Experimental Analysis of Archimedes Screw Turbines

By

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ABSTRACT

EXPERIMENTAL ANALYSIS OF ARCHIMEDES SCREW TURBINES

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This study investigated how power and efficiency in Archimedes screw turbines (AST) is impacted by varying parameters including; length, inner and outer diameter, pitch and number of flights. An index was created that determined how important some variables were over others by testing a number of laboratory sized screws and comparing the resulting power and efficiencies. A MATLAB AST performance model was implemented that included simplified power losses from internal hydraulic friction and outlet submersion to predict experimental results. New experimental techniques were developed and used to investigate fill height and overflow leakage in ASTs. The experiments showed screw rotation speed had an important effect on overflow, however current models are quasi-static and do not include rotation. A revised equation was developed to predict overflow in operating screws that includes the effect of screw rotation speed, and was shown to more accurately predict overflow in laboratory-scale screws.
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Nomenclature

$D_i$  Inner screw diameter (m)
$D_o$  Outer screw diameter (m)
$eff$  Nominal power efficiency calculated by available head (-)
$f$  Bucket volume fill height (-)
$g$  Gravitational constant (9.81 m/s²)
$G_w$  Gap width (m)
$h$  Total head across screw (m)
$h_f$  Friction head loss due to AST trough (m)
$h_o$  Lower basin water level above bottom of AST (m)
$h_{ue}$  Water height above the maximum bucket elevation point (m)
$L$  Screw length (m)
$N$  Number of helical plane surfaces (-)
$Re$  Reynolds Number (-)
$n_b$  Number of buckets in a screw (-)
$P$  Power (W)
$P_{loss, outlet}$  AST exit power loss (W)
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$p$  Hydrostatic pressure on the screw surfaces (Pa)
$P_{avail}$  Available power (W)
$P_w$  Wetted Perimeter (m)
$Q$  Total flow (m³/s)
$Q_b$  Bucket water flow rate (m³/s)
$Q_g$  Gap leakage flow (m³/s)
$Q_o$  Overflow leakage (m³/s)
$R_h$  Hydraulic Radius (m)
$S$  Screw pitch (m)
$T$  Torque created by a single water bucket (N·m)
$T_{tot}$  Torque created by all water buckets in a screw (N·m)
\( V \) Volume of water in a single bucket (m\(^3\))

\( V_{tot} \) Total volume of water in a screw (m\(^3\))

\( v_t \) Transport velocity (m/s)

\( \beta \) Screw angle of inclination (rad)

\( \omega \) Angular velocity of the screw (rad/s)

\( \rho \) Density of water (kg/m\(^3\))

\( \psi \) Relative height of water above the bottom point of AST (-)

\( \eta_T \) Torque loss efficiency coefficient (-)
Chapter 1: Introduction

Archimedes screw generators (ASGs) can be used in low-head hydro sites as a source of micro-hydroelectric power generation. ASGs are an adaptation of the Archimedes screw, a helical surface surrounding a central cylindrical shaft inside a hollow pipe that is used to carry water to higher elevations. Water at higher elevations can be translated down the length of the screw due to the hydrostatic forces from the fluid on the screws surfaces, the resulting rotation of the screw drives a generator that creates efficient, cost effective and clean electricity. Such a device is often referred to colloquially as an Archimedes Screw Turbine (AST), although the AST is actually a quasi-static pressure machine, rather than a true turbine.

![Figure 1-1 Typical Archimedes Screw Pump (Rorres, 2000)](image)

The past few years have seen several attempts to model various aspects of AST operation. Rohmer et al. (2015) developed a quasi-static numerical model to determine the ideal screw. Some more overall performance models (Lubitz at al. 2014, Dellinger et al. 2016) have been reported, in addition, Nuernbergk (2012) has authored a very good German-language textbook that provides a comprehensive treatment of ASTs.
The University of Guelph has 16 experimental laboratory ASTs, all with different parameters including length, pitch, number of flights, and inner and outer diameter. The laboratory set up allows for the control of screw rotational speed and input flow rate. In addition, the laboratory set up is able to output torque from the screw, as well as volume flow rate of water, and inlet and outlet water depths. Testing each screw at a number of fixed flow rates and speeds allows for the development of a dataset that provides a direct comparison for each changing variable.

This data set is then used to create a sensitivity analysis of a current mathematical MATLAB model used to predict mechanical power in the screw. Kozyn (2015) quantified some of these power losses by building off the Lubitz at al. (2014) model. Validation of gap leakage was performed experimentally, bearing losses and hydraulic frictional forces were quantified as translational and rotational stresses on the screw planes and shaft. Experimental outputs from the dataset are used to compare model sensitivity for changing variables.

Since the screws in the laboratory, and most real life ASTs, are encased in a solid trough, fill height of water in a screw and overflow cannot be visually confirmed. The current method for predicting overflow used by Aigner (2008), Nuernbergk and Rorres (2012) and Lubitz et al. (2014) takes volume of water in a bucket, if the volume of water exceeds the bucket size, a fill height above the central shaft water is assumed to flow over the central shaft as water would flow over a v-notch weir. The height of water inside the buckets at any given point is investigated using a static pressure transducer mounted to the bottom of the AST while water is flowing through it. Successful measurement of fill height is used to investigate how changing the inclination angle in the screw affects power and efficiency. The standard screw is set to an angle of 24.5°, the laboratory set up allows for variation of that angle.
Dynamic effects within the screw, such as frictional shear and the translating motion of the weir openings are neglected, potentially leading to discrepancies between the MATLAB model and actual AST overflow. Similarly, there exists slight geometry differences, namely the curved surfaces of the screw. Discrepancies between the model and real-world AST overflow leakage is investigated using the static pressure transducer. A statistical software SPSS is then used to enhance the current overflow equation to include a rotational speed component and amend discrepancies between experimentally calculated, and modelled overflow.

Each experiment allows for a better understanding of ASTs, and overall, aims to accurately predict the power output and efficiency bases on input geometry, volume flow rate and speed. The overall aim is to create a model that can be used in the installation of real-world ASTs, where site conditions can be input, and the ideal screw for that site can be created. As an increasing need for renewable energy grows, the Archimedes screw turbine is an efficient solution for low head hydro sites.
Chapter 2 : Literature Review

2.1 History and Benefits of Archimedes Screw Microhydro Power

Hydropower is considered to be a clean, renewable electricity generation method with low lifecycle greenhouse gas emissions. It has the added bonus of being highly efficient, accompanied by low maintenance costs (Canmet, 2008). There are several different approaches to hydropower, all of which convert potential and kinetic energy in water into electrical energy (Gulliver, 1991). A conventional hydropower-system includes dams, where water is diverted to turn a turbine which powers a generator at a power house. The power from a turbine depends on the incoming volume flow rate of water and the height difference above and below the turbine which is known as head. The amount of available potential energy is proportional to the head (OPG, 2014). A major drawback to conventional hydropower lies in the cost and resources required to plan and build a hydropower plant (Anderson, 2011).

Micro-hydro systems have an output of less than 100kW, and are generally used to supply electricity to small communities (Smith, 1995). Micro-hydro has a much less significant impact on the environment compared to its large hydropower project counterparts because of its ability to use already in place running water or streams instead of a giant reservoir like a dam would (Anderson D., 2011). Many low head dams already in place can be repurposed or restored. In addition, the head and capacity required to power these units are less than larger hydro units, ranging between 5 MW and 50 MW (Schleicher, et al. 2014). In Canada alone, there are an estimated 2000 sites with potential for low head hydro, encompassing sluice gates, irrigation canals, drinking water pressure release valves and municipal wastewater outfalls, as well as in numerous rivers. Unfortunately, low head micro-hydro projects end up being more expensive than large hydropower developments on a per megawatt basis (Canmet, 2008). However, these low
head sites are much more abundant than those for a conventional large hydro system. With the potential sites already in place, micro-hydro seems like a good renewable energy solution, starting with Archimedes Screws.

Archimedes has been accredited with the invention of a screw which lifts water for irrigation and drainage (287-212 B.C.) (Rorres, 2000). Although Archimedes did not reference this as his own invention in his works, many did so for him (Koetsier, 2004). It has been speculated that Archimedes reported knowledge from a trip to Egypt where he had been studying. Since there is no recorded description of water lifting screws prior to his time, and Archimedes was already well established for his inventive, mechanical and mathematical abilities, the name holds as Archimedes Screw.

2.1.1 Archimedes Screw Geometry

An Archimedes' screw consists of a helical surface surrounding a central cylindrical shaft inside a hollow pipe. When used as a pump, the screw is usually turned by a generator or manual labour. As the shaft turns, the bottom end scoops up a volume of water referred to as a bucket. This water will slide up in the spiral tube as the screw turns, until it finally pours out from the top of the screw. The screw pump was used mostly for draining water out of mines or other areas of low lying water. The open troughs and overall design of the screw permits the passage of debris without clogging.

Fig 2-1 shows the incorporated geometries, which includes the outer diameter ($D_o$), inner diameter ($D_i$), pitch ($S$), number of flights ($N$) and length ($L$), and the slope of the central axis relative to the horizontal ($\beta$).
2.1.2 Archimedes Screw Turbines

Archimedes screw turbines (ASTs) can be used at low-head hydro sites as a means of generating electricity. This is accomplished by running the Archimedes screw in reverse, that is, dropping water from the top and allowing the screw to rotate as water descends. This is an economical and efficient way to generate electricity from small streams (Chang K., 2013). The main screw sits in a trough which translates buckets of water down the length of the screw. The screw spins and generates electricity due to hydrostatic pressure from the water on the screws surfaces. As water fills the screw from the inlet at the top of an incline, the pressures on the helical planes of the screw allows for rotation of the screw, this leads to translating buckets of water down the length of the screw to and outlet that is at a lower elevation. There must be a gap between the edge of the screw and the trough, to allow the screw to turn without catching the side, and therefore some water leaks through these gaps. The gap between the helical flights and the trough allows for water drainage when the screw is not turning (Rorres, 2000).

ASTs have many advantages. Unlike most micro-hydro systems, small debris as well as small aquatic creatures can pass through a rotating ASG without much harm to either while
maintaining the function of the ASG itself. In the aquatic industry, Archimedes screws pumps have been utilized for the movement of fish. A study of juvenile salmon in California showed more than 98% of fish passed through the screw unharmed (Mcnabb, et al., 2003). One of the most vital advantages to an AST, is its efficiency. A survey of AST installations in Europe found a mean operational efficiency of 69%, with maximum efficiencies over 75% (Hawle et al., 2012).

In Ontario specifically, there are many potential low head hydro sites where an ASG installation could improve an unused dam and turn them into a worth-while, efficient energy source. Potential AST sites are owned by private landowners, the mining industry, the Ministry of Natural Researches, Ontario Power Generation, conservation groups and local municipalities (CVC, 2011). Figure 2-2 shows approximately 280 sites suitable for ASTs in the province of Ontario, with a total potential for 16 MW of power generation (Kozyn, et al. 2015).

2.2 AST Experimental Research

Some of the first AST experiments were conducted by Brada in the 1990s (Brada, 1999). Since then almost 400 AST units have been installed across Europe, with one in North America, which is grid-connected (Lashofer et al., 2012).
The first attempt to model the power output of an Archimedes Screw turbine, used a simplified two dimensional geometry of the screws helical planes (Muller & Senior, 2009). Neglecting hydraulic energy losses and mechanical frictional losses from the rotational motion, the model was assumed to have steady-state flow conditions. Torque was assumed to be generated by water trapped in the buckets creating hydrostatic pressure force across the plane surfaces. In addition, leakage flow between the planes was accounted for by adapting Nagel's 1962 leakage model. This model seemed to have general agreement with Brada's initial experiments despite not being fully complete.

Lubitz et al. (2014) created a more complete model of the screws performance incorporating the three-dimensional geometry of the rotating screw. Many of the same energy losses were neglected due to the assumption of a steady-state flow, however, gap and overflow leakage were taken into account. This gap leakage model was assumed to be driven by static pressure of the water column above the gap, and the overflow leakage as a result of the inclined cylindrical shaft acting like a v-notch weir. Both leakage models were not yet sufficiently validated.

An attempt to quantify hydraulic losses at the inlet of ASTs was completed by Neurnbergk and Rorres (2012). By analyzing the inlet geometry and flow conditions, and applying a Borda-Carnot model, the upstream head of the inlet channel to the AST was predicted. Under steady state flow, the energy of the flow could be described using a one-dimensional Bernoulli energy equation. The only energy loss was a result of the changing inlet channel geometry and circular trough, which was modelled using the Borda-Carnot energy loss equation.

Originally, the Borda-Carnot energy loss relationship was created for turbulent, incompressible flow in a pipe. It was also derived assuming that rapid expansion of the fluid is
only due to pressure created from the geometry of a pipe (Massey and Ward-Smith, 1998). This relationship produces an energy loss that is proportional to upstream and downstream geometries. There is a general agreement between this equation and Brada's experiment. This model has also yet to be completely verified since losses from transitions are often minute, and inaccuracies in the model would not translate to the overall head prediction.

Another attempt to quantify intake head losses was by building on an analytic model was presented by Fergnani and Bavera, (2016). Looking at variable flow rates and head losses for a constant and variable speed turbine, a comparison was done to determine how head loss effects power output and efficiency. It was concluded that by varying the speeds and incoming flow rate of a screw, there is a cost-effective way to increase efficiency of the screw.

Many frictional power losses have not been examined or incorporated into AST models. For a typical hydro turbine, mechanical, hydropower, impact power and volume power losses have been analyzed (Zeng et al., 2010). In the AST model, mechanical friction has not been incorporated. However, volume power loss corresponding to the leakage losses of an AST has been quantified (Lubitz et al., 2014; Nagel, 1968; Rorres, 2000).

It is important to quantify water impact losses and internal fluid friction losses. Water impact losses are a result of water impacting the leading helical plane while entering the screw. Internal fluid friction is the result of shear stress from the trough and turbine blades. Fluid is moving into an open-channel or pipe, so there would be losses from the friction of the channel or pipe walls. Frictional energy losses along channels or pipes are generally modelled using the Darcy-Weisbach head loss factor (Larock et al., 1999). The Darcy-Weisbach head loss is proportional to the length of the channel, or pipe and flow rate squared, and inversely proportional
to the cross-sectional area. However, the quantification of the hydraulic friction energy losses were not included in any of the early AST models.

Rohmer et al. (2015) developed a quasi-static numerical model to determine the ideal screw. A study was performed on a prototype Archimedes screw that delivered a torque of 250 Nm and had a 0.84 m diameter. The model attempted to correctly predict the efficiency and energy production of the screw based on its rotational speed, geometry and fill height. The model included leakage, friction and overfilling losses. It was concluded that frictional losses change with the size of a screw, and friction losses should be scaled using Darcy-Weisbach friction factor.

Kozyn (2015) quantified some more power losses by building off the Lubitz et al. (2014) model. A validation of gap leakage was performed experimentally, and found that gap leakage models were able to reasonably estimate gap leakage values although there is some type of pumped leakage that needs to be investigated. Other losses tests on a laboratory sized screw were validated against a 7 kW grid-connected ASG. The losses that were investigated included; outlet exit, hydraulic frictional, bearing friction and outlet drag torque power loss. The Borda-Carnot relation was found to reasonably identify power losses at the outlet of the screw with an average of 4.47% of the total power losses. The bearing losses were measured on a dry model and increased with rotating speed. The hydraulic frictional forces were quantified as translational and rotational stresses on the screw planes and shaft. Finally the submergence of the screw outlet was studied. It was found that the deeper the screw outlet is submerged, the greater the power loss at increasing rotational speeds. Even with the current power losses accounted for, there are further refinements remaining, and still more losses to investigate.

Recently there has been increased use of computational fluid dynamics (CFD) to simulate ASTs. Shimomura and Takano (2013), introduce moving particle simulation (MPS) to analyze the
open channel conditions of an AST with a 3D numerical model. Some more attempts to model ASTs using CFD are reported in Schleicher (2012) who used ANSYS CFD tools in order to find the more efficient point for certain model screws. All models displayed a rotational torque distribution. These models were able to show static and dynamic pressures throughout the screw are what drives the screws rotation and therefore, have the capacity to account for some losses.

Waters (2015) also used CFD in order to compare output torque of a variety of ASTs with varying diameter ratio, length, pitch, and number of blades. For each CFD run, the outer diameter of the screw remained at 0.2 m and ran at a speed of 50 RPM, the head difference was varied between 0.2 m and 1.2 m. The CFD results were compared so some experimental samples and an ideal model from Lyons (2014) for confirmation of certain trends. The diameter ratio had a large effect on the performance of the AST, a decrease in diameter ratio increased the torque production. This is theorized to be correct since a smaller inner diameter would allow for a larger volume of water to fill a screws buckets, and give more pressure to turn the screw. As length of the screw increased, the torque did not vary within each individual bucket because the head difference remained constant. Even though a longer screw meant more overall buckets, and more opportunity to create power, there was also the introduction of more friction losses with a longer screw. Ideally, there is a peak value where the screw is long enough to create the most power, but without developing high friction losses. As pitch ratio increased, so did the torque, a larger volume of water was able to provide pressure on a single blade to rotates the screw. In addition, with more water in a bucket, rather than multiple single buckets, there will be less opportunity for friction loss contact. Finally the number of blades had little effect on the torque output for the screw, overall, more flights created less torque. By introducing more blades, even though more buckets are created, the thickness of the screws blades takes away overall water volume for the screw.
Dellinger et al., (2016), included losses in a simulated AST experiment using the open source CFD model OpenFOAM. One set of simulations was run with a fixed rotational speed and changing flow rate, and the other with variable rotational speed and a fixed flow rate. The simulations were compared to experimental results which showed that water at the outlet of the screw had an effect on screws performance. It was also used to verify that losses such as gap leakage and overflow can be modelled using CFD.

Additional losses in the screw includes; bearing losses, inlet losses and outlet losses. These losses are subject to specific AST set ups (Figure 2-3) and are summarized by Fergnani et al. (2016).

![Figure 2-3 AST Power loss summary (Fergnani et al., 2016)](image)

2.3 Literature Review Summary

There are a number of numerical AST power models, with more recent development of CFD models that are being used to predict power outputs of an AST. Although these models are physics-based, most are not complete with power losses that are necessary to accurately predict all the screws effects. By enhancing the Kozyn (2015) model, which already includes; gap leakage, overflow, and simplified inlet and outlet losses, a more complete model can be compared to experimental results.
Chapter 3 : Research Problem Formulation

3.1 Objectives

There are still many unknowns when it comes to quantifying AST power losses. ASTs are very dynamic, and understanding the inner workings and surroundings of a screw will greatly aid in determining the ideal screw style and its running conditions. ASTs are an efficient, environmentally conscious source of hydro power, but it is necessary to develop such a model in order to optimize the design of an AST for a specific location, flow and head. The objective of this study is to break down the screw into some of its individual components in order to enhance the current model. The research project reported in this thesis specifically sets out to:

- Experimentally determine how a screw geometry affects power and efficiency
- Use a sensitivity analysis to understand how a model predicts output efficiency
- Determine the fill height inside a screws bucket by using a pressure transducer
- Investigate how screw inclination angle affects power output and efficiency
- Improve overflow prediction equations using results from the pressure transducer

By analyzing the geometry of the screw, and how water enters, fills, surrounds, and exits the screw predictive models can be enhanced. The basis of this analysis includes taking a set of experimental measurements on a set of screws with different geometry, rotational speed, and incoming flow rate. Many of these factors can be assessed and compared to determine the ideal screw conditions, improve the current model and understand the inner workings of an AST.

This would be an asset when it comes to the installation of real life ASTs as a source of renewable energy. As a relatively new technology, ASTs combine the benefits of environmental
friendliness, robustness, and efficiency which can be utilized at low-head hydro sites across Ontario.

3.2 Geometry

The ideal model, referred to as fh3, uses the geometry of an AST along with flow rate input in order to predict results such as power losses and fill height (Lubitz et al., 2014). Archimedes screws are described as a set of helicoid planes fastened to a central shaft. The screw is usually housed in a closely-fitted enclosure or trough. A gap exists between the trough and the planes, allowing for free rotation of the screw. Fig 3-1 shows the geometry of the screw, which can be defined, based on the outer diameter ($D_o$), inner diameter ($D_i$), pitch ($S$), number of flights ($N$) and length ($L$). The slope of the central axis relative to the horizontal ($\beta$) and angular rotational speed ($\omega$) are also important in properly predicting power.

![Three flighted Archimedes Screw](kozyn2015)

Using these parameters, a model can predict the volume of water in each of the buckets, water that’s entrapped between two helical planes. This, together with the length can be used to find the ideal power and efficiency of a screw.
Another key parameter is the fill height of water in a screws bucket, which is the ratio of water depth and maximum available water depth in a bucket (Lubitz, et al. 2014). The difference between $z_{min}$ and $z_{max}$ is used as the overall water level needed for a full bucket.

$$f = \frac{z_{wl} - z_{min}}{z_{max} - z_{min}}$$  \hspace{1cm} (3-1)

The current model uses cylindrical coordinates to determine values of the fill height. The $z_{min}$ is the lowest point on the helical plane downstream while $z_{max}$ is the “maximum” water level, defined as the fill level where the water level in the bucket intersects with the base of the downstream flight at the top of the inner cylinder ($\theta = 2\pi$), where $f = 1$. The water level is the point where the bucket is actually filled to, an illustration is shown in Figure 3-2.

![Figure 3-2 Water level in bucket description (Kozyn, 2015)](image)

Mathematically, the locations of $z_{min}$ and $z_{max}$ are determined using cylindrical coordinates. The vertical depth is a projection of physical locations on the helical plane surfaces to $z$ which is oriented in Cartesian axis. $w$ is the axis down the center of the shaft which is on an angle $\beta$ from the horizontal.
Ultimately, in order to determine the fill height of water in the screw, the volume of the bucket needs to be determined, based on the incoming flow rate. The volume of water in the bucket assumes the first leading helical plane edge is at the top of the screw. Along $w$, the radial and angular positions of the leading planes are defined:

$$r(w) = r$$  \hspace{1cm} (3-2)

$$\theta(w) = 2\pi \left( \frac{\omega}{S} \right)$$  \hspace{1cm} (3-3)

This means, at any point $(r, \theta)$ the leading helical plane surface $z_1$ and the upstream helical plane, $z_2$ are described as:

$$z_1 = r \cos(\theta) \cos(\beta) - \frac{S\theta}{2\pi} \sin(\beta)$$  \hspace{1cm} (3-4)
\[ z_2 = r \cos(\theta) \cos(\beta) - \left( \frac{S \theta}{2\pi} - \frac{S}{N} \right) \sin(\beta) \]  

(3-5)

This information is used to determine the minimum and maximum points in a bucket which are at 
\[ \theta = \pi, r = \frac{D_o}{2} \text{ and } \theta = 2\pi, r = \frac{D_i}{2}. \]

\[ z_{\text{min}} = \frac{-D_o}{2} \cos(\beta) - \frac{S}{2} \sin(\beta) \]  

(3-6)

\[ z_{\text{max}} = \frac{D_i}{2} \cos(\beta) - S \sin(\beta) \]  

(3-7)

Therefore, the water level in the screw can be described in the vertical direction in terms of fill height:

\[ z_{\text{wl}} = z_{\text{min}} + f(z_{\text{max}} - z_{\text{min}}) \]  

(3-8)

From here, the volume of a bucket can be described as the geometry is shifted down the \( w \)-axis of the shaft.

\[ V = \int_{r=D_i/2}^{r=D_o/2} \int_{\theta=0}^{\theta=2\pi} dV \]  

(3-9)

\[ dV = \begin{cases} 
0 & z_2 > z_{\text{wl}}, z_1 > z_{\text{wl}} \\
\frac{z_{\text{wl}} - z_1}{z_2 - z_1} \frac{S}{N} r dr d\theta & z_2 \geq z_{\text{wl}}, z_1 \\
\frac{S}{N} r dr d\theta & z_2 < z_{\text{wl}}, z_1 > z_{\text{wl}} 
\end{cases} \]  

(3-10)

The volume is also used to calculate the flow of water through the bucket.
\[ Q_b = \frac{NV\omega}{2\pi} \]  

3.3 Power Calculations

In order to predict power and efficiencies for a model, the torque experienced by the helical planes of the screw is calculated. As buckets of water fill, the pressure of water on the planes are what drives the screw and generates torque. For a static case, as used in the model, the pressure in the Lubitz et al. (2014) model can be calculated on the helical surface:

\[ p = \rho g(z_{wl} - z) \]  

The pressure difference between the upstream and downstream portion of the screw can then be used to determine the torque for the surface area and used to determine the torque for a full bucket.

\[ dT = (p_1 - p_2) \frac{S}{2\pi} rd\theta \]  

\[ T = \int_{r=D/2}^{r=D_0/2} \int_{\theta=0}^{\theta=2\pi} dT \]  

The total torque is adjusted for a particular buckets in a specific screw.

\[ T_{total} = T \left( \frac{NL}{S} \right) \]  

This is used to find the output power for a screw.

\[ P = \omega T_{total} \]  

Using the output head, the efficiency of the screw is found using the available head in the screw and the flow rate through the screw.

\[ P_{max} = \rho g L Q \sin \beta \]
\[ \eta = \frac{P}{P_{\text{max}}} \]  

(3-18)

3.4 Power Loss Calculations
3.4.1 Gap and Overflow Leakage

Lubitz et al. (2014) includes the losses from two flow paths when water enters an AST. Gap leakage occurs due to the separation of the screws helical planes and trough. This allows free rotation of the screw, and drainage of some water through the resulting gap. The Lubitz et al. (2014) model calculates gap leakage using a pressure driven equation that takes the static pressure across the gap based on the height of water between buckets. The resulting equation for leakage flow due to the gap is:

\[ Q_g = C G_w (l_w + \frac{l_e}{1.5}) \sqrt{\frac{2gS}{N \sin \beta}} \]  

(3-19)

Where \( g \) is the gravitational constant 9.81 m/s\(^2\), \( G_w \) is the gap width, \( l_w \) is the portion of the gap that is submerged under water and \( l_e \) is the wetted portion upstream from the incoming bucket. Figure 3-4 visually describes the difference between the upper and lower bucket.
Currently, all performance models that include overflow have utilized the same single-equation model. The basis of this model is that AST overflow is analogous to steady-state flow through a triangular weir or spillway. Relationships for this type of weir flow have been empirically derived through dimensional analysis and experimental data. The local geometry at the top surface of an angled screw shaft between the adjacent helical planes is approximately triangular suggesting this model might be appropriate. The overflow leakage \( Q_o \) was expressed by Aigner (2008) and used by both Nuernbergk and Rorres (2012) and Lubitz et al. (2014):

\[
Q_o = \frac{4}{15} \mu \sqrt{2g} \left( \frac{1}{\tan \beta} + \tan \beta \right) (z_{wl} - z_{max})^{\frac{5}{2}}
\]

(3-20)

where \( \beta \) is the angle of the screw in radians, and \( g \) is the gravitational constant 9.81 m/s\(^2\). As in most weir models, the flow over the central shaft is proportional to the height of water above the shaft to a power (in this case 2.5), with a constant \( \mu = 0.537 \), although Rohmer et al. (2016) elected to use a different value of constant \( \mu = 1.09 \). The values of \( z_{wl} \) and \( z_{max} \) are the height of water in a
bucket and the maximum height before overflow begins (f=I). Values for \( z_{\text{min}} \) and \( z_{\text{max}} \) can be calculated theoretically based on the geometry of the screw.

Since the basis of the model is steady-state flow through a triangular weir, dynamic effects within the screw, such as frictional shear and the translating motion of the weir openings are neglected, potentially leading to discrepancies between this model and actual AST overflow. Similarly, there exist slight geometry differences, namely the curved surfaces that might create further discrepancies between these models and real-world AST overflow leakage.

The leakages combined with the bucket flow comes together to create the total flow rare through the AST through its three potential flow paths: (1) flow contained between adjacent helical planes (within the so-called “buckets” formed by the screw), (2) gap flow that occurs between the planes and the trough, and (3) overflow that occurs when the water level in the screw is high enough that water can flow over the central shaft.

\[
Q = Q_b + Q_g + Q_o
\]  

Equations (3-1) to (3-21) summarize the Lubitz et al. (2014) model and is defined as fh3. This model includes power losses from gap leakage and overflow but does not account for other losses that affect the power output of a screw. Some of these additional losses are accounted for in the new model, known as fh5, which includes Equations (3-22) to (3-29), described below.

3.4.2 Hydraulic Friction Power Losses

As water translates down the length of the screw, there are power losses due to the contact between the water and the screw. These losses stem from the translational and rotational nature of the screws movement, and can be subcategorized into drag and rotational direction components.
Furthermore, there are two more ways to separate these losses, which is comparing the frictional loss with the stationary trough, and the central cylindrical shaft and blades.

First focusing on transportation, the Darcy-Weisbach head loss equation for open channel flow is introduced to describe water as it moves from the inlet of the screw to the outlet, aided by gravity and the screws incline.

\[
h_f = f_{DW} \frac{L}{4R_h} \frac{v^2}{2g} \tag{3-22}
\]

Where \( h_f \) is the head loss, and \( v \) as the fluid velocity. \( R_h \) is the hydraulic radius calculated using the cross sectional area and the wetted perimeter \( A/P_w \). The area of the central shaft and the trough were calculated sparely. As water moves down the length of the screw, there is a frictional force created between the screws surface and the water which is where the Darcy-Weisbach friction factor \( f_{DW} \) is used.

\[
f_{DW} = \frac{8gn^2}{R_h^{1/3}} \tag{3-23}
\]

The friction factor used is from Manning’s equation friction factor \( (n) \) due to the comparison of an AST to open channel flow. These values can be converted into a shear stress and multiplied by the number of buckets in a screw along with the wetted areas for the screws trough and central shaft to find frictional force (Kozyn, 2015).

\[
F = f_{DW} \frac{\rho v_t^2}{8} n_b A \tag{3-24}
\]
The overall power loss from the walls is a function of the head loss and flow rate in the screw.

\[ P_{\text{Loss,transport}} = \rho g Q h_f \]  

(3-25)

Next, the rotational power losses come from the rotational nature of an AST. Here, the screws blades rotate in order to trap water into buckets and move them down the length of the screw. Since the central shaft and the blades rotate together, their areas can be separated in order to calculate their losses with more accuracy. The friction from the rotating blades is the dominant loss due to the difference between the rotation of the screw and the transport velocity. Calculating the ideal power loss is much more complicated, due to water in the screw rocking back and forth from blade rotation, uneven distribution of shear stress and general incoming turbulence at the screws inlet.

Starting with the central shaft, the velocity of the central shaft and rotational motion is \( v_{r,c} = r_i w \). The shear stress is calculated using the same component in equation (3-24), but the Darcy-Weisbach friction factor uses \( v_{r,c} \) and \( R_h \) for the central shaft. The resulting power loss equation for the central shaft can then be determined:

\[ P_{\text{Loss,rotation,shaft}} = f_{DW} \frac{\rho v_{r,c}^2}{8} n_b A \]  

(3-26)

The blades are the most difficult challenge, the frictional force of the blades oppose the motion of the flow of water. Figure 3-5 shows a simplified diagram of the shear stress in relational to the rotational screw along the blade.
Figure 3-5 Point on helical plane showing shear stress

The velocity between the blades and the water is not the same, instead the blades are moving in both the radial and orthogonal direction. Since the radial direction is opposing, this is the component to focus on. The power loss for the blades is found by integrating over the entire length of the wetted helical plane and separating the upstream and downstream blades. The final power loss equation uses the previous Darcy-Weisbach friction along with wetted radii from the upstream and downstream positions by their wetted areas.

\[ P_{\text{Loss,blade}} = n_b f_{\text{DW}} \frac{\rho \omega^3}{8} (r_1^3 A_1 + r_2^3 A_2) \]  

(3-27)

These losses are still very simplified and do not include all the intricacies in a fully functioning AST.

3.4.3 Outlet Exit Power Loss

Some screws are submerged at the outlet, this means that water does not flow openly out of the outlet, instead, the bottom buckets are cut off by incoming water. When there is a depth of water at the outlet of the screw, the screw is essentially shortened, meaning there are less buckets
and a power loss. The height of water above the base of the outlet is defined as a ratio in the z-axis:

\[ \psi = \frac{h_o}{D_o \cos(\beta)} \]  

(3-28)

Where \( h_o \) is the actual level of water above the bottom of the screw, compared with the outer diameter of the screw. In addition to higher water levels shortening the length of a screw, water being introduced into the outlet also creates back pressure and drag torques at the outlet of the screw meaning that power losses should theoretically increase as \( \psi \) increases.

The power loss due to higher levels at the outlet is calculated using relationship due to changing head, and does not take back pressure and drag torques into account. The power loss uses the water level at the outlet, \( h_o \), and the output power in the screw to calculate the loss at the exit.

\[ P_{Loss, outlet} = \frac{h_o}{L \sin(\beta) + D_o \cos(\beta)} P \]  

(3-29)

3.5 MATLAB Model

The current model, fh5, is implemented using numerical solutions in MATLAB due to the complexity of the screws geometry. The model is a combination of the Lubitz et al. (2014) which was adapted by Kozyn (2015) to incorporate losses such as gap, overflow, bearing loss, rotational and translational losses. The model, fh5, runs through an iterative process where the flow rate through the screw is input and compared to potential fill heights. The flow rate for a screw is input along with the screws dimensions; the screws angle (\( \beta \)), pitch (S), number of flights (N), inner diameter (\( D_i \)), outer diameter (\( D_o \)), length (L), flow rate (Q), rotational speed (\( \omega \)), gap width (\( G_w \)) constant C, outlet water level (\( h_o \)), radial and angular steps for numerical integration described in
Table 3-1. The model assumes that water is at a steady state as it translates down the length of the screw, experimental and real-life observations visually display a much more turbulent scenario.

This model uses the following inputs in Table 3-1:

<table>
<thead>
<tr>
<th>Parameter in Thesis</th>
<th>MATLAB Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Beta</td>
<td>Inclination angle (rad)</td>
</tr>
<tr>
<td>S</td>
<td>S</td>
<td>Pitch (m)</td>
</tr>
<tr>
<td>N</td>
<td>N</td>
<td>Number of helical planes</td>
</tr>
<tr>
<td>Di</td>
<td>Di</td>
<td>Inner Diameter (m)</td>
</tr>
<tr>
<td>Do</td>
<td>Do</td>
<td>Outer Diameter (m)</td>
</tr>
<tr>
<td>L</td>
<td>l</td>
<td>Screw length (m)</td>
</tr>
<tr>
<td>Q</td>
<td>Q</td>
<td>Flow rate (m³/s)</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>omega</td>
<td>Angular Velocity (rad/s)</td>
</tr>
<tr>
<td>$G_w$</td>
<td>Gw</td>
<td>Gap width (m)</td>
</tr>
<tr>
<td>C</td>
<td>C</td>
<td>Leakage Coefficient (-)</td>
</tr>
<tr>
<td>ho</td>
<td>owl</td>
<td>Height of water at the outlet (m)</td>
</tr>
<tr>
<td>-</td>
<td>rsteps</td>
<td>Radial step size for numerical integration</td>
</tr>
<tr>
<td>-</td>
<td>thetasteps</td>
<td>Angular step size for numerical integration</td>
</tr>
</tbody>
</table>

Model outputs are listed in Table 3-2.

<table>
<thead>
<tr>
<th>Parameter in Thesis</th>
<th>MATLAB Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V$</td>
<td>V</td>
<td>Volume of a single bucket (m³)</td>
</tr>
<tr>
<td>$T_{tot}$</td>
<td>Ttot</td>
<td>Total screw Torque (Nm)</td>
</tr>
<tr>
<td>$P$</td>
<td>P</td>
<td>Total mechanical power (W)</td>
</tr>
<tr>
<td>$P_{avail}$</td>
<td>pavail</td>
<td>Total power available in flow (W)</td>
</tr>
<tr>
<td>-</td>
<td>eff</td>
<td>Power efficiency</td>
</tr>
<tr>
<td>$Q_b$</td>
<td>Qb</td>
<td>Bucket flow (m³/s)</td>
</tr>
<tr>
<td>-</td>
<td>H</td>
<td>Overall head across the screw (m)</td>
</tr>
<tr>
<td>$n_b$</td>
<td>nbuckets</td>
<td>Total number of buckets in the screw</td>
</tr>
<tr>
<td>$T$</td>
<td>T</td>
<td>Torque from a single bucket (Nm)</td>
</tr>
<tr>
<td>$P_{avg}$</td>
<td>pavg</td>
<td>Average wetted perimeter pressure (Pa¹/²)</td>
</tr>
<tr>
<td>$l_w$</td>
<td>lw</td>
<td>Gap wetted perimeter (m)</td>
</tr>
<tr>
<td>$L_e$</td>
<td>le</td>
<td>Gap perimeter wetted only on upstream side</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td></td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
<td></td>
</tr>
<tr>
<td>-</td>
<td>hue</td>
<td>Height of water above 100% fill level (m)</td>
</tr>
<tr>
<td>(Q_l)</td>
<td>Ql</td>
<td>Gap leakage flow (m$^3$/s)</td>
</tr>
<tr>
<td>(Q_o)</td>
<td>Qo</td>
<td>Overflow leakage flow (m$^3$/s)</td>
</tr>
<tr>
<td>(Q)</td>
<td></td>
<td>Total flow (m$^3$/s)</td>
</tr>
<tr>
<td>(P_{L,T})</td>
<td>(P_{Loss,T})</td>
<td>Translational hydrostatic friction power loss (W)</td>
</tr>
<tr>
<td>(P_{L,R})</td>
<td>(P_{Loss,R})</td>
<td>Rotational hydrostatic friction power loss (W)</td>
</tr>
<tr>
<td>(P_{L,o})</td>
<td>(P_{Loss\ Outlet})</td>
<td>Power loss at outlet (W)</td>
</tr>
<tr>
<td>(P_{Shaft})</td>
<td>(P_{Shaft})</td>
<td>Power at shaft with losses (W)</td>
</tr>
</tbody>
</table>
Chapter 4: Experimental AST Comparison

4.1 Experimental Methodology

The University of Guelph Archimedes screw testing system was used for these experiments. This system allows for changes in screw parameters that are not easily accessible in full-scale ASTs, including; slope, upper and lower basin depths, incoming flow rate, rotational speed, and screw geometry.

The AST is situated between an upper basin that supplies water to the screw, and a lower basin that receives water from the screw outlet. Water is circulated through the testing system using a variable speed electric pump and controller that allows precise adjustment of volume flow rate. An Omega FTB740 flow meter in the supply line from the pump continually measures total flow through the system. Since all water passes through the screw, this is a direct measurement of
total flow, $Q$. Water is supplied to the AST upper basin via a secondary, upstream basin containing two 15.2 cm Cipoletti weirs that provide a second independent measurement of total flow. Water depths in all basins are measured using Keller depth gauges within stilling wells.

Screw speed is controlled by a variable frequency drive (VFD) which also functions as a brake. Fill height within the screw is a function of flow and screw rotation speed. Screw rotation speed is measured continuously with a non-contact magnetic switch-based tachometer, and confirmed with a handheld optical tachometer.

A small hole (3mm diameter) that serves as a static pressure port is drilled into the bottom of the middle of the trough of most screws. This port can be fitted with a pressure sensor (Omegadyne PX309). Water depths, rotation speed and pressure are recorded continuously using a National Instruments USB-6009 data acquisition system connected to a computer running a custom-written NI LabView program. Sensors are sampled at high frequency (1000 Hz) and averaged (or totaled in the case of speed) over a one minute interval.

Torque measurements from the screw were determined using an Omegadyne LC703-25 load cell fasted to the screw. One end is attached to a fixed point on the screw support frame, while the other is fastened to an arm rigidly attached to the VFD. This arm is the only mechanical connection that prevents rotation of the VFD. The load cell is installed as a moment distance of 26.5 cm from the screw axis of rotation (Figure 4-2), the rotation of the screw was converted into a torque. From this, the mechanical power at the shaft is determined using the measured torque and angular velocity of the screw.

$$P = \omega \tau$$  \hspace{1cm} (4-1)
4.2 Experimental Procedure

Each of 16 experimental ASTs (Table 4-2) were run at five different flow rate; 6 L/s, 8 L/s, 10 L/s, 12 L/s and 14 L/s, and six different rotational speeds; 20, 30, 40, 50, 60, 80 RPM over outlet fill heights of 0%, 30% and 60% full (Table 4-1). The outlet fill height is defined as depth of water in the lower basin in reference to the outer diameter of the screws outlet, $h_o$. 

![Outlet fill height](image)
For example, at 0%, the water in the lower basin is not high enough to enter the outlet of the screw, at 30% the water level is high enough that it engulfs 30% of the screws outer diameter at the outlet, followed by 60% for the final case. Most screws operate with an outlet water depth close to 60%.

The outlet water level for the screw is measured by taking the angle of the screw and determining the vertical projection of water depth for cases that were 0%, 30% and 60% of the screws outer diameter at the outlet describes in Figure 4-4.

Table 4-1 summarized the experimental parameters and Table 4-2 introduces each experimental screw and its dimensions.

<table>
<thead>
<tr>
<th>RPM</th>
<th>20, 30, 40, 50, 60, 80</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flow Rate (L/s)</td>
<td>6, 8, 10, 12, 14</td>
</tr>
<tr>
<td>Outlet height</td>
<td>0%, 30%, 60%</td>
</tr>
</tbody>
</table>

*Table 4-1 Experimental Parameters*

*Table 4-2 Dimensions of 16 experimental screws*
<table>
<thead>
<tr>
<th>Screw</th>
<th>OD (cm)</th>
<th>ID (cm)</th>
<th>S (cm)</th>
<th>L (cm)</th>
<th>N</th>
<th>ID/OD</th>
<th>S/L</th>
<th>L/S</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>31.58</td>
<td>16.83</td>
<td>44.45</td>
<td>121.92</td>
<td>3</td>
<td>0.53</td>
<td>0.36</td>
<td>2.74</td>
</tr>
<tr>
<td>#2</td>
<td>31.62</td>
<td>16.83</td>
<td>31.75</td>
<td>121.92</td>
<td>3</td>
<td>0.53</td>
<td>0.26</td>
<td>3.84</td>
</tr>
<tr>
<td>#3</td>
<td>31.67</td>
<td>16.83</td>
<td>25.40</td>
<td>121.92</td>
<td>3</td>
<td>0.53</td>
<td>0.21</td>
<td>4.80</td>
</tr>
<tr>
<td>#4</td>
<td>31.69</td>
<td>12.70</td>
<td>31.75</td>
<td>121.92</td>
<td>5</td>
<td>0.40</td>
<td>0.26</td>
<td>3.84</td>
</tr>
<tr>
<td>#5</td>
<td>31.66</td>
<td>12.70</td>
<td>31.75</td>
<td>121.92</td>
<td>4</td>
<td>0.40</td>
<td>0.26</td>
<td>3.84</td>
</tr>
<tr>
<td>#6</td>
<td>31.62</td>
<td>12.70</td>
<td>31.75</td>
<td>121.92</td>
<td>3</td>
<td>0.40</td>
<td>0.26</td>
<td>3.84</td>
</tr>
<tr>
<td>#7</td>
<td>31.62</td>
<td>12.70</td>
<td>31.75</td>
<td>63.50</td>
<td>3</td>
<td>0.40</td>
<td>0.50</td>
<td>2.00</td>
</tr>
<tr>
<td>#8</td>
<td>31.57</td>
<td>12.70</td>
<td>31.75</td>
<td>40.64</td>
<td>3</td>
<td>0.40</td>
<td>0.78</td>
<td>1.28</td>
</tr>
<tr>
<td>#9</td>
<td>31.64</td>
<td>10.16</td>
<td>31.75</td>
<td>121.92</td>
<td>3</td>
<td>0.32</td>
<td>0.26</td>
<td>3.84</td>
</tr>
<tr>
<td>#10</td>
<td>31.61</td>
<td>10.16</td>
<td>44.77</td>
<td>52.07</td>
<td>4</td>
<td>0.32</td>
<td>0.86</td>
<td>1.16</td>
</tr>
<tr>
<td>#11</td>
<td>37.80</td>
<td>16.99</td>
<td>30.20</td>
<td>46.89</td>
<td>4</td>
<td>0.44</td>
<td>0.64</td>
<td>1.55</td>
</tr>
<tr>
<td>#12</td>
<td>37.69</td>
<td>16.89</td>
<td>30.40</td>
<td>61.39</td>
<td>4</td>
<td>0.44</td>
<td>0.50</td>
<td>2.02</td>
</tr>
<tr>
<td>#13</td>
<td>37.69</td>
<td>16.79</td>
<td>30.51</td>
<td>94.69</td>
<td>4</td>
<td>0.44</td>
<td>0.32</td>
<td>3.10</td>
</tr>
<tr>
<td>#14</td>
<td>38.20</td>
<td>16.99</td>
<td>38.30</td>
<td>46.61</td>
<td>4</td>
<td>0.44</td>
<td>0.82</td>
<td>1.22</td>
</tr>
<tr>
<td>#15</td>
<td>38.10</td>
<td>16.79</td>
<td>38.20</td>
<td>61.70</td>
<td>4</td>
<td>0.44</td>
<td>0.62</td>
<td>1.62</td>
</tr>
<tr>
<td>#16</td>
<td>38.61</td>
<td>16.89</td>
<td>38.30</td>
<td>94.89</td>
<td>4</td>
<td>0.44</td>
<td>0.40</td>
<td>2.48</td>
</tr>
</tbody>
</table>

Each experimental run resulted in approximately 30 measurements. For each measurement, the water level in the lower basin was measured using the depth gauge and manually confirmed with a ruler. Water was pumped through the screw at the desired flow rate, being checked with the flow meter and height of water passing over the Cipoletti weirs. The rotational speed of the screw was adjusted using the VFD and confirmed using a handheld tachometer and the magnetic sensor. Once the flow rate, fill height, and lower basin height were set to the desired combination, each sensor was recorded for 60 seconds.
Each screw started testing with the outlet fill height at 60%, 80 RPM rotational speed and 14 L/s flow rate. Each time the sensors were recorded, the rotational speed of the screw was adjusted, keeping the flow rate constant, and approximately two minutes was allocated to allow the system to return to steady state operation at the screws new testing conditions. Lowering of the rotational speed of the screw is repeated for each run, ending at 20 RPM, at which point, the flow rate of water going through the screw is reduced. Once the next desired flow rate is achieved, the rotational rate of the screw is increased incrementally, ending at 80 RPM. In order to increase the flow rate of water through the screw, water is introduced into the system with a hose, and pumped out to decrease the flow rate. After all combinations of flow rates and rotational speeds are recorded, water was drained from the lower basin so the outlet of the screw was 30% full. A new set of measurements is taken using the same procedure described above. The process was repeated a third time to get a set of measurements with the outlet of the screw unsubmerged (0% case).

4.3 Results and Discussion
4.3.1 0%, 30% and 60% Outlet Fill Heights
Increasing outlet water level generally resulted in lower power output for each of the 16 screws. More water at the outlet essentially shortens the screw by filling water into the lower buckets. There is no longer the maximum number of buckets available to turn and create power for the screw.

Figures in Appendix A visually summarize the experimental power outputs for each screw. In all cases, the top surface is power output for the 0% fill case across a range of rotation speed and flow rate. The intermediate surface is the 30% fill case, and the lowest surface is the 60% fill case. The 0% and 30% fill cases are closer in power than the 60% fill cases, which have the largest divergence especially at lower flow rates and higher speeds. Note that in some screws, the 14 L/s
flow rate was not tested due to excessive overflowing at this flow rate, and therefore these results not included in some of the contour plots.

The efficiencies of the screws as a function of outlet depth show the opposite trend. When the outlet basin is filled with water, it lowers the available head. The efficiencies for this analysis are calculated using available hydraulic head (the difference in elevation between the water surfaces in the upper and lower basins), as opposed to the geometric head, which is based on the length and slope of the screw that is normally used. Therefore, partially submerged screws become more efficient with increased outlet fill heights. To calculate maximum experimental efficiency, the head between the inlet of the screw and the outlet was recorded for each experimental run. The screw sat in a fixed frame with the difference between the height of the upper and lower basin as the precise vertical distance between the floors of the upper and lower basin. By taking the actual depth of water in the upper basin and subtracting the depth of water in the lower basin, plus the difference between the two buckets, the experimental head can be calculated and used to find the maximum power available to the screw:

\[ P_{\text{max}} = \rho g (0.60 + d_{UB} - d_{LB})Q \]  \hspace{1cm} (4-1)

where \( g \) is the gravitational constant 9.81 m/s\(^2\) and water density \( \rho \) is taken as 1000 kg/m\(^3\). In the results that follow, the experimental power (torque times rotation speed) is divided by this maximum power in order to calculate efficiency:

\[ \eta = \frac{P}{P_{\text{max}}} \]  \hspace{1cm} (4-2)
For a more detailed analysis, Screws 5 and 16 are investigated individually. This is because at first glance, the longer screws produce more power. These two screws are approximately within the average range of dimensional parameters for the 31.6 cm outer diameter screws (Screw 5) and the 38.6 cm outer diameter screws (Screw 16). A full error analysis for calculated power and efficiency based on experimentally measured values is shown in Appendix B. In the plots that follow, and in the appendix, the error bars for these screws are not included to maintain clarity in the plots. An average power uncertainty of 1.17 W was calculated, and average efficiency uncertainty was 2.8%.

Screw 5 has a flighted length of 121.92 cm, 4 flights, a 31.75 cm pitch and $Di/D_o$ of 0.4. Figure 4-6 shows the experimental power and efficiency of the screw for each speed and flow rate tested. As mentioned previously, maximum power is produced when there is no water at the outlet of the screw (top contour layer), as outlet water level increase, the power produced in each case decreases. In contrast, the experimental efficiency of the screw increases as outlet water level increases, for most cases.
The 6 L/s, 10 L/s and 14 L/s flow rates for Screw 5 results are summarized in Figure 4-7 for more clarity. As the speed of the screw increases, the power reaches a maximum point before beginning to decline. The point of maximum power shifts with changing flow rate; at lower flow rates, maximum power is reached at lower speeds, while at higher flow rates the maximum occurs at higher speeds.
The outlet water level had a clear effect on experimental power output. The 0% and 30% cases agree within uncertainty, and it isn’t until the 60% outlet water level case that these values do not agree within uncertainty. There are power losses associated with each outlet condition; the losses from free falling water, 0% case, and slightly submerged, 30% case, seem to even out. As rotational speed increased, so did the difference between power output from the 0% and 30%, compared to the 60%. At 60%, there is a back pressure on the outlet of the screw that reduces torque due to increased drag forces, and therefore the power of the screw. In addition, at speeds above 50 RPM, it was observed that water was being splashed out of the top of the screw and at the outlet, introducing new loss conditions that are not applicable at lower flow rates and speeds. It is reasonable to conclude that an unsubmerged, or only slightly submerged screw will produce the more power against one that is more than half way submerged at the outlet.
Figure 4-8 shows the trend in efficiency for 6 L/s, 10 L/s and 14 L/s at 0%, 30% and 60% fill. This relationship is not as clear as the power relationship, the point of maximum efficiency still changes with flow rate, however, at high speeds and low flows, the efficiency drastically drops for the 6 L/s, 0% case.

Overall, the efficiency of the screw increases as $h_o$ increases, this is because theoretical power is calculated using the available head in the screw, Equation (4-1). As the water level increases at the outlet of the screw, the head is shortened, and the theoretical maximum power is decreased. Once again the error bars are not included for figure clarity: the analysis in Appendix B produces an average estimated uncertainty of $\pm$ 2.8%. Even though more power is created at lower outlet fill heights, the higher potential for maximum power creates a less efficient screw. At extreme cases, like the 6 L/s case, at high speeds, the power created is so low, that even reducing the available head does not improve efficiency over the 0% and 30% cases. Also to note, since the
speed of a screw affects the amount of water in each bucket, and rotation of a screw relies on hydrostatic forces being exerted on the screws’ blades, it should follow that the maximum points of efficiency should be when these buckets are full (Neurnbergk, 2012).

Similar to Screw 5, Screw 16 also decreases in power as outlet fill height is increased. Screw 16 has a flighted length of 94 cm, 4 flights, 38.3 cm pitch and $D_i/D_o$ of 0.44. Figure 4-9 shows the resulting power and efficiencies at each flow rate and speed. There is an extreme case where the power in the screw becomes negative, at a rotational speed of 80 RPM and low flow rate of 6 L/s, this also has an effect on the relationship in efficiencies.

![Figure 4-9 Screw 16 power and efficiency contour plot for 0%, 30% and 60% fill](image)

Figure 4-9 Screw 16 power and efficiency contour plot for 0%, 30% and 60% fill

Once again the 6 L/s, 10 L/s and 14 L/s flow rates are analyzed in more detail to confirm the power trend summarized by Figure 4-10. The 0% and 30% do not agree within error as frequently as in Screw 5, this may be due to the larger outer diameter of this screw. The size of the outer diameter in this screw is 18% larger than Screw 5, the outlet affects are greater when the screw is slightly submerged. In addition, the maximum power is achieved at a lower RPM than the Screw 5 due to the difference in bucket sizes. Larger buckets require more flow or lower speeds
to reach their full level compared to smaller ones and since Screw 16 has larger buckets than Screw 5.

![Graph showing power output vs RPM for different fill levels and flow rates](image)

*Figure 4-10 Screw 16 0%, 30% and 60% fill for 6 L/s, 10 L/s and 14 L/s*

Overall, as the outlet water level increased, the power of the screw decreased and efficiency increased, the error bars were once again not included for clarity. When there was water present at the outlet of the screw it essentially lost a bucket which previously provided power, and shortened the screw. This lead to conclusion that the length of a screw should also have an effect on the screws power.

The differences in experimental efficiencies for Screw 16 at different flow rates and speeds are not are clear as the power outputs. The same trend that was observed with Screw 5, holds for Screw 16; as the outlet water level increases, the efficiency increases, with exceptions at lower flow rates.
Each 60% fill efficiency dropped off between 30 and 40 RPM, something that was seen with power in Figure 4-10. Mentioned previously, this screw has a larger outer diameter than Screw 5, meaning the size of the screw buckets is larger. It is also shorter than Screw 5, and created less overall power.

There are a set of screws with the same length, comparing all the screws with the same length can help give an initial estimate of how important other factors may be. Since it was shown that the 60% outlet condition consistently provides screws with the maximum power efficiency, and best represent real life ASTs, only the 60% outlet fill heights will be analysed for the remainder of this chapter.
4.3.2 Long Screws 121.9 cm

Screws 1, 2, 3, 4, 5, 6, and 9 have the same 121.9 cm flighted length, each differing in inner diameter, pitch or number of flights. At first glance, it seems as if all these screws are producing very similar power and efficiency outputs. In order to confirm this initial assumption, each screw was compared at the same flow rate (8 L/s). Only the results from the 60% case are shown in Table 4-3, the table includes the dimensions of each screw and the average power and efficiency across all flow rates, excluding 14 L/s, and rotational speeds. The 14 L/s case is not included because Screws 2 and 3 do not have a complete set of data at that flow rate.

![Table 4-3 Screws parameters for 121.9 cm](image)

<table>
<thead>
<tr>
<th>Screw</th>
<th>OD (cm)</th>
<th>ID (cm)</th>
<th>S (cm)</th>
<th>N</th>
<th>ID/OD</th>
<th>L (cm)</th>
<th>Average Power (W)</th>
<th>Average efficiency (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>31.58</td>
<td>16.83</td>
<td>44.45</td>
<td>3</td>
<td>0.53</td>
<td>121.92</td>
<td>26.43 ± 1.36</td>
<td>66 ± 3.1</td>
</tr>
<tr>
<td>#2</td>
<td>31.62</td>
<td>16.83</td>
<td>31.75</td>
<td>3</td>
<td>0.53</td>
<td>121.92</td>
<td>27.98 ± 1.39</td>
<td>67 ± 3.1</td>
</tr>
<tr>
<td>#3</td>
<td>31.67</td>
<td>16.83</td>
<td>25.40</td>
<td>3</td>
<td>0.53</td>
<td>121.92</td>
<td>27.37 ± 1.34</td>
<td>63 ± 2.9</td>
</tr>
<tr>
<td>#4</td>
<td>31.69</td>
<td>12.70</td>
<td>31.75</td>
<td>5</td>
<td>0.40</td>
<td>121.92</td>
<td>27.76 ± 1.39</td>
<td>67 ± 3.1</td>
</tr>
<tr>
<td>#5</td>
<td>31.66</td>
<td>12.70</td>
<td>31.75</td>
<td>4</td>
<td>0.40</td>
<td>121.92</td>
<td>27.80 ± 1.37</td>
<td>67 ± 3.0</td>
</tr>
<tr>
<td>#6</td>
<td>31.62</td>
<td>12.70</td>
<td>31.75</td>
<td>3</td>
<td>0.40</td>
<td>121.92</td>
<td>27.61 ± 1.39</td>
<td>67 ± 3.1</td>
</tr>
<tr>
<td>#9</td>
<td>31.64</td>
<td>10.16</td>
<td>31.75</td>
<td>3</td>
<td>0.32</td>
<td>121.92</td>
<td>27.86 ± 1.40</td>
<td>69 ± 3.2</td>
</tr>
</tbody>
</table>

Figure 4-12 only shows the 8 L/s case in detail since this is the flow rate where the maximum efficiencies occur for each screw.
Screws 2 and 3 reached their maximum power at 50 RPM, with Screw 3 reaching the highest maximum power out of all the screws. The remaining screws reached their maximum peak at 40 RPM. Screw 9 trended above the majority of the screws for each case, having the highest average power and efficiency. Generalizing, all the screws with the same length follow a similar trend for power as a function of rotational speed, and only vary by a maximum of 2 W. The next set of analysis will focus on one changing parameter at a time.

There are several sets of screws with the same physical dimensions except for varying length. These screws were all tested across the same range of fill heights and flow rates, and their output powers and efficiencies can be compared based on varied physical parameters.

4.3.3 Varied Lengths
Out of all 16 experimental screws, three sets have the same outer diameter, inner diameter, pitch, and number of flights, only differing in length. Theoretically, screws with longer lengths allow for more helical blades, which water can interact with and turn the screw, producing more
power. In real life, screws experience power losses due to drag effects each time water interacts with a surface on the screw, a longer screw means there is more potential for power and power losses. Screws with shorter lengths do not have as much length available for water to translate down the length of the screw and turn the screw, however, water is in contact with the screw for a shorter amount of time, and therefore, friction losses should be reduced. From this, it is theorized that although longer screws may produce more power, they may not necessarily be as efficient due to the introduction of more power losses.

The three sets of screws for this length analysis are summarized in Table 4-4 and include three sets of Screws; 6, 7 and 8, each have a three flights, a 31.75 cm pitch and inner outer diameter ratio of 0.40. Screws 11, 12 and 13, which have a 30.4 cm pitch, four flights, and an inner-to-outer diameter ratio of 0.44. Additionally, Screws 14, 15 and 16 have a pitch of 38.3 cm, four flights and an inner outer diameter ratio of 0.44. For each of these sets, the average power and efficiency for the tested flow rates and speeds, across each screw is compared to the changing length (Table 4-4). The average flow rate is 10 L/s and average speed 46.67 RPM. Between the three sets, an overall change in power and efficiency can be cross examined for the ideal combination of screw parameters.

<table>
<thead>
<tr>
<th>Screws</th>
<th>OD (cm)</th>
<th>ID (cm)</th>
<th>S (cm)</th>
<th>N</th>
<th>ID/OD</th>
<th>Lengths (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6,7,8</td>
<td>31.6</td>
<td>12.7</td>
<td>31.8</td>
<td>3.00</td>
<td>0.40</td>
<td>121.9, 63.5, 40.6</td>
</tr>
<tr>
<td>11,12,13</td>
<td>37.8</td>
<td>16.9</td>
<td>30.2</td>
<td>4.00</td>
<td>0.44</td>
<td>46.9, 61.4, 94.7</td>
</tr>
<tr>
<td>14,15, 16</td>
<td>38.2</td>
<td>16.9</td>
<td>38.3</td>
<td>4.00</td>
<td>0.44</td>
<td>46.1, 61.7, 94.9</td>
</tr>
</tbody>
</table>
First, Screws 6, 7 and 8 are compared based on their average power and efficiencies, followed by Screws 11, 12 and 13, and ending with 14, 15, and 16. Each set will be compared against the other at the end of the section.

![Image of Screws 6, 7, 8](image)

**Figure 4-13 Screws 6, 7, 8**

<table>
<thead>
<tr>
<th>Screw</th>
<th>L (cm)</th>
<th>Average Power (W)</th>
<th>Average Efficiency (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>121.9</td>
<td>30.17 ± 1.39</td>
<td>66 ± 3.1</td>
</tr>
<tr>
<td>7</td>
<td>63.50</td>
<td>14.15 ± 0.68</td>
<td>55 ± 2.8</td>
</tr>
<tr>
<td>8</td>
<td>40.64</td>
<td>9.16 ± 0.44</td>
<td>53 ± 3.0</td>
</tr>
</tbody>
</table>

The average power and efficiencies decrease with decreasing length, and do not agree within error according to the results presented in Table 4-5. Contour plots comparing the power and efficiency for each screw are shown in Figure 4-14. The screw with the highest overall power and efficiency, Screw 6, is the top contour surface, followed by screw 7 is the middle surface and Screw 8 is the bottom surface in Figure 4-14.
Figure 4-14 Contour plots of screws 6, 7 and 8 power and efficiency (60% fill)

Screw 8 was the shortest length of this set at 40.64 cm, which yielded an average power of 9.16 W and average efficiency of 53%. By increasing the length of the screw to 63.5 cm (Screw 7), the average power increased to 14.15 W and a 55% average efficiency. Finally, the longest, Screw 6, has a length of 121.9 cm carried a 30.17 W average power and 66% average efficiency. Overall, as the length of the screw increased, so did the power and efficiency of the screw across each measurement. On average, every time the screw was increased by a length of 20 cm, average overall power increased by 5 W and overall efficiency increased by 3.5%.

Screws 7 and 8 both had efficiencies that tend towards zero at low flow rates and high speeds. Specifically, the efficiency at 6 L/s and 80 RPM became negative for Screw 8. This was due to the high speed of the screw, and low flow rate, water was being pumped through the screw and not aiding in its rotation, this caused the power to read as a negative value, and therefore, resulted in a negative efficiency.

It was previously seen that water at the outlet of the screw shortens the screw and therefore, produced less power. This repeated here, as the screw was physically shortened, less power was
created and the screw became less efficient. The next set of screws 11, 12 and 13 are compared to see if this trend is repeated.

![Screws 11, 12, 13](image)

**Figure 4-15 Screws 11, 12, 13**

<table>
<thead>
<tr>
<th>Screw</th>
<th>L</th>
<th>Average Power (W)</th>
<th>Average Efficiency (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>#13</td>
<td>94.69</td>
<td>21.90 ± 1.04</td>
<td>62 ± 2.8</td>
</tr>
<tr>
<td>#12</td>
<td>61.39</td>
<td>12.61 ± 0.66</td>
<td>50 ± 3.1</td>
</tr>
<tr>
<td>#11</td>
<td>46.89</td>
<td>9.22 ± 0.51</td>
<td>48 ± 2.4</td>
</tr>
</tbody>
</table>

Echoing the previous results, the top layer in Figure 4-16 is the longest screw, Screw 13, followed by Screw 12 as the mid layer and Screw 11 as the bottom layer.
Figure 4-16 Contour Power and efficiency plots for screws 11, 12, 13 (60% fill)

Screw 11 had the shortest screw length of 46 cm, the average power and efficiency of 9.22 W and 48%. Screw 12, with a length of 61.39 cm, carried an average power of 12.61 W and 50% efficiency while Screw 13, with a length of 94 cm had 21.90 W average power and 62% average efficiency. Once again, the overall power and efficiency increased when the length of the screw increased. The power increased approximately 5.2 W for every 20 cm length increase, and the efficiency climbed at a much slower rate of 1.5% per 20 cm increase in length.

There were negative power outputs and efficiencies for the shorter Screws 11 and 12. These screws have a larger outer diameter and buckets than the previous set, meaning each flow rate did not fill the bucket as much as the previous set. At 80 RPM and 60 RPM, and 6 L/s and 8 L/s, there was not enough water in the screws buckets to create power, instead water was being carried through by the rotation of the gear motor on the screw. This results in a negative power and efficiency for both Screws 11 and 12.

The longest screw in this set, Screw 13, continued to create the largest power and efficiency, followed by Screw 12, then Screw 11. Screws 14, 15 and 16 have the same three lengths as this set of screws, and only vary in pitch.
Once again, the top contour layer for Figure 4-18 is based on the screw with the longest length. Screw 16 sits on the top, with 15 in the middle and 14 on the bottom. The difference between the power and efficiency of this set of screws and the previous are almost non-existent.
Screw 14 has a length of 46.61 cm with an average power of 6.45 W and average efficiency of 33%. Screw 15 is 76.15 cm in length with an average power of 13.69 W and 41% average efficiency. Screw 16 is the longest in this set at 94.89 cm long with a 20.78 W average power and 57% efficiency. Keeping consistent with the previous two cases, both the maximum and average power readings and efficiencies increased as the length of the screw increased.

All three screws had negative power and efficiencies at higher rotational speeds, above 50 RPM and flow rates below 8 L/s. Compared to the previous set of Screws 11, 12 and 13, Screws 14, 15 and 16 have a pitch that is 8 cm larger. A larger pitch increases the size of screws bucket, and the effect on changing pitch will be analyzed further in section 4.3.6. The explanation for these negative values of power and efficiency follow the last two sets of screws, low flow rates through the screw do not aid in the screws rotations especially at higher speeds.

Overall, as the length of the screw increased, so does the power and efficiency of the screw. This is consistent across each set of screws. All screws examined; 6, 7, 8, 11, 12, 13, 14, 15, and 16 have increased approximately 5 W for every 20 cm increase in length (Figure 4-19). Efficiency began to level off at different points for each screw (Figure 4-20), this may be an indication of the
maximum length conditions of the screw. Basically, for all screws there was power proportional to length but the efficiencies varied depending on the individual screw. Figures 4-19 and 4-20 show a comparison of average power and efficiency for non-dimensional length over pitch.

![Graph of average power and efficiency for non-dimensional length over pitch.](image)

*Figure 4-19 Change in power with non-dimensional screws length (60% fill)*

![Graph of average efficiency for non-dimensional screw length.](image)

*Figure 4-20 Change in efficiency with non-dimensional screw length (60% fill)*
Even though there was a relationship between power and length, there was still a cap when it came to efficiency. A longer screw will create more buckets, and therefore create a greater opportunity for rotation in the screw to create power, however, as the length increases so do power losses. Frictional losses from the water interacting with the screw and weight on bearing are just a few potential losses that are introduced with increased length. To further refine potential losses in the screw, the change of flights is compared.

4.3.4 Flight Number

Screws 4, 5 and 6 differed only in the number of flights. Screw 4 had 5 flights, Screw 5 had 4 flights and Screw 6 had 3 (Table 4-8). The number of flights in a screw is the number of helical surfaces that start at the beginning of the screw: an increased number of flights or blades creates more buckets in a screw with otherwise similar length and pitch. The number of “turns” in the screw for each of the flights is approximately four, a full turn is the amount of times one blade makes a full rotation around the central shaft. This means there are more buckets overall, however, they are smaller in volume due to the amount of flights needed to turn around the central shaft. Smaller buckets do not necessarily aid in turning the screw as well as a few large ones. Each blade also carries along some weight and thickness to the screw, so a large number of blades take up potential volume across the screw. Rorres (2000), predicted more power generation with an increased number of flights due to the screw being able to hold a higher volume of water. This makes additional hydrostatic forces available to turn a screw with more flights.
Figure 4-21 Screws 4 (top), 5 (middle), and 6 (bottom)

Table 4-8 Average power and efficiency for screws 4, 5, and 6 (60% fill)

<table>
<thead>
<tr>
<th>Screw</th>
<th>OD</th>
<th>ID</th>
<th>S</th>
<th>N</th>
<th>ID/OD</th>
<th>L</th>
<th>Power (W)</th>
<th>Efficiency (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>#4</td>
<td>31.69</td>
<td>12.70</td>
<td>31.75</td>
<td>5.00</td>
<td>0.40</td>
<td>121.92</td>
<td>30.20 ± 1.39</td>
<td>65 ± 3.1</td>
</tr>
<tr>
<td>#5</td>
<td>31.66</td>
<td>12.70</td>
<td>31.75</td>
<td>4.00</td>
<td>0.40</td>
<td>121.92</td>
<td>30.42 ± 1.37</td>
<td>65 ± 3.0</td>
</tr>
<tr>
<td>#6</td>
<td>31.62</td>
<td>12.70</td>
<td>31.75</td>
<td>3.00</td>
<td>0.40</td>
<td>121.92</td>
<td>30.17 ± 1.39</td>
<td>66 ± 3.1</td>
</tr>
</tbody>
</table>
Table 4-8 shows that increasing or decreasing the number of flights in a screw does not have a major effect on power or efficiency. The results all agreed within experimental uncertainty and each case is visually summarized in Figure 4-22, where each contour surface layer overlaps one another, not being able to clearly separate them.

![Figure 4-22 Contour power and efficiency plots for screws 4, 5, and 6 (60% fill)](image)

The output power between Screws 4, 5 and 6 were all around $30 \pm 1.4$ W and agreed within experimental uncertainty. The efficiency were also consistent across all three screws at $65 \pm 3.1\%$. It was only the maximum values of power and efficiency that displayed a small upward trend as the number of flights increased. The maximum value of power in each screw was found at 80 RPM and 14 L/s while that maximum efficiency was at the 50 RPM and 8 L/s range.

The number of flights on these screws did not have a major effect on the screws efficiency or power, there was no overall trend. When more flights were added, even though more buckets were created, additional power losses were introduced. Power losses associated with addition flights include inlet impact losses, larger bearing losses, and more internal fluid friction losses. These losses outweighed the benefit of more buckets in a screw, which are necessary for the screws rotation and power generation.
The lack of effect of flight number on power and efficiency could potentially change in larger screws. In a larger screw with higher flow rates, internal fluid friction losses introduced with more buckets may be overpowered by the volume of water travelling through the screw.

4.3.5 Diameter Ratio

The $D_i/D_o$ is the ratio of inner-to-outer diameter in a screw. Only one set of screws had the same set of variables other than the ratio between the inner-to-outer diameter, Screws 2, 6 and 9. Screw 2 had the largest ratio at 0.53, followed by Screw 6 which had a 0.40 ratio and lastly screw 9, with a $D_i/D_o$ of 0.32, summarized in Table 4-9. Theoretically, a larger outer diameter and smaller inner diameter would create buckets of a larger volume, this means that more water is available to turn the screw and create power. The 14 L/s was not included in the averaged values since Screw 2 did not have a complete data set at said flow rate.
Table 4-9 Average power and efficiency for screws 2, 6, and 9 (60% fill)

<table>
<thead>
<tr>
<th>Screw</th>
<th>OD</th>
<th>ID</th>
<th>S</th>
<th>N</th>
<th>ID/OD</th>
<th>L</th>
<th>Power (W)</th>
<th>Efficiency (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>#2</td>
<td>31.62</td>
<td>16.83</td>
<td>31.75</td>
<td>3</td>
<td>0.53</td>
<td>121.92</td>
<td>27.98 ± 1.29</td>
<td>67 ± 3.1</td>
</tr>
<tr>
<td>#6</td>
<td>31.62</td>
<td>12.70</td>
<td>31.75</td>
<td>3</td>
<td>0.40</td>
<td>121.92</td>
<td>27.61 ± 1.39</td>
<td>67 ± 3.1</td>
</tr>
<tr>
<td>#9</td>
<td>31.64</td>
<td>10.16</td>
<td>31.75</td>
<td>3</td>
<td>0.32</td>
<td>121.92</td>
<td>27.86 ± 1.40</td>
<td>69 ± 3.2</td>
</tr>
</tbody>
</table>

Table 4-9 showed an increasing efficiency with decreasing inner diameter, which increases the size of buckets in a screw. In Figure 4-24, Screws 2, 6 and 9 contour layers are indistinguishable since all values agreed within uncertainty while the efficiency was in descending order of 9, 6 and 2.
As $D_i/D_o$ decreased, the maximum power output and efficiency at 8 L/s and 50 RPM increased, keeping in mind each output agreed within experimental uncertainty. The average power and efficiency did not show any particular trend and agreed within experimental uncertainty, with an average power of 27 W and 67% efficiency.

A screw with a smaller $D_i/D_o$ creates a larger bucket volume. It also reduces contact with water in the screw and the central shaft for low flow rates and high rotational speeds. Screw 9 had a smallest inner diameter, and as water travelled down the length of this screw, water did not touch the central shaft at flow rates below 8 L/s above rotational speeds of 50 RPM. Water only interacted with the screws blades and trough, eliminating frictional power losses due to water interacting with the central shaft of the screw. Screw 6 had a slightly larger inner diameter, and water began to interact with the central shaft in cases where it did not for Screw 9, at 8 L/s and 50 RPM. A larger inner diameter created additional frictional power losses, however, Screws 2, 6, and 9 were not different enough to clearly show this trend. Within uncertainty there was not much difference between $D_i/D_o$.

4.3.6 Pitch

Four sets of screws vary in pitch, Screws 1, 2 and 3 have a 44.5 cm, 31.8 cm, and 24.5 cm pitch, each screw is 121.9 cm long, has 3 flights and a $D_i/D_o$ of 0.53. Screws 11 and 14, 12 and 15, and 13 and 16, all have 30.2 cm and 38.3 cm pitches. The pitch is the distance between one flight to the other, lengthwise down the screw, a smaller pitch means that there are more ‘turns’ per flight in any given screw. A turn is the amount of times a helical blade or flight, fully wraps around a screw. More turns means that there are more buckets formed from a particular flight. When a screw has more buckets, there is a larger surface area interaction between water in the screw, and helical planes. Theoretically, more buckets should produce more power, however, with every new bucket
come additional internal fluid friction forces. Also, smaller buckets may not contain the ideal volume of water to sufficiently rotate to screw. Screw 2 did not have a complete set of data at 14 L/s, and therefore, was omitted from all the other results for consistency.

Table 4.10 Screws with changing pitch

<table>
<thead>
<tr>
<th>Screws</th>
<th>OD (cm)</th>
<th>ID (cm)</th>
<th>S (cm)</th>
<th>N</th>
<th>ID/OD</th>
<th>L (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 2, 3</td>
<td>31.6</td>
<td>16.8</td>
<td>44.5, 31.8, 25.4</td>
<td>3</td>
<td>0.53</td>
<td>121.92</td>
</tr>
<tr>
<td>11, 14</td>
<td>37.8</td>
<td>16.9</td>
<td>30.2, 38.3</td>
<td>4</td>
<td>0.44</td>
<td>46.7</td>
</tr>
<tr>
<td>12, 15</td>
<td>37.8</td>
<td>16.9</td>
<td>30.2, 38.1</td>
<td>4</td>
<td>0.44</td>
<td>61.7</td>
</tr>
<tr>
<td>13, 16</td>
<td>37.8</td>
<td>16.9</td>
<td>30.5, 38.3</td>
<td>4</td>
<td>0.44</td>
<td>94.7</td>
</tr>
</tbody>
</table>

It’s difficult to find a relationship between Screws 11 and 14, 14 and 15, and 13 and 16, since there are only two screws to compare in each case. Therefore, only the results from Screws 1, 2 and 3 will be investigated.
Figure 4-25 Screws 1 (top), 2 (middle), and 3 (bottom)

Table 4-11 Average power and efficiency for screws 1, 2 and 3 (60% fill)

<table>
<thead>
<tr>
<th>Screw</th>
<th>OD</th>
<th>ID</th>
<th>S</th>
<th>N</th>
<th>ID/OD</th>
<th>L</th>
<th>Power</th>
<th>Efficiency (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>31.58</td>
<td>16.83</td>
<td>44.45</td>
<td>3.00</td>
<td>0.53</td>
<td>121.92</td>
<td>26.43 ± 1.36</td>
<td>66 ± 3.1</td>
</tr>
<tr>
<td>#2</td>
<td>31.62</td>
<td>16.83</td>
<td>31.75</td>
<td>3.00</td>
<td>0.53</td>
<td>121.92</td>
<td>27.98 ± 1.39</td>
<td>67 ± 3.1</td>
</tr>
<tr>
<td>#3</td>
<td>31.67</td>
<td>16.83</td>
<td>25.40</td>
<td>3.00</td>
<td>0.53</td>
<td>121.92</td>
<td>27.37 ± 1.34</td>
<td>63 ± 2.9</td>
</tr>
</tbody>
</table>

Figure 4-26 shows Screw 2 as the top contour layer, followed by Screw 3 then 1 for power and an order of Screw 2, 1, and 3 for efficiency.
Screw 1 was the least powerful at 26.43 ± 1.36 W, but had better efficiency, 66 ± 3 %, than Screw 3 which was more powerful at 27.34 ± 1.36 W, but the lowest efficiency, 67 ± 2 %. Screw 2 (27.98 ± 1.36 W, 67 ± 3.1 %), had a pitch of 31.75 cm which was a smaller pitch than Screw 1 (S = 44.45 cm) but was not as large as Screw 3 (S = 25.40 cm), and came out as the most power, and efficient screw.

A larger pitch created bigger, fewer buckets, even though each bucket was able to hold a larger volume of water, there was smaller surface of area of water interacting with screw planes. This also meant that there was a reduction in internal fluid friction that normally cause power losses in a screw. The smaller pitch in Screw 3 had the most negative effect on power and efficiency above 60 RPM and 6 L/s. The higher speeds and low flow rate did not allow enough water across each bucket to efficiently operate the screw. At the remaining speeds and flow rates, Screw 3 was able output a larger amount of power, since the volume flow rate was large enough to sufficiently operate the screw. In comparison, Screw 1 had the largest pitch and performed at a higher efficiency than Screw 3, however, the overall power fell short of the other screws. The large pitch reduced internal fluid friction losses that were abundant in Screw 3, but the lack of buckets
limited the amount of power that the screw could ultimately produce. Screw 2 fit directly in between Screws 1 and 3. The 31.75 cm pitch in Screw 2 gives the advantages of reduced internal fluid friction losses, and a sufficient amount of buckets to produce power.

4.5 Overall experimental observations

By analyzing trends across screws with varying physical parameters, some conclusions on overall screw power and efficiency were drawn. The results were presented using an experimental set up, and will be compared to an enhanced model in Chapter 5. These experiments were performed in order to create an overall image of what the ideal screw would look like, and aid in helping manufactures determine where to focus their time when producing ASTs.

Screws that are submerged at the outlet produce less power than screws that are not. Even through less power was created, the efficiency of the screw was stronger when submerged. This result was consistent across all 16 experimental screws. As the screws outlet became submerged, it incurred drag force effects at the outlet. It was harder for the gear motor to turn the screw at the desired speed, and therefore more power was lost. The efficiency of a screws was determined using available head, which is the height between the waters inlet at the screw, and the outlet. Since a screw that is submerged has a lower head than one that is not, the efficiency of the screw became greater.

Increasing length was another import factor when it came to improving power and efficiency. A longer screw length meant there was more available buckets in the screw, and more opportunity to create power. Out of all screws that were compared for length; 6, 7, 8, 11, 12, 13, 14, 15, 16 all have a linear increase in power of approximately 5 W for every 20 cm increase in length. The efficiency was not as clear a relationship, there was a point for each individual screw where the overall average efficiency began to level off. This was an indication that there is a
maximum length that should be used for certain screws to produce the best efficiency. Theoretically, the longer the screw, the more power losses due to drag forces within the screw. The key is to have a screw that is long enough to be able to rotate and create power, but not long enough to be overcome with power losses.

The number of flights did not show a significant increase or decrease in power between 3, 4 and 5 blades. Each screw agreed within error for both average power and efficiency. As the number of blades in a screw increased, the number of buckets in the screw also increased, however, each blade reduces the available volume in the screw, which does not allow the maximum amount of water to aid in screw rotation. The effect of more buckets outweighed the lost volume in this experimental set.

The inner-to-outer diameter ratio is also a bucket changing parameter. As the inner diameter decreased, interactions between incoming flow and the central shaft decreased, allowing for greater hydrostatic forces to turn the screws blades, which should have increased power in the screw. This result was seen on a very small level since all the power outputs and efficiencies agreed within experimental uncertainties.

Changing pitch in the screw also did not drastically change the overall power and efficiency in each screw. Increasing the pitch reduces the number of buckets in a screw, maximizing volume across its flighted length, there are less power losses and a more efficient power production. A smaller pitch allows more buckets to fit in a screw, but also introduced more power loss due to internal fluid friction. In this case, the screw with the middle pitch, was the most power and efficient, Screw 2.
Overall, the outlet condition and length of a screw had the largest effect on power production and efficiency with the pitch, number of flights and inner diameter all changing the amount of buckets, or volume of buckets, that affected the screws in similar ways. From this information, the longest screw with a mid-range pitch and smallest inner diameter should produce the most power and be the most efficient screw, across the 16 experimental screw, this is screw 9.

<table>
<thead>
<tr>
<th>Screw #</th>
<th>OD (cm)</th>
<th>ID (cm)</th>
<th>S (cm)</th>
<th>N</th>
<th>ID/OD</th>
<th>L (cm)</th>
<th>Avg Power (W)</th>
<th>Avg Efficiency (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>#9</td>
<td>31.64</td>
<td>10.16</td>
<td>31.75</td>
<td>3</td>
<td>0.32</td>
<td>121.92</td>
<td>30.4 ± 1.40</td>
<td>67 ± 3.1</td>
</tr>
</tbody>
</table>

These results are kept in mind when looking at future test screws and enhancing a model that can properly predict power output in a screw. This experiment looked at the power output and efficiency of each screw which are important for looking at the screw as a black box. It is still unclear if what is happening inside a screw is accurate when it comes to quantifying power losses. New power losses introduced in Chapter 5 and are applied to these results to create a sensitivity analysis for these screws.
Chapter 5: Sensitivity Analysis Based on Screw Geometry and Adapted Model

5.1 Methods

Following the experimental procedure introduced in Chapter 4, the set screws were tested at specific values of rotation speed and flow, allowing direct comparisons of screws at specific conditions without interpolation of measurements. This also allows examination of the effect of varying one parameter, while holding all others constant, something that is not readily available in most lab settings. Not only was this advantage used to determine optimal screw conditions in terms of average power and efficiency, it can also be used to perform a sensitivity analysis on the new model, \( fh5 \).

Table 5-1 ranks each screw based on their maximum efficiencies for an outlet that is 60% full. As seen in Chapter 4, the maximum efficiency does not necessarily occur at the same flow rate and speed for each screw. It should be noted that the data points collected for each screw were relatively coarse, and jumps in efficiencies occur between each flow rate and speed. This means that realistically, the conditions for maximum efficiency may not occur in the specific testing points. Keeping that in mind, the longer screws still have maximum efficiencies within error (e.g. plus or minus several percent) found in Appendix B.

<table>
<thead>
<tr>
<th>Screw</th>
<th>Max Efficiency</th>
<th>Q (L/s)</th>
<th>RPM</th>
<th>OD (cm)</th>
<th>ID (cm)</th>
<th>S (cm)</th>
<th>N</th>
<th>ID/OD</th>
<th>L (cm)</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.834</td>
<td>8</td>
<td>50</td>
<td>31.66</td>
<td>12.70</td>
<td>31.75</td>
<td>4</td>
<td>0.40</td>
<td>121.92</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>0.832</td>
<td>8</td>
<td>50</td>
<td>31.62</td>
<td>16.83</td>
<td>31.75</td>
<td>3</td>
<td>0.53</td>
<td>121.92</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>0.831</td>
<td>8</td>
<td>60</td>
<td>31.67</td>
<td>16.83</td>
<td>25.40</td>
<td>3</td>
<td>0.53</td>
<td>121.92</td>
<td>3</td>
</tr>
<tr>
<td>9</td>
<td>0.826</td>
<td>8</td>
<td>50</td>
<td>31.64</td>
<td>10.16</td>
<td>31.75</td>
<td>3</td>
<td>0.32</td>
<td>121.92</td>
<td>4</td>
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<td>( \rho )</td>
<td>( \beta )</td>
<td>( Q )</td>
<td>( P_{\text{max}} )</td>
<td>( P_{\text{expt}} )</td>
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<td>16.89</td>
<td>38.30</td>
<td>4</td>
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<td>0.809</td>
<td>8</td>
<td>50</td>
<td>31.62</td>
<td>12.70</td>
<td>31.75</td>
<td>3</td>
<td>0.40</td>
<td>121.92</td>
<td>7</td>
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<td>1</td>
<td>0.794</td>
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<td>40</td>
<td>31.58</td>
<td>16.83</td>
<td>44.45</td>
<td>3</td>
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<td>121.92</td>
<td>8</td>
</tr>
<tr>
<td>13</td>
<td>0.785</td>
<td>12</td>
<td>50</td>
<td>37.69</td>
<td>16.79</td>
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<td>4</td>
<td>0.44</td>
<td>94.69</td>
<td>9</td>
</tr>
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<td>38.10</td>
<td>16.79</td>
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<td>50</td>
<td>37.69</td>
<td>16.89</td>
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<td>4</td>
<td>0.44</td>
<td>61.39</td>
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</tr>
<tr>
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<td>40</td>
<td>38.20</td>
<td>16.99</td>
<td>38.30</td>
<td>4</td>
<td>0.44</td>
<td>46.61</td>
<td>12</td>
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<td>50</td>
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<td>16.99</td>
<td>30.20</td>
<td>4</td>
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<td>46.89</td>
<td>13</td>
</tr>
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<td>10</td>
<td>50</td>
<td>31.62</td>
<td>12.70</td>
<td>31.75</td>
<td>3</td>
<td>0.40</td>
<td>63.50</td>
<td>14</td>
</tr>
<tr>
<td>8</td>
<td>0.686</td>
<td>6</td>
<td>60</td>
<td>31.57</td>
<td>12.70</td>
<td>31.75</td>
<td>3</td>
<td>0.40</td>
<td>40.64</td>
<td>15</td>
</tr>
<tr>
<td>10</td>
<td>0.668</td>
<td>10</td>
<td>50</td>
<td>31.61</td>
<td>10.16</td>
<td>44.77</td>
<td>4</td>
<td>0.32</td>
<td>52.07</td>
<td>16</td>
</tr>
</tbody>
</table>

To calculate experimental efficiency, the maximum available power was calculated for each experimental run using Equation (5-1). Maximum available power was calculated using geometric head instead of hydraulic head (e.g. Equation (4-1)) because the models that were examined (fh3 and fh5) do not allow inputs for the calculation of available head. This is something that can be added to further improve the model.

\[
P_{\text{max}} = \rho g L \sin \beta Q
\]  

(5-1)

This maximum power is used to find the efficiency in each screw by taking the experimental power output from the screw, over the total power available.

\[
\eta = \frac{P}{P_{\text{max}}}
\]  

(5-2)
Each case will compare the experimental results from the screw defined as, and two model results, fh3 and fh5. fh3 model does not contain power losses other than gap and overflow loss, while fh5 contains translational, rotational and outlet condition losses. It is therefore expected that fh3 will over predict the screws efficiencies.

A base case was chosen for each experimental screw of 8 L/s in flow and 50 RPM in rotational speed. This case provided the maximum efficiency for the majority of the screws as seen in Table 5-2, and represents a practical operating condition across all of the screws. All reported efficiencies are taken at the 60% outlet water level mark unless otherwise indicated. This is because the 60% case is more representative of real life screw conditions than the 0% or 30% cases and eliminates the need for additional outlet conditions where water “falls” out of the screw outlet.

5.2 Results and Discussion

Each case was compared at a flow rate of 8 L/s and rotational speed of 50 RPM, described as the base case. The non-dimensional comparisons are; fill height ($f$), inner-to-outer diameter ($Di/Do$), pitch-to-inner diameter ($S/Di$), length-to-pitch ratio ($L/S$), Number of flights ($N$) and outlet water level ($\psi$).

5.2.1 Fill Height

Figure 5-1 shows data from Screw 2 at 50 RPM across the range of tested flows from 6 L/s to 14 L/s, which resulted in a range of fill levels from $f = 0.75$ to $f = 1.45$. 
The peak efficiency was expected to occur at around $f = 1$, at this point the bucket in an AST is full and turning the screw with maximum capability. Figure 5-1 suggests the maximum occurred at approximately $f = 1.2$, this suggests that the current method of fill height measurements may be inaccurate, or the maximum efficiency is actually accompanied by some overflow. Model fh3 over predicted the expected results due to lack of power losses being accounted for. The new model, fh5, takes more losses into account and agrees with experimental results with a slight under prediction but still within error. Model fh5 agreed best at both ends of the fill height, with a larger deviation around $f = 1$, this further suggests that investigation into fill height around that depth needs to be investigated, and will be in Chapter 6.

5.2.2 Diameter Ratio

Experimentally, the efficiency increased with increasing $D_i/D_o$, described in Figure 5-2. The maximum efficiencies occurred at the 8 L/s and 50 RPM base case.
A smaller inner diameter allows for larger volumes of water to translate through the screw without before interacting with the central shaft. Model fh3 did not accurately predict efficiency since gap leakage and overflow losses did not represent all losses associated with changing inner diameter. Model fh5 was able to predict efficiencies with a slight under-prediction. The 0.32 $Di/Do$ was the smallest ratio, at 8 L/s and 50 RPM, water did not interact with the screws central shaft. Model fh5 only added losses due to interactions between the screws blades, trough, gap leakage, overflow and losses at the outlet. At 0.4 and 0.54 $Di/Do$, frictional losses from the central shaft was additionally modelled with fh5. The under-prediction in fh5 suggests that the frictional losses calculated with the screws central shaft are over simplified, causing inaccuracy, or that the fill height is not being correctly measured.

5.2.3 Pitch Ratio

The pitch is the distance between each successive flight, as the pitch increases in a screw, the size of the bucket gets larger. Figure 5-3 shows Screw 1, 2 and 3 at 8 L/s and 50 RPM with pitch ratios varying from 0.2 to 0.37.
Model fh5 strongly under-predicted the efficiency at the lowest pitch ratio, 0.2. As the pitch increased, so did the accuracy of fh5. At the highest pitch ratio, 0.37, the fh5 model and experimental results agreed within uncertainty. Model fh3 over-predicted efficiency in each screw and did not vary with pitch.

Each screw had a maximum efficiency at 8 L/s, however, as the pitch increased, the speed required to maintain maximum efficiency decreased. This is why efficiency decreased with increasing pitch experimentally, but had the opposite effect in model fh5 for the base case. Experimentally, the large pitch created a large bucket that did not have enough volume of water to sufficiently rotate the screw, which decreased efficiency. Model fh5 under-predicted internal frictional losses because the maximum number of buckets were at the lowest pitch, and power losses were calculated based on the interactions between the water and screws bucket surfaces. This further shows that fh5 needs refinement.
5.2.4 Length

<table>
<thead>
<tr>
<th>Screws</th>
<th>Lengths (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6, 7, 8</td>
<td>121.9, 63.5, 40.6</td>
</tr>
<tr>
<td>11, 12, 13</td>
<td>46.9, 61.4, 94.7</td>
</tr>
<tr>
<td>14, 15, 16</td>
<td>46.1, 76.2, 94.9</td>
</tr>
</tbody>
</table>

A longer screw had been more efficient as seen in Chapter 4, there were more buckets available to rotate the screw, which created more power. This also introduced more opportunity for frictional losses. Figures 5-4 and 5-5 show three sets of screws where the only dimension that changed is length at 8 L/s and 50 RPM.

![Figure 5-4 Length Screws 6, 7, and 8: sensitivity 8 L/s 50 ROM (60% fill)](image-url)
Model fh3 did not adjust for power losses due to length of the screw and over predicted the experimental results. Model fh5 was able to predict that length increased the efficiency in each of the screws within experimental uncertainty.

Each screw had a maximum efficiency at different speeds and flow rates. This suggested that there were more intricacies into the length effect on a screw then was presented in fh5. The addition of more buckets added more frictional power losses to the screw. Pitch and number of flights also affected the number of buckets in a screw, however when length was increased, the size of the screws buckets remained the same. Model fh5 was able to predict efficiencies with much more accuracy compared to the investigation of the pitch ratio in section 5.2.3. Overall, fh5 was able to successfully quantify output efficiency within uncertainty for changing length.

5.2.5 Outlet Water Level

The water level at the outlet of the screw is calculated using methods discussed in Chapter 4. The results for Screw 2 at non-dimensional outlet water levels between $\psi = 0$ and $\psi = 0.6$ are summarized in Figure 5-6.
There was a decrease in efficiency as the outlet water level increased, this was because water was being introduced into the bottom of the screw and created back pressure that caused drag forces at the screw outlet. Model fh3 did not include power losses at the outlet, and the predicted efficiency did not change as the outlet condition was changed. The fh3 model was only able to accurately predict efficiency at $\psi = 0$. Model fh5 attempted to quantify losses at the outlet using Equation (3-29) which used the available head in the screw, and the power at the screws shaft. Outlet losses were a major power loss contributor in Chapter 4 and fh5 was able to predict the results within error, slightly trending towards an under-prediction. Since fh5 only used the available head to predict power losses at the outlet, an improvement could be made by quantifying losses due to drag forces caused by back pressure at the outlet.

5.2.6 Number of flights

The number of flights determines the number of buckets in a screw, even though a screw with more flights may have more buckets, the volume of water in each bucket decreases. Since
the blades are not infinitesimally thin, more flights also takes away total volume available in a screw. Figure 5-7 displays Screws 4, 5 and 6 which have 5, 4, and 3 flights.

![Diagram](image)

*Figure 5-7 Flight sensitivity 8 L/s 50 RPM (60% fill)*

Model fh5 was able to predict experimental efficiency within uncertainty, while model fh3 over-predicted the results. In Chapter 4, the number of flights did not have a large effect on the efficiency from screw to screw, this continued to hold in Figure 5-7. Even as the number of flights increased, and more buckets were created, more losses were also created. Each additional blade, or flight, added weight to the screw, which affected bearing losses. In addition, each blade took up space from the overall volume of water available in the screw. On a more detailed level, 4 flights performed the best, from the three screws, followed by 3 flights, then 5 flights for the 8 L/s 50 RPM case. The 4 flighted screw had a enough blades available for hydrostatic forces to rotate the screw, but not so much that it took away from the overall volume of the screw, or too little that there wasn’t enough buckets available for rotation. Finally, each of these screws required the same water level to achieve a full bucket, $f=1$, this further suggested an impact from the physical weight.
and size of the blades were what affected efficiency. Model fh5 was able to successfully predict the experimental results.

5.3 Conclusion

A sensitivity analysis was able to show how closely models fh3 and fh5 predicted experimental efficiencies. There are still improvements that need to be made in model fh5, such as an inclusion of bearing losses, pitch effect, and proper quantification of power losses at the outlet. In addition, the translational and rotational power losses in the model were simplified and still did not fully account for the dynamic motions inside a rotating screw. Overall, the new model showed agreement with the experimental results within error, trending towards a slight under-prediction in most cases. Since fh3 and fh5 both predict the fill height within a screws buckets in the same manner, the fill height analysis in the next three chapters will proceed with comparisons to model fh5.
Chapter 6 : Validation of Bucket Fill Measurements

6.1 Introduction

The fill height in an operating Archimedes screw is an important parameter affecting its performance, yet it is not easily observed. Kozyn and Lubitz (2014) visually observed fill height in a small laboratory screw that was installed within a transparent plastic tubular trough. However, most real world ASTs (including the laboratory screws examined in this study) are encased in a solid trough meaning that the fill height in a ASTs bucket cannot be visually observed with any precision while the screw is operating, and an alternative method of directly measuring fill height was needed.

6.2 Measurement of Fill Height

In practice, the fh3 and fh5 models require fill height as an input, and the resulting flow through the screw is a model output. Since in practice, flow is usually available but fill height is not known a priori, the models are normally implemented using an iterative process where the flow rate through the screw is calculated across a range of potential fill heights, and results are interpolated to find the expected fill height at the needed flow rate.

6.2.1 Improved Fill Height Parameter

Fill height variable, \( f \), calculated using Equation (3-4) from Chapter 3 is not precise because the intersection of the water surface plane and \( \theta = 2\pi \) does not correctly represent the highest point on the screws central shaft. Due to the cylindrical and helical nature of an AST, the true vertical maximum fill that falls at \( \theta = 2\pi \) is not necessarily at the very top of the screw, but should fall slightly underneath. Figure 6-1 shows the new position of overflow, Point A, compared to when it was previously measures, Point B.
By taking the derivative of Equation (3-4), with respect to theta, the local maximum and minimum based on the geometry of the screw can be found. This new adjustment is the offset due to the cylindrical nature of the screw and can be used to find the offset at the minimum, \( \pi \), and maximum, \( 2\pi \).

\[
\frac{dz_1}{d\theta} = 0 = r\cos\beta (-\sin\theta') - \frac{S}{2\pi}\sin\beta
\]  \hspace{1cm} (6-1)

\( \theta' \) is the offset that will be used in an adapted version of Equation (3-4) to create \( z_1' \) and represented the new location of \( z_{max} \) due to the cylindrical nature of the screw.

\[
\theta' = \sin^{-1}\left(\frac{-S}{2\pi\sin\beta} \cdot \frac{\sin\beta}{r\cos\beta}\right)
\]  \hspace{1cm} (6-2)

\[
z_1' = r\cos(\theta + \theta')\cos\beta - \frac{S(\theta + \theta')}{2\pi}\sin\beta
\]  \hspace{1cm} (6-3)

\( z_{max}' \) is calculated by subtracting \( z_1' \) with \( r_0 \) and \( \theta = \pi \) and \( r, \theta = 2\pi \). This is applied to the experimental results from here on out and will be defined as \( z_{max}' \). The differences between the previously measured values of \( z_{max} \) and the new \( z_{max}' \), are summarized in Table 6-1.
Table 6-1 Comparison of adjusted $z_{\text{max}}$ values for slope 24.5°

<table>
<thead>
<tr>
<th>Screw</th>
<th>Original $z_{\text{max}}$ (m)</th>
<th>New $z'_{\text{max}}$ (m)</th>
<th>Change (m)</th>
<th>Change (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.1281</td>
<td>0.1247</td>
<td>-0.0034</td>
<td>-2.6294</td>
</tr>
<tr>
<td>2</td>
<td>0.1546</td>
<td>0.1529</td>
<td>-0.0017</td>
<td>-1.1070</td>
</tr>
<tr>
<td>3</td>
<td>0.1680</td>
<td>0.1669</td>
<td>-0.0011</td>
<td>-0.6491</td>
</tr>
<tr>
<td>4</td>
<td>0.1361</td>
<td>0.1354</td>
<td>-0.0007</td>
<td>-0.5467</td>
</tr>
<tr>
<td>5</td>
<td>0.1360</td>
<td>0.1352</td>
<td>-0.0007</td>
<td>-0.5509</td>
</tr>
<tr>
<td>6</td>
<td>0.1358</td>
<td>0.1351</td>
<td>-0.0008</td>
<td>-0.5554</td>
</tr>
<tr>
<td>7</td>
<td>0.1358</td>
<td>0.1351</td>
<td>-0.0008</td>
<td>-0.5554</td>
</tr>
<tr>
<td>8</td>
<td>0.1356</td>
<td>0.1348</td>
<td>-0.0008</td>
<td>-0.5615</td>
</tr>
<tr>
<td>9</td>
<td>0.1243</td>
<td>0.1246</td>
<td>+0.0002</td>
<td>0.1944</td>
</tr>
<tr>
<td>10</td>
<td>0.0972</td>
<td>0.0978</td>
<td>+0.0006</td>
<td>0.6490</td>
</tr>
<tr>
<td>11</td>
<td>0.1867</td>
<td>0.1858</td>
<td>-0.0009</td>
<td>-0.4772</td>
</tr>
<tr>
<td>12</td>
<td>0.1853</td>
<td>0.1844</td>
<td>-0.0009</td>
<td>-0.4837</td>
</tr>
<tr>
<td>13</td>
<td>0.1846</td>
<td>0.1837</td>
<td>-0.0009</td>
<td>-0.4799</td>
</tr>
<tr>
<td>14</td>
<td>0.1717</td>
<td>0.1703</td>
<td>-0.0014</td>
<td>-0.7962</td>
</tr>
<tr>
<td>15</td>
<td>0.1705</td>
<td>0.1692</td>
<td>-0.0013</td>
<td>-0.7762</td>
</tr>
<tr>
<td>16</td>
<td>0.1731</td>
<td>0.1718</td>
<td>-0.0013</td>
<td>-0.7411</td>
</tr>
</tbody>
</table>

For a slope of 24.5°, the maximum change in $z_{\text{max}}$ is in Screw 1, with a 2.6% decrease, the lowest change being Screw 9 which carries a 0.2% increase. Overall, the changes in $z_{\text{max}}$ do not seem significant enough to greatly affect the experimental results, however, a small change in $z_{\text{max}}$ propagates through the entire fill height analysis of a screw. The minimum and maximum values are key in accurately determining the fill height of water in the screw, which dictates how much power the screw can produce. From now on, these new adjusting $z_{\text{max}}$ values will be used to
determine fill height. Fill height with the newly adjusted values will be defined as \( f' \) and the \( z_{max} \) as \( z'_max \).

6.2 Experimental Procedure

Two experimental procedures were used to determine fill heights. The first was concurrent with Chapter 4, the second was extensive testing of Screws 2 and 16. For the second procedure, flow was increased in increments of 0.3 – 1 L/s for rotational speeds from 0 to 50 RPM, while fill height was recorded. The lower basin was always maintained at a water level below the bottom of the screw outlet (0%), to ensure consistent outlet loss effects.

6.3 Bucket Depth Measurements

The static pressure at the bottom of a bucket is proportional to the depth of water in a bucket. It should be noted that in a turning screw, this local depth will repeat continuously as each bucket translates over a fixed point on the trough. A static pressure port was included in the bottom of the screw troughs at mid-length and a fast-response pressure sensor (Omegadyne PX309) was used to continuously record the static pressure. This method was first tested in a smaller laboratory screw which had an acrylic tube casing (described in Kozyn and Lubitz 2014), allowing for the fill height of the buckets to be verified visually. Successful results in this screw are included in Appendix C. The pressure sensor was calibrated before testing using a controllable depth water column to provide known static pressure at the sensor, the calibration is found in Appendix D.
A constant depth of water exists at the pressure sensor in a static screw. When the screw is turning, each time a screw blade passes the pressure sensor the depth of water in each of the buckets fluctuates in a range of approximately 2 cm height, corresponding to the step-down height between adjacent buckets. An example is shown in Figure 6-3.
An average depth of water inside the bucket is taken as the average height of water recorded. Additionally, since the top of the pressure sensor does not rest directly on the inner casing of the AST it has an offset. The water height is based off visually observing when the water depth is at \( f = 1 \) in a slowly turning screw, and other heights are determined as offsets relative to that baseline depth. Once the average depth of water in the bucket is offset, this provides a value for \( z_{wl} \) which can be used with \( z'_{\text{max}} \) calculated directly from the screws geometry, to determine a value for \( f' \).

6.4 Results and Discussion

To validate that the pressure sensor correctly predicted the fill height of water in the screw, the actual water level in the screws buckets was compared to the water depth predicted by \( \text{fh5} \). This is done for both experiments. Firstly, Screws 1, 2, 3, 4, 5, 6, 9 and 15 were all analyzed together since tests of these screws followed the same experimental procedure. The static pressure based on the output voltages from the PX309 pressure transducer, which will be called the measured pressure, were converted into depths assuming a static water column. These depths were divided by the maximum water level required for a full bucket, \( z'_{\text{max}} \) in the screw to determine non-dimensional fill height. Comparisons of fill height measured using the pressure sensor, and the corresponding fill height predicted using the \( \text{fh5} \) model, are shown in Figure 6-4.
A fill height error analysis calculated an average uncertainty of $f'$ of $\pm 0.072$ (details are reported in Appendix C.) For each screw in Figure 6-4, the fill height between the pressure sensor and the model agreed within error for fill heights approximately between $0 < f' < 1$. Every screw varied slightly but on average above $f' = 1$, the reading from the pressure sensor and $f_h5$ began to diverge, with the model predicting a greater fill height than was measured by the pressure sensor. There were a few reasons as to why the model was over predicting fill height. Experimentally, as the bucket of a screw filled with water the gap width between the helical planes and the trough did not stay consistent, instead, it was observed that the gap became slightly wider on the sides of the screw forming more of an elliptical trough as opposed to a cylindrical one. In addition, as the speed of the screw increased, water began to splash out of the top of the screw, lowering the water level.
in the screws buckets. Above fill heights of \( f' = 1 \), the flow of water through the screw exceeded the maximum volume of the bucket, this is overflow. Overflow is a power loss that does not aid to the rotation of the screw. The impact of how overflow is modeled in fh5 is discussed below, and a more detailed analysis of overflow on Screws 2 and 16 will be the focus in Chapter 8.

Higher resolution data was acquired for Screws 2 and 16 than for the other screws. Data for these two screws was compared side by side to analyze how different screw dimensions affected the increase of fill height for buckets of a different size and shape. Fill heights from model fh5 were compared to fill heights from the pressure transducer, shown in Figure 6-5. Unlike the previous data set (of more screws, but with less data for each screw), each change in rotational speed is compared separately due to the assumption that speed affects how the screws buckets fill.
Figure 6-5 Screws 2 and 16 measured pressure compared with modelled

Model fh5, and measured pressure agreed within error for Screw 2 until a divergence at approximately $f' = 1.2$, while Screw 16 diverged just below $f' = 1$. This suggests that the size of a bucket had an effect on fill height. Screw 2 is smaller than Screw 16 outer diameter, pitch, and number of flights. Screw 2 has smaller buckets than Screw 16 and as the water filled Screw 2, the model was able to predict the buckets fill height with greater accuracy. This may have also been due to a more even gap width distribution around the trough of Screw 2.

Using a pressure transducer was an accurate was to measure fill height in a screw with an average uncertainty of $\pm 0.072$. When $f' < 1$ water was trapped between the helical planes of the screw and the increase in water was accurately predicted using the current methods in model fh5. When $f' > 1$, the fill height in the bucket was dependent on flow rate, rotational speed, and screws.
dimensions. Rotational speed was not used when calculating fill height in model fh5. The fill height being inaccurate for \( f' > 1 \) is a problem when measuring the overflow.

The fill height being inaccurate over \( f' = 1 \) is a problem for measuring the overflow, however, the data for cases where the water level is less than full show that the pressure sensor can be used to accurately measure water depth in operating Archimedes screws. This was confirmed both visually in the small laboratory screws, and using comparisons to model predictions.

This new method of measuring fill height is used to investigate the effect of how fill changes with inclination angle. In the previous experiments, the screws were always set at an angle of approximately 24.5°, there should be confirmation as to why this inclination angle was chosen.
Chapter 7: Screw Slope Effect

7.1 Experimental Procedure

Up until this point, experimental tests were executed at an inclination angle of 24.5°. This was defined as the ideal screw angle since it allowed the screw to lay directly in between both the upper and lower basin so that enough water could be pumped through the screw without flooding the gear motor. It is also within the range of normal screw slope seen in installations of full-scale ASGs. Now that fill heights in a screws bucket can be measured using a pressure transducer, the way a bucket fills can be compared at multiple inclination angles.

Screw 2 was tested in this study at two speeds and three angles, the angles were chosen to give a good range of test slopes while working within the restrictions of the laboratory set up. If the screws angle was too shallow, the gear motor that regulates the screws speed becomes at risk of being submerged in the upper basin at higher flow rates. For angles that were too steep, there becomes a risk of not enough water available in the lower basin to pump to the upper basin and feed into the screw. Screw 2 was held at two speeds, 30 RPM and 50 RPM, and flow rates in the screw were increased incrementally from 0 to 15 L/s to obtain a full range of flow rates.

7.2 Results and Discussion

Screw 2 was tested at three different angles for two speeds to determine if the angle of the screw had an effect on fill height and power. The fill height is examined against non-dimensional flow rate (Equation 7-1), which is proportional to the ASTs angular velocity and uses the speed of the screw, \( \omega \), flow rate, \( Q \) and dimensions related to the screws cross sectional area (Figure 7-1).

\[
Q_{nd} = \frac{8Q}{\omega (D_o^2 - D_i^2)}
\]  
(7-1)
Between both cases, the fill height within the screw increased in a similar fashion, and all the measured pressures agreed within error. The steepest angle, 29 degrees, predicted a slightly higher fill height as the flow rate of the screw is increased. This was followed by the 24.5 degree and then the 20 degree slope. This followed the theoretical expectation that a steeper angle would tilt the bucket, causing a lower flow rate to produce a higher fill height and \textit{visa versa}. Although visually, these angles did not seem to have a large effect on fill height, the small differences were enough to effect the power and efficiency of the screw.

A power and efficiency analysis was performed on Screw 2. For 30 RPM and 50 RPM separately, power output and efficiency was compared against non-dimensional flow rate for each inclination angle. The result was that the ideal angle, 24.5°, resulted in the largest power output and efficiency summarized in Figures 7-2 and 7-3. Efficiency was calculated using Equation (5-2).
For both rotational speeds, the $24.5^\circ$ screw angle resulted in the best experimental power output. This verified that $24.5^\circ$ was a good estimate for inclination angles in lab experiments. The
29° case also produced reasonable power and efficiency results, however, there was a drop in efficiency non-dimensional flow rates below 0.5. The 20° sloped screw had the largest loss in power and efficiency across both speeds and each flow rate.

This aligned with the theory that since ASTs use pressure of water acting down on its helical blades for rotation, a screw with a steeper angle was able to cover more surface area to push down on the screws blades in order to allow it to rotate. A shallow angle, like 20°, did not have the maximum use of water on the helical blades.

If a screw was placed horizontally, there would be no way for water to move down the length of the screw after the inlet. It would also not be the most efficient to have a screw directly vertical since water would leak through the ASTs gaps, and slide down the helical surfaces without creating efficient buckets to trap water and move down the screw. Therefore, the ideal angle for these screws is 24.5°.

As the angle of Screw 2 changed so did the head, this was because the length of the screw remained constant across all angles. It is possible to keep head constant for changing angles as long as the length of the screw is adjusted as well.

7.3 Inclination Angle Summary

Inclination angles had an effect on a screws power and efficiency. At 24.5°, each studied case produced both the most power and greatest efficiency. In terms of fill height, the angle of the screw produced very slight variation in how the screw filled with increasing flow rate, landing within uncertainty. This effect would be more evident in screws with larger buckets and more extreme inclination angles. For the purposes of fill height, and how it related to overflow, explored in the next chapter, the inclination angle remained at the ideal 24.5°.
Chapter 8: Adaptation of Overflow Equation in an AST

Overflow occurs when the amount of water in a ASTs bucket exceeds its total volume. This overflow is defined using a non-dimensional fill height, \( f' \), where \( f' = 1 \) is a 100% filled bucket. At any value above 1, some water from the bucket will flow over the top of inner cylinder into the next bucket down the chute: this is considered overflow. A fill height analysis was presented in Chapter 6, where it was observed that the fh5 model prediction of fill height and the measured fill height began diverge around values of \( f' > 1 \), which is the condition where overflow begins. Initial experiments suggested that the overflow model used in fh5 (Equation 3-20) may not correctly predict overflow in the lab screws (Songin and Lubitz, 2016). Due to this divergence, an attempt to correct the current overflow model is presented by using the pressure transducer to calculate overflow and compare it to the output from the current model.

8.1 Current Model

The current method of predicting overflow (Equation 3-20) is based on an assumption that overflowing water can be modeled as an angled, v-notch weir (Aigner, 2008). This method is the only specific model of overflow known to the author, and it has been used by Neurnbergk and Rorres (2012) and Lubitz et al. (2014). Since the screws central shaft and blade create a triangular shape, the v-notch weir flow model seemed appropriate. The flow of water over the shaft to the power of 5/2 models flow over the central shaft, and the inclination angle takes into account the orientation of the screw. To recap from Chapter 3, the Aigner (2008) overflow leakage \( (Q_o) \), contains a discharge coefficient, \( \mu \), which is approximately 0.537 due to a triangular distribution, is modelled as:
\[ Q_o = \frac{4}{15} \mu \sqrt{2g} \left( \frac{1}{\tan \beta} + \tan \beta \right) (z_{wl} - z'_{max})^{5/2} \]  

(8-1)

The values of \( z_{wl} \) and \( z'_{max} \) are the height of water in a bucket and the maximum height before overflow begins \((f = 1)\). Values for \( z'_{min} \) and \( z'_{max} \) can be calculated theoretically based on the geometry of the screw using Equation (6-3).

The difference between \( z'_{min} \) and \( z'_{max} \) is used as the overall water level needed for a full bucket. The bucket fill height can therefore be calculated as the ratio of water depth inside the bucket to the maximum fill level, with the adjusted values from Chapter 6.

\[ f' = \frac{z_{wl} - z'_{min}}{z_{wl} - z'_{max}} \]  

(8-2)

Currently, overflow is determined by applying the fill height of water in the screw, to Equation (8-1). In order for \( fh5 \) to predict \( f' \), an iterative process is used: the flow of water is input, the model calculates the volume of water in a bucket, and based on the dimensions of the screw, predicts the fill height in the bucket. Since the basis of the model is steady-state flow through a triangular weir, dynamic effects within the screw, such as frictional shear and the translating motion of the weir openings are neglected, potentially leading to discrepancies between this model and actual AST overflow. Similarly, slight geometry differences exists, namely the curved surfaces that might create further discrepancies between these models and real-world AST overflow leakage. The current model does not account for dynamic effects within the screw, including frictional shear and rotation of the screw, and that the overflow occurs across a moving, curved surface instead of a sharp edged weir. A theoretical and experimental investigation of overflow was performed for a turning, laboratory-scale AST and compared to the results to the existing model.
8.2 Overflow Theory

As water travels down a rotating Archimedes Screw, there are a variety of dynamic effects taking place in each bucket. These effects depend on the screw’s rotational speed, the slope of the screw, and the size and shape of the bucket. Rotating buckets entrap water between helical blades and translate water down the screw. This means that as water enters the inlet of the screw the rotation of a blade cuts through the incoming flow and begins transferring the water down at its own speed, causing a disruption in the flow of water, and potential disruption at the inlet stream. From this point, water translates down the length of the screw using three flow paths; (1) buckets, (2) gap flow, and (3) overflow. Each flow path has an effect on the other. Dynamic effects in the buckets stems from the inflow of water, the effect of shear stress on the blades and the rotation of the blades causing induced currents. As water leaks through the gaps between the blades and the trough, this introduces a change in water dynamics in the buckets from the gap base. The focus in this chapter is to clarify how screw dynamics affect overflow. As water begins to overflow over the screws central shaft and pours into the downstream bucket, the water surface is interrupted by falling water. The current model for overflow is accurate enough for water flowing through a static, sharp crested weir. Since the screw is rotating, there is reason to believe that additional overflow is not being account for, and therefore being quantified properly. This is important because overflow is a loss, this flow does not aid in the rotation of the screw, and therefore does not contribute to power generation.

Overflow is not evenly distributed along the length of the screw. As a screw fills with water, the buckets in the center of the screw overflow first, followed by the buckets closer to the bottom of the screw and ending with buckets around the inlet. This is due to the geometry of the screw and other effects that have yet to be investigated. As overflow leaks from one bucket to another, the water fills the next bucket down and therefore overflow becomes part of the screws
bucket volume. Once the water travels down to the middle bucket, there is a maximum point before water fills over to the other side of the screw. This was independently seen in CFD analysis presented by Dellinger (2016) in Figure 8-1. Eventually, once enough water, all the buckets begin overflowing.

Since it has been seen experimentally determined that even though overflow is occurring more power can be made, it is important to properly quantify the screws flow in this flow regime. The reason more power is made during overflow is because the greater water depth within the bucket produces more torque than at lower water levels. The dynamics of water in a screws bucket suggests that overflow has additional beyond those in Equation (8-1), such as shear friction that causes waves.
8.2.1 Shear Stress

As water fills the bucket of a rotating screw, there is a power loss due to transport friction and rotational friction (Kozyn, 2015). This means there is a shear stress between the water surface and the central shaft, and the water surface and the rotating blades. There have been many investigations performed on submersed rotating cylinders where water flowing towards the cylinder had the ability to create vortices (da Silva et al. 2011, Kumar et al. 2011). The rotation of the cylinder affected the lift and drag forces on the cylinder. As water approached the rotating surface, viscous drag between the water and cylinder allow for the redirection of flow around the top or bottom on the cylinder.

The rotating cylinder can be considered a moving boundary condition, where the screw is a partial boundary, and the water surface is open channel flow. Liquid interactions with moving surfaces have been mostly solved by modifying the Blasius boundary-layer approximation (Antyshev, 1985). Blasius looked at the boundary layer equation for two-dimensional laminar flow over a parallel semi-infinite plate. The free stream velocity over the plate is held constant which resulted in constant pressure at the edge of the boundary layer, resulting in the moment equation for governing flow.

\[
\frac{u}{\partial x} + v \frac{\partial v}{\partial y} = \nu \left( \frac{\partial^2 u}{\partial y^2} \right)
\] (8-3)
Boundary layers along continuous and finite moving plates have been greatly studied by Sakiadis (1961). For laminar flow, momentum thickness and drag were larger on a continuous plate and smaller for a finite plate, and boundary layer characteristics were determined by the boundary conditions. It was also concluded that as a fluid travels along a moving surface, its velocity is equal to the velocity of the solid surface for laminar free steam flow (Merkin and Needham, 1985). Menu and Tavoularis (2007) placed a moving channel on the top of a closed loop water channel with a free surface. For a smooth moving plate of infinite length, and uniform free stream, the characteristics of the boundary layer depends only on the relative velocity between the plate and the free stream (Menu and Tavoularis, 2007).

Combining cylindrical surfaces as a moving boundary condition, lift from a rotating cylinder and open channel flow was visualized by Morton (1983). A rotating cylinder in a water channel was seen to produce a strong lifting force as water flowed past the surface. Further, Nesic and Caroll (2002) used a rotating cylinder partially submerged in water to study water-wetting at various speeds. This phenomenon has not yet been studied in ASTs to the author’s knowledge.

![Figure 8-3 Cylinder partially submerged in water a) 0 rpm, b) 500 rpm (Nesic and Caroll, 2002)](image)

As the depth of water in the screws bucket approaches $f' = 1$, the shear stress between the water and the rotating surfaces causes water to be dragged up over the central shaft and side of the
blades, creating some overflow before the bucket has reached or exceeded $f'' > 1$. Water that was originally measured as part of the buckets volume, is lost to overflow and no longer aids in the rotation of the screw. The rotating blades create an uneven shear on the water in the buckets, and blade shear causes secondary currents in the buckets that are not in the direction of transport. These currents are not negligible at high rotational speeds, and can also cause overflow.

8.2.2 Surface Dynamics

The second cause of inaccurate overflow prediction by the current model is the surface dynamics of water in the buckets, which stems from uneven shear stress and screw rotation. As the screw rotates, surface waves develop in each of the screws buckets. In the static case, the surface water level is assumed relatively steady. In rotating cases, for each rotation there is a pulse of water that rocks between the trough of the screw towards and central shaft, and back. When the depth of water in the bucket approaches $f'' = 1$, water is ‘rocked’ over the screws central shaft into the downstream bucket. Figure 8-4 shows screen shots from a video of overflow beginning in a lab AST. From the left, water begins to approach the middle of the central shaft, as the screw rotates further water is pulsed over the central shaft.

Figure 8-4 Progression of water pulsing over screws central shaft
Steady waves over a cylinder can be modelled using open channel flow. (See for example Shen et al. 1989). A wave passing over a stationary semi-circle, or bump in an open channel will cause the water surface profile to slightly increase at the face of the semicircle in order to traverse it.

![Figure 8-5 Flow over a stationary semi-circle (Anh and Hosoda, 2007)](image)

Water pulsing over the central shaft causes premature overflow and is more dominant as the speed of the screw increases. Water is no longer overflowing due to the volume of the screw, but being pushed over as the speed of the screw increases. The currents created from the rotating screw is potentially a large cause of overflow under-prediction.

Both the shear stress and surface currents in the screw theoretically suggest that there is a current under-prediction of overflow for a rotating screw. The new equation for overflow is expected to predict overflow as a function of water level \(z_{wl}\), angle of the screw \(\beta\) and rotational speed \(\omega\).

\[
Q_o = f(f, \beta, \omega)
\]  

(8-4)

A rotating screw contains a plethora of moving parts, dynamic and shear effects, and to the authors knowledge there has been no attempt to come up with a theoretical overflow formula that
accounts for these effects. This investigation does not attempt to incorporate all these theoretical effects mathematically, but instead uses experimental results to determine a new or adjusted overflow equation, since Equation (8-1) assumes the water in the screws buckets is static. In order to determine a new equation for overflow, or a correction for the current overflow model, experiments were performed on two lab sized ASTs to find a relationships between rotational speed, and overflow.

8.3 Experimental Procedure

Two laboratory scale screws, Screw 2 and Screw 16, (summarized in Table 8-1), were tested with a range of total flow rates including those that produced overflow.

<table>
<thead>
<tr>
<th>Screw</th>
<th>OD (cm)</th>
<th>ID (cm)</th>
<th>S (cm)</th>
<th>N</th>
<th>ID/OD</th>
<th>L (cm)</th>
<th>L/S</th>
<th>z’_{max} (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>#2</td>
<td>31.62</td>
<td>16.83</td>
<td>31.75</td>
<td>3</td>
<td>0.53</td>
<td>121.92</td>
<td>3.84</td>
<td>15.47</td>
</tr>
<tr>
<td>#16</td>
<td>38.61</td>
<td>16.89</td>
<td>38.30</td>
<td>4</td>
<td>0.44</td>
<td>94.89</td>
<td>2.48</td>
<td>17.33</td>
</tr>
</tbody>
</table>

Starting at no flow through the screw, flow was increased in increments of 0.3 – 1 L/s for rotational speeds from 0 to 50 RPM, data including bucket fill height was recorded. The experimental procedure for these screws was the same as that was used to measure the fill height in Chapter 6. The amount of overflow that could be produced at higher flow rates was limited since increasing the rotational speed resulted in lower fill height in each of the buckets. In addition, the highest possible flow from the pump was 14 L/s, which also limited the overflow that could be achieved at higher rotational speeds.

During each measurement, the flow rate, rotational speed of the screw, height of water at the inlet, and bucket fill height were recorded over a 60 second time period and averaged. The
lower basin was always maintained at a water level below the bottom of the screw outlet, to ensure consistent outlet loss effects. Measurement of bucket fill height is described in Chapter 6.2, and a full error analysis is presented in Appendix E. The average uncertainty was found to be plus or minus $\delta f = 0.072$.

8.4 Results and Discussion
8.4.1 Overflow Measurements

A pressure transducer provided a measure of water depths inside the screw which can be converted to fill heights, $f'$. As the fill height inside the screw increased so did the flow rate of water. Since the maximum fill height without overflow ($f' = 1$) was determined visually, and the fill height correction is a representation of the visual overflow, the fill height is labelled as $f$. Once the buckets began to overflow, the rate at which flow rate increased changed, the change in increasing flow rate is the transitional point where $f = 1$ shows in Figure 8-6. By fitting an equation for $Q = f(f)$ between $0 < f < 1$, flow rates excluding overflow above $f = 1$ were extrapolated and compared to the actual recorded flow rate. This fit equation included both bucket flow and gap flow. Since gap flow was continuous smoothly varying function primarily of gap length, it was reasonable to use the curve fit to account for the relatively small increase in gap flow when $f > 1$. The remaining difference between the extrapolated flow rates and the actual measured flow rates when $f > 1$ was then the overflow $Q_o$. Figures 8-6 through 8.9 show the measured fill heights as a function of flow rate.
The measured pressure was also compared to non-dimensional flow rate in Figure 8-7, showing the collapsing effect of rotational speed on fill heights.
There was a sharp change in increasing fill height at approximately $f = 1.2$, read by the pressure sensor. For each speed, the fill height of water in the bucket linearly increased with increasing flow rate as the fill level approached $f = 1$. The increase of water level was steepest for the stalled case and slowest at the highest rotational speeds. After the bucket reached its maximum volume, the level at which the depth of water increased with increasing flow rate began to plateau. Especially at higher speeds above 60 RPM, the amount of water that splashed out of the top of the screw visually observed to increase. In addition, the uneven nature of the gap between the screws trough and the helical blades meant as the flow rate increased, more water began to leak out of the sides of the trough.
Similarly to Screw 2, the fill height of water for Screw 16 increased linearly until there was a sharp change at \( f = 1 \) (Figure 8-8). Unlike Screw 2, the fill height increases as a much higher rate while overflowing. Screw 16 had a larger outer diameter, less water was able to splash out of the top of the trough, trapping water between buckets in a more efficient manner. Figure 8-9 shows how the non-dimensional flow rate collapsed for each fill height.

![Figure 8-8 Overflow analysis for screw 16](image)

Here the collapse is more distinct compared to Screw 2 in Figure 8-7; this is possibly due to the larger size of the screw. Two investigations of possible overflow relationships for these screws are presented below. Although it is theorized that speed will have an effect on the overflow, first a simple adjustment to the constant \( \mu \) in Equation (8-1) was investigated. Then, using a statically software program SPSS, a relationship between the overflow results and rotational speed was investigated.
8.4.2 Adjustment of Overflow Equation Constant

Model fh5 uses overflow equation uses overflow equation (8-1). An investigation was done to see if this equation had the potential to properly predict overflow with the simple adjustment of constant $\mu$. Rohmer et al. (2015) suggested that using $\mu = 1.09$ would theoretically give better agreement with experimental results than the commonly used value of $\mu = 0.537$. Since it has been concluded in Chapter 6 that the model inaccurately predicts the fill height in the screw for values of $f > 1$, the actual fill heights from the pressure sensor are plugged into Equation (8-1). Screw 16 is chosen for this investigation due to the large amount of experimental points that were taken for the 0 RPM case. Figure 8-10 shows the experimentally determined overflow and the overflow predicted by equation (8-1) with different values of constants $\mu$ for Screw 16. Non-dimensional overflow is just the measured overflow divided by the total flow rate shown in Equation (8-5).

$$Q_{o,nd} = \frac{Q_o}{Q}$$  \hspace{1cm} (8-5)
Equation (8-1) was used to calculate overflow along with constants $\mu = 0.537$, Rohmers suggested $\mu = 1.09$, and a new experimentally determined coefficient of $\mu = 3.76$. This new constant was determined by back-calculating $\mu$ from the 0 RPM results. The error bars were not shown in Figure 8-10 for clarity but there is an average uncertainty of $\pm 0.072$. When the screw was not rotating, there were no rotational effects that were theorized to affect the overflow results. It is also apparent in Figure 8-10 that there was a speed dependence to $Q_o/Q$ that could be accounted for simply by modifying the constant $\mu$. The most significant under-prediction of overflow occurs at relatively low fill heights and higher speeds. These are the conditions when the film flow effect of a rotating lower surface (in the form of the central cylinder and adjacent flight) would be expected to be greatest. Equation (8-1) is based on the assumption of a sharp-edged weir, which specifically neglects shear interactions between the flowing water and the bounding surfaces defining the weir, so this effect of friction of water on the adjacent surfaces would be entirely absent in predictions by Equation (8-1).
At higher fill levels, overflow may occur across the entire centre shaft, in which case, pitch S would also be a factor. The data from this experiment only allowed consideration of the effect of fill and speed, however, these other variables were also important. As rotational speed increased, so did the slope of the experimental overflow results. Equation (8-1) with an adjusted constant $\mu = 3.76$ did an adequate job predicting overflows at low rotational speeds, but as the speed of the screw increased so did the rate at which fill heights increased. This suggested that as fill heights increases, the height of water over the overflowing blade also increases. Even when the buckets were not full, there was a loss due to the rotation of the screw that is not accounted for in Equation (8-1).

The results showed that a single equation based on a static model of weir flow is incapable of predicting overflow in rotating screws because of a lack of inclusion of any dynamic effects in an operating screw. Since there is not one constant that is appropriate across the flow rates, it is theorized that the overflow equation is also speed dependent, and so is ultimately a function of fill height, rotation speed and slope.

8.4.3 Investigation of New Overflow Equation
A new adjustment to Equation (8-1) was derived using a linear regression tool with statistical software SPSS [IBM SPSS Statistics 24]. Figure 8-11 shows the difference between the overflow calculated by the fh5 model and the observed experimental overflow.
Both Screws 2 and 16 are represented in Figure 8-11 and there is generally an under-prediction of overflow by fh5. At low speeds (corresponding to low $Qo$), fh5 predicted experimental overflow with greater accuracy: the key is to find a relationship for overflow when speed increases.

SPSS was used to develop a non-dimensionalized equation that predicts overflow using flow rate and rotational speed. Non-dimensional overflow is calculated using Equation (8-5), non-dimensional flow rate with Equation (7-1) while the non-dimensional speed is found using the Muysken limit, an empirical limit where frictional losses surpass power losses (Equation (8-6)).

$$RPM_M = \frac{50}{D_o^{2/3}}$$

(8-6)

The Muysken limit changes with changing outer diameter and is used along with the measured RPM in each screw to find a non-dimensional value for speed, $\omega_{nd}$. 
\[ \omega_{nd} = \frac{RPM}{RPM_M} \] (8-7)

Each screw was first considered separately to determine if the effect of screw geometry greatly influenced the estimated overflow. Equations (8-8) and (8-9) were fit using incoming flow rate and speed resulting in \( R^2 \) values of 0.988 and 0.967 for Screws 2 and 16 and are compared in Figure 8-12.

\[
Q_{o,nd,2} = 0.84 Q_{nd} + 0.61 \omega_{nd} - 0.83 \quad (z_{wl} > z_{max}) \tag{8-8}
\]

\[
Q_{o,nd,16} = 0.61 Q_{nd} + 0.35 \omega_{nd} - 0.46 \quad (z_{wl} > z_{max}) \tag{8-9}
\]

*Figure 8-12 Qo corrections for screws 12 and 16*
There is an additional constant in Equations (8-8) and (8-9) when using the non-dimensional flow rate and speed to predict overflow. The constants in Equation (8-8) are larger than the constants in Equation (8-9) suggesting that the geometry of the screw had an additional effect that has not been accounted for. Combining data from both screws to find a universal equation also yielded a constant, and had an $R^2$ of 0.969.

$$Q_{o,nd} = 0.712Q_{nd} + 0.271\omega_{nd} - 0.544 \ (z_{wl} > z_{max}) \quad (8-10)$$

![Figure 8-13 Qo correction using flow rate and rotational speed](image)

Equation (8-10) improved accuracy at higher flow rates, but has a constant of 0.544 that might not be theoretically expected. The constant may be some other factor that affects how overflow is predicted. The equation accounted for some of the screws geometry due to Equation (7-1), the non-dimensional equation used for flow rate. This equation may also change with screw sizes beyond the experimental range that was tested. As the size of the screw gets larger, surface tension effects would be expected to have less of an effect on the overall power production and
efficiency of the screw. These corrections are able to be used in small scale laboratory screws, when data from the pressure transducer is unavailable.

SPSS was used to create another overflow equation that utilized the existing static overflow equation. Equation (8-1) already accounts for screw geometry and angle, and should scale with the other experimental screws. Equation (8-11) can be used to predict overflow when data from the pressure transducer is available for the screws. An additional component of non-dimensional speed is added onto Equation (8-1) in order to correct the difference between $fh5$ and the actual experimental overflow. This equation is only valid when $z_{wl} > z_{max}$.

$$\frac{Q_o}{Q} = 1.085 \left( \frac{4}{15} \mu \sqrt{2g} \left( \frac{1}{\tan \beta} + \tan \beta \right) \frac{(z_{wl} - z'_{max})^{5/2}}{Q} \right) + 0.238 \omega_{nd}$$

$$R^2 = 0.947$$

Equation (8-11) used results from both Screw 2 and 16 and has several key components: the non-dimensional overflow, which contains some of the screws dimensions, the inclination angle of the screw, the fill height of the screw, and the non-dimensional speed. The coefficient for the non-dimensional overflow is nearly 1, suggesting that the main overflow adjustment lies in the rotational speed of the screw. As the speed of the screw increased, so did the overflow. Figure 8-14 shows how the new overflow equation predicts the experimental results.
Equation (8-11) accurately predicts experimental overflow within uncertainty at higher overflows above 0.2. Below 0.2. The new equation under-predicts the overflow for some cases. There is also a slight curvature to the results in Figure 8-14, suggesting a polynomial may better predict overflow. Fitting the above results with a polynomial only increases the $R^2$ values by 0.01, and results in an addition constant, and it therefore not investigated. Overall, the conclusion is that overflow can be predicted more accurately by including the rotational speed as a component.

8.5 Summary
For the laboratory sized screws at the University of Guelph, it has been determined visually and through readings of measured pressure, that the current static assumption for overflow is not accurate. This is important since overflow is considered a power loss, and developing a model that accurately predicts power in a screw, must also include predicting power losses. Generally, as the speed of the screws investigated increased, so did the amount of overflow. Error in overflow is
especially difficult to quantify, since the screws trough is open, allowing water to not only overflow from one bucket to another, but to splash out of the screw, decreasing the overall amount of water aiding screw rotation.

From a theoretical stand point, this splashing and rocking of water is occurring due to secondary currents caused by the rotating blades of the screw. In addition, surface tensions between the screws rotating surfaces and water have not been accounted for. Even though a theoretical mathematical equation was not developed to quantify all these losses, an introduction into understand another dynamic part of an AST was presented.

An important remaining uncertainty is how overflow scales with screw size. Equation (8-1) may be more effective in full scale screws, where greater overflow volumes would mean reduced impacts due to interaction with the rotating screw surfaces. Development of a universal overflow equation will require determining the impacts of scale on overflow. A multi-scale set of overflow data at different rotational speeds would be useful for deriving an overflow equation including the effects of AST geometry, angle, speed and fill height.
Chapter 9 : Conclusions and Recommendations

This thesis looked at a variety of laboratory scale ASTs with varying parameters and tested them at a set of flow rates, inclination angles and speeds in order accurately characterize power and efficiency. The ability to test this many screws with a flexibility of slope, input flow and rotational speeds is something that is not available in any other known laboratory. In addition, new techniques for measuring the amount of water in a screws bucket were used to resolve a discrepancy in the way overflow is measured. A well rounded model is key to the advancement of real-world ASTs, since they are relatively new source of renewable energy. A summary of the finding from the research in this thesis include:

- Comparing length, outlet submergence, inner diameter, pitch and number of flights, length has the greatest effect on the power output and efficiency of a screw.
- By including translational, rotational, and outlet power losses in the current model, the ability to predict power and efficiencies was achieved with much greater accuracy than in earlier models that did not include these effects.
- A pressure transducer tapped to the bottom of a solid AST through was able to accurately read fill heights in screws buckets.
- The inclination angle of a screw did not have a large effect on how the screw fills, but will affect the output power and efficiency of a screw, leaving the ideal screw angle in the range of 24.5°.
- Overflow was not being correctly modelled when assuming static water in the screw, and rotational component was added to resolve discrepancies.

Starting with power losses, the current model established by Lubitz et al. (2014) known as fh3, included losses due to gap leakage, pressure difference between screws buckets, and overflow,
based on the static amount of water in the screw. The enhanced model, \( fh5 \), introduced losses due to water at the outlet of the screw, and rotation and translational shear stress losses. Outlet losses stem from typical screw condition, after water flows through an AST is exits into a lower basin, at times the water level is high enough that is enters into the outlet of the screw. When this happens, it creates a shear stress between the rotating blades and the water’s surface. In addition, as water enters the outlet of the screw, it lowers the available head and essentially shortens the screw. Rotational and translational losses along the enclosed trough, the rotational central shaft and the blades is another large power loss that has only recently been accounted for. By including these into the new model, a more accurate prediction of power and efficiency is theoretically able to be achieved.

A database was created for each of the 16 experimental screws, which were tested for a consistent set of flow rates, rotational speeds and outlet fill heights (summarized in Table 4-1). Testing each screw at the same combinations of conditions provided the opportunity to compare and contrast the effect of each parameter individually. Across all the screws, the outlet fill heights showed a consistent trend. No water at the outlet of the screw consistently showed a greater result in power compared to water being filled into the outlet. Increasing water level at the outlet of the screw, effectively shortened the screw and introduced new frictional power losses. This suggested that length has an effect on power and efficiency. An investigation into length confirmed this, power increased linearly with length against three different sets of screws that were tested. An increase of approximately 5 W was observed for every 20 cm increase in length. Efficiencies followed an increasing trend as well, but plateaued suggesting that even though more power is created with length, there are still power losses associated with the internal workings of the screw. Some geometric parameters did not have a large effect on power or efficiencies, these included
number of flights and the diameter ratio, which had average power and efficiencies across test ranges that agreed within error. Even though theoretically more flights means more buckets, it also means that there is less water in each bucket, more frictional losses, and less overall volume due to the thickness of the screws blades. Similarly, even though a smaller inner diameter provides larger buckets, the weight of water has an effect on screw bearing losses. There is a similar effect with pitch where larger buckets does not necessarily translate to a better screw. The key is to combine all these parameters together to determine the ideal screw, one that is long enough to have a sufficient amount of buckets, which are large but also have enough blades available to successfully turn the screw.

A sensitivity analysis of the outputs fh3 and fh5 was performed to summarize any power loss improvements that were made. The efficiencies for screws that were 60% submerged at the outlet were compared experimentally along with both models. The new model, fh5 resolved most outputs within error, trending as an under-prediction to the experimental results. The reason for the under-prediction is the generalization in the power loss equations. Most of the power loss equations still assume that water is static in the screws buckets, moving surface water is a factor in the screws losses.

Even with the experimental results, it is difficult to break down the inner dynamics of a screw due to its inaccessibility. Since ASTs are usually enclosed in a solid encasement, confirming the fill height in a screw, and how water interacts at the screws surface is not visually possible. By drilling a hole along the bottom of the screws enclosure, and tapping it with a pressure transducer, it was theorized that the static pressure reading could be converted into a depth of water in a bucket. The pressure sensor was tested in a small-scale AST with clear encasement to confirm accurate reading of the sensor and then introduced to the laboratory scale screws. It was concluded that the
fh5 accurately predicts fill height in a screw up until a maximum point, where water then begins to overflow. This sparked an investigation into the validity of how overflow is measured. By accurately being able to read the fill height of water in a screws bucket, an investigation on inner screw interactions was investigated in more detail.

Screw 2 was tested at three different inclination angles, 20°, 24.5° and 29°, the resulting power, efficiencies and fill height in the buckets were recorded to determine the optimal slope. It was concluded that the optimal angle of 24.5° that has been being used thus far, predicts the greatest power and efficiency even though the other angles do not have a largely visible effect on how fill height in the screw increased. The 29° slope also displayed reasonably good results, while 20° was the worst slope in terms of efficiency.

Finally, prediction of overflow in an AST was modelled by a v-notch weir equation that assumed static flow through a screw. Since the screw is rotating and water at the surface is constantly moving, it was theorized that rotation was a key component to overflow. Trying to adapt a constant in the current overflow equation, \( \mu = 3.76 \) helped resolve overflow discrepancies in low RPM cases, but failed to accurately predict overflow at more reasonable experimental speeds. Two Screws, 2 and 16, were tested to show that rotational speed does have an effect on overflow both visually and experimentally. The experimental overflow was for to equations using SPSS. The resulting equation took the results from the current overflow equation and added a component of speed. Creating a new overflow equation based on theorized dynamics in the screw was not done, instead, a general equation that encompassed the speed based discrepancy was put forth. It is also possible that this new overflow equation scales with screw size but it was not possible to test this aspect. In a larger AST, larger amounts of water flowing over the screws central shaft overpowers
any frictional power losses and the current static equation may do an adequate job predicting overflow in larger screws.

It is recommended that a deeper investigation into power losses is done. Most of the losses utilized in fh5 are still simplifications, assuming static water moving through an AST. Now that the measured pressure can be used to determine overflow, and show that there is a rotational effect on water in the screw, a more complete analysis can be investigated. In addition, a theoretical model based on mathematics can be attempted for the overflow equation. These screws have been tested experimentally, predicted with a model, a CFD analysis on the screws would create a complete AST model that can be translated into the real-world.
References


Appendix A: Power and Efficiency Charts
Figure A-1 Contour plots for power, RPM and flow rate for all 16 screws
Figure A-2 Contour Efficiency RPM and Flow rates for 16 experimental screws
Appendix B: Error analysis for power and efficiency

The error analysis for Power was based on the error of each component from the equation below:

\[ P = \omega T \] (B-1)

\( P \) is the overall power generated from the screw, \( \omega \) is the speed of the screw in rad/s and \( T \) is the torque. The speed measurements are taken with a magnetic switch that turns off each time the screw rotates, as described in chapter 4. The torque measurements are taken by a load cell that measures the force needed to break the screw from the gear motor. The torque is the force recorded by the load cell multiplied by 0.265 m which is the length between the center of the gear motor, or shaft, and the load cell.

In order to find the error, an analysis was done on each individual component of the power equation and a standard error analysis equation was applied.

\[ \delta P = \sqrt{\left( \frac{\partial P}{\partial \omega} \delta \omega \right)^2 + \left( \frac{\partial P}{\partial T} \delta T \right)^2} \] (B-2)

Below is a table of uncertainties for each component in the power equation.

<table>
<thead>
<tr>
<th>( \delta \omega )</th>
<th>( \delta T )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 rad/s</td>
<td>0.22* Nm</td>
</tr>
</tbody>
</table>

*average based on error analysis of Torque

\[ \delta P_{avg} = 1.16 \, W \] (B-3)
A similar error analysis was performed on the efficiency of each screw. The efficiency uses the experimental power and its error found previously, with the addition of theoretical maximum power.

\[
\eta = \frac{P}{P_{\text{max}}} \quad \text{(B-4)}
\]

\[
P_{\text{max}} = \rho g Q (L \sin \beta) \quad \text{(B-5)}
\]

The theoretical maximum power has additional components of uncertainty including the measured length of the screw, the flow rate, and inclination angle. The uncertainty in the flow rate, previously estimated at 5% (Kozyn, 2015). The length of the screw has an error of 0.001 m from Stanley Tylon measuring tape, while the angle has an error of 1 degree, or 0.0017 rad. Density of water 998 kg/m³ and 9.81 m/s² are held as constants. These new uncertainties are accounted for in the standard error equation:

\[
\delta \eta = \sqrt{\left(\frac{\partial \eta}{\partial P} \delta P\right)^2 + \left(\frac{\partial \eta}{\partial L} \delta L\right)^2 + \left(\frac{\partial \eta}{\partial \beta} \delta \beta\right)^2 + \left(\frac{\partial \eta}{\partial Q} \delta Q\right)^2} \quad \text{(B-6)}
\]

A table of uncertainties describes the estimated errors of each component.

<table>
<thead>
<tr>
<th>(\delta P)</th>
<th>(\delta L)</th>
<th>(\delta \beta)</th>
<th>(\delta Q)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.16*</td>
<td>0.001 m</td>
<td>0.0017 rad</td>
<td>5%</td>
</tr>
</tbody>
</table>

*average based on error analysis of Power above

\[
\delta \eta_{\text{avg}} = 0.03 \quad \text{(B-7)}
\]
Appendix C : Measured Pressure Confirmation

Before a static pressure transducer was used to measure fill heights in the enclosed screw, it was measured in a smaller screw with a clear encasement to confirm its ability to measure depths with accuracy. A burette and retort stand combination were used to calibrate the pressure transducer. The pressure transducer was fastened to the bottom of the burette and water was slowly introduced to the top.

![Retort stand and pressure sensor for sill confirmation](image)

Water was measured manually at different heights above the pressure sensor and the corresponding voltage was recorded using a data acquisition reader and code,
DAQ_Aquisition_KSongin.m script in MATLAB. 10 second runs were recorded and the average of the resulting voltages were plotted against the measured depths to find a calibration curve.

![Graph showing calibration curve]

\[ y = 0.1752x - 0.0823 \]

Figure C-2 Calibration curve of confirmation fill height

Where \( y \) was the depth of water above the pressure sensor and \( x \) is the voltage output from the pressure sensor. The dimensions of the small scale screw are summarized below:

Table C-1 Small-scale screw dimensions

<table>
<thead>
<tr>
<th>Length</th>
<th>Inner Diameter</th>
<th>Outer Diameter</th>
<th>Pitch</th>
<th>Flights</th>
<th>Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>58 cm</td>
<td>8 cm</td>
<td>14 cm</td>
<td>15 cm</td>
<td>3</td>
<td>25°</td>
</tr>
</tbody>
</table>
The ASG was situated in a clear plastic encasement between two basins, on an angle of 25°. Water was run through a hose into a basin on the floor which has a 37 cm diameter 221 cm long bypass pipe connecting it to another basin each with a 50 gallon capacity. One of the floor basins allow for water to be pumped up into the upper basin and discharge from the lower basin to exit into the other flow basin. Once enough water has filled the upper basin and begins to flow into the entrance of the screw, the screw should rotate freely, translating water down the screws shaft. If the screw does not spin or there is a hitch in the rotating blades, the screw may be adjusted with an allen key by adjusting the bearings. In addition, there is no break on the screw, the screw is slowed down by means of a leather belt wrapped around the shaft of the inner diameter of the screw where the RPM sensor is. The RPM sensor is attached at the top crossbar of the screws metal frame in order to avoid contact with water. The pressure sensor is attached to the pipe fitting on the bottom of the screws encasement.
A variety of pictures were taken with alternating flow rate and screw RPM and the depths provided by the pressure sensor were compared to pictures for verification.

Each time the screw rotates, a magnet passes the RPM sensor. The magnet turns the sensor on, recording a voltage of about 5.5 V. In order to determine the RPM of the screw, the number of times the magnetic switch was turned on is counted over a time frame of 5 seconds and multiplied by 12 in order to give the number of rotations in one minute.
Calculating the depth of water inside the screw while the screw is rotating was also performed by hand. Each time the blade passes over top of the pressure sensor, it creates a linear curve output between each of the blades. Since the screw has three flights, three blade passages can also be used to determine the RPM of the screw. The pressure sensor recorded raw voltages as the screw was rotating for 60 seconds. The output gives the same pattern shown in Figure 3, and is the depth of water in the screw as the blades rotate. The maximum values correspond to the leading edge of the blade and the minimum values to the end of the blade. Since the pressure sensor is attached to the screw perpendicular to the encasements surface, the measurements of depth are directly from the top of the pressure sensor to the water line.
A picture of each run was taken with a Cannon Rebel camera in order to confirm the height measurements of water against the pressure sensors predictions. The raw data should be able to be super imposed onto the photos and mimic the same trend.
This confirmed that the pressure sensor is able to accurately measure the depth of water inside an ASG. Therefore, for cases where the water level in the ASG is unknown due to a lack of visual evidence. The water height may still be predicted, this can be helping when determining at which point there is over flow in a screw.
Appendix D: Pressure Sensor Calibration

The Omegadyne PX309 pressure sensor was calibrated by fastening the pressure sensor to the base of a clear bucket. The bucket was filled with water and the depth of water was measured with a ruler.

The voltage output at each depth of water by the pressure sensor was recorded and plotted to create a calibration curve. This relationship is then used to determine the depth of water in the screws bucket when visual confirmation is not possible.
Figure D-2 Pressure transducer calibration curve

$y = 0.1209x + 0.0101$
Appendix E: Fill height error

The error analysis for the fill heights was based on the error of each component in the following equation:

\[
f = \frac{d - o_f}{z_{\text{max}}}
\]  

(E-1)

\(d\) is the raw depth of water in a screw bucket given by the pressure sensor, \(o_f\) is the offset of the pressure sensor due to the tap the sensor is screwed in to, and \(z_{\text{max}}\) is the height required for \(f = 1\) conditions. The errors for each component depend on their measurement tool. The raw depth of water is measured by the Omegadyne PX309 pressure sensor, with a manuafactures error of 2%. The offset depth is measured with calipers, however, since the pressure transducer is submerged in water there is possibility for an air bubble that may offset the actual measurements of depth in a bucket, 0.5cm. Calculations for \(z_{\text{max}}\) are based on the screws geometry found in equation (3-4), the error on these components, inner diameter, outer diameter, pitch are 0.5 cm with the slope having an error of 1 rad/s.

<table>
<thead>
<tr>
<th>(\delta d)</th>
<th>(\delta o_f)</th>
<th>(\delta z_{\text{max}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>2%</td>
<td>0.5 cm</td>
<td>0.5 cm</td>
</tr>
</tbody>
</table>

Table E-1 Fill height error

In order to provide these errors, an analysis was done on each individual component and a standard error analysis equation was used. The result is an average error in fill height of 0.072.

\[
\delta f = \sqrt{\left(\frac{\partial f}{\partial d}\delta d\right)^2 + \left(\frac{\partial f}{\partial z_{\text{max}}}\delta z_{\text{max}}\right)^2 + \left(\frac{\partial f}{\partial o_f}\delta o_f\right)^2}
\]  

(E-2)

\[
\delta f = 0.072
\]  

(E-3)
Appendix F: Overflow Error

Overflow was measured using the following equation:

\[
Q_o = \frac{4}{15} \mu \sqrt{2g} \left( \frac{1}{\tan \beta} + \tan \beta \right) (z_{wl} - z'_{max})^{5/2}
\] (F-1)

Where (F-1) is the standard static v-notch weir equation. The error in inclination angle \( \beta \) being one degree. The error in water level \( z_{wl} \) stemmed from the error calculated in fill height, resulting in \( \pm 0.5 \) cm and the maximum bucket fill height at \( \pm 0.05 \) cm.

\[
\delta Q_o = \sqrt{\left( \frac{\partial Q_o}{\partial \beta} \delta \beta \right)^2 + \left( \frac{\partial Q_o}{\partial z_{wl}} \delta z_{wl} \right)^2 + \left( \frac{\partial Q_o}{\partial z'_{max}} \delta z'_{max} \right)^2}
\] (F-2)

\[
\delta Q_o = 0.05 \text{ L/s}
\] (F-3)

Experimental overflow uncertainty was a direct result of error in fill height (\( \pm 0.072 \)) and flow rate (\( \pm 5\% \)) measurements. The overflow was experimentally extrapolated using those two factors.

\[
\delta Q_o = \sqrt{\left( \frac{\partial Q_o}{\partial Q} \delta Q \right)^2 + \left( \frac{\partial Q_o}{\partial f} \delta f \right)^2}
\] (F-4)

\[
\delta Q_o = 0.03 \text{ L/s}
\] (F-5)