from the $\gamma$-ray data gated on the 718 keV photopeak. The corresponding $\chi^2/\nu$ of each fit is given in the lower panel.

With the statistical consistency of the 562 individual cycles confirmed, the pile-up and dead-time corrected data from the individual cycles within each run were summed and a single fit for each run was performed. The deduced $^{10}$C half-life for each run is summarized in Table 4.2 and plotted in Figure 4.13. The dead-time and pile-up corrected data from each run were then summed and a single fit to the entire data set was performed. The corresponding fit to the summed data is shown in Figure 4.14 and yields a $^{10}$C half-life of $19.2969 \pm 0.0052$ s, where the uncertainty quoted is purely statistical.
Table 4.2: Half-lives obtained for $^{10}$C from each individual run at the 8π spectrometer. The fit function includes the $^{10}$C activity and a free background parameter.

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</table>
$T_{1/2}^{(10}{\text{C}}) = 19.2970 \pm 0.0052 \text{ s}$

$\chi^2/\nu = 1.19$

Figure 4.13: The run-by-run $^{10\text{C}}$ half-life determinations. The blue, magenta, and black data points correspond to applied dead times of 40 $\mu$s, 27 $\mu$s, and variable, respectively, while the circles, squares, and triangles correspond to shaping times of 0.5, 1.0, and 2.0 $\mu$s, respectively.

4.1.3 Contaminants

Although, in general, contamination from other $\beta$ decaying sources is greatly reduced in $\gamma$-ray photopeak counting due to the highly selective nature of the $\gamma$-ray gate, the possibility of time-dependent sources of contamination within the $\gamma$-ray gate of interest due to bremsstrahlung from the stopping of endpoint $\beta$ particles or the Compton scattering of higher energy $\gamma$-rays must be considered. During the $^{10\text{C}}$ half-life experiment with the $^{10\text{C}}^{16\text{O}}$
Figure 4.14: The summed pile-up and dead-time corrected data (black) and the best fit result (red) from the entire γ-ray counting experiment with the 8π spectrometer.

beam, contamination from radioactive $^{13}$N, in molecular form, was identified as an in-beam contaminant in the 4π gas counter. Further details will be discussed in Section 4.2.3, but here we note that since the β decay of $^{13}$N proceeds exclusively to the ground state of $^{13}$C, there is no contribution from Compton scattering of γ-rays from the $^{13}$N decay within the 718-keV photopeak gate. However, the possibility still remains that a time-dependent contribution from positron annihilation photons summing with bremsstrahlung from the stopping of the $\beta^+$ particles from $^{13}$N decay could contaminate the 718-keV photopeak gate. In order to quantify the possible contribution from this effects, GEANT4 simulations were performed.
Figure 4.15: Simulated energy spectra deposited in the $8\pi$ HPGe detectors from the decays of a) $^{10}$C and b) $^{13}$N. Although the absence of any $\gamma$-ray photopeaks in the $^{13}$N decay spectrum removes the possibility of contamination from Compton scattering of higher energy $\gamma$-rays, there remains the possibility of a time-dependent contribution from positron annihilation combined with bremsstrahlung under the 718-keV photopeak.
Simulations of both the $^{10}$C decay and the $^{13}$N decay in the $8\pi$ spectrometer were generated in order to determine the relative contributions of $^{13}$N to $^{10}$C within the 718 keV photopeak gates used in the analysis. In total, a sample consisting of 10 million decay events were simulated for each of $^{10}$C and $^{13}$N. The resulting energy spectra recorded in the $8\pi$ HPGe detectors are shown in Figure [4.15].

The Geant4 simulations were used to calculate the contribution of $^{13}$N decays within the 718-keV photopeak gate. As a conservative approach, the number of $^{13}$N decay events that deposit energy in an HPGe detector within the 710-725 keV window were used to set the upper limit, since this corresponds to the energy gate used for the subset of data recorded with the 0.5 $\mu$s shaping time and was the largest gate used. The corresponding energy region used to set the limits is shown in Figure [4.16] where only 1 in $10^5$ simulated $^{13}$N decays, or 0.06% relative to the $^{10}$C decays, deposits an energy within window.

The relative contribution of 0.06% obtained from these simulations assumes an equal number of $^{10}$C and $^{13}$N decays, since 10 million events were generated for each nucleus. The experimental beam rate of $^{10}$C was, however, much larger than the beam rate of $^{13}$N. The relative beam intensities of $^{13}$N and $^{10}$C was determined from the $\beta$ counting experiment discussed in Section 4.2. Since the absolute beam rates can vary significantly from run to run due to, for example, changes in the tune of the RIB or changes in the proton beam intensity from the cyclotron, the relative intensity of $^{13}$N to $^{10}$C was used. Although the observed relative intensity of $^{13}$N to $^{10}$C was more consistent, large fluctuations were still observed. For instance, the average relative $^{13}$N to $^{10}$C intensity was determined to be 0.35% from the $\beta$ counting experiment, while the maximum relative intensity was observed to be 0.72%. Thus, to be conservative, the maximum relative intensity of $^{13}$N to $^{10}$C (0.72%), as determined from the $\beta$ decay data, was used to set the limits on the $8\pi$ $\gamma$-ray data.
Figure 4.16: Number of counts in the 718-keV region of the γ-ray spectrum. Each of the $^{10}\text{C}$ (red) and $^{13}\text{N}$ (blue) spectra were generated with 10 million events in GeANT4. A relative contribution of $^{13}\text{N}$ to $^{10}\text{C}$ of 0.06% is obtained from this simulation in the 710 – 725 keV region.

The initial activity of $^{13}\text{N}$ relative to $^{10}\text{C}$, $\frac{A_N(t=0)}{A_C(t=0)}$, at the start the 8π counting period was calculated using the maximum relative beam intensity of $R_N/R_C = 0.72\%$ using:

$$\frac{A_N(t = 0)}{A_C(t = 0)} = \frac{R_N}{R_C} \left( e^{-\lambda_N t_{cool}} \right) \left( 1 - e^{-\lambda_N t_{on}} \right) / \left( e^{-\lambda_C t_{cool}} \right) \left( 1 - e^{-\lambda_C t_{on}} \right),$$

where $t_{on}$ is the length of time in which the beam was implanted onto the 8π tape system and $t_{cool}$ represents the time following the implantation before measuring the decay. At the 8π spectrometer $t_{cool} = 0$.

Since the initial activity of $^{10}\text{C}$ can be obtained from the fit to the summed data, an upper limit on the initial activity of $^{13}\text{N}$ can be determined. Considering the suppression of
Figure 4.17: Chop plot of all summed data taken with a shaping time of 0.5 µs.

$^{13}$N in the 718 keV gate by $10^{-5}$, the initial activity at the start of the decay, for the summed data, that fall within our energy window was determined to be 0.0030 c/s. Including $^{13}$N as a parameter in the fitting procedure, with its intensity fixed at this upper limit, results in a difference in the fitted $^{10}$C half-life of $<10^{-5}$ s, which is statistically insignificant. Therefore, no contaminant was included in the final analysis of the $8\pi$ γ-ray gated data.
Figure 4.18: Chop plot of all summed data taken with a shaping time of 1.0 $\mu$s.

### 4.1.4 Rate Dependent Effects

As described in Chapter 3, performing a chop plot analysis can help to identify possible sources of rate dependent contamination in the data set. A set of chop plots for each of the shaping times is given in Figure 4.17, 4.18, and 4.19. The chop plot for the entire summed dataset is given in Figure 4.20. Since each point in the chop plot comes from the same dataset, the data are highly correlated, and are thus not expected to be scattered about the mean. No evidence of any systematic rate dependence is apparent from these chop plots.
Figure 4.19: Chop plot of all summed data taken with a shaping time of 2.0 μs.
4.1.5 Systematics

In order to investigate any possible systematic effects arising from the chosen running conditions, the data were also grouped according to the different experimental running conditions. Following the completion of a run, the shaping time on the spectroscopic amplifier, as well as the applied dead-time, were varied. The systematic groupings of shaping times and dead-times, along with the $\chi^2/\nu$ of the individual groupings, are shown Figure 4.21.
Figure 4.21: Grouping of the $^{10}\text{C}$ half-life data from the $8\pi$ spectrometer according to the different experimental running conditions. The systematic grouping of the shaping times yields a $\chi^2/\nu = 2.01$ and is used to assign a systematic uncertainty of 0.0052 s to the half-life measurement.

Assigning a systematic uncertainty is done following the procedures of the Particle Data Group (PDG) \[1\]; if any of the independent systematic groupings have a $\chi^2/\nu > 1$, the statistical uncertainty is inflated. This inflation is done by multiplying the statistical uncertainty by the square root of the largest $\chi^2/\nu$. For the $\gamma$-ray counting experiment, the $\chi^2/\nu$ corresponding to the grouping of the different shaping times was the largest of the two groupings, and the statistical uncertainty of 0.0052 s was thus inflated by $\sqrt{2.01}$. The
resulting $^{10}\text{C}$ half-life obtained from the $\gamma$-ray photopeak counting experiment with the $8\pi$ detector was therefore:

$$T_{1/2}(^{10}\text{C}) = 19.2969 \pm 0.0052_{\text{stat.}} \pm 0.0052_{\text{sys.}} \text{ s}$$

$$= 19.2969 \pm 0.0074 \text{ s.} \quad (4.2)$$

4.2 The $^{10}\text{C}$ Half-Life via Direct $\beta$ Counting

The second $^{10}\text{C}$ half-life measurement was performed by direct $\beta$ counting. Data was taken with two different $4\pi$ proportional gas counters, which are colloquially referred to as the “old” and the “nes” detectors. The designs of these two gas counters are nearly identical; the only practical difference being the thickness of the anode wire. The different wire thickness leads to different plateau region for the two gas counters: the plateau region of the old counter was measured prior to the experiment to be between 2700-2800 V whereas the plateau region of the new counter was measured between 2400-2600 V as shown in Figures 2.15 and 2.16.

4.2.1 Dead-time Measurements

Before accurate half-life measurements can be extracted from the observed activity, the fixed dead-times applied following each event must be measured. Nominal dead-times of 3 $\mu$s and 4 $\mu$s are enforced by two gate and delay generators that are used with the two individual MCS modules, also colloquially referred to as the “old” and the “new” MCS. The actual dead-times were measured via the source and source-plus-pulser method \cite{62} which was described in Chapter 3. A 15 kBq $^{90}\text{Sr}$ source was used for all of the dead-time measurements.
Table 4.3: Summary of the dead-time runs taken with the $4\pi$ gas counters. A 15 kBq $^{90}$Sr source was used for all measurements.

<table>
<thead>
<tr>
<th>Run</th>
<th>Pulser Rate (kHz)</th>
<th>Bias Voltage (V)</th>
<th>Threshold (mV)</th>
<th>New MCS ($\mu$s)</th>
<th>Old MCS ($\mu$s)</th>
<th>Dead-time (3 $\mu$s)</th>
<th>Dead-time (4 $\mu$s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>917</td>
<td>5</td>
<td>2700</td>
<td>70</td>
<td>3</td>
<td>4</td>
<td>2.969(29)</td>
<td>3.975(33)</td>
</tr>
<tr>
<td>918</td>
<td>5</td>
<td>2700</td>
<td>95</td>
<td>3</td>
<td>4</td>
<td>3.000(17)</td>
<td>4.010(19)</td>
</tr>
<tr>
<td>919</td>
<td>5</td>
<td>2750</td>
<td>95</td>
<td>3</td>
<td>4</td>
<td>3.029(14)</td>
<td>4.037(15)</td>
</tr>
<tr>
<td>1083/1085</td>
<td>10</td>
<td>2400</td>
<td>95</td>
<td>4</td>
<td>3</td>
<td>2.988(16)</td>
<td>3.996(16)</td>
</tr>
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<td>10</td>
<td>2450</td>
<td>70</td>
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<td>3</td>
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<td>2500</td>
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<td>3.014(19)</td>
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</table>

The running conditions for each of the dead-time measurements taken before and after the $^{10}$C half-life experiment, as well as the deduced dead-times for each run, are summarized in Table 5.1.

The dead-times calculated from the individual runs are plotted in Figure 4.22. To investigate any systematic biases, the dead-time measurements were grouped according to the different running conditions. A plot of the systematic groupings of the dead-time measurements is given in Figure 4.23. The bias voltage systematics yielded the largest $\chi^2/\nu$ and therefore the statistical uncertainty was inflated by the square root of the corresponding $\chi^2/\nu$. The deduced dead-times and uncertainties, which include both the systematic and statistical uncertainties, were thus measured to be:

$$\tau(3 \, \mu s) = 3.0039 \pm 0.0079 \, \mu s$$  \hspace{1cm} (4.3)

$$\tau(4 \, \mu s) = 4.0127 \pm 0.0080 \, \mu s.$$  \hspace{1cm} (4.4)
Figure 4.22: Run by run dead time measurements with each gate and delay generator. The black (blue) data points represent data taken with the old (new) MCS module.
Figure 4.23: Summary of systematics investigated for the dead-time measurements. The data points corresponding to the threshold, MCS module, and pulser rate are identical as no other combination of these systematics were done. All dead-time measurements were taken with a 15 kBq source.
4.2.2 Run Settings

The individual run settings for the $^{10}$C half-life measurements with the $4\pi$ gas counters are summarized in Tables 4.4, 4.5 and 4.6 where each of the different tables corresponds to a different dwell time used during the experiment.

Table 4.4: Summary of run settings for runs taken with a dwell time of 1.0 seconds. All of the data in this table were taken with the old gas counter.

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<th>Bias Voltage (V)</th>
<th>Threshold Voltage (mV)</th>
<th>New MCS (µs)</th>
<th>Old MCS (µs)</th>
<th>Beam Type</th>
<th>Beam-On Time (s)</th>
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Table 4.5: Summary of run settings for runs taken with a dwell time of 1.2 seconds. All of the data in this table were taken with the $^{10}\text{C}^{16}\text{O}$ beam.

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Table 4.6: Summary of run settings for runs taken with a dwell time of 1.4 seconds. All of the data in this table were taken with the $^{10}\text{C}^{16}\text{O}$ beam and with the old gas counter.

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<th>Old MCS (µs)</th>
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4.2.3 Contaminants

During the experimental running time, a long-lived contaminant was identified in the $A = 26$ beam. In order to identify it, a run optimized to measure it’s half-life was performed. In this run, the beam was implanted for 30 minutes in order to build up a sample of the long-lived contaminant. Following this implantation, the beam was allowed to “cool” for 5 minutes in order to ensure that the $^{10}\text{C}$ activity had decayed to a negligible level. The sample was then moved into the $4\pi$ counter and the decay was measured for 950 seconds.
Figure 4.24: Fit to the data from the run used to identify the long-lived contaminant in the $A = 26$ beam. With a measured half-life of $T_{1/2} = 9.97 \pm 0.08$ mins., the contaminant was identified as $^{13}$N.

The dead-time corrected data from this run and the corresponding fit is shown in Figure 4.24. The measured half-life of the contaminant was determined to be $T_{1/2} = 9.97 \pm 0.08$ minutes. This half-life is consistent with $^{13}$N whose literature half-life is $9.9670 \pm 0.0037$ minutes [76]. Although $^{13}$N would not, by itself, be delivered to the experimental stations as part of the $A = 26$ beam, it could be present in the $^{10}$C$^{16}$O beam in a molecular form such as HC$^{13}$N, $^{13}$C$^{13}$N, or $^{13}$N$_2$. The mass differences between $^{10}$C$^{16}$O and each are given in
Table 4.7: Mass difference of potential species in the \(A=26\) molecular beam. The mass difference quoted is taken with respect to the mass of \(^{10}\text{C}^{16}\text{O}\). See text for details.

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<th>Mass Difference (MeV)</th>
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<td>(\text{HC}^{13}\text{N})</td>
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<td>(^{26}\text{mAl})</td>
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Table 4.7 The mass difference of each possible contaminant falls within the resolving power of the mass separator, whose resolution for this experiment was \(\delta_m/m \approx 1/1000\).

Although there was no evidence for additional sources of contamination in the \(A = 26\) molecular beam from the NiO production target used in this experiment, a few additional contaminants that have been observed in past \(A = 26\) RIB experiments at TRIUMF-ISAC \(^{77}\)\(^{79}\) produced from SiC production targets were considered. First, contributions from a contaminant of surface ionized \(^{26}\text{Na}\) were investigated. Since nearly all of the \(^{26}\text{Na}\) \(\beta\) decays (99\%) are followed by a 1809-keV \(\gamma\)-ray \(^{77}\), its presence should be identifiable by the presence of this \(\gamma\)-ray photopeak in the \(8\pi\) \(\gamma\)-ray data. Since no such peak was observed in the full \(\gamma\)-ray singles energy spectrum, the data were further constrained to optimize the potential signal to background of this peak. First, due to its short half life (\(T_{1/2} = 1.07128(25)\) s \(^{77}\)), only the data collected during the first 7.2 s (first 6 bins at 1.2 s/bin) of the implantation period were used. At this point, the \(^{26}\text{Na}\) was fully saturated while the contribution from the primary \(^{10}\text{C}\) activity (\(T_{1/2} \sim 19\) s) was reduced. Additionally, only the \(\gamma\)-\(\beta\) coincidence data within this time was used to further reduce background contribution.
Figure 4.25: Gamma-ray spectrum for $\gamma$-$\beta$ coincidences during the first 7.2 s of implantation. In the inset, a zoom in of the window around 1809 keV is shown.

The corresponding $\gamma$-$\beta$ coincidence energy spectrum recorded with the $8\pi$ spectrometer during the first 7.2 s of implantation for the entire dataset collected during the experimental running time is shown in Figure 4.25.

In the inset of Figure 4.25, the energy range corresponding to the 1809 keV photopeak from $^{26}$Na decay is shown. Within this energy range, only two counts were observed from the entire data collected, which strongly suggests that no $^{26}$Na was present in the beam. For completeness, however, these two counts were used to set a limit on the activity of $^{26}$Na.
and its effect on the $^{10}\text{C}$ half-life was determined. First, the observed two counts need to be corrected for the efficiency of the ZDS, which was measured to be $\sim 10\%$, and the photopeak efficiency of the $8\pi$ spectrometer, which is $\sim 0.7\%$ at 1.8 MeV. The beam rate of $^{26}\text{Na}$ was then calculated using Equation 4.5, which set an upper limit of $\leq 0.9$ ion/s in the beam.

\[
N_i = \int_0^t R_i (1 - e^{-\lambda_i t}) dt
\]

\[
N_i = \frac{R_i}{\lambda_i} (\lambda_i t - e^{-\lambda_i t} - 1)
\]  

The data from the $\beta$ counting experiment was subsequently refit with the inclusion of $^{26}\text{Na}$ using this deduced upper limit. The resulting change in the $^{10}\text{C}$ half-life when the $^{26}\text{Na}$ was included in the fit is given in Table 4.8. No statistically significant change in the half-life was observed.

Table 4.8: Effect on the half-life due to potential contaminants. The absolute difference is taken with the $^{10}\text{C}$ half-life when no additional contaminant other than the $^{13}\text{N}$ is included in the fitting procedure. No reduction in the upper limit on the $^{26m}\text{Al}$ contamination due to the different magnet settings for $^{26m}\text{Al}$ and $^{10}\text{C}^{16}\text{O}$ is considered for the numbers given below.

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<th>Absolute Difference (s)</th>
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</tbody>
</table>

Finally, potential contamination from $^{26m}\text{Al}$ which, again, has been previously observed in other $A = 26$ RIB experiments at TRIUMF produced from SiC production targets was investigated. The $\beta$ decay of $^{26m}\text{Al}$ proceeds exclusively to the ground state of $^{26}\text{Mg}$[63],

[63]
and therefore an upper limit cannot be obtained from the $\gamma$-ray data. Since no $^{26m}$Al could be observed in the $^{10}$C $\beta$ activity data, the magnet settings were changed to the optimal values for the mass of $^{26m}$Al in order to maximize the delivery of any $^{26m}$Al that was being extracted from the NiO target.

First, the re-tuned beam was sent to the $8\pi$ spectrometer to identify any new isotopes present with the new magnet settings. The $\gamma$-ray energy spectrum for this run is given in Figure 4.26 and reveals an additional $\gamma$-ray photopeak at 613 keV that was not observed in the data taken with the original magnet settings optimized for the mass of $^{10}$C$^{16}$O. This photopeak was identified to originate from the decay of $^{78}$Br, which could be present in the beam as a $3^+$ charge state, as its charge-to-mass ratio is 26, and would result from interactions of the primary proton beam with the Ta metal of the target holder.

The fit to the decay curve for the $^{26m}$Al run taken with the $4\pi$ proportional gas counter therefore included contributions from $^{26m}$Al ($T_{1/2} = 6346.02 \pm 0.54$ ms [3]), $^{78}$Br ($T_{1/2} = 387.0 \pm 2.4$ s [80]), and $^{10}$C. The $^{26m}$Al run taken with the gas counter was composed of 10 cycles, and the decay curve and corresponding fit for this run is shown in Figure 4.27. An initial $^{26m}$Al intensity of $A(t = 0) = 46(21)$ c/s was deduced from the fit to the data taken from this run. Using this value, the beam rate of $^{26m}$Al was calculated using:

$$R = \frac{A(t = 0)e^{\lambda_{cool}}}{(1 - e^{-\lambda_{on}})}$$

(4.6)

to be $R(^{26m}$Al$) = 6.4(29)$ ions/s. To be conservative, this value was used as the upper limit of the $^{26m}$Al beam rate in studying its potential effect on the $^{10}$C half-life measurements.
Figure 4.26: Gamma-ray spectrum for $\gamma$-$\beta$ coincidences corresponding to the data taken with the magnet settings optimized to the mass of $^{26m}$Al. A zoom in on the energy range between 400 – 800 keV is given in the inset and reveals the three photopeaks that are observed in this spectrum at 511 keV, 613 keV, and 718 keV. The new peak at 613 keV that arises at these magnet settings is consistent with the energy of a $\gamma$-ray which originates from the $\beta$ decay of $^{78}$Br. This species could be present at these settings as a $3^+$ charge state as its charge to mass ratio is 26.
Figure 4.27: Best fit results for the GPS run in which the magnet settings were optimized for $^{26m}$Al.

It should be noted that this upper limit was obtained using data in which the beam was re-tuned to the mass of $^{26m}$Al, and thus likely represents a significant overestimate of the true $^{26m}$Al activity in the data used in the $^{10}$C half-life analysis. This is demonstrated by the fact that the deduced intensity of $^{10}$C in the fit to these data is reduced by three orders of magnitude at the new magnet settings optimized from the $^{26m}$Al compared to the original settings optimized for $^{10}$C$^{16}$O. Nonetheless, to be conservative, no suppression factor is applied to the $^{26m}$Al beam rate calculated from the data with the $^{26m}$Al magnet settings.
For each cycle in the $^{10}$C half-life analysis, the activity of $^{26m}$Al at the start of the decay was calculated using the corresponding beam-on and cool times for each run from Equation 4.6. The contribution from $^{26m}$Al was then fixed at this upper limit in the fit to the summed data. The effect on the half-life is given in Table 4.8. A small change in the half-life was observed. This difference, however, if added in quadrature with the statistical uncertainty, is statistically insignificant. Moreover, reducing the contribution of $^{26m}$Al by a factor of 10 due to the different magnet settings during the $^{10}$C$^{16}$O data collection, which is significantly less than the suppression of $^{10}$C which was observed to be 3 orders of magnitude between the magnet settings results in no change in the $^{10}$C half-life was obtained to the number of decimal places quoted. Therefore, no further contribution from $^{26m}$Al was considered in the final analysis.

It should be noted that, although past experiments have observed significant contamination from $^{26}$Na and $^{26m}$Al in the $^{A}=26$ RIBs, these experiments were performed using a SiC production target to extract the $^{A}=26$ RIBs. In this experiment, a NiO target was used to extract the $^{A}=26$ beam and, therefore, it is not entirely surprising that these contaminants did not appear to be present in the RIB in any significant quantities.

Contribution from $^{78}$Br, which was identified in the $\gamma$–ray spectrum when the magnet settings were optimized to the mass of $^{26m}$Al was also considered. A beam rate of $R(^{78}$Br$)=186(70)$ c/s, deduced from the Figure 4.27, was used to calculate the relative activity at the start of the $\beta$ decay counting period for the $^{10}$C$^{16}$O experimental data and was included as a fixed parameter in the fitting procedure. Inclusion of a $^{78}$Br contaminant in the fitting procedure led to a small shift in the deduced $^{10}$C half-life, however, led to negative initial intensities of $^{13}$N. This is not only, in general, unphysical, but is also known to be incorrect since the $^{13}$N contaminant was already measured in the $A=26$ beam. Moreover, the
absence of the 613 keV photopeak in the $\gamma$-ray spectrum during the experiment (i.e. the magnet settings tuned to $^{10}\text{C}^{16}\text{O}$) suggests that there is no contamination from $^{78}\text{Br}$. Again, reducing the contribution of $^{78}\text{Br}$ by a factor of 10 due to the different magnet settings during the $^{10}\text{C}^{16}\text{O}$ data collection, which is significantly modest in comparison to the suppression of $^{10}\text{C}$ which was observed to be 3 orders of magnitude between the magnet settings results in no change in the $^{10}\text{C}$ half-life at the level of precision quoted for this result. The contribution from $^{78}\text{Br}$ was thus not considered in the final analysis.

The remaining analysis was thus performed considering a single contaminant of molecular $^{13}\text{N}$ in the $^{10}\text{C}^{16}\text{O}$ beam.

### 4.2.4 Half-life Analysis

The data obtained from the MCS modules were binned in 500, 450, 400 bins per cycle with each bin being 1.0 s, 1.2 s, or 1.4 s in duration, respectively, resulting in measurements of the decay times between 500-560 seconds. A summary of the run settings is given in Tables 4.4, 4.5, and 4.6. The decay activity was fit to a function with a primary decay of $(^{10}\text{C})$, a contaminant $(^{13}\text{N})$, and a constant background. The half-life of $^{13}\text{N}$ was fixed to its literature value of 9.9670(37) mins. $^{76}$ and the initial intensities of $^{10}\text{C}$ and $^{13}\text{N}$, as well as the half-life of $^{10}\text{C}$, were left as free parameters.
Figure 4.28: The deduced cycle-by-cycle $^{10}$C half-lives and the corresponding $\chi^2/\nu$ of the fit to the data from each individual cycles.

In total, 647 individual cycles were recorded with the gas counter with a total of 598 cycles retained in the final analysis, corresponding to 94% of the total data acquired during the experiment. Cycles with a significant drop in counts ($\leq$ 50% counts compared to the average counts per cycle within the same run) were removed from the analysis as these likely corresponded to a loss of beam during the cycle. Such reductions in the count rate resulted in the rejection of 17 cycles. The remaining 630 cycles were individually fit. Of these cycles, an additional 32 cycles were removed from the analysis due to large residuals.
Figure 4.29: The run-by-run $^{10}$C half-life measurements. The open (solid) circle data points represent runs taken with a $^{10}$C ($^{10}$C$^{16}$O) beam and the black (blue) data points represent the data taken with old (new) gas counter.

(between 6 − 20σ), corresponding to noise spikes in either the decay or background region, as depicted in Figure 3.6. The cycle-by-cycle $^{10}$C half-life measurements, as well as the $\chi^2/\nu$ of each fit, for the remaining 598 good cycles are shown in Figure 4.28.

Following the assessment of the individual cycles, the dead-time corrected cycles from each run were summed and fits to the individual runs were performed. The corresponding run-by-run $^{10}$C half-life measurements are shown in Figure 4.29 and listed Table 4.9.
Table 4.9: Half-life obtained for $^{10}$C for the individual runs. The fit functions includes a $^{10}$C component, a $^{13}$N contaminant, and a free background component.

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<th>$T_{1/2}$ (s)</th>
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The dead-time corrected data from each run with common dwell times were then summed in order to perform a single fit for each dwell time. For the 1.0 second dwell time data, the $^{10}\text{C}$ and $^{10}\text{C}^{16}\text{O}$ data were summed separately due to the fact that there is no $^{13}\text{N}$ contaminant.
present in the subset of the data taken with the $^{10}$C atomic beam. The corresponding fit for each dwell time is shown in Figures 4.30, 4.31, 4.32, and 4.33. The final result is taken to be the weighted average of these four measurements. A plot of the individual measurements and the resulting weighted average is shown in Figure 4.34.

Figure 4.31: The summed dead-time corrected data (black) and the best fit (red) from the subset of data taken with the $^{10}$C$^{16}$O molecular beam and dwell time of 1.0 s/bin and measured with the $4\pi$ gas counter. The bin by bin residuals are shown in the inset.
Figure 4.32: The summed dead-time corrected data (black) and the best fit (red) from the subset of data taken with the $^{10}$C$^{16}$O molecular beam and dwell time of 1.2 s/bin and measured with the $4\pi$ gas counter. The bin by bin residuals are shown in the inset.

\[ T_{1/2} = 19.3022(30) \text{ s} \]
\[ \chi^2/\nu = 1.11 \]
\[ \mu = 0.004; \sigma = 1.05 \]
Figure 4.33: The summed dead-time corrected data (black) and the best fit (red) from the subset of data taken with the $^{10}$C$^{16}$O molecular beam and dwell time of 1.4 s/bin and measured with the $4\pi$ gas counter. The bin by bin residuals are shown in the inset.

$T_{1/2} = 19.3033(39) \text{ s}$

$\chi^2/\nu = 1.05$
Figure 4.34: Deduced $^{10}$C half-lives from the fits to the summed data for the different dwell times and beams. The red solid and dashed lines represent the weighted average and the $\pm 1\sigma$ uncertainty, respectively.

4.2.5 Rate Dependent Effects

A set of chop plots for each of the summed datasets were generated and are shown below in Figures 4.35, 4.36, 4.37 and 4.38. None of these chop plots exhibit any statistically significant rate dependent behaviour.
Figure 4.35: Chop plots for the subset of data taken with the $^{10}$C atomic beam with the $4\pi$ gas counter. Chop plots are extended to approximately 3.5 times the $^{10}$C half-life.
Figure 4.36: Chop plots for the subset of data taken with the $^{10}$C$^{16}$O molecular beam and dwell time of 1.0 s/bin and measured with the $4\pi$ gas counter. Chop plots are extended to approximately 3.5 times the $^{10}$C half-life.
Figure 4.37: Chop plots for the subset of data taken with the $^{10}$C$^{16}$O molecular beam and dwell time of 1.2 s/bin and measured with the $4\pi$ gas counter. Chop plots are extended to approximately 3.5 times the $^{10}$C half-life.
A possible source of rate dependence occurs if the initial rate in the gas counter is too high. While maximizing the rate in the gas counter offers the advantage of increased statistics, the performance of the gas counter becomes compromised at higher rates due to the onset of space-charge effects [55]. The initial rate in the gas counter was therefore typically kept at, or below, 10 kHz. The initial rate in the gas counter for some of the cycles, however, reached as high as 15 kHz. In order to investigate the effect of the initial rate on the deduced $^{10}\text{C}$ half-life, the highest rate data were systematically removed and the data were refit. This was
done by imposing an upper limit on the rate and removing any bins which exceed this rate. Beginning with an initial upper limit on the rate of 5 kHz (which includes at least a portion of the data from all of the cycles), the upper limit was increased in 500 Hz increments and the half-life was recalculated. The results of this analysis are shown below in Figure 4.39.

![Figure 4.39: A plot of the $^{10}$C half-life for varying upper limits on the initial rate. For increased statistics, only leading channels whose rates are above the upper limit, rather than the entire cycle, are removed from the half-life determination. Each data point in this plot thus contains all of the data to the left of it.](image)

To further investigate any possible rate dependence in the data, the $^{10}$C half-life was plotted as a function of the initial rate. A weighted linear regression was performed and
a slope of $58(83) \times 10^{-8}\text{ s}^2$ was obtained. This slope is consistent with zero and implies that there is no dependence of the measured $^{10}\text{C}$ half-life on the initial rates used in this experiment. These results are shown in Figure 4.40.

Figure 4.40: A plot of the $^{10}\text{C}$ half-life versus the initial $^{10}\text{C}$ activity. A slope of $58(83) \times 10^{-8}\text{ s}^2$ was obtained from a weighted linear regression meaning that no rate dependence of the half-life is observed. Due to the large number of cycles, the data were grouped in initial rate bins of 500 Hz.

4.2.6 Systematics

The systematic grouping of the data according to the different experimental running conditions is shown in Figure 4.41. In this experiment, a wide variety of experimental parameters were varied between runs.
The different parameters include: the bias voltage applied to the gas counter, the discriminating threshold voltage, the different dwell times, the different radioactive beams ($^{10}$C and $^{10}$C$^{16}$O), the use of two different gas counters, as well as the varied initial rate in the detector. It should be noted that the large range of bias voltages used in this analysis originates from the fact that two different gas counters, whose plateau regions differ, were used in the analysis. The plateau region of the new gas counter was measured to be in the range of 2400 – 2600 V while the plateau region of the old gas counter was measured to be between 148
2700 – 2800 V.

Additionally, the data from each cycle was dead-time corrected using the measured values given in Equation 4.4 and in order to determine the effect of the dead-time of the deduced $^{10}\text{C}$ half-life, the dead-times were varied by $\pm 1\sigma$ from their central values and the data was refit with the new dead-time correction. Moreover, since the $^{13}\text{N}$ half-life that was used in the fitting procedure was fixed to the central value obtained from literature [76], the data was also re-fit by changing the $^{13}\text{N}$ half-life by $\pm 1\sigma$. The corresponding change in the $^{10}\text{C}$ half-life upon varying these parameters within their $\pm 1\sigma$ ranges are shown in Figure 4.42. No effect on the $^{10}\text{C}$ half-life was observed due to the uncertainty in the literature value of the $^{13}\text{N}$ half-life [76]. However a small change in the $^{10}\text{C}$ half-life was obtained when the dead-times were varied by $\pm 1\sigma$. Taking half of the difference between these two values gives an uncertainty contribution of $\pm 0.0004$ s resulting from the uncertainty in the measured dead-times. Adding this systematic uncertainty in quadrature with the statistical uncertainty yields no change in the quoted uncertainty.

Since none of the groupings in Figure 4.41 resulted in a $\chi^2/\nu$ greater than one, no systematic uncertainty originating from the different running conditions was applied to the data. The run-by-run distribution, however, yielded a $\chi^2/\nu = 1.02$. Inflating the statistical uncertainty by $\sqrt{1.02}$ yields no change in the quoted uncertainty. Therefore, the deduced $^{10}\text{C}$ half-life obtained from the weighted average of the summed data given in Figure 4.34 yields the final result from the $\beta$ decay counting experiment of:

$$T_{1/2}({^{10}\text{C}}) = 19.3009(17) \text{ s}, \quad (4.7)$$

where the systematic uncertainties discussed above have been included in quadrature but are ultimately negligible compared to the statistical uncertainty. With a precision of $\pm 0.009\%$,
this result represents the single more precise superallowed half-life measurement reported to date and the first to achieve a precision better than $10^{-4}$.

Figure 4.42: Fitted $^{10}$C half-life obtained when varying the $^{13}$N half-life and dead-time values within their $\pm 1\sigma$ values.

4.3 Updated Scalar Current Limit

An updated ideograph of the $^{10}$C half-life measurements including the $\beta$ and $\gamma$-ray counting measurements reported here is shown in Figure 4.43. The improved agreement between the $^{10}$C half-life measurements is qualitatively demonstrated in Figure 4.43 by the single
peaked nature of the ideograph and represents a significant improvement in the confidence that should be placed in this important half-life compared to the prior situation depicted in Figure 4.1. Quantitatively, the 6 most precise $^{10}$C half-life measurements shown in Figure 4.43 give $\chi^2/\nu = 1.90$ in comparison to $\chi^2/\nu = 2.65$ prior to the inclusion of the measurements reported in this thesis. Following the procedures of Ref. [30] for the scaling of the uncertainty results in a new world-average half-life for $^{10}$C of $T_{1/2} = 19.3015 \pm 0.0025$ s, a factor of three improvement in precision compared to the previously adopted world average [3].

Including this updated $^{10}$C half-life, as well as recent improvements in the $^{14}$O superallowed $Q_{EC}$ value [81] and branching ratio [82], the reciprocal of the corrected $\mathcal{F}t$-values for the 14 precisely determined superallowed decays [3] are plotted versus $\gamma\langle W^{-1} \rangle$ in Figure 4.44. The slope of this plot yields $b_F = -0.0018 \pm 0.0021$ for the Fierz interference term, which includes a small systematic uncertainty contribution from the transition-dependent radiative corrections [3]. This new result remains fully consistent with the absence of weak scalar currents, yielding $C_S/C_V = +0.0009 \pm 0.0011$ for the ratio of weak scalar to vector couplings under the assumption of left-handed neutrinos. The reduction in the uncertainty on $b_F$ from the value of $\pm 0.0026$ that has been reported in all surveys of the superallowed data for the past decade [3,30] results primarily from the recent reduction in the $^{14}$O superallowed $\mathcal{F}t$-value uncertainty [81,82], while the high-precision $^{10}$C half-life measurements reported here removes the possibility of a significant ($\sim 0.5\sigma$) shift in the central value of $b_F$ raised by the inconsistencies in the previous $^{10}$C half-life measurements shown in Figure 4.1.
Further improvements in the limit placed on $b_F$ from the superallowed $\beta$-decay data will require either an improved branching ratio measurement for $^{10}\text{C}$ or further improvements in the nuclear structure dependent theoretical corrections [3,15] which currently dominate the uncertainties in the $Ft$ values for all of the other 9 superallowed decays with experimental $ft$ values determined to better than $\pm 0.15\%$. 

Figure 4.43: Individual $^{10}\text{C}$ half-life measurements [68–74], including the two measurements reported in this thesis which have been published in Ref. [75]. The black curve is the sum of normal curves from the six most precise measurements, with the shaded band representing the new $^{10}\text{C}$ world average half-life of $19.3015 \pm 0.0025 \text{ s}$ calculated from these six measurements.
\( b_F = -0.0018 \pm 0.0021 \)

\( \mathcal{F}_0 = 3070.8 \pm 1.8 \text{ s} \)

\( \chi^2/\nu = 0.44 \)

Figure 4.44: Plot of \((\mathcal{F}t)^{-1}\) as a function of \(\gamma\langle W^{-1} \rangle\) for the 14 most precisely measured \(0^+ \rightarrow 0^+\) superallowed emitters. The slope yields an updated value of \(b_F = -0.0018 \pm 0.0021\) for the Fierz interference term. The red solid and dashed lines represent the slope and ±1σ uncertainty, respectively.
Chapter 5

$^{22}\text{Mg}$ Half-life Measurement via $\beta$ Counting

In the most recent survey of the world superallowed data [3], the constancy of the 14 transitions with $\mathcal{F}t$ values measured to a precision of 0.3% or better was used to confirm the CVC hypothesis at the level of $10^{-4}$. This constancy, and the resulting world-average superallowed $\mathcal{F}t$ value used to determine the weak vector coupling constant, $G_V$ and the up-down element of the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix, $V_{ud}$, are, however, dependent on the theoretical approach used to calculate the isospin symmetry breaking corrections, $\delta_C$. Due to the strong model dependence of the $\delta_C$ corrections, many different theoretical approaches have been investigated [15–27]. A subset of $\delta_C$ corrections calculated from different theoretical approaches was shown in Figure 1.4 and illustrates the large model dependence between the different theoretical approaches. The large model dependent variations in the $\delta_C$ corrections are problematic since they directly affect the Standard Model tests of the CVC hypothesis, the determination of $V_{ud}$, and the test of
CKM unitarity.

As discussed in Chapter 1, the model dependence of the superallowed $\mathcal{F}t$-values arising from the different theoretical approaches to the $\delta_C$ corrections was historically taken into account by assigning a systematic uncertainty to the world average $\mathcal{F}t$ value. In the most recent evaluation of the world superallowed data \cite{3}, however, only the SM-WS $\delta_{C2}$ corrections were used to calculate the $\mathcal{F}t$ values and no model dependent systematic uncertainty arising from the $\delta_C$ corrections was assigned to the $\mathcal{F}t$ value. This choice was motivated by the better agreement with the CVC hypothesis obtained with the SM-WS $\delta_{C2}$ corrections as well as a recent measurement of the $^{38}\text{Ca}$ superallowed branching ratio \cite{33} which resulted in a better agreement between the experimentally determined ratio of $ft$ values for the “mirror” superallowed transitions $^{38}\text{Ca} \rightarrow ^{38m}\text{K}$ and $^{38m}\text{K} \rightarrow ^{38}\text{Ar}$ with the calculated $ft$ ratio using the SM-WS approach, as shown in Figure 1.9.

As shown in Figure 1.8, however, if only the 9 most precisely measured superallowed transitions, with $\mathcal{F}t$ values determined to 0.15% or better, are retained one obtains $\mathcal{F}t_{WS} = 3072.20(62)_{stat}(36)\delta'_{R}$ with $\chi^2/\nu = 0.67$ and $\mathcal{F}t_{HF} = 3071.43(76)_{stat}(42)\delta'_{R}$ with $\chi^2/\nu = 1.00$. Both of these sets of $\delta_C$ calculations thus yield $\mathcal{F}t$ values that are fully consistent with the CVC hypothesis, but with central values that differ by 0.77 s, which is equivalent to the statistical uncertainty of the entire world superallowed data set added in quadrature with the estimated theoretical uncertainty on the $\delta_C$ and $\delta_{NS}$ corrections within each model. Since the larger $\chi^2/\nu$ value for the CVC test with the HF $\delta_{C2}$ corrections is associated with the five least precisely measured $\mathcal{F}t$ values, namely $^{22}\text{Mg}$, $^{34}\text{Ar}$, $^{38}\text{Ca}$, $^{62}\text{Ga}$, and $^{74}\text{Rb}$, an improvement in the precision of the $\mathcal{F}t$ values for these nuclei is critical to test the validity of these $\delta_C$ corrections. The uncertainty in the $\mathcal{F}t$ values for the two high-Z superallowed emitters, $^{62}\text{Ga}$, and $^{74}\text{Rb}$, is dominated by the uncertainty in theoretical corrections. The
uncertainty in the $\mathcal{F}t$ value for the lower-$Z$ $T_z = -1$ emitters, $^{22}\text{Mg}$, $^{34}\text{Ar}$, and $^{38}\text{Ca}$, on the other hand, is dominated by the experimentally determined $ft$ values. Therefore, an improvement in the experimentally measured $ft$ values for these cases will be critical to further test the different $\delta_C$ corrections.

For the case of $^{22}\text{Mg}$ decay shown in Figure 5.1, the uncertainty in the $\mathcal{F}t$ value is dominated by the experimental uncertainties in the branching ratio and half-life measurements. The currently adopted half-life of $^{22}\text{Mg}$, $T_{1/2} = 3.8752 \pm 0.0024 \text{ s}$ [3], is determined from two measurements, $T_{1/2} = 3.8755 \pm 0.0012 \text{ s}$ [83] and $T_{1/2} = 3.857 \pm 0.009 \text{ s}$ [84]. The disagreement between these two measurements is depicted in Figure 5.2 which, with a $\chi^2/\nu = 4.0$,
leads to an inflation in the uncertainty of the adopted world-average half-life for $^{22}$Mg by a factor of 2 [3]. A measurement of the $^{22}$Mg half-life was therefore performed in order to address the discrepancy between the two previous measurements reported in literature.

The $^{22}$Mg half-life experiment was performed at TRIUMF’s Isotope Separator and Accelerator (ISAC) facility. A 40 $\mu$A beam of 480 MeV protons from TRIUMF’s main cyclotron impinged on a SiC production target to produce spallation products. The target was coupled to the ion guide laser ion source (IGLIS) to produce intense beams of laser-ionized $^{22}$Mg while suppressing surface ionized contaminants, such as $^{22}$Na, by a factor of $10^5$ to $10^6$ [17].

Figure 5.2: $^{22}$Mg half-life measurements from Refs. [84] and [83]. The $\chi^2/\nu = 4.0$ leads to an inflation in the uncertainty of the weighted average by a factor of two.
The high-resolution mass separator was then used to select a beam of $A = 22$ products which included $^{22}\text{Mg}$ at $\sim 10^5$ ions/s and a remaining contaminant of $^{22}\text{Na}$ at $\sim 10^4$ ions/s which was delivered to a $4\pi$ proportional gas counter as a 30 keV ion beam.

### 5.1 Dead-time Measurements

Dead-time measurements, using the source and source-plus-pulser technique previously described in Chapter 3, were taken before and after the $^{22}\text{Mg}$ half-life experiment in order to accurately determine the dead-times imposed by the two gate and delay generators used with the two different MCS modules. These measurements were all performed using a 20 kBq $^{90}\text{Sr}$ source and a 10 kHz pulser. The detector bias voltage and CFD threshold voltage were varied between the different dead-time runs in order to investigate any systematic dependence. A summary of the settings used in the individual runs, as well as the corresponding measured dead-time values, are given in Table 5.1.

The run-by-run dead-time measurements are shown in Figure 5.3. To investigate any systematic effects, these measurements were grouped according their different experimental running conditions and the resulting $\chi^2/\nu$ of the groupings were calculated. Plots of the systematic groupings are shown in Figure 5.4. Since none of the systematic groupings yield a $\chi^2/\nu > 1.00$, the statistical uncertainty was inflated by the square root of the run-by-run $\chi^2/\nu$, which, for both gate and delay generators was $\chi^2/\nu = 1.07$. Including this small systematic uncertainty, the measured dead-time values were determined to be:

$$
\tau(3 \mu s) = 2.9827 \pm 0.0032 \mu s
$$

$$
\tau(4 \mu s) = 3.9978 \pm 0.0031 \mu s.
$$

(5.1)
Table 5.1: Summary of the run settings for the different dead-time measurements for the $^{22}\text{Mg}$ half-life experiment. The dead-time measurements from runs 10-24 were taken prior to the $^{22}\text{Mg}$ half-life experiment while the dead-time measurements from runs 122-146 were taken after the experiment. All of the measurements were taken using a 20 kBq $^{90}\text{Sr}$ source and a 10 kHz pulser.

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<th>Threshold (mV)</th>
<th>New MCS Dwell Time (s)</th>
<th>Old MCS Dwell Time (s)</th>
<th>Dead-time (3 $\mu$s)</th>
<th>Dead-time (4 $\mu$s)</th>
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τ = 2.9827(31) µs
χ²/ν = 1.07

τ = 3.9978(30) µs
χ²/ν = 1.07

Figure 5.3: Individual dead-time measurements for (a) the 3 µs and (b) 4 µs dead-times. The black (blue) data points correspond to the data taken before (after) the ²²Mg half-life experiment.
Figure 5.4: Systematic grouping of the data for the (a) 3 µs and (b) 4 µs dead-time measurements. The different applied bias voltages and discriminating threshold voltages are represented by the black and blue data points, respectively.
5.2 Half-life Analysis

The $^{22}\text{Mg}$ half-life experiment was performed in cycles mode. A cycle was comprised of a beam-on time in which the beam was implanted onto the tape for 0.6-0.7 s to build up a sample of $^{22}\text{Mg}$. In order to avoid space-charge effects in the $4\pi$ gas counter, a 1 s wait time was added before moving the sample into the gas counter in order to limit the rate in the gas counter to \( \leq 10 \) kHz. The decay was measured using two MCS modules which recorded the same data but with the different ($\sim 3\mu s$ and $\sim 4\mu s$) dead-times applied. The data was taken with two different dwell times of 0.36 s and 0.40 s and, with a total of 250 bins per cycle, corresponded to total decay times of 90 s and 100 s, respectively, or roughly 25 half-lives of $^{22}\text{Mg}$. The run settings for the individual cycles are given in Table 5.2.

A total of 740 cycles were recorded during the beam time. Of these cycles, those with a noticeable drop in counts were removed from the analysis as these correspond to a loss of beam during the cycle. These cycles typically had $\leq 50\%$ counts compared to the average counts per cycle within the same run. This criteria removed 38 cycles. An additional 21 cycles were removed from the analysis due to individual channel large residuals ($\geq 6\sigma$) which corresponded to noise spikes in the detector. Following cycle rejection, a total of 681 good cycles remained, corresponding to 97% of the total data acquired during the experimental running time.

A dead-time correction using the measured dead-times given in Equation 5.1 was applied to the data and a fit to each individual cycle was performed. The corresponding $^{22}\text{Mg}$ half-life along with the $\chi^2/\nu$ of the fit to the data for the good cycles are shown in Figure 5.5. In the lower panel of Figure 5.5, two small “dips” in the $\chi^2/\nu$ distribution is observed. Upon further investigation into this feature, it was determined that the decrease in the $\chi^2/\nu$ originates from
Table 5.2: A summary of the run settings for the $^{22}$Mg half-life runs.

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<td>100</td>
<td>0.36</td>
<td>4</td>
<td>3</td>
<td>20</td>
<td>12</td>
</tr>
<tr>
<td>72</td>
<td>2450</td>
<td>100</td>
<td>0.36</td>
<td>4</td>
<td>3</td>
<td>9</td>
<td>5</td>
</tr>
<tr>
<td>74</td>
<td>2450</td>
<td>100</td>
<td>0.36</td>
<td>4</td>
<td>3</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>75</td>
<td>2450</td>
<td>100</td>
<td>0.36</td>
<td>4</td>
<td>3</td>
<td>18</td>
<td>18</td>
</tr>
</tbody>
</table>

the increased effective degrees of freedom. For Gaussian statistics, the number of degrees of freedom, $\nu$, is simply the number of data points minus the number of free parameters
Figure 5.5: The cycle-by-cycle half-life measurements for $^{22}$Mg. The corresponding $\chi^2/\nu$ of the fit to the data for the individual cycles is shown in the lower panel. The black and blue data points represent the data taken with a dwell time of 0.40 s and 0.36 s, respectively.

in the fit function and a $\chi^2/\nu$ equal to one indicates a good fit to the data. However, in order to account for the low number of counts in the background part of the decay region, the number of degrees of freedom must be redefined in order to provide a more appropriate assessment of the goodness of the fit to the data [65]. For such a scenario, the number of degrees of freedom must be altered to account for the fact that bins with no counts do not contribution any degrees of freedom [65].
Figure 5.6: A plot of the (a) background rate and (b) degrees of freedom for the individual cycles. The black and blue data points represent the data taken with a dwell time of 0.40 s and 0.36 s, respectively.
The background rate and degrees of freedom for the individual cycles are plotted in Figures 5.6a and 5.6b, respectively. With the revised definition of \( \nu \), the degrees of freedom can be seen to vary slightly between cycles, and, in particular, are slightly increased for the cycles in which the background rate was increased. The increase of \( \nu \) results in a lower \( \chi^2/\nu \) for these cycles. The resulting fit to the data for these cycles were, however, good and as such these cycles were kept for the final analysis.

Table 5.3: \(^{22}\)Mg half-life measurements from the individual runs. The data was fit with a decaying \(^{22}\)Mg component and a free background component.

<table>
<thead>
<tr>
<th>Run</th>
<th>( T_{1/2} ) (s)</th>
<th>Error (s)</th>
<th>( \chi^2/\nu )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.8798</td>
<td>0.0026</td>
<td>1.07</td>
</tr>
<tr>
<td>2</td>
<td>3.8769</td>
<td>0.0028</td>
<td>0.98</td>
</tr>
<tr>
<td>3</td>
<td>3.8685</td>
<td>0.0028</td>
<td>0.91</td>
</tr>
<tr>
<td>4</td>
<td>3.8752</td>
<td>0.0028</td>
<td>0.99</td>
</tr>
<tr>
<td>5</td>
<td>3.8739</td>
<td>0.0028</td>
<td>0.91</td>
</tr>
<tr>
<td>6</td>
<td>3.8736</td>
<td>0.0028</td>
<td>0.73</td>
</tr>
<tr>
<td>7</td>
<td>3.8744</td>
<td>0.0029</td>
<td>1.04</td>
</tr>
<tr>
<td>8</td>
<td>3.8731</td>
<td>0.0029</td>
<td>0.96</td>
</tr>
<tr>
<td>9</td>
<td>3.8750</td>
<td>0.0033</td>
<td>0.92</td>
</tr>
<tr>
<td>10</td>
<td>3.8753</td>
<td>0.0035</td>
<td>1.17</td>
</tr>
<tr>
<td>11</td>
<td>3.8742</td>
<td>0.0033</td>
<td>0.91</td>
</tr>
<tr>
<td>12</td>
<td>3.8772</td>
<td>0.0033</td>
<td>0.88</td>
</tr>
<tr>
<td>13</td>
<td>3.8725</td>
<td>0.0033</td>
<td>1.03</td>
</tr>
<tr>
<td>14</td>
<td>3.8731</td>
<td>0.0032</td>
<td>1.04</td>
</tr>
<tr>
<td>15</td>
<td>3.8776</td>
<td>0.0032</td>
<td>1.05</td>
</tr>
<tr>
<td>16</td>
<td>3.8730</td>
<td>0.0033</td>
<td>0.99</td>
</tr>
<tr>
<td>17</td>
<td>3.8729</td>
<td>0.0033</td>
<td>0.91</td>
</tr>
<tr>
<td>18</td>
<td>3.8755</td>
<td>0.0034</td>
<td>0.96</td>
</tr>
<tr>
<td>19</td>
<td>3.8707</td>
<td>0.0034</td>
<td>1.01</td>
</tr>
<tr>
<td>20</td>
<td>3.8733</td>
<td>0.0035</td>
<td>1.02</td>
</tr>
<tr>
<td>21</td>
<td>3.8647</td>
<td>0.0035</td>
<td>0.96</td>
</tr>
<tr>
<td>22</td>
<td>3.8751</td>
<td>0.0037</td>
<td>1.01</td>
</tr>
<tr>
<td>23</td>
<td>3.8678</td>
<td>0.0046</td>
<td>0.96</td>
</tr>
<tr>
<td>24</td>
<td>3.8774</td>
<td>0.0043</td>
<td>1.01</td>
</tr>
</tbody>
</table>
Figure 5.7: Run by run half-life measurements for $^{22}$Mg. The corresponding $\chi^2/\nu$ of the fit for each run is given in the lower panel.

Following the assessment of the individual cycles, the dead-time corrected data from each run were summed and a single fit were performed. The resulting $^{22}$Mg half-life deduced from each run and the corresponding $\chi^2/\nu$ of the fit are given in Table 5.3. The run-by-run half-life distribution is shown in Figure 5.7.

Finally, the dead-time corrected data from each run taken with common dwell times were summed. The fits to the summed datasets for the 0.40 s and 0.36 s dwell times are shown in Figure 5.8.
Figure 5.8: Fit to the summed data taken with a dwell time of (a) 0.40 s and (b) 0.36 s. The bin by bin residuals are shown in the lower panels.
5.2.1 Contaminants

The measured $^{22}\text{Mg}$ half-lives given in Table 5.2 and shown in Figures 5.5, 5.7, and 5.8 were obtained by fitting the dead-time corrected data to a function that included a single decaying component of $^{22}\text{Mg}$ and a constant background component. Although the IGLIS ion source suppresses the surface ionized $^{22}\text{Na}$ contaminant in the beam by a factor of $10^5-10^6$ [47], this contaminant was still delivered to the experimental station at a rate of approximately $10^4$ ions/s. Since $^{22}\text{Na}$ has a half-life of $T_{1/2} = 2.6029(8) \text{ y}$ [85], the activity from $^{22}\text{Na}$ can be very well approximated by a constant over the $\sim 100 \text{ s}$ decay period and was thus accounted for in the constant background component of the fit function. However, in order to ensure that the $^{22}\text{Na}$ activity had no effect on the deduced $^{22}\text{Mg}$ half-life, the constant background component in the fit was replaced with an exponentially decaying function with the half-life of $^{22}\text{Na}$. The data was subsequently refit and the resulting change in the $^{22}\text{Mg}$ half-life was at the $10^{-9}$ s level. This difference is completely negligible compared to the statistical precision of the measured $^{22}\text{Mg}$ half-life. Moreover, since the $\beta$ decay of $^{22}\text{Mg}$ proceeds to $^{22}\text{Na}$, the possibility of contamination from the “grow-in” activity of $^{22}\text{Na}$ as the daughter of $^{22}\text{Mg}$ decay could also affect the deduced $^{22}\text{Mg}$ half-life. The data was once again re-fit with an additional term corresponding to the grow-in activity of the $^{22}\text{Na}$ daughter produced by the $^{22}\text{Mg}$ decay. The resulting change in the $^{22}\text{Mg}$ half-life was at the $10^{-10}$ s level and is also entirely negligible compared to the statistical uncertainty.

Contributions from several other potential in-beam isobaric contaminants that would not be resolved by the mass separator were also considered, although with the combination of much lower ionization efficiency for these contaminants from the highly selective laser ionization scheme combined with the surface ion suppression provided by IGLIS, none were expected to be present in the beam. During the experimental running period, the $A =$
Figure 5.9: The $\gamma$-ray energy spectrum in $\beta$-$\gamma$ coincidence recorded with the GRIFFIN spectrometer during the $^{22}\text{Mg}$ experiment. The absence of characteristic $\gamma$-rays from potential isobaric contaminants was used to set limits on contributions to the activity in the gas counter during the half-life measurements.

$^{22}\text{RIB}$ was also delivered to the GRIFFIN spectrometer. The $\gamma$-ray energy spectrum in $\beta$-$\gamma$ coincidence that was recorded with GRIFFIN is shown in Figure 5.9. The lack of characteristic photopeaks from potential isobaric contamination suggests that no additional contaminants were present in the $A = 22$ RIB. Nonetheless, the GRIFFIN $\beta$-$\gamma$ coincidence spectrum was used to place limits on possible contamination from other species, in particular, $^{22}\text{O}$, $^{22}\text{F}$, $^{44}\text{K}^{2+}$ (whose charge to mass ratio is 22), $^{21}\text{Na}$ and $^{21}\text{F}$ (which could be delivered
in the $A = 22$ beam as $H^{21}Na$ and $H^{21}F$ molecules, respectively).

An upper limit on the intensity from the individual contaminants was determined by fixing the full-width at half-maximum (FWHM) and centroid of the $\gamma$-ray photopeaks at the expected locations corresponding to the characteristic $\gamma$-ray energies from the contaminants and fitting the “peak” areas at these energies. The energy of each characteristic $\gamma$-ray that was used in the analysis is given in Table 5.4. For the unphysical cases in which the central value of the fitted peak area was negative, the Gaussian probability distribution was integrated over the physical region of positive counts and the $1\sigma$ upper limit was deduced by determining the number of counts corresponding to 68% of the area of the positive count region of the probability distribution.

Table 5.4: Investigation of potential contaminants in the $A = 22$ radioactive ion beam. The upper limit on the contaminant corresponds to its activity relative to $^{22}$Mg at the beginning of the counting period. The change in the $^{22}$Mg half-life is relative to the deduced $^{22}$Mg half-life when no contaminants are included in the fitting function.

<table>
<thead>
<tr>
<th>Potential Contaminant</th>
<th>Half-life ($s$)</th>
<th>$\gamma$-ray Energy (keV)</th>
<th>Upper Limit on the Activity Relative to $^{22}$Mg</th>
<th>Change in the $^{22}$Mg Half-life ($s$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{22}$O</td>
<td>2.250(90)</td>
<td>918</td>
<td>$1.1 \times 10^{-4}$</td>
<td>-0.00010</td>
</tr>
<tr>
<td>$^{22}$F</td>
<td>4.230(40)</td>
<td>2082</td>
<td>$1.4 \times 10^{-6}$</td>
<td>$5.6 \times 10^{-7}$</td>
</tr>
<tr>
<td>$^{21}$Na</td>
<td>22.448(8)</td>
<td>351</td>
<td>$1.5 \times 10^{-6}$</td>
<td>0.00004</td>
</tr>
<tr>
<td>$^{21}$F</td>
<td>4.158(20)</td>
<td>1395</td>
<td>$9.9 \times 10^{-6}$</td>
<td>0.00001</td>
</tr>
<tr>
<td>$^{44}$K</td>
<td>1327.8(114)</td>
<td>1157</td>
<td>$1.7 \times 10^{-8}$</td>
<td>$4.2 \times 10^{-8}$</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td>0.00011</td>
</tr>
</tbody>
</table>

The fitted peak areas were then used to place upper limits on the activity of the individual contaminants and were included as a fixed parameter in the half-life fitting procedure in order to investigate if there was any effect on the measured $^{22}$Mg half-life. The corresponding changes in the $^{22}$Mg half-life when the individual contaminants were included in the fit are
given in Table 5.4. The resulting changes in the $^{22}$Mg half-life from the inclusion of each contaminant were added in quadrature and the resulting change of 0.00011 s was included as a systematic uncertainty in the final result.

### 5.2.2 Rate-dependent Effects

In order to avoid space charge effects in the gas counter, the beam-on and cool times were adjusted to ensure that the maximum rate in the gas counter never exceeded 14 kHz. Individual cycles were, however, recorded over a broad range of initial rates in the gas counter, between 6 kHz and 14 kHz, by varying the beam-on and cool times. A weighted linear regression of the $^{22}$Mg half-lives from the individual cycles as a function of the initial count rate in the detector was performed in order to investigate whether the rates in the gas counter used in this experiment introduced any bias in the deduced $^{22}$Mg half-life. A plot of the $^{22}$Mg half-life from the individual cycles as a function of the initial count rate is shown in Figure 5.10. The corresponding linear regression results in a slope of $(21 \pm 31) \times 10^{-8}$ s$^2$ and is fully consistent with zero indicating that the $^{22}$Mg half-life was not affected over the initial count rates in the gas counter used in this experiment.

As discussed in Chapter 3, additional rate dependence investigations were performed by removing leading channels and re-fitting the remaining data. The resulting chop plots for the summed datasets taken with dwell times of 0.40 s and 0.36 are shown in Figure 5.11. No statistically significant change in the $^{22}$Mg half-life was observed with removal of up to 30 leading channels, corresponding to approximately 3 half-lives of $^{22}$Mg or the removal of approximately 87.5% of the data.
Figure 5.10: The $^{22}$Mg half-life as a function of the initial rate in the gas counter. The slope from a weighted linear regression is shown and red and is consistent with zero.
Figure 5.11: Chop plots for the summed data taken with dwell times of (a) 0.40 s and (b) 0.36 s. The removal of 30 channels corresponds to approximately 3 half-lives of $^{22}$Mg. No trends indicative of systematic rate-dependent effects are observed in either of the datasets.
5.2.3 Systematics

Figure 5.12: Grouping of the $^{22}$Mg half-life measurements according to the different experimental running conditions. The systematic grouping of the bias voltages yields a $\chi^2/\nu = 1.44$ and is used to assign a systematic uncertainty of 0.00043 s to the half-life measurement.

As shown in Figure 5.12, both of the MCS modules and the two different dead-time values yield half-life results in complete agreement. Each of the adopted run-by-run half-lives thus corresponds to the average of the deduced $^{22}$Mg half-lives from the two MCS modules.

Following the completion of each run, the experimental running conditions were varied in order to investigate potential systematic effects arising from the choice of running conditions. The grouping of the data according to the different running conditions, which included
different bias voltage (2350 V, 2400 V, 2450 V, and 2500 V) applied to the gas counter and different CFD threshold voltages (70 mV, 85 mV, 100 mV, and 115 mV) are shown in Figure 5.12. While the grouping by threshold settings yields $\chi^2/\nu = 0.39$, the grouping by bias voltage gives $\chi^2/\nu = 1.44$. Although a $\chi^2/\nu$ of 1.44 for three degrees of freedom is expected 23% of the time for statistically independent data, we follow the conservative approach recommended by the Particle Data Group [1] and inflated the statistical uncertainty by $\sqrt{1.44}$, associated with a possible systematic effect associated with the detector bias. This results in a systematic uncertainty of 0.00043 s.

Finally, the dead-times were varied within $\pm 1\sigma$ of their measured values and the data re-fit in order to investigate any variations in the $^{22}\text{Mg}$ half-life. The resulting change in the half-life of 0.00004 s was included as an additional systematic uncertainty. Adding the statistical uncertainty in quadrature with the systematic uncertainties from the addition of contaminants, the uncertainty in the dead-time measurements, and the bias voltage systematic, yields the final $^{22}\text{Mg}$ half-life result [87]:

$$T_{1/2} = 3.87400 \pm 0.00065_{\text{stat}} \pm 0.00043_{\text{sys}} \pm 0.00011_{\text{contam}} \pm 0.00004_{\text{DT}} \text{ s}$$

$$= 3.87400 \pm 0.00079 \text{ s.} \quad (5.2)$$

### 5.3 Updated $ft$-value

An updated plot that includes the new $^{22}\text{Mg}$ half-life measurement reported here is shown in Figure 5.13. Since the measurement from Ref. [84] is now more than a factor of 10 less precise than the result from the current work, it was not included in the weighted-average. Therefore, the updated world average is calculated using the two most precise $^{22}\text{Mg}$
Figure 5.13: $^{22}\text{Mg}$ half-life measurements from Refs. [84], [83], and the result presented in this thesis [87]. The updated world average half-life and $\chi^2/\nu$ quoted here are calculated using only the two most precise measurements.

measurements, namely the result presented here [87] and that of Ref. [83], leading to the world-average half-life result of $T_{1/2}(^{22}\text{Mg}) = 3.87445 \pm 0.00069$ s with a $\chi^2/\nu$ of 1.1. This represents an improvement in the precision of the world average half-life for $^{22}\text{Mg}$ by a factor of 3.5.

An updated $^{22}\text{Mg}$ $ft$-value using the results from Ref. [3], the updated $^{22}\text{Mg}$ half-life presented in this chapter, as well as an updated $Q$-value measurement of $4781.40 \pm 0.22$ keV [88], was calculated to be $ft = 3051.0 \pm 6.9$ s. The $^{22}\text{Mg}$ half-life measurement reported here was

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essential for resolving the discrepancy between the previous two $^{22}$Mg half-life measurements [83,84] reported in literature. With the resulting factor of 3.5 improvement in the precision of the updated world-average half-life for $^{22}$Mg, the uncertainty in the $^{22}$Mg $ft$-value is now completely dominated by the uncertainty in the $^{22}$Mg superallowed branching ratio. An improvement in the precision of this branching ratio is therefore essential to provide a stringent test of the SM-HF and SM-WS $\delta_C$ correction for the superallowed emitter $^{22}$Mg. A novel experimental approach to precisely measure this branching ratio will be briefly discussed in Chapter 6.
Chapter 6

Conclusions and Future Work

Although superallowed Fermi $\beta$ decay transitions between $T = 1$, $J^\pi = 0^+$ isobaric analogue states represent only a small fraction of all nuclear $\beta$ decays, high-precision measurements of the $Ft$ values for these transitions play an important role in testing fundamental properties of the Standard Model. These decays have been critically evaluated for nearly 50 years [3,28,30,89–91] and progress in both the experimental and theoretical studies of these decays continue to provide the most precise avenue to test the CVC hypothesis and, in combination with $V_{us}$ and to a lesser degree $V_{ub}$, the unitarity of the CKM matrix, two fundamental pillars of the Standard Model. Although $V_{ud}$ obtained from the superallowed data has remained consistent within uncertainty over the past several decades, while its precision has steadily improved [3], this has, to some extent, resulted from the cancellation of significant changes and improvements in both the experimental measurements [92] and theoretical corrections [15] with opposite effects on the world-average $Ft$ value. Furthermore, significant changes in the determination of $V_{us}$ [93,94] which is combined with $V_{ud}$ to test unitarity of the quark mixing matrix continues to motivate studies of such fundamental properties of
the Standard Model. Moreover, searches for new physics beyond the Standard Model as it is known today remains of foremost interest and can be investigated using high precision measurements of low-energy nuclear $\beta$ decays as well as in the high-energy regime \cite{95}.

Of the 20 superallowed $Ft$ values which have been critically evaluated \cite{3}, 14 have currently been determined to a precision of 0.30\% and the constancy of the $Ft$-values of these 14 superallowed transitions have provided stringent tests of the Standard Model as well as possible extensions to it \cite{3}. Since these tests are determined using the corrected superallowed $Ft$ values, it is not only important to have precise measurements of the experimental $ft$ values, but also stringent tests of the theoretical models used to calculate the $Ft$-values. In fact, the uncertainty in the individual $Ft$-values for all of the $T_z = 0$ superallowed emitters among the 14 precisely measured cases are currently dominated by these theoretical calculations. This is, however, not the case for most of the $T_z = -1$ superallowed emitters such as $^{10}$C, $^{22}$Mg, $^{34}$Ar, and $^{38}$Ca where experimental uncertainties continue to dominate the final $Ft$ value uncertainties and thus improved experimental measurements remain critical to testing the theoretical models used to calculate the isospin symmetry breaking corrections.

In the current thesis, new high-precision half-life measurements were performed for two of these $T_z = -1$ superallowed emitters, $^{10}$C and $^{22}$Mg. In each case, the new high-precision results presented here resolved significant discrepancies among the previously reported half-life measurements and led to improvements in the world-average half-lives by factors of $2.5 - 3.5$.

The two $^{10}$C half-life measurements presented in this thesis were motivated by the significant disagreement between the previous $^{10}$C half-life measurements \cite{68} \cite{71}. The two $^{10}$C half-life measurements presented in this thesis were performed using two different methods: $\gamma$-ray photopeak counting using the $8\pi$ $\gamma$-ray spectrometer and direct $\beta$ counting using a $4\pi$ continuous flow proportional gas counter. These independent half-life measurements yielded
values of $T_{1/2}(\gamma) = 19.2969 \pm 0.0074$ s and $T_{1/2}(\beta) = 19.3009 \pm 0.0017$ s that were consistent with one another, and, most notably, the $^{10}$C half-life measurement using the $\beta$ counting technique was measured to a precision of 0.009%, which is the most precise superallowed half-life measurement reported to date \cite{75}. Including these measurements with the evaluated world-average $^{10}$C half-life data resulted in improved consistency in the dataset and resulted in an improvement in the uncertainty of the world-average $^{10}$C half-life by nearly a factor of three, to $T_{1/2}(^{10}$C) = $19.3015 \pm 0.0025$ s with a $\chi^2/\nu = 1.90$ among the 6 most precise measurements.

The half-life measurement of $^{22}$Mg was also motivated by the disagreement within the previous world dataset. In the work presented in this thesis, the $^{22}$Mg half-life was measured with a $4\pi$ proportional gas counter and determined to be $T_{1/2}(^{22}$Mg) = $3.87400 \pm 0.00079$ s \cite{87}. This result is in agreement with, but more precise than the other precise measurement of $T_{1/2}(^{22}$Mg) = $3.8755 \pm 0.0012$ s reported in Ref. \cite{83}. Including the new measurement reported here in the evaluated $^{22}$Mg world average half-life dataset results in an improvement in the weighted average by more than a factor of three to $T_{1/2}(^{22}$Mg) = $3.87445 \pm 0.00069$ s with a $\chi^2/\nu$ of 1.1 between the two most precise measurements.

6.1 The $^{10}$C Superallowed Branching Ratio and Improving the Limit on $b_F$

With the discrepancy between the $^{10}$C half-life dataset resolved in this thesis, an improvement in the precision of the $^{10}$C superallowed branching ratio is now the only remaining experimental quantity that can significantly improve the limit on $b_F$ set by the superallowed data as depicted in Figure \ref{fig:6.1}. Currently, the $^{10}$C branching ratio is dominated by two
The relative uncertainty of the individual contributions to the $\mathcal{F}t$ value of $^{10}$C. The uncertainty in the $\mathcal{F}t$ value is currently dominated by the uncertainty in the branching.

precision measurements, both performed over two decades ago [96,97]. The $\beta$ decay scheme of $^{10}$C is shown in Figure 6.2. With only two $\gamma$-rays following the $\beta$ decay of $^{10}$C to its daughter $^{10}$B, and an entirely negligible 2nd forbidden $\beta$ decay branch to the $^{10}$B ground state, the branching ratio can be determined by measuring the ratio of intensities of the two $\beta$-delayed $\gamma$-rays from the decay. This requires measuring the relative $\gamma$-ray detection efficiencies between the 718-keV $\gamma$-ray and the 1022-keV $\gamma$-ray with high-precision. The
superallowed branching ratio is then given by:

\[ BR(0^+ \rightarrow 0^+) = \frac{I(1022)}{I(718)} = \frac{Y(1022) \epsilon(718)}{Y(718) \epsilon(1022)}, \]  

where \( Y \) represents the number of photopeak counts observed for each of the \( \gamma \)-rays, and \( \frac{\epsilon(718\,\text{keV})}{\epsilon(1022\,\text{keV})} \) represents the relative photopeak efficiencies for detecting the 718-keV \( \gamma \)-ray relative to the 1022-keV \( \gamma \)-ray. This method is complicated, however, by the fact that the observed rate of the 1022-keV \( \gamma \)-ray from the decay of \( ^{10}\text{C} \) must be distinguished from the pile-up of two 511-keV \( \gamma \)-rays, originating from electron positron annihilation, which will contaminate the 1022 keV photopeak. Thus, both a high relative efficiency and accurate calibration of the

Figure 6.2: Decay scheme for the \( \beta \) decay of \( ^{10}\text{C} \) to \( ^{10}\text{B} \).
Figure 6.3: Updated limit on the Fierz interference term extracted from the superallowed \( F_t \) values given an improvement on the \( ^{10}\text{C} \) branching ratio. The hypothetical \( ^{10}\text{C} \) \( F_t \) data assumes a new measurement of the superallowed branching ratio in which the central value remains unchanged but is determined to a precision of 0.1%

pile-up must be performed in order to obtain a branching ratio measurement to the required precision of 0.1% or better. At this time, an improvement in the \( ^{10}\text{C} \) superallowed branching ratio is actively being addressed using the same approach pioneered in Ref. [96] in which the relative efficiencies for the 718 keV and 1022 keV \( \gamma \)-rays of interest in \( ^{10}\text{B} \) is directly determined by populating the transition in a 1:1 ratio via a \( ^{10}\text{B}(p,p') \) reaction. The goal of a recent \( ^{10}\text{C} \) branching ratio experiment with the NUBALL array at the ALTO facility in France [98] is to reduce the systematic uncertainty from 0.25%, as obtained in Refs. [96]
and $\text{He}_7$ to 0.1%. Figure 6.3 shows the impact that this single measurement of the $^{10}$C branching ratio at the ±0.1% level could have on the limits on $b_F$ set by the superallowed data where, for illustration, we have assumed the central value remains unchanged but the precision of the world average $^{10}$C superallowed branching ratio is improved by a factor of 1.7. This hypothetical result would improve the precision of the updated limit on $b_F$ of $-0.0018 \pm 0.0021$ reported in this thesis to $-0.0026 \pm 0.0019$. Any further improvement in the precision of $b_F$ determined from the superallowed data would then rely entirely on improvements to the theoretical correction terms that dominate the uncertainties of most of the other precisely determined $F_t$ values.

6.2 The $^{22}$Mg Superallowed Branching Ratio and Isospin Symmetry Breaking Corrections

With the discrepancy between the $^{22}$Mg half-life measurements resolved in the current thesis, an improvement in the superallowed branching ratio for $^{22}$Mg is essential to improve the uncertainty in the $F_t$ value in order to provide a stringent test of the SM-HF and SM-WS $\delta C$ corrections. The relative uncertainty of the contributions to the $^{22}$Mg $F_t$ value is shown in Figure 6.4. The $\beta$ decay scheme of $^{22}$Mg is shown in Figure 6.5. Similar to the case of $^{10}$C, there is a negligible 2nd forbidden $\beta$ decay branch to the ground state of $^{22}$Na and the superallowed branch can be determined from the relative intensities of the $\beta$ delayed $\gamma$-rays. A significant challenge, however, results in precisely determining the relative efficiencies between the key 74 keV and 583 keV $\gamma$-rays. Currently, the superallowed branching ratio for $^{22}$Mg has only been measured to a precision of ±0.23% using a precisely calibrated HPGe detector [83], which is well below the benchmark precision of ±0.10% for determining the
Figure 6.4: The relative uncertainty of the individual contributions to the $\mathcal{F}t$ value of $^{22}$Mg. The uncertainty in the $\mathcal{F}t$ value is currently dominated by the uncertainty in the branching superallowed $ft$ value. During the same experimental running period as the $^{22}$Mg half-life measurements reported in this thesis, a branching ratio measurements for $^{22}$Mg was performed through a novel technique [99] that capitalizes on the very high $\gamma$-$\gamma$ coincidence efficiency of the GRIFFIN spectrometer [57] to directly measure the relative efficiencies of the 74 keV and 583 keV $\gamma$-rays in coincidence with the 1280 keV transition populated in $\approx 5.5\%$ of $^{22}$Mg decays. The analysis of these data is currently underway.

The impact of the $^{22}$Mg branching ratio measurement with a projected precision of 0.1% is shown in Figure 6.6. In this plot, the current central value of the $^{22}$Mg branching ratio
remains unchanged while the measurement improves the precision of the $F_t$ value. The resulting world-average $F_t$ values using both the Woods-Saxon and Hartree-Fock $\delta_C$ corrections become:

\[
F_t(WS) = 3072.44(61)_{stat}(36)_{\delta_R} \quad \chi^2\nu = 0.64 \quad (6.2)
\]
\[
F_t(HF) = 3071.17(73)_{stat}(42)_{\delta_R} \quad \chi^2\nu = 1.60. \quad (6.3)
\]

In this hypothetical scenario, the increase of the $\chi^2/\nu$ for the Hartree-Fock $\delta_C$ corrections from 1.26 to 1.60 would strengthen the argument made in Ref. [3] that the Hartree-Fock
Figure 6.6: $F_t$ values for the 14 most precisely measured $T = 1$ superallowed transitions. The upper (lower) plot use the WS (HF) $\delta_C$ calculations used in the evaluation of the $F_t$ data. The hypothetical $^{22}$Mg $F_t$ values show the result of a branching ratio measurement with an estimated precision of 0.1% with no change in central value.

The calculations do not form a consistent and should not be included in the evaluation of the $F_t$ values. However, if the $^{22}$Mg branching ratio were to be measured at its current $-1\sigma$ value, with an uncertainty of 0.1%, as depicted in Figure 6.7, the resulting world-average $F_t$ values...
would become:

\[ \mathcal{F}_t(WS) = 3072.23(61)_{\text{stat}}(36)_{\delta_R} \quad \chi^2 \nu = 0.48 \]  
\[ \mathcal{F}_t(HF) = 3071.92(73)_{\text{stat}}(42)_{\delta_R} \quad \chi^2 \nu = 1.19. \]  

(6.4)  
(6.5)

In this scenario, the results obtained using the Hartree-Frock \( \delta_C \) corrections would be entirely consistent with the CVC hypothesis and the rejection of these calculations in the superallowed evaluated dataset would appear unjustified. An improvement in the precision of the \(^{22}\text{Mg}\) branching ratio measurement can thus clearly help to further constrain the theoretical models used to calculate the \( \delta_C \) corrections in superallowed Fermi \( \beta \) decays.

### 6.3 Additional \( T_z = -1 \) Superallowed Measurements

In addition to the \(^{22}\text{Mg}\) branching ratio, experimental constraints on the \( \delta_C \) corrections should also focus on the other two least precisely measured \( T_z = -1 \) emitters of the 14 high-precision cases, namely \(^{34}\text{Ar}\) and \(^{38}\text{Ca}\). A breakdown of the relative uncertainty based on the individual components required to calculate the \( \mathcal{F}_t \) is given in Figures 6.8 and 6.9 and highlights that, for both cases, the uncertainty in the \( \mathcal{F}_t \) value is dominated by experimental uncertainties. Currently, the uncertainty in the \( ft \) value for \(^{34}\text{Ar}\) is dominated by the branching ratio, which consists of a single measurement with a precision of 0.26\% \[100\]. Moreover, the \(^{34}\text{Ar}\) half-life is also dominated by a single high-precision measurement \[101\]. The daughter of the \( T_z = -1 \) superallowed emitter \(^{34}\text{Ar}\) is \(^{34}\text{Cl}\), which is a \( T_z = 0 \) superallowed emitter. In a given decay chain, for the special case where the ratio of parents and daughter decay constants is \( \lambda_P = 2\lambda_D \), the total activity will be distributed according to the half-life of the
Figure 6.7: $\mathcal{F}t$ values for the 14 most precisely measured $T = 1$ superallowed transitions. The upper (lower) plot use the WS (HF) $\delta_C$ calculations used in the evaluation of the $\mathcal{F}t$ data. The hypothetical $^{22}\text{Mg}$ $\mathcal{F}t$ value shows the result of a branching ratio measurement with the estimated precision of 0.1% and the central value at -1σ of its current value.

daughter and is therefore insensitive to the decay of the parent. For this particular decay chain, the half-life of $^{34}\text{Cl}$ is 1.81 times longer than that of $^{34}\text{Ar}$. Although this factor is not identically two, simulations of the decay of $^{34}\text{Ar}$ reveals that the total $\beta$ decay activity largely resembles that of the single exponential decay of the daughter $^{34}\text{Cl}$ [56]. Thus, due to the selective nature of the $\gamma$-ray photopeak method, a measurement of the $^{34}\text{Ar}$ half-life via $\gamma$-ray counting would provide a critical independent test of the single half-life measurement reported, which was performed using the $\beta$ counting method and relied entirely on an
Figure 6.8: The relative uncertainty of the individual contributions to the $F_t$ value of $^{34}$Ar. The uncertainty in the $F_t$ value is currently dominated by the uncertainty in the branching.

An external constraint on the ratio of $^{34}$Ar to $^{34}$Cl nuclei at the start of the counting period for its precision [101]. Both the branching ratio and the half-life of $^{34}$Ar will be measured with the high-precision GRIFFIN spectrometer at the ISAC facility [102].

Additional tests of the isospin symmetry breaking corrections can be obtained through high precision measurements of the $ft$-values of superallowed mirror transitions. These transitions can provide a particularly sensitive test of the $\delta_C$ corrections since the uncertainty in the difference of the $\delta_C$ corrections between mirror decays are much smaller than those in the absolute $\delta_C$ values themselves. Thus far, only two cases have been measured with
Figure 6.9: The relative uncertainty of the individual contributions to the $Ft$ value of $^{38}\text{Ca}$. The uncertainty in the $Ft$ value is currently dominated by the uncertainty in the branching. comparable precision, $^{38}\text{Ca} \rightarrow ^{38m}\text{K}$ and $^{38m}\text{K} \rightarrow ^{38}\text{Ar}$, as well as, $^{34}\text{Ar} \rightarrow ^{34}\text{Cl}$ and $^{34}\text{Cl} \rightarrow ^{34}\text{S}$. Of these two measurements, the $A = 38$ $ft$ measurements favour the SM-WS $\delta_C$ correction, whereas the $A = 34$ $ft$ measurements favour the SM-HF $\delta_C$ correction. The potential impact of an improvement in the $^{34}\text{Ar}$ branching ratio and half-life is shown in Figure 6.10. In this illustration, the current central values of the $^{34}\text{Ar}$ half-life and branching ratio are assumed to remain unchanged with improvements in their precision to 0.04% and 0.06%, respectively. With such improvements, the ratio of $ft$ values between the $A = 34$ mirror transitions strongly favour the SM-HF over the SM-WS $\delta_C$ corrections. Two additional
Figure 6.10: A plot of the ratio of the superallowed $ft$ values for the mirror transitions given improvements in the $^{34}$Ar half-life and branching ratio measurements. In this hypothetical situation, the half-life and branching ratio measurements are assumed to remain at their current world-average central values with improvements in their precision to 0.04% and 0.06%, respectively.

$T = 1$ superallowed mirror transition candidates exist: the $A = 26$ mirror transition nuclei, $^{26}$Si and $^{26m}$Al, as well as the $A = 42$ mirror transition nuclei, $^{42}$Ti and $^{42}$Sc. Currently, the precision of the branching ratios for both $^{42}$Ti and $^{26}$Si must be significantly improved in order to provide a sensitive test of the isospin symmetry breaking correction [3]. Preliminary results for the case of $^{26}$Si decay have been presented in Refs. [34] and [35] while the $^{42}$Ti superallowed branching ratio remains highly uncertain.
6.4 Conclusions

The study of superallowed Fermi $\beta$ decays remains an extremely active field, both experimentally and theoretically. The continued developments and improvements of experimental techniques not only provide improvements in the $ft$ values, but also help constrain theoretical corrections, and continue to provide more stringent tests of the Standard Model. The results of this thesis have highlighted the capability of performing high-precision half-life measurements for the $T_z = -1$ superallowed emitters $^{10}\text{C}$ and $^{22}\text{Mg}$ at TRIUMF-ISAC. Experimental measurements of the other quantities required to determine the superallowed $ft$ values, namely the branching ratio and the $Q$-value, are also possible at TRIUMF. With the ability to produce a wide variety of high-intensity RIBs, many superallowed half-life measurements [75, 78, 79, 103–108] using the $4\pi$ proportional gas counter and $8\pi$ spectrometer, branching ratio measurements [109–113] with the $8\pi$ spectrometer and GRIFFIN, as well as high-precision $Q$-value measurements [88, 114] with the TITAN Penning-trap, have been (or will be) included in the evaluation of the world superallowed data [3]. The combination of these intense radioactive ion beams provided by ISAC and the state-of-the-art experimental facilities at TRIUMF will continue to push the frontiers of high-precision superallowed Fermi $\beta$ decay measurements for the foreseeable future.
Bibliography


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