Investigation of excited $0^+$ states populated in the $^{162}\text{Er}(p, t)$ reaction

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ABSTRACT

*Investigation of excited $0^+$ states populated in the $^{162}\text{Er}(p, t)$ reaction*

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Interpreting the nature of excited states in well-deformed nuclei has been an ongoing challenge in our understanding of nuclear structure. Some of the approaches that have been implemented to interpret the occurrence of low-lying excited $0^+$ states include vibrational excitations in β-phonons and γ-phonons, as well as pairing excitations. A further complication is the presence of shape coexistence which can increase the number of low-lying states, and if the shapes undergo mixing, the spectroscopic signatures can become ambiguous. An example of such a case is in the rare-earth region, where there is a rapid change in the ground state shape from $N = 88$ to $N = 92$. One of the difficulties in resolving the nature of these states is that there is a paucity of data, particularly for excited $0^+$ states, in the rare earth region.

Two-neutron transfer reactions are ideal for probing $0^+ \rightarrow 0^+$ transitions in deformed nuclei. One of the intriguing features of the rare-earth region are the strongly-populated $0^+$ states that emerge in both $(p, t)$ and $(t, p)$ two-neutron transfer reactions that have been interpreted as signatures of shape coexistence and pairing isomerism. In the present work, excited $0^+$ states have been studied via the $^{162}\text{Er}(p, t)^{160}\text{Er}$ reaction at the Maier-Leibnitz Laboratory in Garching, Germany using 22 MeV and 24 MeV proton beams and the reaction products were momentum-analyzed with the Q3D magnetic spectrograph.

In this work, sixty-nine levels were observed and angular momenta were assigned for
thirty-eight of these states. In total, seven excited $0^+$ states were assigned, six of which were unknown until the present work and one of which was not definite. The cross sections of these low-lying excited $0^+$ states, with the $0^+_2$ state population around 18% of the ground state strength, suggests a special character for this state which is inconsistent with a $\beta$-vibration interpretation. The similarity of the population of the $0^+_2$ state in $^{160}$Er with low-lying $0^+$ states in $N = 90$ isotones suggests that the nature of these excitations are quite similar, with the possibility that the $0^+_2$ state is an example of shape coexistence. Results of the relative population of the excited $0^+$ states in $^{160}$Er will be presented, and placed into context with similar experiments in the $N = 90$ region.
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Chapter 1

Introduction

1.1 Nuclear Shell Model

Although the strong interaction is responsible for binding quarks into nucleons, it can also describe, in a nuclear regime, the effective force responsible for binding nucleons together. When the strong force is taken in its latter context, it is called the nuclear force. To be able to accurately describe the motions of nucleons in a nucleus, one must describe the nucleus as a quantum many-body fermionic system, where nucleons interact via the nuclear force. In general, the many-body Schrödinger equation must be solved, where the Hamiltonian is given by a kinetic term and two-body, three-body, and higher-order interaction terms, by

\[
\hat{H} = \sum_{i=1}^{A} -\frac{\hbar^2 \nabla_i^2}{2m} + \sum_{i>j}^{A} \sum_{j=1}^{A} v_{ij}(\vec{r}_i, \vec{r}_j) + \sum_{i>j}^{A} \sum_{j>k}^{A} \sum_{k=1}^{A} v_{ijk}(\vec{r}_i, \vec{r}_j, \vec{r}_k) + \ldots \tag{1.1}
\]

for a nucleus consisting of \( A \) nucleons. However, even in a simplified picture, where the nuclear force is assumed to be dominated by two-body interactions, the interactions between
pairs of nucleons are still quite complex, and the Schrödinger equation becomes too computationally involved to be solved exactly beyond light nuclei. Because of this complexity, one of the most adapted approaches in modelling nuclei is a phenomenological one, called the shell model.

In its simplest form, the shell model describes how the motions of nucleons are governed by assuming that the particles are non-interacting. Under this assumption, the interaction terms in the Hamiltonian described in Equation 1.1 can be replaced by a short, attractive, mean potential which is produced by the remaining nucleons. This mean field is usually introduced with a simple potential like an infinite well potential or a spherically symmetric quantum harmonic oscillator, however, a far more accurate description of the shell model potential can be achieved by using a potential with a Woods-Saxon form.

The nuclear shell model is commonly explained using the analogy to an atomic system, where electrons fill discrete “shells” with different occupation numbers following the Pauli principle. Atoms with completely occupied electron shells tend to have the highest ionization energy and are thus the most stable. The presence of nuclear shells is quite clearly evidenced by the large discontinuities between the two-nucleon separation energies with their predicted separation by the semiempirical mass formula, which doesn’t account for shell structure. When the difference is plotted, large discontinuities are observed, as shown in Figure 1.1, which indicate regions of increased stability. These regions of stability occur at specific proton and neutron numbers, called “magic numbers”, which occur when the nucleon number is equal to 2, 8, 20, 28, 50, 82, 126, and so on. When both proton and neutron number are magic, such as in the $^{40}$Ca nucleus, the nucleus is dubbed “doubly-magic”.

One more term in the shell model potential is included to properly reproduce the observed magic numbers. If the orbital angular momentum of the level is non-zero, the addition of
Figure 1.1: Comparison between experimentally determined two-nucleon separation energies with the semiempirical mass formula predictions from [1]. The circled values are the nuclear magic numbers.

A “spin-orbit” term in the shell model potential splits each subshell into \((2j+1)\)-degenerate levels. The fitting of the strength of this term allows for an exact reproduction of the observed magic numbers. A nuclear shell model diagram with an appropriate potential including a spin-orbit term is shown in Figure 1.2.

The shell model relies on the simplification that only the valence nucleons are active, while all filled shells in the potential are inert. This greatly reduces the number of degrees
Figure 1.2: Nuclear shell model orbitals from a Woods-Saxon potential with (right) and without (left) the inclusion of a spin-orbit interaction [2]. The spin-orbit term splits the Woods-Saxon energy levels into \((2j + 1)\)-degenerate levels that have \(j = l \pm \frac{1}{2}\). The magic numbers which represent major shell closures are circled.

of freedom in shell model calculations and allows the model to describe the microscopic characteristics of many light to medium-mass nuclei, such as the spins and parities of odd-A nuclei ground states [1]. However, shell model calculations become quite computationally
involved for deformed nuclei. Away from shell closures, the valence nucleons tend to have coherent single-particle motion, giving rise to collective excitations [3]. For this reason, an alternative model is used to describe these collective motions in deformed nuclei such as the ones in the $N = 90$ region investigated in this thesis.

1.2 Collective Models

Although the shell model has enjoyed tremendous success at describing the properties of light nuclei, it is not very successful at describing the behaviour of heavier nuclei, particularly those away from the closed shells. In fact, some heavier nuclei have low-lying excited states that are better described as collective motions, where the motion of many nucleons are considered rather than the single-particle orbits [4]. The first collective model was developed by Bohr and Mottelson [5], which models the nucleus as a charged liquid drop of constant density, where the parameterized nuclear surface can experience surface vibrations, deformations and rotations.

1.2.1 Collective Vibrations

The shape of the nuclear surface in the collective model is described by a set of collective time-dependent coordinates, $\alpha_{\lambda\mu}(t)$, which parameterize the spatial distribution of nucleon density [4]. This parameterization uses spherical harmonics to describe the radius of the nucleus at a point, $(\theta, \phi)$, on the nuclear surface at a time, $t$, by

$$R(\theta, \phi, t) = R_0 \left(1 + \sum_{\lambda=0}^{\infty} \sum_{\mu=-\lambda}^{\lambda} \alpha_{\lambda\mu}^* (t) Y_{\lambda\mu}(\theta, \phi) \right)$$

(1.2)
where \( R(\theta, \phi, t) \) describes the time-dependent nuclear radius, \( R_0 \) is the average radius of the nucleus with zero deformation, and \( \alpha_{\lambda\mu}(t) \) are the time-dependent expansion parameters which describe the variation of the nuclear surface. The \( \mu \) subscript varies from -\( \lambda \) to \( \lambda \), which defines the multipolarity of the vibration or deformation, whose shapes are plotted in Figure 1.3. The parity of the mode is given by \((-1)^\lambda\). The \( Y_{\lambda\mu}(\theta, \phi) \) functions are the spherical harmonics which describe the orbital motion of the nucleons for given angular momentum, \( \lambda \), and magnetic quantum number, \( \mu \), which satisfy

\[
Y^*_{\lambda\mu}(\theta, \phi) = (-1)^\mu Y_{\lambda-\mu}(\theta, \phi)
\]  

where the parameterizations for the nuclear surface must also satisfy the same property such that

\[
\alpha^*_{\lambda\mu} = (-1)^\mu \alpha_{\lambda-\mu}.
\]

Figure 1.3: Illustration of the multipole modes of vibration adapted from [4] with \( \alpha_{\lambda\mu} = n\alpha_{\lambda} \) for \( n = 0, 1, 2, 3 \) and \( \lambda = 1, 2, 3, 4 \). The \( \lambda = 0 \) is a constant, high-energy monopole vibration indicating nuclear compression, \( \lambda = 1 \) is the dipole centre-of-mass shift and \( \lambda = 2, 3, 4 \) represent quadrupole, octupole and hexadecupole vibration, respectively. The dipole vibration term is ignored, because it represents a shift in the centre of mass of the nucleus.
To ensure that $R(\theta, \phi, t)$ is rotationally invariant under a change of coordinates from $(\theta, \phi) \rightarrow (\theta', \phi')$, the parameters in the rotated system must also map to the rotated coordinates $(\theta', \phi')$. A Wigner D-matrix can be defined that transforms the quantity $\alpha_{\lambda\mu'}$ into a rotated set of coordinates with

$$\alpha'_{\lambda\mu} = \sum_{\mu} D^{(\lambda)}_{\mu\mu'} \alpha_{\lambda\mu'}.$$  \hspace{1cm} (1.5)

In collective vibrational motion, small vibrations of the nuclear surface are created which are measured in discrete quanta called phonons, which carry energy $\hbar \omega$. Although thus far we have only introduced the formalism for vibrations of spherical nuclei, quadrupole-deformed nuclei can also vibrate in two distinct ways. If $\mu = 0$, the projection of $\mu$ on the phonon angular momentum symmetry axis preserves axial symmetry. These types of vibrations are called $\beta$ vibrations. If $\mu = 2$, axial symmetry is not preserved, the nucleus can still undergo a non-symmetric vibration called a $\gamma$ vibration. Figure 1.4 shows a diagram of $\beta$ vibrations and $\gamma$ vibrations along their symmetry axis.

![Figure 1.4: Schematic diagram of a $\beta$ vibration and $\gamma$ vibration of a deformed nucleus (dashed line) compared to its spherical counterpart (solid line) along its symmetry axis. The $\beta$ vibration preserves axial symmetry, while the $\gamma$ vibration does not. The symmetry axis is shown to be out of the page.](image-url)
1.2.2 Collective Rotations

When a nucleus has a permanent deformation instead of a spherical shape, it is said to be a deformed nucleus. Permanent deformation is reflected in a non-zero electric quadrupole moment which is common to the rare-earth and actinide nuclei [1]. The level of deformation is quantified by a deformation parameter, $\beta$, which is related to the intrinsic quadrupole moment of the nucleus. Assuming a pure quadrupole deformation, which is the only type of permanent deformation to play a large role in low-lying collective excitations of rare-earth nuclei, the five expansion parameters become

$$
a'_{2\pm 1} = 0 \quad a'_{20} = \beta \cos(\gamma) \quad a'_{2\pm 2} = \frac{1}{\sqrt{2}} \beta \sin(\gamma)$$

(1.6)

where the quadrupole deformation parameter, $\beta$, is defined as [4]

$$
\beta^2 = \sum_{\mu} |\alpha'_{2\mu}|^2.
$$

(1.7)

Because the orientation of the intrinsic frame with respect to the laboratory frame can be described using three Euler angles, three of the $a'_{2\mu}$ parameters can be set to zero [3,6]. The two remaining parameters are typically chosen to be the Hill-Wheeler coordinates $\beta, \gamma$ [7]. Using this convention, $\gamma$ represents the degree of axial symmetry, where $0^\circ \leq \gamma \leq 60^\circ$ is sufficient to describe all quadrupole-deformed nuclear shapes, from a $\gamma = 0^\circ$ prolate spheroid to $\gamma = 60^\circ$ oblate spheroid [3].

The change in the nuclear radius due to the collective rotation can be described as

$$
\Delta R = R_0 \sqrt{\frac{5}{4\pi}} \beta \cos \left( \gamma - \frac{2\pi}{3} k \right) \text{ for } k=1,2,3,...
$$

(1.8)
where the $\beta$ deformation parameter can be written in terms of the $\epsilon_2$ deformation parameter, which is used to plot Nilsson orbitals in Figure 1.6, for $\gamma = 0^\circ$, by

$$\beta = \sqrt{\frac{\pi}{5}} \left[ \frac{4}{3} \epsilon_2 + \frac{4}{9} \epsilon_2^2 + \frac{4}{27} \epsilon_2^3 + \frac{4}{81} \epsilon_2^4 + \cdots \right]$$

(1.9)

assuming a pure quadrupole deformation.

### 1.2.3 Quadrupole Deformation

Deformation in a nucleus can induce an electric quadrupole moment. This term can be described in the intrinsic frame as

$$Q_0 = \frac{3}{\sqrt{5\pi}} Z R_0^2 \beta (1 + \frac{2}{7} \sqrt{\frac{5}{\pi}} \beta + \cdots)$$

(1.10)

where $Z$ is the atomic number of the nucleus, $R_0$ is the static nuclear radius, and $\beta$ is the deformation parameter. Experimentally, the quadrupole moment must be measured in the laboratory frame. For a nucleus with total angular momentum, $J$, and having a projection of the angular momentum on the symmetry axis, $K$, this laboratory quadrupole moment becomes

$$Q = Q_0 \frac{3K^2 - J(J+1)}{(2J+3)(J+1)}.$$  

(1.11)

When a nucleus de-excites, it emits a $\gamma$-ray which carries off angular momentum $\lambda$. For electric quadrupole ($E2$) transitions, this can be expressed in terms of the deformation
parameter, $\beta$, and the intrinsic quadrupole moment, $Q_0$, by

$$B(E2; J_i \rightarrow J_f) = \frac{5}{16\pi} Q_0^2 e^2 \langle J_i K_i 20 | J_f K_f \rangle^2$$

(1.12)

where $\langle J_i K_i 20 | J_f K_f \rangle$ is the Clebsch-Gordon coefficient which specifies the angular momentum coupling of the initial and final states. Because the strong interaction is much stronger than the electromagnetic interaction for scales relevant in nuclear physics, the electric distribution of the nucleons can be measured without perturbing the nuclear distribution [1].

One of the key features which indicate the onset of collectivity in a nucleus is that the $2^+_1$ state energy is drastically reduced in nuclei away from closed shells compared to those at shell closures [6]. As the energy of the $2^+_1$ state decreases, the $B(E2; 0^+_1 \rightarrow 2^+_1)$ reduced transition probability described in Equation 1.12 generally increases, so this quantity is used as a measure of collectivity in a nucleus [3].

Another useful observation is that as the energy of the $2^+_1$ state decreases, the $E_{4^+_1}/E_{2^+_1}$ ratio increases. Near shell closures, nuclei have $E_{4^+_1}/E_{2^+_1} < 2$, and in the spherical vibrator model, $E_{4^+_1}/E_{2^+_1} \sim 2.0$. In many mass regions, nuclei pass through a rapid transitional region between the vibrational and rotational limits [6]. Using Expression 1.16, the rotational limit for axially symmetric quantum rotors can be derived to be $E_{4^+_1}/E_{2^+_1} \sim 3.33$. Both $B(E2; 0^+_1 \rightarrow 2^+_1)$ and $E_{4^+_1}/E_{2^+_1}$ systematics for rare-earth nuclei in the $N = 90$ region are plotted in Figure 1.5.

The similarities between the Er isotopes and other nuclei in the $N = 90$ region and its significance in terms of the experimental motivation are revisited in Section 1.4.
Figure 1.5: $B(E2; 0_1^+ \rightarrow 2_1^+)$ and $E_{4+}/E_{2+}$ systematics as a function of neutron number in the $N = 90$ region. The dramatic increase in both quantities in the $N = 90$ region suggests that these isotopes lie in a transitional region of rapid shape change. The trends across the Er isotopic chain have a striking similarity to the systematics of other isotopes in this region.

1.3 Deformed Shell Model

While the shell model has enjoyed some success in describing the microscopic characteristics of spherical nuclei, using the model to calculate single-particle states of deformed nuclei becomes much less practicable as shell model calculations increase in complexity. This is why the deformed shell model, otherwise known as the Nilsson model, was introduced, which is an independent-particle model for non-spherical nuclei. The deformed shell model uses a combination of the single-particle formalism that was developed for the shell model in
Section 1.1 and the formalism for collective excitations on a spherical nucleus described in Section 1.2 to describe the motions of deformed nuclei, with only the valence nucleons active.

Far from closed shells, a nucleus minimizes its energy by adopting a permanent deformation. To accomplish this, the nucleus rearranges its single-particle levels, breaking its spherical symmetry. As a consequence, spherical potentials are no longer appropriate in modelling deformed nuclei, so an ellipsoidal potential is generally used in order to describe shell effects [1]. The potential in the deformed shell model is based on an anisotropic harmonic oscillator, given by

\[ V(r) = \frac{m}{2} (\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2) \] (1.13)

where \( m \) is the mass of the nucleon which is confined with frequencies \( \omega_x, \omega_y, \) and \( \omega_z \). These frequencies can be written as

\[ \omega_k = \omega_0 \frac{R_0}{a_k} \quad \text{for} \quad k = x, y, z \] (1.14)

where the \( a_k \) variables represent half of the length of the ellipsoid axes.

The Nilsson diagram in Figure 1.6 plots the single-particle energies for deformed nuclei as a function of the deformation parameter of the nucleus, \( \epsilon_2 \). The single-particle energies split in the presence of a deformed field, and a residual Hamiltonian is responsible for state mixing which creates the curvature of the lines. In the Nilsson model, the rotation of the nucleus as a whole is ignored because it contributes far less to the deformation of the potential than the rotation of one nucleon around the nucleus.

Because of the splitting of nuclear states with non-zero angular momentum into two substates when the spin-orbit potential is included, the orbital angular momentum, \( l \), is no
Figure 1.6: Nilsson Diagram for nuclei between the N=82 and N=126 shell closures [8]. The $^{160}$Er nucleus lies in a deformation region of $\epsilon_2 \sim 0.3$. 
longer a good quantum number. The Nilsson wave functions are defined by

\[ \Psi_{\Omega[N_n\Lambda]} = \sum_{l,j} c_{Nlj} \psi_{Nlj} \]  

(1.15)

where \( \psi_{Nlj} \) are the spherical wave functions, and \( c_{Nlj} \) are the expansion coefficients. At this point it becomes important to define the symmetry axis for the nucleus, because the energy levels in the deformed shell model are no longer degenerate. The states split into \( j + \frac{1}{2} \) distinct orbitals\(^2\), where the energy of the orbital depends on the components of \( J \) along the symmetry axis [1]. We define \( \Omega \) and \( \Lambda \) to be the projections of the total angular momentum of the single-particle, \( j \), and orbital angular momentum, \( l \) of the orbits on the symmetry axis, \( z \). The \( N \) quantum number represents the total number of anisotropic harmonic oscillator shells, and \( n_z \) is the number of oscillator shell quanta along the symmetry axis. The single-particle orbitals for prolate and oblate shapes are shown in Figure 1.7.

\[ \text{Figure 1.7: Single-particle orbits (} j = \frac{7}{2} \text{) and their positive projections along symmetry axis (} \Omega = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2} \text{) for prolate (left) and oblate (right) shapes. Adapted from [1].} \]

\( ^2 \)Each of these orbits are still doubly-degenerate in \( \pm \Omega \)
The kinetic energy of the quantum rotor can be described in terms of its angular momentum, \( J \), and its moment of inertia, \( \mathcal{J} \), and the projection of \( J \) on the symmetry axis, \( K \), as

\[
E = \frac{\hbar^2}{2\mathcal{J}} (J(J+1) - K(K+1))
\]

which reduces to the symmetric quantum rotor for \( K^\pi = 0^+ \) bands.

A more realistic form of the Nilsson potential can be obtained by modifying the anisotropic harmonic oscillator potential described in Equation 1.13 [9]. The inclusion of a \( \vec{l} \cdot \vec{s} \) spin-orbit term allows for the reproduction of the magic numbers. In addition, a \( \mathcal{L}^2 \) term is also included, where \( \langle \mathcal{L}^2 \rangle_N = N(N+3)/2 \) is the average \( l^2 \) in a harmonic oscillator with \( N \) shells. The role of the \( l^2 \) term in the potential is two-fold: it allows the potential to reproduce more realistic results at large distances, and smoothes out the oscillator potential to resemble one between a square well and harmonic oscillator [9].

1.4 Shape Coexistence and Motivation

The promotion of particles across a shell gap may produce an excited state with a different level of deformation than the ground state, which can be preferred to the normal-ordering of Nilsson orbitals [10]. In the case that a nucleus has two states which have a different intrinsic deformation, are reasonably closely spaced, and remain distinct, the nucleus is said to possess shape coexistence. One of the manifestations of shape coexistence would be the presence of excited states which aren’t usually expected, which can include excited \( 0^+ \) states.

For several decades, two-neutron transfer reactions, such as \((p,t)\) and \((t,p)\) reactions have been the method of choice to study excited \( 0^+ \) states. Two-neutron transfer reactions probe
the pairing interaction between nuclear wave functions, where if a nucleus contains strong pairing correlations, its $0^+ \rightarrow 0^+$ transition strength is greatly enhanced. One of the key findings was that in these reactions, the cross sections of the first excited $L = 0$ excitation ($0^+_2$) and ground states ($0^+_1$) were comparable in $N = 88 - 90$ nuclei, where a rapid onset of deformation emerges [10]. This rapid onset of deformation in $N = 90$ nuclei is often interpreted as an example of shape coexistence [10], which manifests itself in various regions across the chart of nuclides, as shown in Figure 1.8.

Although the original motivation for studying $N \sim 90$ nuclei in the context of shape coexistence is its similarities to the $N = 60$ region [10], many interesting features have been observed in $N = 90$ nuclei which further motivate interest in this region in nuclei such as pairing isomeric bands in $^{152}$Sm, $^{154}$Gd [11, 12] and the possibility that the first excited $0^+$ band, based on the $0^+_2$ state, forms a "second vacuum" and the role of the $^{11/2^-}[505]$ configuration in this state in $^{154,155}$Gd [13, 14]. The observed systematic trends in the $B(E2; 0^+_1 \rightarrow 2^+_1)$ value and $E_{4^+_1}/E_{2^+_1}$ ratio, plotted in Figure 1.5, highlights the similarity between the isotopes in the aforementioned rare-earth nuclei, as well as the Nd and Er isotopic chains.

Classically, deformed heavy nuclei are observed to have $\beta$-vibrational and $\gamma$-vibrational bands which result from collective low-lying excitations [5]. However, the nature of low-lying excited $0^+$ states is still highly debated [10, 15]. Part of the problem in discerning the nature of this state is that there exists a paucity of data, particularly in heavy nuclei, for these low-lying $0^+_2$ states.

Figure 1.9 compares the relative strength of low-lying $0^+$ and $2^+$ states in $N = 90$ nuclei $^{154}$Gd and $^{152}$Sm through (p,t) and (t,p) reactions. One of the key features observed is the asymmetry between the strong population of low-lying excited $0^+_3$ states between both
two-neutron transfer reactions, while the strength of the $0^+_2$ remains relatively high in both reactions.

Figure 1.8: Mass regions which exhibit evidence of shape coexistence [10]. In this work, the N=90 region (Region K) is examined.

The motivation for investigating the Er isotopes, particularly $^{160}$Er, in the context of shape coexistence is the similarity between the Er isotopic chain and other rare-earth isotopes such as Gd, Sm and Nd, shown in Figure 1.5. In this thesis, a $^{162}$Er$(p,t)$ experiment is performed and excited state cross sections and angular momenta are reported, with specific emphasis on the strength and interpretation of excited $0^+$ states.
Figure 1.9: Excited $0^+$ state strength in $^{154}\text{Gd}$ and $^{152}\text{Sm}$ in (p,t) and (t,p) reactions [16]. Of particular interest is the asymmetry between the (p,t) and (t,p) strength of the $0^+_3$ state, while the $0^+_2$ state is strongly populated in both reactions.
Chapter 2

Direct Nuclear Reactions

For many decades, nuclear reactions have been used to probe the properties of nuclei. The first such observation of a nuclear reaction was by Ernest Rutherford [1], in the famous experiment investigating the elastic scattering of α-particles on a gold foil [17], which led to the discovery of the atomic nucleus. The discovery resulted in an interest in the field of nuclear reactions by experimental physicists and uncovered many different species of reaction products and mechanisms.

In general, nuclear reactions are classified into three categories - direct, pre-equilibrium, and compound reactions. The classification of the interaction depends on the amount of nucleons involved in the interaction and the time scale on which the reaction occurs. These reaction mechanisms are illustrated in Figure 2.1.

A direct reaction occurs when a projectile collides with a nucleus and interacts with only a few nucleons, which are typically near the Fermi surface. These peripheral processes occur in about $10^{-22}$ s, which is the required amount of time for the projectile to traverse the nucleus [18, 19]. Many types of direct reactions exist, such that each outcome from the
interaction between the projectile and target is defined as an \textit{exit channel}, or \textit{partition}. The simplest example of a direct reaction is elastic scattering, where the projectile and target are left in the same initial state and do not exchange energy. This is referred to as the elastic channel. Many non-elastic channels also exist, such as inelastic scattering, nucleon transfer reactions, breakup reactions and charge-exchange reactions. Each of these reaction channels is opened due to a different reaction mechanism (defined by the number of nucleons which are transferred, Q-values, excitation of a residual nucleus, etc.) and each channel can probe different components of the nuclear wave function.

![Figure 2.1: Diagram of direct, compound and pre-equilibrium reaction mechanisms [20].](image)

In a \textit{compound} reaction, the projectile strikes a target, forming an excited and unstable compound nucleus. The projectile exchanges energy with many nucleons as it scatters in the nucleus [18], and after a sufficient amount of time, as nucleons continue to undergo collisions, one or more nucleons (most often neutrons, which does not feel the Coulomb force) may gain
enough energy to break free of the nucleus if the energy of the compound system is above the n-emission channel threshold. The entire process usually occurs on a time scale of up to $10^{-16}$ s [19]. Compound reactions are generally modelled using a statistical approach, where if the nucleus is formed with a high excitation energy, the density of states is very high, such that many states are excited from an incident beam energy. From Bohr’s independence hypothesis, this statistical characteristic implies the compound nucleus “loses memory” of the way which it was formed, and the subsequent decays will be independent of the initial reaction [19, 21]. Many applications of these types of reactions exist, including the use of chain compound reactions to generate nuclear power and to build nuclear weapons.

A pre-equilibrium reaction is an intermediate process between direct and compound reactions, but is not well-described by a combination of the two types. Like a direct reaction, the nucleus emits a particle or photon, but the projectile still interacts with many nucleons, and after some time will form a compound nucleus which reaches statistical equilibrium. [20,22]. These reactions, such as deep inelastic scattering and fragmentation reactions, generally occur on a time frame between that of direct and compound processes.

### 2.1 Differential Cross Section

To quantify the intrinsic probability of a scattering event, we define the cross section to be the probability of measuring an ejected particle at a certain energy and angle $(\theta, \phi)$ from the beam axis. To derive the formalism of the differential cross section in terms of the scattering amplitude, as depicted in Figure 2.2, we first start by deriving an expression for the incident and scattered wave functions. A plane wave with velocity $\vec{v} = \hbar \vec{k}/\mu$ at a
Figure 2.2: Diagram describing the incident flux $j_{inc}$ and scattered flux $j_{scat}$ defined into solid angle $\Delta \Omega$ for a nuclear reaction.

Position, $\vec{r}$, can be expressed as

$$\psi_{inc}(\vec{r}) = \frac{1}{\sqrt{V}} e^{i\vec{k} \cdot \vec{r}}$$  \hspace{1cm} (2.1)$$

where the plane wave function normalization is chosen to be the three-dimensional particle in a box. An interaction with a spherical potential, $V(r)$, will produce a scattered spherical wave which will radiate outward at a distance, $r$, with a velocity, $v'$, and wave number, $k'$, given by [21]

$$\psi_{scat}(\vec{r}) = \frac{1}{\sqrt{V}} f(\theta, \phi) e^{ik' r}.$$  \hspace{1cm} (2.2)$$

The asymptotic wave function well outside the interaction region (which is valid if the potential is short-ranged) [21] is a superposition of the incident plane wave and the scattered
spherical waves,

\[ \psi(\vec{r}) = \psi_{\text{inc}} + \psi_{\text{scat}} \]
\[ = \frac{1}{\sqrt{V}} \left[ e^{i\vec{k} \cdot \vec{r}} + f(\theta, \phi) \frac{e^{i\vec{k}' \cdot \vec{r}}}{r} \right] \]

where \( f(\theta, \phi) \) is a scattering amplitude of the spherical wave with respect to the plane wave. The amplitude is now taken to be an overall normalization factor, because the difference in the amplitude of the scattered wave can be included in \( f(\theta, \phi) \). It is convenient to introduce the quantum mechanical description of the probability current density, also known as flux, in units of particles/time/area, given by

\[ \vec{j} = \Re \left\{ \psi^* \left( -\frac{i\hbar}{\mu} \nabla \right) \psi \right\} \]

where the incident flux using Equations 2.1 and 2.5 can be written quite simply as

\[ \vec{j}_{\text{inc}} = \frac{\hbar}{\mu} \vec{k}. \]

The corresponding scattered flux using Equations 2.2 and 2.5 has a more complicated expansion but can be expressed by

\[ \vec{j}_{\text{scat}} = \frac{\hbar |f(\theta, \phi)|^2}{\mu r^2} + \mathcal{O}(r^{-3}) \]

where the \( \mathcal{O}(r^{-3}) \) term is neglected because it is small in the limit that \( r \to \infty \).\(^1\)

\(^1\)To simplify the expression, the terms that emerge multiplying an exponential factor of the form \( e^{\pm i\vec{k} \cdot (1 - \cos \theta)} \), where \( \theta \) is the angle between \( \vec{k} \) and \( \vec{r} \), are neglected as they rapidly fluctuate for physical measurements at large distances and average to zero [23].

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The initial scattered flux expression given in Equation 2.7 does not satisfy the requirement that flux must remain independent of the radius when integrated over the surface of a sphere [21, 23]. To rectify this, a $1/r^2$ factor is multiplied to the scattered flux expression to obtain Equation 2.7. Furthermore, the expression for the scattered flux should account for the solid angle coverage of the detector, which is measured in units of steradian (sr). Since the solid angle subtended by a detector of effective area $da$ at a distance $r$ from the scattering centre is given by

$$d\Omega = \frac{da}{r^2},$$

(2.8)

the scattered flux through an area $da$ can be written, in units of particles/time, as

$$j_{\text{scat}} \cdot da = r^2 j_{\text{scat}}.$$

(2.9)

The differential cross section can now be written as the ratio of outgoing angular flux to the incoming flux of a reaction [21] as

$$\left( \frac{d\sigma}{d\Omega} \right)_{\text{lab}} = \frac{j_{\text{scat}} \cdot da}{j_{\text{inc}}}$$

$$= \frac{k'}{k} |f(\theta, \phi)|^2.$$

(2.10) (2.11)

There are two specific cases which are of interest. In the case of elastic scattering, there is no change of momentum and thus ($k = k'$), which simplifies the cross section to the square of the scattering amplitude. However, in general, collisions are not elastic ($k \neq k'$). Since the scattering amplitude was an arbitrary factor to normalize the scattered flux with respect to the incident flux, it is common practice to redefine the scattering amplitude to
include the $\sqrt{k'/k}$ factor, so that the cross section can be written in terms of only the scattering amplitude. Furthermore, when no angular momentum is transferred, or is there is no polarization of the particles, the azimuthal dependence of the scattering wave function vanishes. As a result, for the two-neutron transfer reaction presented in this thesis where the incident protons are unpolarized, the scattering amplitude is written as $f(\theta, \phi) \rightarrow f(\theta)$. With these substitutions, the differential cross section expression becomes

$$
\left( \frac{d\sigma}{d\Omega} \right)_{lab} = |f(\theta)|^2.
$$

(2.12)

For transfer reactions on heavy nuclei, it is convenient to define the barn (b), equal to $10^{-28}$ m$^2$, to describe the magnitude of cross sections. In this work, differential cross sections are expressed in units of mb/sr to agree with the conventions used by FRESCO (see Section 2.3.3).

## 2.2 Non-Relativistic Kinematics

It is simplest to describe the products produced during a direct reaction as

$$
a + A \rightarrow b + B.
$$

(2.13)

In forward kinematics, this describes the interaction between a projectile, $a$, which is an accelerated beam comprised of a light incident particle (usually n, p, d, t or α), with a target, $A$, which is taken to be a heavy, stable nucleus which can be evaporated onto a thin foil. The reaction produces an ejectile, $b$, which is studied and whose energy and energy deposit can be used to identify the particle type, and residual recoil nucleus, $B$, which is
of spectroscopic interest. The reaction is generally written in short as $A(a, b)B$. From a theoretical standpoint, is more convenient to express cross section in the centre-of-mass frame rather than the laboratory frame. Since the number of detected particles is invariant of the frame of reference, the laboratory and centre-of-mass frames are related by

$$\frac{d\sigma}{d\Omega_{\text{lab}}}\,d\Omega_{\text{lab}} = \frac{d\sigma}{d\Omega_{\text{cm}}}\,d\Omega_{\text{cm}}.$$  

(2.14)

Because the centre-of-mass frame is chosen to be independent of the recoil angle $\phi$, the cross section can be written as

$$\frac{d\sigma}{d\Omega_{\text{lab}}}\,d(\cos \theta_{\text{lab}}) = \frac{d\sigma}{d\Omega_{\text{cm}}}\,d(\cos \theta_{\text{cm}}).$$  

(2.15)

The conversion factor between these two formalisms can be derived from conservation of momentum and energy laws. Equating the components of the velocity from Figure 2.4

Figure 2.3: Laboratory and centre-of-mass kinematics of the $^{162}\text{Er}(p, t)$ reaction.
Figure 2.4: Diagram of the kinematics of a transfer reaction exit channel. The capital letters indicate laboratory coordinates and the lowercase letters represent centre-of-mass coordinates.

\[ v_b \sin \theta_{cm} = V_b \sin \theta_{lab} \]  
\[ v_b \cos \theta_{cm} + V_{cm} = V_b \cos \theta_{lab} \]

where the centre-of-mass velocity can be derived in terms of the reduced mass between particles \(a\) and \(A\) to be \(V_{cm} = m_a V_a / (m_a + m_A)\). At a beam energy of 24 MeV, the non-relativistic regime is valid for incoming protons with a velocity of 0.22c. For convenience, \(\gamma\) is defined in terms of \(E_{cm} = m_a E_{lab} / (m_A + m_a)\) and the Q-value as

\[ \gamma = \left[ \frac{m_b m_a}{m_A m_B} \frac{E_{cm}}{E_{cm} + Q} \right]^{1/2} \]
such that the angle $\theta_{lab}$ is related to its centre-of-mass counterpart and $\gamma$ by

$$\tan \theta_{lab} = \frac{\sin \theta_{cm}}{\gamma + \cos \theta_{cm}}.$$  (2.19)

The differential cross section is related to the centre-of-mass angle by

$$\frac{d\sigma}{d\Omega_{lab}} = \frac{d\sigma}{d\Omega_{cm}} \left( \frac{(1 + 2\gamma \cos \theta_{cm} + \gamma^2)^{3/2}}{|1 + \gamma \cos \theta_{cm}|} \right)$$  (2.20)

which can be used to compare the experimental cross sections, which are measured in the laboratory frame, to the DWBA calculations, which are typically calculated in the centre-of-mass frame.

### 2.3 Scattering Theory

The scattering problem in nuclear reaction theory consists of solving the time-independent Schrödinger equation for the outgoing quantum-mechanical wave functions for a given interaction potential. Let the model space be

$$\Psi_{model} = \sum_i \psi_i(R_i) \phi_i$$  (2.21)

where $\phi_i$ is written as $\phi_i = \phi_{ai}\phi_{Ai}$ for convenience. The $\phi_{ai}$ and $\phi_{Ai}$ variables represent the bound state wave functions of the projectile $a$, and target, $A$, at positions $\vec{R}_{ai}$ and $\vec{R}_{Ai}$ and masses $m_{ai}$ and $m_{Ai}$ respectively, and $\psi_i(R_i)$ is the wave function describing the relative motion at $R_i = R_{ai} - R_{Ai}$ [24]. The task is then to solve the model Schrödinger equation...
where the scattering wave function satisfies

\[ [\mathcal{H} - E]\Psi_{\text{model}} = 0 \]  \hspace{1cm} (2.22)

where \( \mathcal{H} \) is the total effective Hamiltonian which contains the projectile-target kinetic energy and the internal nuclear Hamiltonians for the projectile and target, and \( E \) is energy for the model space. By projecting \( \Psi_{\text{model}} \) onto basis states \( \phi_j \) as is done in [25] where \( i \) and \( j \) are different states in the same nucleus, one can derive the Schrödinger equation which couples the \( \psi_i(R_i) \) wave functions together using the Hamiltonian, \( H_j \), and the kinetic energy, \( E_j \), for each channel \( j \) by

\[ [E_j - H_j]\psi_j(R_j) = \sum_{i \neq j} \langle \phi_j | \mathcal{H} - E | \phi_i \rangle \psi_i(R_i) \]  \hspace{1cm} (2.23)

which can be solved using iterative methods. The matrix element \( \langle \phi_j | \mathcal{H} - E | \phi_i \rangle \) can be expressed in two different forms, called post form if the Hamiltonian for the exit channel is used and prior form if the Hamiltonian for the entrance is used. These Hamiltonians can be written as

\[
\langle \phi_j | \mathcal{H} - E | \phi_i \rangle = \left\{
\begin{array}{ll}
H_j - E_j + V_j & \text{post form} \\
H_i - E_i + V_i & \text{prior form}
\end{array}
\right.
\]  \hspace{1cm} (2.24)

where \( i \) is the initial channel, \( j \) is the final channel and \( V_i \) and \( V_j \) are some effective interaction potentials. In the case of elastic scattering, where there is a spherical potential, the partial wave expansion method converges rapidly and is used to calculate the scattering amplitude. In the case of a transfer reaction, however, the partial wave expansion does not always have
rapid convergence. When population to intermediate channel is weak, or when the couplings
between channels are weak, the Distorted-Wave Born Approximation (DWBA) becomes a
useful tool to evaluate transition amplitudes to determine the cross section.

2.3.1 Distorted Wave Born Approximation

Although the solution to Schrödinger equation is quite complicated, the Born approx-
imation can greatly simplify the scattering problem so that the transition matrix element
may be calculated. If a scattering potential is weak, perturbation theory can be used to
expand the potential into a sum called the Born series, where each term in the Born series
is related to the order, or number of steps, of the scattering process.

The Born approximation assumes that the scattering potential does not significantly al-
ter the wave function inside the potential, so that the Born series can be truncated at the
first term (one-step DWBA) or second term (one + two-step DWBA) and remain valid.
A one-step Born approximation describes a single-step event from entrance to exit chan-
nels, while higher-order approximations represent events described by multiple steps, using
some intermediate reaction channel. Furthermore, only couplings between channels in one
direction are considered, instead of a complete coupled-channel calculation.

In a first-order DWBA calculation, as a result of post-prior equivalence, the system of
equations in Equation 2.24 are equivalent, such that the transition amplitude matrix (T-
matrix) element can be described more simply as

\[ T^{DWBA}_{ji} = \langle \psi_j^+ \phi_j | V | \phi_i \psi_i^+ \rangle \]  

(2.25)

where \( V \) is the interaction potential. If the one or few-nucleon transferred particle in a direct
transfer reaction is defined to be $\nu$, such that the target, $A$, can be written in terms of the residual nucleus, $B$, as $A = B + \nu$, and the ejectile, $b$, can be written in terms of the projectile, $a$ as $b = a + \nu$, then the interaction potential can be approximately written as

$$V \approx \begin{cases} V_{va} + U_{aB} - U_i \quad \text{prior} \\ V_{vb} + U_{aB} - U_j \quad \text{post} \end{cases}$$

(2.26)

where $V_{va}$ and $V_{vb}$ are the real binding potentials for the entrance and exit channels. $U_{aB}$ is the core-core optical model potential which describes the scattering between $\nu$ and $a$ and finally $U_i$ and $U_j$ are the optical model potentials in the entrance and exit channels. The expressions $U_{aB} - U_i$ and $U_{aB} - U_j$ are called remnant terms and are revisited in Section 2.3.3.

The asymptotic incoming and outgoing wave functions $\psi^{-}$ and $\psi^{+}$ depend on their relative distance, $\vec{r}$, and momentum, $\vec{k}$, can be described by

$$\psi^{\pm}(\vec{k}, \vec{r}) \xrightarrow{r \to \infty} e^{i\vec{k} \cdot \vec{r}} + f(\theta, \phi) e^{\pm ikr}$$

(2.27)

where the incoming and outgoing wave functions are related by

$$\psi^{-}(\vec{k}, \vec{r})^* = \psi^{+}(-\vec{k}, \vec{r}).$$

(2.28)

The differential cross section can be expressed in terms of the transition amplitude matrix (T-matrix) elements, $T(\vec{k}_j, \vec{k}_i)$, as

$$\left(\frac{d\sigma}{d\Omega}\right)_{i \to j} = \frac{\mu_j \mu_i}{2\pi\hbar^2} \left(\frac{k_j}{\vec{k}_i}\right) |T(\vec{k}_j, \vec{k}_i)|^2$$

(2.29)
where the T-matrix elements is related to the scattering amplitude in Equation 2.12 by

\[ T(k_j, k_i) = \frac{-2\pi \hbar^2}{\mu_j} f_j(\theta, \phi) \]  

(2.30)

where \( f_j(\theta, \phi) \) is the scattering amplitude of the exit channel defined in Equation 2.12.

### 2.3.2 Optical Model Potential

To reduce the complexity of the many-body scattering problem, in scattering theory, the interactions between the nucleons in the target are replaced by an average potential experienced by the nucleons called the **optical model potential**. The optical model potential is named as such due to the resemblance of its solutions to those of light incident on a partially opaque glass sphere [1], which can be described in terms a real component, which models the refraction of the light, and an imaginary component, which describes its attenuation. In the case of a two-neutron transfer, this approximation is equivalent to a neutron pair being diffused with an absorbing well. The optical model potential introduced in Equation 2.26 can thus similarly be expressed as

\[ U(r, E) = V(r, E) + iW(r, E) \]  

(2.31)

where the real component \( V(r, E) \) represents the nuclear interactions and \( iW(r, E) \) represents the flux lost to the reaction exit channel. Many forms of the optical model potential exist, and special care must be taken to choose the appropriate form in FRESCO calculations (see
Section 2.3.3). Using the form of [26], $U(r, E)$ can be described as

$$U(r, E) = -V_v(E)f(r, R_v, a_v) - iW_v(E)f(r, R_v, a_v) + i4a_D W_D(E) \frac{d}{dr} f(r, R_D, a_D)$$

$$+ \left(V_{SO}(E) + iW_{SO}(E)\right) \left(\frac{\hbar}{m_e c}\right)^2 \vec{l} \cdot \vec{\sigma} \frac{1}{r} \frac{d}{dr} f(r, R_{SO}, a_{SO}) + V_c(r).$$

(2.32)

The volume terms can be isolated into real and imaginary components $V_v(E)$ and $W_v(E)$, because at high beam energies ($E > 20$ MeV/u), the nucleons closer to the center are probed and so it is expected that nucleon-nucleon interactions in the center of the nucleus to contribute to the absorption term [1]. Since the nuclear force is short, these terms should have a radial dependence on the density distribution of the nucleus. The form which generally best describes the smearing of the density function of nuclei near the surface is the Woods-Saxon shape shown in Figure 2.5, described by

$$f(r, R_i, a_i) = \frac{1}{1 + \exp[(r - R_i)/a_i]}$$

(2.33)

where $r$ is the radial coordinate, $R_i = r_i A^{1/3}$ is the mean nuclear radius parameter in fm, where $A$ is the mass number and $r_i$ is typically around 1.25 fm, $a_i$ is the surface diffuseness parameter in fm, and $i$ is an index denoting the type of interaction which is either volume (V), surface (D) or spin-orbit (SO).

At a low incident beam energy ($E < 20$ MeV/u), the density of states is very low and many-nucleon collisions would be suppressed, because the Pauli exclusion principle forbids nucleons to be scattered into low-energy states. As a result, a potential of Woods-Saxon form would not be appropriate in describing the surface and spin-orbit potentials. Instead it is an adopted practice to consider $W_D(E)$ as a surface-peaked potential, in the form of a Woods-Saxon derivative shown in Figure 2.5. Since the transfer reactions in this work are
in the intermediate energy regime, both imaginary surface and volume terms are calculated. The $V_{SO}(E)$ and $W_{SO}(E)$ terms are the real and imaginary spin-orbit terms, which are also chosen to be surface-peaked because the core nucleons should have zero spin-density [1]. The $(\hbar/m_{\pi}c)^2$ factor is the square of the reduced pion Compton wavelength which is $\sim 2.0$ fm for spin-orbit potentials and $\vec{l} \cdot \vec{\sigma}$ is the scalar product of the intrinsic and orbital angular momentum operators.

Unfortunately, the definition of optical model parameters differ from set to set. One of the most prevalent differences lies in the spin-orbit term, where some authors define the spin-orbit term to be $\vec{l} \cdot \vec{\sigma}$ to be $\vec{l} \cdot \vec{S}$, where the Pauli and spin matrices are related by

\[
\vec{S} = \frac{\vec{\sigma}}{2}
\]  

(2.34)

This difference will introduce a factor of 2 in nuclei which are not spin-1/2, so for the optical model potential corresponding to say, a deuteron, the spin-orbit optical model potential well depth is adjusted by a factor of 2 to ensure consistency.

The volume well depths, radii and diffuseness parameters are adjusted to best fit a compilation of elastic scattering data for various mass ranges and energies. Depending on the choice of data sets used during the fitting procedure, different optical model potential parameters sets can be calculated. The optical model which best describes the data is further discussed in Chapter 4.

For charged projectiles, the Coulomb term is also included to account for the scattering
of charged spheres, of charge $z$ for the projectile and $Z$ for the target,

$$V_c = \frac{Zze^2}{4\pi\varepsilon_0 r} \quad r \geq R_c$$

$$= \left( \frac{3}{2} - \frac{r^2}{2R_c^2} \right) \frac{Zze^2}{4\pi\varepsilon_0 R_c} \quad r \leq R_c$$

Given that charge radius $R_c$ has a radial dependence on the charge density distribution, the charge radius is described by $R_c = r_cA^{1/3}$.

### 2.3.3 FRESCO

To be able to extract the angular momentum from angular distributions, theoretical predictions for the shapes of these distributions must first be produced. FRESCO (Finite-Range with Exact Strong COuplings) is a general-purpose reaction code developed by Ian
Thompson since 1983 capable of performing coupled-channel calculations [27]. FRESCO uses the Distorted-Wave Born Approximation (DWBA) to solve the coupled-channel equations and uses mass, angular momentum, parity, optical model potential and numerical integration parameters to calculate cross sections and create angular distributions. The DWBA method is further described in Section 2.3.1.

The FRESCO code uses Fortran 90 or Fortran 95 to read an input file with a specific format to parse the reaction parameters. An example of this input file is illustrated in Figure 2.6. The first line of the input file is reserved for a comment, then the “NAMELIST” tag indicates the beginning of the namelist section where the numerical integration parameters, (&fresco), incoming and outgoing nuclei information (&partition), optical model parameters (&pot), wave function overlaps (&overlap) and coupling between these wave functions (&coupling) are defined.

In the &fresco section, rmatch defines the radius at which the wave function gets matched to its asymptotic form which increases systematically by a step size hcm. The minimum and maximum total angular momenta are also defined as ji and jf. The angular distributions are calculated from a minimum angle thmin to maximum angle thmax in step size thinc. Other variables define the convergence and integration parameters whose definitions can be found in [28].

The next section &partition defines the masses and nucleon number of the projectile and target involved in the reaction. Two reaction mechanisms can be used in a multi-nucleon transfer. The first process is when more than one nucleon is transferred in a single step (called a simultaneous or 1-step process) where only the incoming and outgoing reaction channels must be defined. The second process is if the nucleons are taken to be transferred altogether and sequentially with a relative amplitude to one another. This second type of transfer is
Figure 2.6: FRESCO input file for a two-neutron transfer reaction.
called a \textit{1+2-step} or \textit{sequential} process, and in these types of reactions, an intermediate partition is also defined with a Q-value relative to the incoming partition. Because the intermediate Q-value for the (p,d) reaction is very large in comparison to the Q-value for the (p,t) reaction, the reaction mechanism is taken to be dominated by the simultaneous component. However, the input file lines which would be required in a multi-step process are included and indicated in red in Figure 2.6. For more information on single and multi-step DWBA, see [21,25]. The related line in the \\texttt{&states} section for each partition defines this Q-value, excitation energy and the angular momentum and parity to each state defined.

The \texttt{&pot} section includes the optical model potentials where the incoming wave function interacts with, each indexed by a \texttt{kbpot} value. These potentials are generally of type 1 (a volume nuclear interaction), type 2 (surface nuclear interaction) or type 3 (spin-orbit potential for the projectile). Each optical model potential parameters indicate the depth of the potential well in MeV, reduced radius in fm and the diffuseness in fm and are indexed by \texttt{p(1) - p(3)} for the real terms and \texttt{p(4) - p(6)} for the imaginary terms, respectively.

An estimate for the optical model for a scattering potential can be obtained by fitting experimental data for a range of nuclei and energies by varying the coefficients of a global optical potential. Although an optical model fit to a specific nucleus may better reproduce experimental data for a given nucleus. The proton optical model potentials used are Koning and Delaroche [26], Becchetti and Greenlees [29], Menet et al. [30], Morillon et al. [31], Walter et al. [32], and Varner et al. [33]. For the optical model fits, see Chapter 5.

To be able to reproduce the reaction in FRESCO, the quantum states in initial and final states must be defined in the \texttt{&overlap} section. In this section, \texttt{ic1} defines the composite nucleus partition and \texttt{ic2} defines the core partition. These are defined for the projectile if \texttt{in=1} or target if \texttt{in=2}. Here, \texttt{kind} = 0 indicates the spin of the core is ignored and the
\[(l, sn), j\] coupling is used, where \(sn\) is the spin of the fragment, \(l\) is the relative orbital angular momentum of the state, \(j\) is the total angular momentum and the number of radial nodes is defined to be \(nn\) (in FRESCO this is equivalent to the quantum number, \(n\), which starts at 1) [28]. The term \(be\) specifies a positive binding energy of each bound state, where \(isc=1\) allows the potential well depth to be changed to reproduce the binding energy dependent on the potential well depth.

For a two-neutron transfer, a \&twont section must also be included to define the two-neutron transfer amplitude. Due to the high mass number of the nucleus which ranges beyond the scope of shell model calculations, no two-neutron amplitude information can be calculated, and thus these amplitudes must be taken as unity. The validity of this assumption is further discussed in Section 5.4.

Finally, the \&coupling section defines the type and strength of the couplings between each partition involved in the transfer. In particular \(kind = 7\) defines a finite-range transfer, from the partition defined by \(ic\) from to the partition defined by \(icto\). Here, \(ip1=0\) or \(ip1=1\) depending on if the post or prior form of the interaction respectively, and \(ip2=-1\) if a full complex remnant should be used in the integration. The remnant refers to the difference between the core-core optical potential and the initial channel potential defined in \&pot. More information on the post/prior forms and the remnant term can be found in Section 2.3.1. The term \(ip3\) refers to the potential index which should be used as the potential between the cores, unless \(ip3=0\) in which case the interaction potential will default to that in the partition of the projectile core. [34]

The \&cfp section, or coefficients of fractional parentage, defines the strengths of these overlap functions. These are again defined for the projectile if \(in=1\) and target if \(in=2\), from state \(ia\) to state \(ib\) in the partition defined in \&coupling using the overlap indexed by
kn. These amplitudes, $a$, are the square root of the spectroscopic factor, and are defined as unity in the case of a simultaneous transfer. In a sequential transfer, the amplitudes are also defined as unity except the amplitude associated with the overlap of the deuteron with the triton which is set to $a = 1.176$ to reproduce the proper radial overlap between the deuteron and the triton [35].

FRESCO outputs the calculated angular distributions in output files which are compatible with Xmgrace plotting software [36] in files beginning in “fort.” All angular distributions are output in fort.16. The fort.201 file contains the elastic scattering cross sections, and individual cross sections calculated from each line in &state defined in the exit channel &partition are printed in fort.201-fort.210, beginning with the elastic scattering exit channel in fort.201.

### 2.3.4 Elastic Scattering

If no angular momentum is transferred and the relative kinetic energy during a collision of two nuclei is unchanged, the process is called **elastic**. In an elastic scattering process, contributions from Rutherford scattering and nuclear effects lead to constructive or destructive interference in the spectrum. Since the target thickness is estimated by the manufacturer, elastic scattering data can be used to provide a correction to the target thickness, and select the optical model potential described in Section 2.3.2 that best fits the entrance channel. The scattering amplitude can be found by using a partial wave expansion, where the spherical symmetry of the potential permits the partial wave functions to be expanded using Legendre polynomials corresponding to different angular momenta to be calculated separately. Solving Schrödinger’s equation

\[
[\hat{T} + V - E]\psi(R, \theta) = 0
\]  

(2.37)
with $\hat{T} = -\hbar^2 \Delta^2/2\mu$. By expanding the wave function into partial waves, the resulting wave function can be written as

$$\psi(R, \theta) = \sum_{L=0}^{\infty} (2L + 1)i^L P_L(\cos \theta) \frac{u_L(R)}{kR}$$

(2.38)

where $u_L(R)/kR$ is the radial part of the wave function. The solution for the scattering amplitude is

$$f(\theta) = i \frac{2}{2k} \sum_{L} (2L + 1)(1 - e^{2i\delta}) P_L(\cos \theta)$$

(2.39)

where $\delta$ is the phase shift associated with each partial wave. For a pure Coulomb potential given by

$$V(R) = \frac{Ze^2}{R}$$

(2.40)

the Rutherford cross section is calculated to be

$$\frac{d\sigma}{d\Omega} = \frac{Ze^2}{4E} \frac{1}{\sin^2\left(\frac{\theta}{2}\right)}$$

(2.41)

where $E = \hbar^2 k^2/2\mu$. Since the scattering potential can be written as the sum of the nuclear and Coulomb components, the phase shifts should also be related. Let

$$\delta_{nc} = \delta_c + \delta_n$$

(2.42)

where $\delta_c$ is the Coulomb phase shift, $\delta_n$ is a pure nuclear phase shift and $\delta_{nc}$ is the total elastic scattering phase shift. If this is substituted into Equation 2.39 an expression relating
the scattering amplitudes is obtained

\[ f_{nc}(\theta) = f_c(\theta) + f_n(\theta) \]  \hspace{1cm} (2.43)

where \( f_c(\theta) \) is the Coulomb scattering amplitude between two particles interacting via a coulomb potential and \( f_n(\theta) \) is the Coulomb-distorted nuclear amplitude. The total elastic scattering cross section can be written as

\[ \frac{d\sigma}{d\Omega} = |f_{nc}(\theta)|^2 \]  \hspace{1cm} (2.44)

which agrees with the result from Equation 2.12 which is valid for elastic collisions. The elastic scattering angular distributions were calculated using FRESCO (see Section 2.3.3) and are plotted in Chapter 5. Since the difference in optical model lies in the nuclear component of the interaction, the most appropriate way to test an optical model’s agreement to data is to plot the “Ratio to Rutherford” cross section angular distribution. This quantity compares the nuclear components of the elastic scattering data by normalizing to the cross section from Rutherford scattering.
Chapter 3

Experiment

3.1 MLL Facility

All of the work presented in this thesis was performed at the Maier-Leibnitz-Laboratorium (MLL) facility on the joint campuses of Ludwig-Maximilians-Universität München and Technische Universität München in Garching, Germany. One of the key features of the facility is its 14 MV tandem Van de Graaff generator which provides the facility with polarized and unpolarized accelerated stable beams, further discussed in Section 3.2.2. The negative ion beams are supplied by a Stern-Gerlach atomic beam source discussed in Section 3.2.1. The post-accelerated beams are then deflected into an experimental hall using a 90° analyzing magnet. Because the level density in heavy nuclei is so high, excellent energy resolution is needed to resolve discrete states in the residual nuclei. The Quadrupole-3-Dipole (Q3D) magnetic spectrometer is an excellent choice as it provides both excellent energy resolution as well as covers low angles required to create angular distributions. For \( L = 0 \) transfers in particular, these angular distributions peak in cross section at 0°, so the capability of the
Q3D to probe low angles is also necessary. The Q3D uses magnetic fields to focus and deflect the reaction products on a focal plane detector developed for the Q3D at the MLL which is discussed in Section 3.6. A diagram of the floor plan of the MLL for the experimental setup showing the experimental components is shown in Figure 3.1.

Figure 3.1: Diagram of MLL floor plan of the MLL [37], highlighting the key features in this experimental setup.
3.2 Beam Delivery

3.2.1 Ion Source

The Stern-Gerlach ion source was first developed for the Munich MP-tandem Van de Graaff by Ralf Hertenberger to provide the MLL facility with both polarized and unpolarized ion beams until 2014. The Stern-Gerlach setup uses an atomic beam source with electron-cyclotron resonance ionization and a cesium vapour jet target, which are further discussed in the following sections. A diagram of the setup can be found in Figure 3.2.

Figure 3.2: The components of the Stern-Gerlach polarized ion source at the MLL as depicted in [38].

The first component of the Stern-Gerlach design is the atomic beam source (ABS). Molecular hydrogen or deuterium gas is dissociated in this chamber due to electron scattering in a cold plasma created by using a high radiofrequency circuit [38]. An atomic beam is then
created by adiabatic expansion of dissociated hydrogen or deuterium gas into a chamber through a nozzle which is cooled down to 80 K. The second section of the ABS reduces beam scatter and subsequently increases the beam intensity by about 20% [38]. Four permanent FeNdB Stern-Gerlach sextupole magnets electronically polarize the beam by filtering the spin of the electrons and selecting only those whose hyperfine state is aligned with the magnetic field [39]. Adiabatic radiofrequency transitions (ARF) can be used to force electrons into unoccupied Zeeman-splitted hyperfine states to convert electronic polarization into nuclear polarization.

The ABS system is used in conjunction with a 2-step charge-exchange process instead of the less-efficient one-step charge-exchange technique at the MLL. The 2-step charge exchange process consists of two parts. First, an ECR plasma is used to ionize atomic beams of hydrogen and deuterium at rates of $6.4 \times 10^6 \text{ H/s}$ and $5 \times 10^6 \text{ D/s}$ with an efficiency of a few percent [38]. Pancake coils and six FeNdB magnets confine the plasma using magnetic fields and maintain polarization of the beam. The Stern-Gerlach source can also provide unpolarized beams of $\text{H}^-$, $\text{D}^-$, $\text{^3He}^-$ and $\text{^4He}^-$ by using a separate gas inlet into the ECR.

This ionized beam is guided with axial magnetic fields towards the cesium target in the charge exchange section of the source. Using two-electron pickup from a dense cesium vapour jet, a negative ion beam is produced with an efficiency of about 30% [38]. The Stern-Gerlach ion source produces high-intensity unpolarized $\text{H}^-$ and $\text{D}^-$ beams of 50 $\mu$A as well as polarized beams at an intensity of 1 - 2 $\mu$A with a polarization of about 80% [38]. In this experiment, an unpolarized $\text{H}^-$ beam was produced at a rate of $\sim 2 \mu$A to avoid compromising the thin $^{162}\text{Er}$ target.
3.2.2 Tandem Van de Graaff accelerator

Van de Graaff generators are electrostatic accelerators, first designed by R. Van de Graaff in 1931 [40], which use electrostatic fields to accelerate ions.

Figure 3.3: Schematic diagram of MP-tandem Van de Graaff generator adapted from [41] currently in use at the MLL facility.

In the two-stage or tandem Van de Graaff design shown in Figure 3.3, the voltage is applied twice to achieve twice the beam energy, for singly-charged ions, as its single-accelerating counterpart. A negatively charged beam, produced from the ion source described in Section 3.2.1, is accelerated towards a high voltage (HV) terminal in the centre of the accelerator. The high-voltage in the terminal, which is a large electric conductor, is maintained by transporting positive charges on an insulating rubber belt called the charging chain. The H\(^{-}\), D\(^{-}\) beams are ionized by a stripper foil, which removes two electrons, and the residual positive ions are electrostatically repelled away from the HV terminal [1], producing a secondary stage
of acceleration. The positive ions now have a kinetic energy of 2eV, where V is the potential on the HV terminal, which is a maximum of 14 MV for the tandem at the MLL. One of the limitations of these types of electrostatic accelerators are tandem “sparks”, when the tandem experiences electrical breakdown of the atmosphere surrounding the HV terminal. To reduce this breakdown, the tandem is kept at a pressure of 10-20 atm of sulfur hexafluoride (SF$_6$) gas which is selected because of its superior electrostatic insulating properties. In 2010, the MP-tandem Van de Graaff generator at the MLL was operated at 12 MV to produce a 24 MeV proton beams which were delivered to the Q3D (Section 3.4). In 2014, the tandem was not able to achieve this HV potential and was thus operated at 11 MV to produce 22 MeV proton beams.

### 3.2.3 Faraday Cup

In order to measure the beam current directly without compromising the integrity of the beam, a conducting Faraday cup is placed 0° behind the target in the target chamber of the Q3D. Because a negligible amount of the beam interacts with the target, a Faraday cup can provide a measure of the unscattered beam incident on the target. A current integrator connected to the Faraday cup counts the total number of ions deposited on the conductor, and outputs the value of this current into a variable called “Scaler 1” which is related to the total charge on the conductor. However, this value must be scaled by the full-scale current on the cup of 2 µA for a particle rate of 1000 counts/s, such that the total charge is

$$Q = Scaler1 \times \frac{2 \mu A}{1000 s^{-1}} \quad (3.1)$$
Since the ions are singly positively charged, the number of beam particles \( N_b \) will be related to the total charge \( Q \) and the elementary charge \( e \) by

\[
N_b = \frac{Q}{e}.
\]  

(3.2)

This quantity is used to determine the experimental cross section and is revisited in Chapter 4.

### 3.3 Target

Due to the low natural abundance of \(^{162}\text{Er}\) of 0.14(1)% [42], isotopical high-purity samples cannot be commercially purchased. A \(^{162}\text{Er}\) sample, that achieved a final enrichment of over 99%, was purchased from the Stable Isotopes Division of the Oak Ridge National Laboratory and was further enriched by R. Leonard using the isotope separator at Florida State University and deposited directly onto thick carbon foils. The targets are mounted to a target ladder, which is housed in a high-vacuum target chamber. The target ladder at the MLL can hold up to a maximum of five targets and is shown in Figure 3.4.

![Figure 3.4: Photograph of 4 targets mounted on an MLL target ladder, taken December 12th, 2016. The last target position is occupied by a collimator (far left in the figure) to collimate the beam if required.](image-url)
The target had an isotopic purity above 99%, which was estimated from the isotopic impurities found using elastic scattering data and is further discussed in Section 4.6. Because of an offset which was introduced in the solid angle to the Q3D, the average target thickness could not be extracted for the 24 MeV data set collected in 2014. However, the same target was used in the 2014 data set so that the target thickness was extracted by normalizing the 22 MeV data to the optical model potential. The average target thickness extracted from the 22 MeV elastic scattering data was $61(3)\mu g/cm^2$.

To choose an optical model potential, the 24 MeV elastic scattering data was used because a larger angular distribution was collected, and the relative normalization of each angle is trusted. In order to account for the uncertainty related to the target thickness determination and due to the choice of optical model potential, a conservative systematic error of 5% is included for quoted differential cross section measurements.

### 3.4 Q3D magnetic spectrograph

The Quadrupole-3-Dipole (Q3D) is a compact, large-dispersion magnetic spectrograph which focuses reaction ejectiles onto a focal plane detector using magnetic field. The scattered ejectiles enter the Q3D through an aperture which can be used to limit the beam intensity to the Q3D. The solid angle of this aperture is described in Section 3.5. The scattered beam then passes through a quadrupole magnet, which focuses the scattered reaction ejectiles vertically. The quadrupole magnet is often called a quadrupole lens due to its similarity to an optical lens which uses glass to focus an object [1].

The multipole element, which is located after the first dipole magnet, produces quadrupole, hexpole, octupole or decapole fields which correct for the reaction kinematics of the outgoing
particle, so that the energy is independent of the angle of the Q3D [43, 44]. This correction is required due to the large acceptance of the Q3D of about 6°.

The three dipole magnets use a magnetic field to separate the reaction products by energy and bend them onto a focal plane detector. Each magnetic component of the Q3D are dependent on the reaction kinematics, and as such, are unique to the reaction taking place. The Q3D interface calculates and sets the required magnetic fields for optimal resolution on the focal plane given the Q-value of the reaction and masses of the projectile and target. A schematic of the components of the Q3D are illustrated in Figure 3.5.

The Q3D is mounted on a semi-circular iron track which allows the spectrograph to be rotated through 5-150° with respect to the beam axis. A schematic diagram of this geometry is shown in Figure 3.7. This rotation can be used to create angular distributions, which are an important tool in studying angular momentum transferred in a nuclear reaction. In the photograph in Figure 3.6, the target chamber, quadrupole magnet, and first dipole magnet of the Q3D can be seen.

### 3.5 Determination of Effective Solid Angle

The solid angle available to the Q3D is controlled using two sets of micrometers to limit the scattered beam in the horizontal (x) and vertical (y) direction which obscure the diamond-shape aperture to the Q3D. The spectrograph has a solid angle coverage of 10-15 msr [43]. To a good approximation, the solid angle coverage of the detector can be written as

\[ \Delta \Omega = \frac{\Delta A}{R^2} \]  

(3.3)
for a differential area of a spherical detector. In the limit that \( R \) is large, however, this expression is valid in describing the solid angle available to the Q3D. The solid angle of the aperture shown in Figure 3.8 can be expressed as

\[
\Delta A = \begin{cases} 
4xy, & \frac{x}{A} + \frac{y}{B} \leq 1 \\
4xy - (x - A + Ay/B)(y - B + Bx/A), & \frac{x}{A} + \frac{y}{B} > 1
\end{cases}
\] (3.4)

where \( A = 31.5 \text{ mm}, B = 36.75 \text{ mm}, \) and \( R = 354.8 \text{ mm}. \) Generally, only the x-slits are opened or closed to control the solid angle during an experiment to reduce the chance of systematic error and to eliminate the calibration of the slits in both directions. However, in the elastic scattering data set, both the x-slits and the y-slits were opened during the course of the experiment. However, in any instance where these slits were both opened, 2 data sets were taken at the same angle where the x-slits were changed keeping the y-slits constant, and the data could be re-normalized.
3.6 Focal Plane Detector

Once the reaction products have been momentum separated with the Q3D, they are focused onto a focal plane detector. The focal plane detector consists of two proportional counters and a plastic scintillator to identify the reaction products and measure the position of the particle on the focal plane. A schematic diagram showing a cross-sectional area of the focal plane is shown in Figure 3.9.

As the reaction products pass through the entrance foil of the focal plane detector, they enter the chamber of a proportional gas counter. As these heavy charged particles pass through the 500 mbar isobutane in the chamber [48], they ionize electrons from the gas. In a gas counter, an electric field will increase these electron’s energy as it collides with the gas as it drifts towards an anode wire. If this electric field strength is sufficient, the electrons which are accelerated by the field can gain enough kinetic energy to in turn ionize other gas atoms,
producing more electrons, and this process is called a Townsend avalanche. Although gas-filled counters can reach up to $10^5$ secondary events, the electric fields are chosen such that the number of secondary events are on the same order as the number of primary events [1], and these types of counters are so-called proportional gas counters.

As the avalanche of electrons drift towards the anode wire, the charge collected on the wire is proportional to the energy loss of the charged particle as it interacts with the gas. By integrating this current produced on the anode wire, the total charge from the liberated electrons can be determined. A second proportional gas chamber is used measure the vertical alignment of the focal plane detector using two anode wires with independent signals.

At the same time, the charge avalanche induces a positive charge deposit on a cathode
strip foil. The cathode strip foil is a segmented plane of 272 cathode strips 3 mm long with 0.5 mm spacing between the strips [46], located behind the second proportional gas chamber. The horizontal position is achieved by the single-strip cathode charge distributions [48]. Each hit on a cathode strip above a threshold on a strip is digitized and if 3-7 adjacent strips are above this adjustable threshold, a trigger is generator to enable the readout of the ASIC’s. These hits are recorded by an application-specific integrated circuit (ASIC) the position of the strips and hit number is read out to the $M A R_A B O U$ data acquisition. The multiplicity of the $0^{th}$ channel of these strips are related to the dead time of the ASICs. The signal from the anode wires is proportional to the summed particle energy on the wires, which allows for particle identification as shown in Figure 3.10. The pulses are fit on-line with a centre-of-gravity fit, which is much faster, but are then re-fit in the offline analysis with a Gaussian distribution to reduce the systematic error and produce a more accurate fit.

A plastic scintillator then stops the reaction products and measures their rest energy. The rest energy combined with the anode wire signal creates a $\Delta E$-E telescope and can
be used to identify the reaction ejectile with high resolution. An example of this particle identification is shown in Figure 4.1.

The focal plane is 1.8 m long, with an active length of 0.89 m [46], and has a non-uniform curvature. Although the curvature allows for a larger energy range of data to be taken during a run, it does complicate the analysis as a polynomial fit must be performed to the data to calibrate channel number read out from the ADC to the particle’s energy. The energy calibration is calculated in Section 4.3.

Because of the finite range of momentum that the focal plane can handle, data must be collected for discrete energy ranges called momentum bites. The energy range accepted by the focal plane is approximately 8% of the ejectile energy. For example, if the proton beam
energy is 24 MeV and $Q = -7.9298$ MeV for the $^{162}$Er$(p,t)$ reaction, the focal plane will cover a momentum range $\sim 1.29$ MeV per momentum bite. Three momentum bites of data were taken to cover an excitation energy range up to approximately 3 MeV.

### 3.7 Experiments Performed

The work presented in this thesis is an amalgamation of data from two distinct experiments performed on the same target. In July of 2010, a $^{162}$Er$(p,t)$ experiment was performed
at a beam energy of 24 MeV. An angular distribution from 5-55° was collected for two momentum bites of data, spanning 0-1.3 MeV and 1.4-2.7 MeV. A third low-momentum bite of data was collected at 30° to test for significant population of high energy states up to 3.8 MeV.

In February of 2014, an angular distribution from 6-45° was collected for 0.0-1.3 MeV and 2.6-3.7 MeV momentum bites, with two runs at 35° in between to ensure proper overlap. However, the beam energy of 24 MeV initially used was unable to be achieved in 2014, so a proton beam energy of 22 MeV was used instead. Due to the higher quality nature of the 2010 data, the low-momentum bite energy calibration is quoted for the 24 MeV data but the angular distributions and cross sections are quoted for the 22 MeV data for the third momentum bite, whose angular distribution was not collected at 24 MeV.
Chapter 4

Analysis

The analysis of the data can be split into an online component performed during the experiment, and the offline component of the analysis which was completed at the University of Guelph. The $MAR_{BOU}$ data acquisition software package developed for the tandem accelerator at the MLL is used during the online analysis for event building, as well as data collection, readout, and storage [49].

The HistPresent software was used to view spectra and place gates on outgoing particle energy on-line, so that the data collected during the course of an experiment can be monitored. In the offline component of the analysis, new gates on the data are set on a run by run basis. Once the gates are saved, the data is replayed with the selected gates in place to reduce the background of the spectrum, and the charge distributions from the cathode strips and re-fit with a Gaussian fit instead of a centre-of-gravity fit which was done in the online analysis to reduce computation time. The particle spectra, energy spectra, and various other spectra with systematic checks of the beam and focal plane are saved into ROOT ¹ files which were used to replay the data offline.

¹ROOT is a data storage and analysis software developed by CERN [50].
4.1 Particle Identification

Particle identification at the focal plane is possible because an incoming particle’s energy is monitored by the two proportional counters as well as a thick plastic scintillator. As the reaction ejectiles passes through the first proportional counter, with a single anode wire, the signal will read an energy $\Delta E_1$. As it passes through the second proportional counter, it will read a summed energy $\Delta E$ over the two anode wires. Finally, as the ejectiles hits the scintillator the rest of its energy is deposited and the scintillator reads on a rest-energy signal $E$. A $\Delta E - E$ telescope can identify the type of particle that produced the signal, so by placing gates around the type of reaction product, which are tritons in this case, the level of particle impurity in the energy spectra can be reduced. Furthermore, using the signals from the first and second proportional counter a $\Delta E - \Delta E_1$ gate can be placed to further reduce the impurities from beam scatter of different reaction particle types. These two particle gates are shown in Figure 4.1.

The large rate of deuterons in the focal plane detector are from $^{13}\text{C}(p,d)$ reaction whose Q value of -2.7 MeV lies within the energy region in focus for the $^{162}\text{Er}(p,t)$ reaction at a beam energy of 24 MeV focused at an excitation energy of 2.7 MeV.

4.2 Peak Fitting

The particle spectrum obtained was fit using a software called FitPic (fp). The software was originally developed to fit $\gamma$-ray spectra acquired from Ge(Li) detectors, but the fitting form is general enough that it can be adapted to fit charged particle spectra [51]. The form
Figure 4.1: $\Delta E - E$ spectrum (top) and $\Delta E - \Delta E_1$ spectrum (bottom) with (right) and without (left) an energy gate on outgoing tritons in $^{162}$Er$(p,t)$ reaction at 30°.

of the equation used to fit the peaks is

$$y = a + bZ + cZ^2 + dZ^3 + \sum_i \frac{A_i}{\sqrt{2\pi}\sigma} \exp \left[ -\frac{1}{2} \left( \frac{Z - Z_i}{\sigma} \right)^2 \right] \cdot \left[ 1 + P_3 \left( \frac{Z - Z_i}{\sigma} \right)^4 + P_4 \left( \frac{Z - Z_i}{\sigma} \right)^{12} \right]$$

$$+ \frac{A_i}{\sigma} \left[ 1 - \frac{1}{(Z - Z_i/\sigma)^2 + 1} \right] \cdot \left[ P_2 + P_5 \exp \left( \frac{Z - Z_i}{\sigma} P_6 \right) \right]$$

(4.1)

where the first four terms represent the background function on which the spectrum is fit, $A_i$ is the area of the Gaussian-fitted peak of width $P_1$ and with a centroid located at $Z_i$.

The next three expressions represent three different types of tails which can be applied to the Gaussian function: polynomial, constant or exponential. $P_3$ and $P_4$ are polynomials of
order 3 and 12, respectively, $P_2$ is a constant tail to the fit, and $P_5$ is the coefficient of the exponential tail, with a range $P_6$. Each of these parameters $P_i$, $i=1,2..6$, can be selected to be fixed, free or interpolated.

If a parameter is fixed, this means that the coefficient is set to a constant value. If the parameter is free, it will be adjusted to reproduce the least $\chi^2$ between the fit function and the data. FitPic minimizes the $\chi^2$ according to groups, which is a set of peaks which have the same shape and are fit with the same background. The interpolation setting will find a trend of the parameter across every group and fit the variable to the interpolation. For example, if the $\sigma$ parameter was found to vary quadratically across channel number, which is quite typical, then $\sigma$ can be set to the quadratic fit and the remaining fit parameters can be varied. An example of a spectrum analyzed in FitPic is shown in Figure 4.2. In this work, the constant tail parameter was set to zero because it is not applicable for the energy spectrum of charged particles. The exponential tail was only used when required, usually only on highly-populated peaks below 1 MeV.

4.3 Energy Calibration

To determine the degree of the polynomial which best describes the energy dependence of the focal plane, an F-test is performed. An F-test determines whether the addition of a higher-order term in a polynomial is significant within a 5% confidence interval. The form of the F-value is as follows

$$F = \frac{\chi^2(m) - \chi^2(m + 1)}{\chi^2(m + 1) / N - m - 1}$$

(4.2)
Figure 4.2: Sample fits using FitPic software for the $^{162}$Er$(p, t)$ reaction at 30$°$ and 24 MeV beam energy. The colours have been inverted in the image for better clarity.

where $N$ here represents the number of data points and $m$ the number of terms in the polynomial, such that a quantity $\chi^2(k)$ would have $N - k$ degrees of freedom [52]. Since the focal plane is curved, the fit to relate the channel number, $x$, to the energy of a peak, $E(x)$, must be in a second or third-order polynomial. An F-test was conducted to determine that the focal plane curvature is most appropriately described by a third-order polynomial. The fit to the data is thus described by

$$E(x) = a + bx + cx^2 + dx^3$$  \hspace{2cm} (4.3)
where the a, b, c, d coefficients of the energy calibration are determined by reducing the $\chi^2$ fit of the channel numbers of the fitted peaks to the literature energy levels of $^{160}\text{Er}$ [53] for each angle.

To test the quality of a data fit, the least $\chi^2$ method is defined for independent measurements $x_i, y_i$ as

$$
\chi^2 = \sum_{i=1}^{N} \frac{[y_i - f(x_i)]^2}{\sigma_i}
$$

(4.4)

where $\sigma_i$ is the standard deviation of the errors in $y_i$, and $f(x_i)$ is the value of the fit from the polynomial which equivalent to Equation 4.3. However, for fits to data which are second-order polynomials or greater, such as the one in Equation 4.3, the coefficients of the fit function are correlated, and thus the covariance matrix must be calculated. The $\chi^2$ can be written in terms of the covariance matrix, $V_{ij}$, as

$$
\chi^2 = \sum_{i,j=1}^{N} (x_i - f(x_i))[V_{ij}]^{-1}(x_i - f(x_i))
$$

(4.5)

where Equation 4.4 is recovered when the measurements are independent as the covariance matrix is diagonal [54]. The MINUIT2 program, derived from the Fortran-base MINUIT by Fred James which is used for statistical data analysis [55] is employed to perform the $\chi^2$ minimization to determine the energy calibration parameters in Equation 4.3 and the covariance matrix to quantify the error associated to the fit.

The energy calibration fit functions, covariance matrices and reduced $\chi^2$ values are quoted
Figure 4.3: Energy Calibration of three momentum bites at a proton beam energy of 24 MeV. The literature data points used in the energy calibration are from [53].

in Equations 4.3, 4.3 and 4.3.

\[
E_1(x) = 1362 - 0.6598x + 4.911 \times 10^{-5}x^2 - 1.857 \times 10^{-9}x^3
\]

\[
V_{ij} = \begin{pmatrix}
1.4 \times 10^{-01} & -1.9 \times 10^{-04} & 6.5 \times 10^{-08} & -4.1 \times 10^{-12} \\
-1.9 \times 10^{-04} & 3.2 \times 10^{-07} & -1.3 \times 10^{-10} & 1.1 \times 10^{-14} \\
6.5 \times 10^{-08} & -1.3 \times 10^{-10} & 6.1 \times 10^{-14} & -8.4 \times 10^{-18} \\
-4.1 \times 10^{-12} & 1.1 \times 10^{-14} & -8.4 \times 10^{-18} & 1.8 \times 10^{-21}
\end{pmatrix}
\] (4.6)

\[
\chi^2_{\text{red}} = 0.65
\]

\[
E_2(x) = 2738 - 0.8108x + 1.577 \times 10^{-4}x^2 - 2.131 \times 10^{-8}x^3
\]

\[
V_{ij} = \begin{pmatrix}
2.5 \times 10^{+00} & -1.5 \times 10^{-03} & -1.4 \times 10^{-07} & 1.4 \times 10^{-10} \\
-1.5 \times 10^{-03} & 1.5 \times 10^{-06} & -2.4 \times 10^{-10} & -4.4 \times 10^{-14} \\
-1.4 \times 10^{-07} & 2.4 \times 10^{-10} & 3.3 \times 10^{-13} & -8.7 \times 10^{-17} \\
1.4 \times 10^{-10} & -4.4 \times 10^{-14} & -8.7 \times 10^{-17} & 3.5 \times 10^{-20}
\end{pmatrix}
\] (4.7)

\[
\chi^2_{\text{red}} = 1.53
\]

65
4.4 Cross section measurements

The experimental differential cross section can be describing the probability of a transfer reaction scattering a particle into direction \((\theta, \phi)\) can be written as

\[
\frac{d\sigma}{d\Omega} = \frac{N_c}{LT\Delta\Omega N_b N_t \epsilon}.
\] (4.9)

In Expression 4.9, \(N_c\) represents the total integrated counts in a peak, \(LT\) represents the total live time related to the detector and data acquisition system, \(N_b\) is the number of beam particles provided by the faraday cup and \(N_t\) is the number of particles in the target. The number of particles in the target is extracted from the elastic scattering fit to an optical model potential and is calculated in Section 4.5. The efficiency, \(\epsilon\), is the efficiency related to the Q3D and particle gating, however, the efficiency is taken to be 100% because these errors are orders of magnitude smaller than the systematic error introduced to account for optical model potential dependence and the error in the target thickness determination from elastic scattering.

There are two sources of dead time in this experiment which must be accounted for in the cross section. One of these dead times is due to the data acquisition system (DAQ),
and the other related to the dead time of the detector (DET). Dead time occurs when a signal is received, but the data cannot be recorded because the system is busy processing the previous event. The dead time directly affects the cross section described in Equation 4.9 and must be corrected for. However, this effect can be accounted for if the proportion of missed events is known.

If the DAQ is busy, the current integrator increments a quantity called Scaler 3 when the event was not recorded. Dividing this by Scaler 1, which is related to the total integrated charge described in Section 3.2.3, the dead time of the DAQ,

$$DT_{\text{DAQ}} = \frac{\text{Scaler3}}{\text{Scaler1}}$$  \hspace{1cm} (4.10)

is obtained. The dead time due to the detector is related to the analog-to-digital converter. If charge was collected on a cathode strip before the signal had been digitized, the application-specific integrated circuit (ASIC) described in Section 3.6 would recognize this and would send a signal to increment the first bin of the analog-to-digital converter (ADC). The ADC has 2550 channels, where the first bin represents “Channel0”. The total number of counts sent to the ADC can be found by integrating the number of counts in the entire spectrum for a given run. The dead time of the detector, provided that the dead times are small, can be described by

$$DT_{\text{DET}} = \frac{N_{\text{Channel0}}}{N_{\text{total}}}$$  \hspace{1cm} (4.11)

Since the proportion of dead time and live time in either system must equal unity, the total
live time can be expressed by

\[ LT = (1 - DT_{DAQ}) \cdot (1 - DT_{DET}) \] (4.12)

which is calculated for each run in this work.

### 4.5 Elastic Scattering

During a reaction experiment, elastic scattering data is also collected. The reason for this is two-fold:

1. Normalization of the elastic scattering angular distribution to an optical model potential, described in Section 2.3.2, can determine the optimal entrance channel optical model potential parameters best suited for the reaction kinematics.

2. Although an estimate of the target thickness is given by the target manufacturer, a normalization of the elastic scattering data to an optical model potential provides a high-accuracy correction to the target thickness measurement.

The number of particles in the target \( N_t \) is given by

\[ N_t = \frac{\rho t N_A T}{M \cos \phi} \] (4.13)

where \( \rho \) is the density of the target material, \( t \) is the target thickness, where \( \rho t \) is estimated by the manufacturer, \( T \) is the correction factor applied to the target thickness from the optical model potential normalization of the elastic scattering data, \( M \) is the molar mass of the target, and \( N_A \) is Avogadro’s number. The \( \cos \phi \) factor accounts for the angle of the
target relative to the beam axis. For elastic scattering reactions, the target angle is generally kept as half the Q3D angle up to a maximum of 30-40°. A schematic of the target angle with respect to the beam axis is shown in Figure 3.7.

In this work, a full angular distribution of $^{162}\text{Er}(p,p)^{162}\text{Er}$ elastic scattering data was collected and the angular distribution was normalized to several global optical model potentials. Since these global optical model potentials are often derived from data sets which include measurements that had insufficient resolution to separate the ground state from the first excited state, especially in deformed nuclei, there can be a bias in the optical model which overestimates the elastic cross section at large angles, where the cross section for the inelastic population of the $2^+_1$ level can become relatively large. To properly compare the angular distribution, a coupled-channel fit should be performed, however, a fairly good approximation is achieved by simply adding the inelastic cross section of the $2^+_1$ state to that of the ground state.

### 4.6 Spectrum Impurities

To have confidence that the observed peaks in the energy spectrum are energy levels in $^{160}\text{Er}$ rather than levels from a contaminant, three regimes where impurities can arise were considered:

1. **Isotopic contaminants:** Although the $^{162}\text{Er}$ target is estimated to be over 99% pure by the manufacturer, isotopic contaminants may be present in the target. The stable Er isotopes which were considered are $^{164}\text{Er}$, $^{166}\text{Er}$, $^{168}\text{Er}$ and $^{170}\text{Er}$. Considering the elastic scattering spectrum, the amount of isotopic contaminants in the residual nucleus from $^{162}\text{Er}$ and/or $^{164}\text{Er}$ is only around 1%. In addition, each higher-mass isotope has a
Figure 4.4: Elastic scattering angular distribution using the sum of ground state and first excited state cross sections, with normalization applied. The proton optical model potentials that were considered were [26], [33], [29], [31] and [32]. The optical model potential which best represents the elastic scattering data was chosen to be Becchetti and Greenlees.

higher Q-value, such that the strength of the ground state of a high-mass contaminant would be pushed off the spectrum of the focal plane at the momentum setting set for a $^{162}$Er target.

2. High-mass contaminants: High-mass contaminants might also be present in the sample. Some potential candidates are A=162 isotopes of Lu, Yb, Tm, Ho, Dy, Tb, Gd and Eu. However, of these candidates, the only stable isotope is $^{162}$Dy, while the other isotopes have half-lives on the order of seconds or minutes and would not be present in the target sample. However, since the Q-value of $^{162}$Dy$(p, t)$ is less negative than that of $^{162}$Er$(p, t)$, all energy states below 1.4602 MeV will be out of focus on the focal
plane. As such, there was no evidence of impurity contributions from $^{162}$Dy.

3. Low-mass contaminants: An example of a low-mass contaminant which might be present in the spectrum is the natural carbon foil backing the target sample, which presents itself as $^{12,13}$C. Another source of background could be $^{14}$N or $^{16}$O from atmospheric contaminants in the target chamber. The location of these peaks can be determined from the reaction kinematics for elastic scattering of these low-mass contaminants shown in Figure 4.5. The low-mass contaminants found were $^{28}$Si, $^{16}$O, $^{14}$N, $^{12}$C, $^{13}$C and $^{11}$B, which are shown in Figure 4.6.

![Figure 4.5: Elastic scattering kinematics showing location of contaminant ground states relative to the ground state of $^{160}$Er for different angles.](image)

Figure 4.5: Elastic scattering kinematics showing location of contaminant ground states relative to the ground state of $^{160}$Er for different angles.
Figure 4.6: Elastic scattering kinematics showing contaminant angular dependence. Because the Q3D fields are set for the kinematics of the desired nuclear reaction, all other reactions will be out of focus on the focal plane, so the wide impurity peaks appear as to be out of focus. The coloured arrows show the direction in which the contaminant peaks are pushed out of the spectrum at high angles, and the dots represent the energies at which the peaks would appear from the reaction kinematics.
Chapter 5

Results and Discussion

5.1 Angular Distributions

Although two-neutron transfer reactions such as the \((p, t)\) reaction are not sensitive to the single-particle \(j\), they do probe the total orbital angular momentum, \(L\), which was transferred to the triton by the di-neutron pair. Furthermore, different states with the same transferred angular momentum will be probed in a similar way. The resulting angular distribution has a unique shape, which is in principle Q-value dependent, but is identifiable. By collecting cross sectional data at different angles, an angular distribution can be created to assign orbital angular momenta to excited states. In particular, two-nucleon transfers such as the \((p, t)\) reaction employed in this work yield distinctive \(L = 0\) angular distributions. For this reason, even excited states up to \(\sim 3\) MeV can be unambiguously identified if they are \(L = 0\) transfers.

At higher excitation energy and angular momenta, however, there is significantly poorer agreement of the DWBA calculation to the data. This is expected, as for higher angular
momentum transfers, there is more likely to be coupled-channel effects which involve multi-step processes, where there is strong coupling between states. For this reason, only those angular momenta with $L \neq 0$ angular distributions that can be assigned with a very high degree of certainty are reported. However, since the context of the work focuses on excited $0^+$ states, states populated with $L = 0$ transfer are reported with a high degree of confidence. The angular distributions for all states populated in the experiment can be found in Appendix A1.

5.2 Assigned States and Parities

The energy-calibrated triton spectrum collected for the $^{162}\text{Er}(p, t)$ reaction at a beam energy of 24 MeV and a Q3D angle of $30^\circ$ is shown in Figure 5.1. Three momentum bites of data were required to observe the excitation energy range between 0 MeV and 3.2 MeV. The angular momenta determined for some of the more prominent peaks are labelled in the figure. The members of the ground state band up to $J^\pi = 8^+$ and another low-lying $K^\pi = 0^+$ band head member at an energy of 894 keV can clearly be seen. What is interesting to note is that although this $0^+_2$ state is very strongly populated with respect to the ground state, all observed excited $0^+$ states beyond the first excited state are very weakly populated relative to the ground state. There are no known contaminants which can be deduced from the spectrum, and the purity of the target is estimated to be over 99%.

In total, sixty-nine energy levels were observed and angular momenta for thirty-five of these states were assigned. A summary of all the levels is found in Table 5.1.

---

1It is worth clarifying that the angular distributions are plotted with the least $\chi^2$ to the data, however, when a relative cross section is extracted, the DWBA calculations are normalized to the $5^+$ data point.
Figure 5.1: Energy-calibrated spectrum collected for $^{162}\text{Er}(p, t)$ reaction between 0 MeV and 3.2 MeV at 30° with the angular momentum of some of the more prominent peaks labelled. Note the strength of the excited 0$^+$ states.

5.2.1 894 keV State

The 894 keV state was previously the only (tentative) excited 0$^+$ assignment, and has been confirmed as a 0$^+$ state in this work, from both the 24 MeV beam energy and 22 MeV beam energy angular distributions. Further discussion of the excited 0$^+$ strength and possible interpretation is described in Section 5.6.
Table 5.1: Energy and Angular Momentum Assignments for observed energy levels in the first and second momentum bite of collected $^{160}$Er($p$,t) data at a beam energy of 24 MeV. The literature values for energy and parities are from the Nuclear Data Sheets [53].

<table>
<thead>
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<th>$E_{ex}(keV)$</th>
<th>$J_{ex}$</th>
<th>$K_{ex}$</th>
<th>$E_{lit}(keV)$</th>
<th>$J_{lit}$</th>
<th>$E_{ex}(keV)$</th>
<th>$J_{ex}$</th>
<th>$E_{lit}(keV)$</th>
<th>$J_{lit}$</th>
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<td>3299(5)</td>
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</tbody>
</table>
5.2.2 988 keV State

The total angular momentum of the 988 keV state could not be assigned through comparison to angular distributions. The literature assignment of this state is a $3^+$ state, which is thought to be the second member of a $\gamma$-vibrational band. The FRESCO angular distributions calculations cannot handle odd-J even-parity states using a 1-step DWBA transfer because these are forbidden transitions under the assumption that the neutron pair couple to $S = 0$, and thus this state was assumed to be correctly assigned as a $3^+$ state.

5.2.3 1231 keV State

Because the focal plane is not resolved within 1 keV, the energy assignment of the 1231 keV state is somewhat ambiguous. The literature assigns two states within a very short energy range: 1229.3(4) keV as an $8^+$ state and 1230.3(2) keV as an $2^+, 3^+, 4^+$. Because the $8^+$ member of the ground state band should not be very strongly populated, and this state is quite strongly populated in the $(p,t)$ reaction (in fact, it is more strongly populated than the $6^+$ member of the ground state band), it is assigned as the $4^+$ member of the $K^\pi = 0^+_2$ band.

5.2.4 1317 keV State

While performing the experiment, an overlap between each momentum bite is usually selected such that a few states can be resolved in both momentum bites, to ensure that a full spectrum is obtained. However, because the region near $E = 1300$ keV is vacant of all peaks except the proposed $5^+ \gamma$-vibrational state at 1316.7(2) keV, which is expected to only be very weakly populated, the decision was made to not take an overlap containing this peak in
order to maximize the statistics with the beam time and to reach the highest possible energy in the second momentum bite. Thus, the 1317-keV state [53] is not assigned in this work.

5.2.5 1375 keV State

The indication that the 1375 keV state is part of a $K^\pi = 0^- \rightarrow$ band is motivated by the similarity of the $^{160}$Er level scheme [53] to the $^{162}$Er level scheme [56]. The $^{162}$Er data suggests that the $K^\pi = 0^- \rightarrow$ band head ($1^-$) and the $3^-$ member of the band lie at 1352.17(5) keV and 1356.77(7) keV, respectively, which would likely not be resolved in this work and form a doublet. It is very likely that the same excitation is present in $^{160}$Er at 1375 keV. Since the $1^-$ state is not strongly populated in this work (further discussed in Section 5.2.6), the only the $3^-$ member is expected to be populated with any strength.

5.2.6 1474 keV State

The 1474 keV state is tentatively labelled as a $1^- \rightarrow$ state, which may be a $K^\pi = 1^- \rightarrow$ band head. In principle, a $J^\pi = 1^- \rightarrow$ coupling is allowed, however, this is only possible if the neutrons occupy the $i_{13/2}$ and $h_{11/2}$ orbitals. These orbitals are thought to be furthest from the Fermi surface, so the state should thus only be weakly populated. The the cross sections quoted in Appendix A for the 1474 keV state are indeed quite weak, and thus a tentative assignment of this state as a $J^\pi = 1^- \rightarrow$ transfer is made.

5.2.7 1584 keV State

Although the literature [53] describes the 1584 keV state to have either a 1, 2$^+$ state, a $J^\pi = 1^+ \rightarrow$ state is very weakly populated because of the selection rule that the spins of the
di-neutron pair couple to $S = 0$, as described in Section 5.2.6. Despite this limitation, the 1584 keV state does agree very well with a $J^\pi = 2^+$ angular distribution, and thus this state is assigned as a $2^+$.

5.2.8 Uncertainty in Angular Momentum Assignments

It is worth noting that some of the members of the third and fourth $K^\pi = 0^+$ bands, as well as the second and third $K^\pi = 2^+$ bands are speculative and based on similarities in the $^{160}$Er band structures to those of $^{162}$Er and $^{152}$Sm. Some of the systematics that were considered were the energy spacings between rotational bands and the energy differences between $K^\pi = 2^+$, $K^\pi = 0^-$ bands in neighbouring nuclei and $^{160}$Er, where the structures were similar. Despite the uncertainty in relying on band structures in neighbouring nuclei to assign angular momenta, most levels had angular distributions which agreed with the calculated angular momentum from the DWBA calculations. The angular momenta which were not assigned with complete certainty are clearly labelled in Table 5.1.

5.3 Relative Cross Section Strength

In a single-nucleon transfer reaction, the single-particle orbitals populated by the addition or removal of a nucleon can directly probe the single-particle orbitals involved in the transfer. This angular distribution can be compared to the DWBA calculation to obtain the probability of a single-particle orbital population, called a spectroscopic factor. However, in a two-neutron transfer, although the total angular momentum of the di-neutron pair is known, an absolute value for the single-particle total angular momentum, $j$, cannot be extracted.

To report the strength of the excited $0^+$ states relative to the ground state, it is common
practice to take a ratio of the DWBA-normalized excited state to the DWBA-normalized ground state. By doing this, a Q-value correction is applied to the data to account for the difference in kinematics of the reactions. The relative cross section strength, $S$, is defined by

$$S = \frac{\left(\frac{d\sigma}{d\Omega}\right)_{\text{lab}}^{0^+_x}}{\left(\frac{d\sigma}{d\Omega}\right)_{\text{dwba}}^{0^+_x}} \div \frac{\left(\frac{d\sigma}{d\Omega}\right)_{\text{lab}}^{0^+\text{gs}}}{\left(\frac{d\sigma}{d\Omega}\right)_{\text{dwba}}^{0^+\text{gs}}} \quad (5.1)$$

where the differential cross sections are listed in the centre-of-mass frame. In this work, these calculations were done using FRESCO, described in Section 2.3.3, concentrating on the strength of the excited $0^+$ states. The strength of all assigned excited $0^+$ states can be found in Table 5.2.

<table>
<thead>
<tr>
<th>$E_{\text{exp}}(keV)$</th>
<th>$S(0^+_x/0^+_1)(%)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100(0)</td>
</tr>
<tr>
<td>894</td>
<td>18(1)</td>
</tr>
<tr>
<td>1279</td>
<td>1.0(1)</td>
</tr>
<tr>
<td>1528</td>
<td>1.0(1)</td>
</tr>
<tr>
<td>1864</td>
<td>1.4(5)</td>
</tr>
<tr>
<td>1930</td>
<td>0.10(3)</td>
</tr>
<tr>
<td>2032</td>
<td>1.7(1)</td>
</tr>
<tr>
<td>2129</td>
<td>0.7(1)</td>
</tr>
</tbody>
</table>

Table 5.2: Energy and relative strengths of observed excited $0^+$ states, normalized to the $5^+$ point. One of the key findings is the highly populated $0^+_x$ state, which is a key signature of nuclei with shape coexistence [10].

In total, seven excited $0^+$ states were assigned, six of which were unknown until the present work and one of which was not definite. The relative cross section strengths are reported in Table 5.2. Unfortunately, due to the high level density in the third momentum bite taken with a 22 MeV beam energy, no spin assignments were made in the third momentum bite of data. However, from the shape of the angular distributions and weak population of
these states, it is very unlikely that any of these excited states can be assigned as excited $0^+$ states.

5.4 Single-Particle Orbital Choice

One ambiguity which has not yet been addressed is the choice of single-particle orbital which was used in the DWBA calculations to extract the relative strength of excited $0^+$ states. As discussed in Chapter 1, because the model space of shell-model calculations is too large to yield sensible results for two-neutron cfp values, which describe the probability of removing two particles from a single-particle orbital, an alternative approach must be taken to address the generality of the choice of single-particle orbital used in DWBA calculations.

Although the magnitude of the cross sections are dependent on the single-particle orbital choice, for any transferred angular momentum, the shape of angular distributions does not vary greatly, and the relative cross section is truly independent of the choice or orbital, which is demonstrated in Figure 5.2. This justifies the claim that the choice of single-particle orbital is inconsequential, because the relative strengths of the cross sections are quoted and not the absolute cross sections.

The $2f_{7/2}$ orbital was selected as the single-particle content of the di-neutron pair in DWBA calculations due to the proximity of the orbital to the Fermi surface in the $N = 92$ region. By looking at the Nilsson diagram in Figure 1.6, in the deformation region of $\epsilon_2 = 0.3$, it is clear that in a $N = 92$ nucleus the valence shell is either the $2f_{7/2}$ orbital or the $1i_{13/2}$ orbital. By consulting the Nuclear Data Sheets, it is clear that there are no low-lying positive-parity states, so it was determined that the $2f_{7/2}$ orbital is the most appropriate orbital to which to transfer the di-neutron pair.
There are however, a few states for which assuming the strength is pure $2f_{7/2}$ is not possible. For transfers beyond $L = 6$, the $1i_{13/2}$ orbital must be used to achieve this angular momentum. Furthermore, to transfer to negative-parity states, a combination of the $2f_{7/2}$ and $1i_{13/2}$ is used. For more information, see the FRESCO input file in Figure 2.6.

Figure 5.2: Single-particle orbital transfer strength into $0^+_1$ (top) and $2^+_1$ (bottom) states of $^{160}$Er. Note although the magnitude of the angular distributions cross sections varies with single-orbital choice, the shape of the angular distribution is nearly independent of the orbital. These calculations were performed assuming the cfp = 1 for each orbital.
5.5 Error Discussion

There were several sources of systematic error associated with the calculated cross sections as well as the relative cross sections. This section will discuss these sources of error and their treatment.

5.5.1 Error in Cross Section Calculations

1. Error in target thickness due to solid angle correction: The cross sections reported in this thesis have errors associated in the peak fitting process, estimated by FitPic, the errors associated in the determination of the live time of the detector and data acquisition, and the error in the target thickness. However, a systematic error could arise in the calculation of the target thickness which is dependent on the choice of optical model.

A calibration to determine the micrometer slit offset was not performed in the experiment in 2010 using the 24 MeV proton beam energy, and thus the absolute target thickness could not be extracted for this data set. Instead, when a slit setting was changed, two data points for the same angle were taken, which means a relative normalization could be performed. However, there are two ways of performing this normalizations, either assuming the effective solid angle at low angles was correct, or higher angles. These normalizations achieved drastically different results in terms of the effective target thicknesses, either 62 $\mu g/cm^2$, or 81 $\mu g/cm^2$, respectively. A secondary elastic scattering data set was taken at 22 MeV with the same target in 2014 to resolve the target thickness. The target thickness of the 22 MeV data set was calculated to be 61 $\mu g/cm^2$, which differs from one of the normalization methods less
than 2%. However, since the error solid angle for the 24 MeV effective target thicknesses cannot be estimated, this error is inflated to be certain that there was no change in target thickness between the 24 MeV and 22 MeV runs which was not accounted for.

2. Error in target thickness due to choice of optical model potential: The error in the target thickness associated to the choice of optical model potential was always less than 1% at the 15° normalization data point in the elastic scattering.

A conservative systematic error of 5% is included in the cross section calculations to account for the uncertainty in the target thickness.

5.5.2 Error in Relative Cross Section Calculations

1. Error due to choice of optical model potential: The relative cross sections quoted had at most a relative difference for various optical models of 8%. However, since the relative cross sections agreed with one another within error, this error is assumed to be accounted for in the systematic treatment of the optical model dependence in the cross section calculations and is thus not considered in this analysis.

2. Error due to choice of normalization angle: The two-neutron transfer data was normalized at the 5° data point because it had the largest cross section and was the least sensitive to the precise location of the minima. The percent difference in relative cross sections between 5° and 6° for 24 MeV and 22 MeV ground state is estimated at 3%. This error is much smaller than the systematic error already associated with the target thickness, which likely plays a role in the difference in relative cross section for these
two energies and is thus not included in this thesis.

3. Error due to choice of single-particle orbital: An investigation was performed to quantify the errors in the relative cross sections for first few excited $0^+$ states for different orbitals. Apart from the $1i_{13/2}$ and $1h_{11/2}$ orbitals, which from Figure 5.2, the relative cross sections of the first and second excited $0^+$ states seem to be fairly independent of the choice of orbital as predicted. The estimated error in the choice single-particle orbital in the relative cross section is always less than 2% compared to the $2f_{7/2}$ orbital, which is negligible with respect to the errors already assigned to the relative cross sections.

5.6 Interpretation of Excited $0^+$ States

The strength of excited $0^+$ states relative to the ground state strength are listed in Table 5.2. The focus of this work is to investigate the strength of the $0^+_2$ in $^{160}$Er, which was found to have a relative strength of 18(1)%. All other excited $0^+$ states were found to be very weakly populated, with a relative strength of 2% or less. In the $N = 90$ region, the strongly-populated $0^+_2$ state is classically associated with a $\beta$ vibration, but this interpretation has recently been brought into question [10, 15]. The observation that the $0^+_2$ state is the only excited $0^+$ which is strongly populated suggests another interpretation should be presented. It is proposed that this state could be a subtle example of shape coexistence. To further motivate this claim, several systematic trends were scrutinized in order to examine similarities to the $N = 90$ shape-coexisting isotones.

One of the systematic trends which was investigated was the strength of the $0^+_2$ state
compared to other $N = 92$ rare-earth isotones Gd and Dy, which is plotted in Figure 5.3.

This comparison was strongly motivated by the similarity in the relative strength of the $0_2^+$ strength in $N = 90$ isotones $^{154}$Gd and $^{152}$Sm, which both demonstrated a highly populated $0_2^+$ state in $(p,t)$ and $(t,p)$ reactions, shown in Figure 1.9 [16]. It was found that the $0_2^+$ state is consistently and strongly populated along $N = 92$ isotones, although the strength was not quite as high as in the $N = 90$ nuclei. This systematic tendency of highly populating the $0_2^+$ state in $(p,t)$ reactions in both $N = 90$ and $N = 92$ nuclei suggests that the mechanism responsible for strongly populating the $0_2^+$ state might be playing a similar role in both regions.

The systematics of the $0_2^+$ state strength were also investigated with respect to the Er isotopic chain. If the strength of the $0_2^+$ state is compared to other Er isotopes, as shown in Figure 5.4, it is clear that beyond $^{164}$Er, there is a sudden shift of the $0^+$ strength away

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2The referenced nuclear data sheets for increasing $A$ are [53,56,60–68]
Figure 5.4: Systematics of $0^+_2$ strength (left) and total $0^+$ strength (right) across Er isotopes. Data for $^{160}$Er is from this work, $^{162,164}$Er are from [57] and $^{166,168}$Er are from [58] and [59], respectively. The total $0^+$ strength is estimated by summing the centroid of strength which seems to fragment over several excited $0^+$ states, particularly in $^{166,168}$Er which are further away from the $N = 90$ region.

from the $0^+_2$ state towards higher-energy states. However, there is conclusively no decrease in total excited $0^+$ strength, as shown in the right-hand side of Figure 5.4.

There have been several publications emphasizing the role that the $\nu_1^{-11/2}$ [505] orbital may be playing in the structure of the low-lying $0^+$ states. Kulp et al. identified a pairing isomeric band in $^{154}$Gd and $^{152}$Sm. In both nuclei, the pairing isomeric band was assigned to the $0^+_3$ band, that was observed to have a dramatically different (and smaller) moment of inertia than other bands. More recently, Sharpey-Schafer et al. have investigated a series of nuclei in the $N = 90$ region, and have suggested that the first excited $0^+$ band, based on the $0^+_2$ state, forms a "second vacuum", also invoking the argument that the $\nu_1^{-11/2}$ [505] configuration is playing a dominant role in this state as well. Allmond et al. [69] have offered
Figure 5.5: Similarity between the energy of the centroid of $0^+$ strength in Er isotopes (red) and the energy of the $\nu_{11/2}^-[505]$ orbital in the odd-A nuclei (black) from the Nuclear Data Sheets.

An explanation of this apparent ‘over counting’ of the $\nu_{11/2}^-[505]$ configuration, pointing out that other configurations, such as the $\nu_{1/2}^+[400]$ orbital, can be involved as well. Allmond et al. speculate that the $0_2^+$ state may involve the $(\nu_{11/2}^-[505])^2 + (\nu_{1/2}^+[400])^2$ configuration, whereas the $0_1^+$ state may involve the $(\nu_{11/2}^-[505])^2 + X^2$, with $X$ being some other orbital.

As was stated above, the two-neutron-transfer reaction does not enable the determination of the single-particle orbit involved. It is noted, however, that the $0_2^+$ level is very strongly populated in the $(p,t)$ reaction, despite being built on the high-$j$ $\nu_{11/2}^-[505]$ orbital. It can be conjectured that the other configurations involved in the $0^+$ wave function, that can include the low-$j$ $\nu_{1/2}^+[400]$ orbital, are responsible for the observed strength. If the $\nu_{11/2}^-[505]$ orbital is an important configuration for the $0^+$ states in the $N = 92 - 100$ Er isotopes, it might be expected the locus of enhanced $0^+$ strength might follow the energy of the $\nu_{11/2}^-[505]$.
orbital as observed in the odd-A isotopes. Figure 5.5 shows the excitation energy locus of the strongly populated $0^+$ states in the Er isotopes as well as twice the excitation energy of the $\nu_{1/2}^{-}[505]$ orbital in the odd-mass Er chain. As can be seen, there is a strong similarity in shape of the energy curves, lending support to the suggestion that the $\nu_{1/2}^{-}[505]$ orbital is playing a central role in the wave functions of the $0^+$ states that have strong populations in the $(p,t)$ reaction. It appears that the $\nu_{1/2}^{-}[505]$ is in fact playing a key role in this pairing mechanism in the $N = 92$ region, which agrees with the postulated role in the $N = 90$ region.
Chapter 6

Conclusions

The nature of excited states in well-deformed rare-earth nuclei has been the subject of great debate in nuclear structure physics, with possible interpretations varying from vibrational excitations in $\beta$-phonons and $\gamma$-phonons to pairing excitations. However, low-lying excited $0^+_2$ state can also be a key signature of a shape coexisting state, and the exact nature of these states, particularly in the $N = 90$ region, is still ambiguous.

A $^{162}\text{Er}(p,t)^{160}\text{Er}$ reaction was performed at the Maier-Leibnitz Laboratory in order to study the nature of excited $0^+$ states. These two-neutron transfer reactions such as $(p,t)$ and $(t,p)$ area excellent probes of the pairing component of nuclear wave functions and can shed light on the nature of excited $0^+$ states. The angular distributions collected were compared to the DWBA calculations to determine the angular momenta of states below 3 MeV, and the relative strength of excited $0^+$ states. In this work, sixty-nine energy levels were observed and angular momenta for thirty-eight of these states. In total, seven excited $0^+$ states were assigned, and of these states six new states were assigned.

The new data introduced in this thesis can provide us with several insights towards the
interpretation of exited $0^+_2$ states in the rare earth region. The finding that the $0^+_2$ state is very strongly populated with 18% of the ground state strength, with higher excited $0^+$ states having a relative strength of less than 2%, suggests that this state might have another interpretation other than the classically adopted $\beta$ vibration.

A comparison of the strength of this state in the $N = 90$ isotones suggests that the mechanism responsible for this state in nuclei with $N = 90$ might also be playing a key role in the $N = 92$ region. Furthermore, by comparing this state to other isotones in the $N = 92$ region, it strongly suggests that this mechanism is also responsible for the highly-populated $0^+$ states in Gd, Nd. Since one of the key features of shape coexistence in $N = 90$ nuclei is the presence of a strongly-populated $0^+_2$ state [10], the presence of this spectroscopic feature directly motivates the claim that the Er isotopic chain, particularly $^{160}$Er, might also be examples of nuclei possessing of shape coexistence.

If the systematics of this state are plotted for the Er isotopic chain, beyond $^{164}$Er, the strength of the $0^+_2$ state disappears, however, the total strength populating excited $0^+$ states remains relatively high. Furthermore, the excitation energy locus of the population strength increases in energy, matching well that of the $h_{11/2}^{-}[505]$ in odd-A isotopes. This trend further motivates the suggestion that the $h_{11/2}^{-}[505]$ orbital might be playing a role in these $(p,t)$ reactions. While the orbital is very close to the Fermi surface at $N = 88$, which marks the beginning of a transitional region, beyond $N = 96$, there is only weak population of the low-lying $0^+_2$ state.

The suggestion that the mechanism involved in the high $0^+_2$ strength is linked to the $h_{11/2}^{-}[505]$ orbital is quite convincingly demonstrated by the argument that the energy of the centroid strength of $0^+$ states tracks extremely well with the energy of the $h_{11/2}^{-}$ orbital in the odd-A Er isotopes. This claim alongside the suggestion that the same orbital plays a key
role in $N = 90$ nuclei has led to the suggestion that the $h^{11\frac{2}{3}}[505]$ orbital plays a key role in
the $(p,t)$ two-neutron transfer into $^{160}$Er.

To be able to conclusively determine whether the Er isotopic chain is in fact a subtle
example of shape coexistence should be evidenced by other experimental measurements. In
particular, to be able to conclusively determine whether the $0^+_2$ state is a shape coexisting
state, electric monopole $E0$ transition strengths as well as $E2$ transition strengths would be
required. Exploration of the Er isotopic chain in the $N = 90$ region, which is impossible in
direct transfer reactions in forward kinematics due to the unstable nature of $^{160}$Er, would be
a natural starting point to look at spectroscopic features of shape coexistence.
Appendix A: Angular Distributions

The cross sections obtained in the $^{162}$Er$(p,t)$ transfer and possible angular momenta assignments are plotted below. The optical model potential for the incoming protons was chosen to be Becchetti and Greenlees [29].
Figure A.1: Angular distributions from the $^{162}$Er($p,t$) reaction at a proton beam energy of 24 MeV.
Figure A.2: Angular distributions from the $^{162}$Er($p,t$) reaction at 24 MeV beam energy.
Figure A.3: Angular distributions from the $^{162}$Er($p, t$) reaction at 24 MeV beam energy.
Figure A.4: Angular distributions from the $^{162}$Er($p, t$) reaction at 24 MeV beam energy.
Figure A.5: Angular distributions from the $^{162}$Er($p$, $t$) reaction at 24 MeV beam energy.
Figure A.6: Angular distributions from the $^{162}$Er($p,t$) reaction at 24 MeV beam energy.
Figure A.7: Angular distributions from the $^{162}\text{Er}(p,t)$ reaction at 24 MeV beam energy.
Figure A.8: Angular distributions from the $^{162}$Er($p,t$) reaction at 24 MeV beam energy.
Figure A.9: Angular distributions from the $^{162}$Er($p$,t) reaction at 24 MeV beam energy.
Figure A.10: Angular distributions from the $^{162}$Er($p,t$) reaction at 24 MeV beam energy.
Figure A.11: Angular distributions from the $^{162}$Er($p$,t) reaction at 24 MeV beam energy.
Figure A.12: Angular distributions from the first momentum bite of the $^{162}$Er($p,t$) reaction at a proton beam energy of 22 MeV. The angles with two cross sections data points correspond to a renormalization of the ground state cross-sections in two separate runs, one with higher statistics and one with lower statistics.
Figure A.13: Angular distributions from the third momentum bite of the $^{162}$Er$(p,t)$ reaction at 22 MeV beam energy. The angles with two cross sections data points correspond to a renormalization of the ground state cross-sections in two separate runs, one with higher statistics and one with lower statistics.
Figure A.14: Angular distributions from the third momentum bite of the $^{162}$Er($p,t$) reaction at 22 MeV beam energy.
Bibliography


