Magnetic and Porous Media Thermoacoustic Systems: Modeling and Experimentation

By

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A Thesis presented to The University of Guelph

In partial fulfilment of requirements for the degree of Doctor of Philosophy in Engineering

Guelph, Ontario, Canada © Md. Shariful Islam, January, 2017
ABSTRACT

MAGNETIC AND POROUS MEDIA THERMOACoustIC SYSTEMS: MODELING AND EXPERIMENTATION

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In this thesis, magnetic and porous media thermoacoustic systems have been investigated by developing rigorous mathematical models and subsequent experiments. The inherent irreversibility of the thermoacoustic stack is analyzed in the presence of a magnetic field applied across the stack perpendicular to the direction of fluid oscillations to improve the stack efficiency. Two different types of stacks are considered for the analytical modelling: a porous medium coupled with a thick solid plate and a multi-plate stack with a transverse magnetic field.

For the porous medium coupled with a thick solid plate stack, the analytical expressions of the fluctuating velocity and temperature of the oscillating fluid are derived from the governing Darcy momentum and energy equations. Consequently, the simplified analytical expressions for the Nusselt number, heat flux, work flux, entropy generation rate, and the efficiency are derived and presented graphically. It is observed that the thermoacoustic irreversibility can be minimized by increasing the applied magnetic field resulting in increased efficiency of the system.

For the multi-plate thermoacoustic stack, the effects of magnetic field on the heat transfer are analyzed using complex Nusselt number. The unsteady-compressible-viscous forms of the continuity, momentum, and energy equations are used to derive the analytical
solution for the fluctuating velocity and temperature. Then, the simplified analytical solutions for the complex Nusselt number are derived. The first order analytical equations for the energy, heat, and work fluxes for a thermoacoustic refrigerator are also derived and presented graphically. In the absence of a magnetic field, all of these simplified analytical expressions are compared with the data available in the literature and an excellent agreement is observed.

Finally, an experimental setup is designed and constructed which is utilized to measure the performance of porous medium thermoacoustic refrigerator and heat pump systems. The stack length and position of a prototype thermoacoustic refrigerator are optimized using numerical, analytical, and experimental analysis. For an optimal stack length (175mm) and position (42mm from the acoustic driver’s end) at a constant frequency and drive ratio, the maximum temperature of 88.9 °C at the hot end and -8.5 °C at the cold end of the stack are achieved. The maximum cooling capacity achieved is 17.85 watts at 5.42% coefficient of performance relative to the Carnot’s coefficient of performance. The results from the thermoacoustics theories developed in this thesis can be potentially applied to design the next generation thermoacoustic systems.
DEDICATION

This thesis is dedicated to my mother Billkis Banu
ACKNOWLEDGEMENTS

First of all, I would like to express my sincere gratitude to my supervisor Dr. Shohel Mahmud for giving me the opportunity to work in this area. I want to thank him for his knowledge, advice, patience, and restless effort to guide me and that made my research achievement possible.

I would also like to thank my co-supervisor Dr. Mohammad Biglarbegian for his insightful comments and useful suggestions. He was always there to listen and give guidance.

I would also like to thank my committee member, Dr. Roydon Fraser, for his advice and enormous support during this research.

I would also like to thank the graduate coordinator, Dr. Animesh Dutta, for his valuable comments during the qualifying exam and continuous support.

I would also like to thank Dr. Syeda Tasnim for her support and help. I would also like to thank Mike Speagle, Joel Best, Hong Ma, and Phil Watson for their cooperation during my research period. I would also like to thank Laurie Gallinger for administrative help.

I would also like to thank all members of Advanced Energy Conversion and Control Lab, University of Guelph, Muath Alomair, Ronil Rabari, Raihan Siddique, Manar Al-Jethelah, Kaswar Jamil, Yazeed Alomair, and Rakib Hossain for their cooperation and feedback in group presentation.

Finally, I would like to express my deepest appreciation to my parents, my son Aaryan Sharif, Raiyan Sharif, and my wife Rukhsana Liza. Their endless support and encouragement made this thesis possible.
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**NOMENCLATURE**

- \( a \): Velocity of sound, m·s\(^{-1}\)
- \( \mathbf{B} \): Magnetic induction, Wb·m\(^{-1}\)
- \( C_p \): Specific heat of the fluid at constant pressure, J·kg\(^{-1}·K\(^{-1}\)
- \( C_{sm} \): Specific heat of the solid matrix, J·kg\(^{-1}·K\(^{-1}\)
- \( Da \): Ratio of permeability of the porous medium and viscous penetration depth,
  \[ = \frac{K}{\delta_v^2} \]
- \( \text{DR} \): Drive ratio, \( = \frac{p_0}{p_m} \)
- \( D_h \): Hydraulic diameter, m
- \( E \): Electrical field intensity, Volt·m\(^{-1}\)
- \( \varepsilon_z \): Energy flux density, W·m\(^{-2}\)
- \( \mathbf{F}_{em} \): Electromagnetic volume force, N·m\(^{-3}\)
- \( f \): Frequency of oscillation, Hz
- \( f_r \): First Rott’s function of thermoacoustics,
  \[ = \frac{\tanh((1+i)\sqrt{1+Ha_s^2/2iS_w})}{(1+i)\sqrt{1+Ha_s^2/2iS_w}} \]
- \( f_k \): Second Rott’s function of thermoacoustics,
  \[ = \frac{\tanh((1+i)\sqrt{PrS_w})}{(1+i)\sqrt{PrS_w}} \]
- \( h \): Convective heat transfer coefficient, W·m\(^{-2}\)
- \( \text{Ha}_s \): Hartmann number, \( = B_s\delta_v\sqrt{\sigma_s/\mu} \)
- \( i \): Complex number, \( = \sqrt{-1} \)
- \( \mathbf{J} \): Current density, amp·m\(^{-2}\)
- \( K \): Permeability of the porous medium, m\(^2\)
- \( k \): Thermal conductivity, Wm\(^{-1}·K\(^{-1}\)
- \( k_s \): Solid wall thermal conductivity, W·m\(^{-1}·K\(^{-1}\)
- \( k_{sm} \): Thermal conductivity of solid matrix, W·m\(^{-1}·K\(^{-1}\)
- \( k_f \): Thermal conductivity of the fluid, W·m\(^{-1}·K\(^{-1}\)
\( L_s \) Stack length, m

\( L_{sn} \) Normalized stack length (= \( 2\pi f L_s / a \))

\( Nu \) Complex Nusselt number

\( Nu_{av}' \) Complex Nusselt number considering space averaged temperature as reference temperature when \( \Gamma_0 \rightarrow 0 \).

\( Nu_{av\_inv}' \) Complex Nusselt number considering space averaged temperature as reference temperature for an inviscid fluid when \( \Gamma_0 \rightarrow 0 \).

\( Nu_{av\_inv}' \) The boundary layer limit of \( Nu_{av\_inv}' \) when \( \Gamma_0 \rightarrow 0 \).

\( Nu_{av}\_av'' \) Complex Nusselt number considering space averaged temperature as reference temperature when \( \Gamma_0 \rightarrow \infty \).

\( Nu_b \) Complex Nusselt number considering bulk mean temperature as reference temperature.

\( Nu_b' \) Complex Nusselt number considering bulk mean temperature as reference temperature when \( \Gamma_0 \rightarrow 0 \).

\( Nu_b\_inv' \) Complex Nusselt number considering bulk mean temperature as reference temperature for inviscid fluid when \( \Gamma_0 \rightarrow 0 \) and \( Ha_\delta \rightarrow 0 \).

\( Nu_b\_inv\_\infty' \) The boundary layer limit of \( Nu_b\_inv' \) when \( \Gamma_0 \rightarrow 0 \) and \( Ha_\delta \rightarrow 0 \).

\( Nu_b'' \) Complex Nusselt number considering bulk mean temperature as reference temperature when \( \Gamma_0 \rightarrow \infty \).

\( Nu_b\_inv'' \) Complex Nusselt number considering bulk mean temperature as reference temperature for inviscid fluid when \( \Gamma_0 \rightarrow \infty \) and \( Ha_\delta \rightarrow 0 \).

\( P \) Pressure, N\( \cdot \)m\(^{-2} \)

\( p_m \) Mean pressure, N\( \cdot \)m\(^{-2} \)

\( \nabla p \) Pressure gradient, N\( \cdot \)m\(^{-3} \)

\( p_0 \) Fluctuating pressure amplitude, m

\( \dot{Q}_2 \) Second order local heat flux, W\( \cdot \)m\(^{-3} \)
\( Q_{cn} \) normalized cooling power, \( = \dot{Q}_c / p_m aA \)

\( \dot{S}_{\text{gen}}^m \) Volumetric entropy generation rate, \( \text{W} \cdot \text{m}^{-3} \cdot \text{K}^{-1} \)

\( \dot{S}_{\text{gen}} \) Total entropy generation, \( \text{W} \cdot \text{K}^{-1} \)

\( S \) Dimensionless entropy generation

\( S_w \) Swift number, \( = y_0 / \delta_v \)

\( \tilde{S}_w \) Modified swift number, \( = y_0 / \delta_k \)

\( T \) Temperature of the fluid, K

\( T_{s1} \) Temperature inside the solid wall, K

\( \nabla T \) Temperature gradient, \( \text{K} \cdot \text{m}^{-1} \)

\( T_w \) Wall temperature, K

\( T_{\text{ref}} \) Reference temperature, K

\( t \) Time, s

\( u \) Axial velocity component, \( \text{m} \cdot \text{sec}^{-1} \)

\( U \) Dimensionless axial velocity, see Eq.(24)

\( V \) Velocity vector, \( \text{m} \cdot \text{sec}^{-1} \)

\( v \) Transverse velocity, \( \text{m} \cdot \text{sec}^{-1} \)

\( \dot{w}_2 \) Second order work flux, \( \text{W} \cdot \text{m}^{-3} \)

\( \dot{y}_{\dot{z}} \) Work flux density, \( \text{(W/m}^2) \).

\( W_n \) The normalized acoustic power, \( = \dot{W} / p_m aA \)

\( x \) Axial direction, m

\( X_s \) The distance from the acoustic driver’s end to the mid length of stack, m

\( X_{sn} \) Normalized stack position, \( = 2\pi f X_s / a \)

\( \Delta x \) Length of the porous medium and solid wall, m

\( y \) Transverse direction of the porous medium, m

\( \bar{y} \) Transverse direction of the solid wall, m

\( y_0 \) Half channel width, m

\( Y \) Dimensionless transverse distance, \( = y / \delta_v \)
Greek symbols

$\alpha_f$ Thermal diffusivity of the fluid, $= k / \rho_f C_p$

$\beta$ Thermal expansion coefficient, K$^{-1}$

$\delta_v$ Viscous penetration depth, $= \sqrt{2\nu / \omega}$

$\delta_k$ Thermal penetration depth, $= \sqrt{2\alpha_f / \omega}$

$\mu$ Dynamic viscosity of the fluid, N·m$^{-2}$·sec

$\nu$ Kinematic viscosity, m$^2$·sec$^{-1}$

$\sigma$ Porous medium heat capacity ratio (see. Eq. (8a))

$\sigma_k$ Electrical conductivity of the fluid

$\phi$ Porosity (=void volume/total volume)

$\Gamma$ Ratio of $\nabla T_m$ and $\nabla T_{cr}$.

$\Gamma^*$ Ratio of $\nabla T_m$ and $\nabla T_{cr}$ in the absence of magnetic field,
$= \nabla T_m K \rho_m C_p / T_m \beta \mu f$

$\rho$ Density of the fluid, kg·m$^{-3}$

$\rho_m$ Mean density of fluid, kg·m$^{-3}$

$\rho_{ms}$ Mean density of the solid, kg·m$^{-3}$

$\rho_{sm}$ Density of the Solid matrix, kg·m$^{-3}$

$\Psi$ Viscous dissipation function

$\alpha_f$ Thermal diffusivity of the fluid, m$^2$·sec$^{-1}$

$\alpha_s$ Thermal diffusivity of the solid wall, m$^2$·sec$^{-1}$

$\beta$ Thermal expansion coefficient, K$^{-1}$

$\delta_v$ Viscous penetration depth, $= \sqrt{2\nu / \omega}$

$\delta_k$ Thermal penetration depth, $= \sqrt{2\alpha_f / \omega}$

$\delta_{ln}$ Normalized thermal penetration depth ($= \delta_k / y_0$)
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<td>$\rho$</td>
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<tr>
<td>$\tau$</td>
<td>Time period, $=2\pi/\omega$</td>
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<td>$\eta$</td>
<td>Efficiency</td>
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<tr>
<td>$\lambda$</td>
<td>Acoustic wavelength, m</td>
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<tr>
<td>$\varepsilon$</td>
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<td>$\Theta$</td>
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<tr>
<td>$\omega$</td>
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<tr>
<td>$\rho$</td>
<td>Density of the fluid, kg·m$^{-3}$</td>
</tr>
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<td>$\tau$</td>
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<tr>
<td>$\nabla T_{cr}$</td>
<td>Critical temperature gradient, K·m$^{-1}$</td>
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<td>$\Gamma_0$</td>
<td>Temperature gradient ratio, $=\frac{\nabla T_{cr}}{\nabla T_m}$</td>
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**Subscripts and Superscripts**

- 1: First order variable
- $\infty$: Free stream value
- m: Mean value
- sm: Properties correspond to solid matrix material of the porous medium
- w: Value at the interface of the solid wall and porous medium
- cr: Critical value
- a: Adiabatic oscillation
- FF: Fluid friction
- HT: Heat transfer
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<td>s1</td>
<td>Value inside the solid wall</td>
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<tr>
<td>Mag</td>
<td>Magnetic force</td>
</tr>
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<td>~</td>
<td>Complex conjugate</td>
</tr>
<tr>
<td>-</td>
<td>Time average value</td>
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Chapter 1
INTRODUCTION

A thermoacoustic system converts energy from thermal to acoustic (engine mode) or from acoustic to thermal (refrigerator/heat pump mode). The thermal energy received by the stack of the thermoacoustic system converts into acoustic energy by using thermal and hydrodynamic interactions between the oscillating fluid and the stack solid wall (Swift [1988]).

A typical thermoacoustic system consists of three major elements: a stack and two heat exchangers placed inside a resonance tube. The stack is the most important element of a standing wave thermoacoustic system. The stack can be a single solid surface (single-plate), porous media, porous media inside the parallel plate, or a set of parallel solid surfaces of the same length (multi-plate). The two heat exchangers are typically attached on both sides of the stack. Figures 1.1 (a) and (b) show a comparison between conventional and thermoacoustic engines. In a thermoacoustic engine, a temperature gradient can be created across the stack by using appropriate thermal loads on the heat exchangers. The fluid inside the resonant tube starts oscillating if this temperature gradient exceeds a critical value (Swift [1988]). The fluid oscillation will create a thermoacoustic wave if it exceeds the frictional and other losses inside the resonant chamber. This wave can be converted into other forms of energy (e.g., electricity) using proper transducers (e.g., piezoelectric transducer) (Nouh et al. [2014]). In a thermoacoustic refrigerator, the input is the standing waves that can be created by a suitable acoustic driver (e.g., the loudspeaker) (see Fig. 1.1(d)). A temperature difference between the two heat exchangers can be obtained if the temperature gradient across the stack is lower than a critical value (Jaworski and Mao [2013]). Figures 1.1 (c) and (d) present a comparison between a conventional refrigerator and a thermoacoustic refrigerator.

The first practical thermoacoustic apparatus was built in Los Alamos National Laboratory and the University of California, USA, by Wheatley et al. [1983], and was based on Rott’s thermoacoustic theories. Since then, thermoacoustic systems are used mostly for cryogenic applications (Wheatley et al. [1986] and Hofler [1986]). Garrett et al. [1993]
Figure 1.1: (a) Classical heat engine, (b) thermoacoustic engine, (c) classical refrigerator, and (d) thermoacoustic refrigerator

built and tested a thermoacoustic refrigerator for the space applications at the Naval Postgraduate School (NPS) in Monterey, California. The cooling capacity of the device was 4 watts, with a temperature span of 80 K. Johnson et al. [2000] built a thermoacoustic refrigerator with a larger cooling capacity (10kW) for a naval ship. Adeff and Hofler [2000] built a thermoacoustic refrigerator by coupling it with a solar power-driven thermoacoustic prime mover at the Monterey NPS. The cooling capacity of this refrigerator was 2.5 watts, with a temperature span of 17.7 ºC. Tijani [2001] built a loudspeaker-driven thermoacoustic refrigerator with a cooling capacity of 4 watts that can produce a low temperature of -65 ºC. More recently, thermoacoustic systems are being developed for potential applications such as heat exchanger (Paek et al. [2005], de
Jong et al. [2014]), engine (Mumith et al. [2014], El-Rahman and Rahman [2014], Hariharan et al. [2015]), gas mixture separator (Geller and Swift [2009], He et al. [2013]), power generator (Lili et al. [2013]), cancer cell detector (Zhu and Popovic [2011]), and tumor detector (Fu et al. [2014]). From these examples, it is evident that the thermoacoustic devices have been used for numerous applications.

Renewable energy and low-cost energy sources, such as geothermal energy (Haddad et al. [2014]), solar energy (Shen et al. [2009], Pan et al. [2014]), cooking stove waste heat (Chen et al. [2012]), automotive waste heat (Gardner and Howard [2009]) and industrial waste heat (Yang et al. [2014], Mumith et al. [2014]), can be used to drive thermoacoustic systems. A wide variety of combustible fuels such as natural gas, biofuel, methane, alcohol, gasoline and fuel oil can also be used to run the systems. Unlike vapor compression refrigeration systems, thermoacoustic refrigerators do not need any compressors, sliding seals, lubricants or expansion valves, since the thermoacoustic refrigeration system works well at a higher frequency and only a single frequency of sound is required for its proper operation. Hence, it is easier to control noise and vibration compared to vapor compression refrigeration systems. Therefore, thermoacoustic refrigerators are even quieter than vapor compression refrigeration systems. Both thermoacoustic refrigerators and engines use environmentally benign working fluids such as air or inert gas. The favorable characteristics of a thermoacoustic system can be summarized as follows:

- Reliable and simple in design,
- Contains no moving parts,
- Causes minimal pollution,
- Low manufacturing, maintenance and operational costs,
- Low level of noise and vibration,
- Flexible in terms of energy sources (gas, solar power, etc.).

A typical thermoacoustic engine has more power density compared to a typical automobile engine (Swift [1988]). As an example, a thermoacoustic engine which operates at a mean pressure 10 bar, frequency 1000 Hz, and mach number 0.01 produces power density of 8 W/cm³ compared to a typical automobile engine’s (100 hp, 1 ft³) power density 3W/cm³ (Swift [1988]). These features motivated numerous researchers
(Swift [1988], Garrett et al. [1994], Backhaus and Swift [1999], Reid and Swift [2000], Liu and Garrett [2006], Berson et al. [2011], Qiu et al. [2012], Swift et al. [2014], Yu et al. [2014], and Zhao and Li [2015]) to investigate the performance of thermoacoustic systems over the past few decades.

The relatively poor energy conversion efficiency, however, is a major drawback of the thermoacoustic system. The coefficient of performance (COP) of a vapor compression refrigeration system is about 50% of Carnot COP (Swift [2002]), whereas the COP of different thermoacoustic refrigerators reported by Garrett et al. [1993] and Johnson et al. [2000] is 16%, 8%-17%, 6.6% and 19% of the Carnot COP, respectively. The thermal efficiency of a large diesel engine is about 40% (Swift [2002]). A travelling waves-based thermoacoustic engine delivered 30% thermal efficiency, which corresponds to the 41% of Carnot efficiency (Backhaus et al. [1999]). However, thermoacoustic technology is the youngest of the heat engine cycles (Garrett [1993]). More investigations are required to improve its efficiency.

1.1. MOTIVATION AND THESIS CONTRIBUTIONS

The thermoacoustic effects are produced due to the hydrodynamic and thermal interaction of the oscillating compressible fluid with the stack solid wall. The sound waves or the temperature difference in the stack solid plate creates oscillation in the gas parcel. The gas parcel undergoes adiabatic compression and expansion while moving along the plate and exchanges heat between the gas parcel and the solid plate at a constant pressure. After adiabatic compression, if the plate temperature is lower than the gas parcel’s temperature, the heat transfer occurs from the gas parcel to the plate. On the other hand, after the adiabatic expansion, if the plate temperature is higher than the gas parcel’s temperature, the heat transfer occurs from the plate to gas parcel [Swift [1988]]. These heat flow from the gas parcel to the stack plate or vise versa do not occur instantinuously. There is a time lag of heat flow between the stack wall and the oscillating fluid which is known as time phasing. These time phasing affects the fluctuating velocity, temperature, and pressure of a thermoacoustic system, which ultimately influences the thermoacoustic system’s efficiency. Swift et al. [1985] observed through theoretical studies that the time phasing can be controlled by using
external magnetic fields to increase a thermoacoustic system’s performance. Wheatley et al. [1986] used an external magnetic field to control the time phase of a natural heat engine where liquid sodium was used as a working fluid. Afterwards, a few number of articles were published that considered the improvement of thermoacoustic prime mover/refrigerator using an external magnetic field. Mahmud and Fraser [2005c] developed analytical models of a single plate thermoacoustic system to investigate the influence of a magnetic field on heat flux, work flux, and the operating conditions. They further estimated the heat transfer by incorporating magnetic field perpendicular to the direction of oscillating fluid flow in a porous stack. Mahmud and Fraser [2005c] found that addition of magnetic field was capable of enabling larger stack dimensions and hence enhanced heat transfer rate from the stack element. Larger stack dimensions are useful for improving heat exchanger efficiency, thus improving overall thermoacoustic system efficiency when a magnetic field was placed perpendicular to the direction of flow.

The flow and thermal characteristics of the working fluid inside thermoacoustic systems and their interaction with the stack wall control the systems’ performance in relation to heat flux, work flux, entropy generation, heat transfer rate, energy flux density and overall efficiency. Despite the recent progress in thermoacoustic engines and refrigerators, there are many gaps in the literature that need to be addressed in order to better predict the performance and design thermoacoustic engines and refrigerators.

The thesis contributions are as follows:

1. Developed analytical solutions for fluctuating velocity, temperature, Nusselt number, entropy generation, heat flux, work flux and efficiency of a thermoacoustic system considering stack as a porous medium attached to a thick solid plate and a transverse magnetic field is applied across the stack. Analyzed the effect of magnetic field on fluctuating velocity, temperature, Nusselt number, entropy generation, heat flux, work flux, and efficiency.

2. Derived the analytical solutions for the complex Nusselt number of a multi-plate thermoacoustic system with a transverse magnetic field. Analyzed the effect of magnetic field on the complex Nusselt number.
3. Developed analytical expression for the energy, work, and heat fluxes of a multi-plate thermoacoustic refrigerator in the presence of transverse magnetic field. Described the effect of magnetic field on the energy, work, and heat fluxes.

4. Designed and developed an experimental setup of a thermoacoustic refrigerator using porous medium as a stack material to investigate its performance. Analytical, numerical, and experimental work were carried out to find the optimal stack length and position to achieve the optimal coefficient of performance relative to Carnot coefficient of performance (COPR).

1.2. Thesis Organization

The rest of the thesis is organized as follows:

Chapter 2 presents a brief literature review. The literature is reviewed according to the following aspects: porous thermoacoustic systems, magnetic thermoacoustic systems, heat transfer inside the thermoacoustic systems, energy, heat, and work fluxes of thermoacoustic refrigerator, and experimental work on thermoacoustic refrigerator.

Chapter 3 describes modeling and analysis of a thermoacoustic system considering stack as a porous medium attached to a thick solid plate and a transverse magnetic field is applied to the system. In this chapter, the effect of magnetic field on fluctuating velocity, temperature, Nusselt number, entropy generation, heat flux, work flux, and efficiency are described.

Chapter 4 introduces the modeling and analysis of a multi-plate thermoacoustic system considering a transverse magnetic field across the stack. It shows the effect of magnetic field on the complex Nusselt number and how the magnetic field effect on the heat transfer rate in a thermoacoustic system.

Chapter 5 finds analytical solutions for the energy, work, and heat fluxes of a multi-plate thermoacoustic refrigerator where transverse magnetic field is applied across the stack.
the effect of magnetic field on the energy, work, and heat fluxes are also described in this chapter.

Chapter 6 presents the experimental results. The design procedures of an optimize thermoacoustic refrigerator are described in this chapter. The effect of drive ratio and the stack length on the performance of the refrigerator are also presented in this chapter.

Chapter 7 draws conclusions and summarizes the results of this thesis. It also suggests possible future work.

1.3. Publications From This Research

Chapters 3 and 4 have been published in peer-reviewed international journals. Chapters 5 is currently under review.

Articles published in refereed journals


Chapter 2
LITERATURE REVIEW

2.1 INTRODUCTION

Different approaches are presented in the literature to understand and improve the performance of thermoacoustic systems (Rott [1980], Wheatley et al. [1986], Swift [1988], Swift [1992], Poese and Garrett [2000], Tijani [2002], Roh et al. [2007], and Mahmud and Pop [2011]) over the years. Swift [1988] found that if the stack material is poorly thermally conductive, it will achieve proper phasing between the oscillating fluid and the solid wall and eventually increase the efficiency. Till now the choice of stack material has been very limited. For example, the wire mesh stack, the plastic roll stack, a metal or ceramic honeycomb stack having square and hexagonal channel sections, and the pin stack (Swift [2002]). Each type of stack configuration has its own advantages and disadvantages. Porous media are used as a stack to increase the heat transfer area. However, the porous media increase thermoacoustic irreversibility (Tasnim et al. [2011b]). Thermoacoustic irreversibility can be minimized by increasing external magnetic force (Mahmud and Fraser [2006b]). The overall efficiency of a thermoacoustic device largely depends on the heat transfer rate between the stack and the oscillating fluid. Furthermore, thermoacoustic system’s efficiency can be improved by properly modulating heat transfer between the stack and the oscillating fluid. The dimensionless Nusselt number is used to measure the convection heat transfer coefficient—increasing Nusselt number implies an increase in the convection heat transfer rate. In this context, the study on dimensionless Nusselt number is important in order to improve the thermoacoustic system’s efficiency. Therefore, the literature is reviewed on Porous thermoacoustic systems, Magnetic thermoacoustic systems, Heat transfer inside the thermoacoustic systems, and Experimental work. Thus, the research gaps, objectives and sub-objectives are determined.

The rest of this chapter is organized as follows: Section 2.2 presents literature on Porous thermoacoustic systems, Section 2.3 describes literature on magnetic thermoacoustic systems, Section 2.4 reports the research gap and objectives on the porous magnetic thermoacoustic systems, Section 2.5 presents literature on heat transfer inside the
multiplate thermoacoustic systems, and consequently the literature on energy, heat, and work fluxes are reviewed on Section 2.6. Section 2.7 describes the research gaps objectives on multiplate magnetic thermoacoustic system. Section 2.9 presents literature on experimental work, Section 2.10 describes the research gaps in Section 2.9 and define the research objectives, and the chapter summary is given in Section 2.10.

2.2 POROUS THERMOCOUSTIC SYSTEMS

Several authors use the porous media as a stack material to improve the performance of the thermoacoustic system as it increases the heat transfer area. Roh et al. [2007] developed thermoacoustic theories for a random porous medium. Their theories included development of thermal and viscous functions, wave, and temperature distribution equation. Jensen and Raspet [2010] developed an analytical model for fibrous porous materials.

Mahmud and Fraser [2009] developed thermoacoustic theories that approximated stack as a porous medium embedded inside two thin solid walls. They used transient Darcy-Brinkman momentum equation to model the fluctuating velocity inside the porous medium. They developed analytical model for the wave equation, fluctuating velocity, temperature, complex Nusselt number, and energy flux.

Mahmud and Pop [2011] extended Mahmud and Fraser’s [2009] work by identifying modes of operations by observing the energy field variation with the dimensionless Darcy number. They observed heat pump, prime mover, and useless modes of operation at a higher temperature gradient. However, at a lower temperature gradient only heat pump and useless modes of operation were observed.

Tasnim and Fraser [2009] considered porous medium stack where Darcy momentum equation was used to model the flow field. The problem was formulated as a conjugate heat transfer problem with non-zero wall thickness. The analytical expressions for the oscillating temperature, complex Nusselt number, and energy flux density were obtained. The non-conjugate thermoacoustic modeling of a Darcy porous medium is available in
Tasnim et al. [2013]. Tasnim et al. [2011b] extended Tasnim and Fraser’s [2009] work by analyzing irreversibility in terms of entropy generation associated with viscous and heat transfer effects inside the porous stack region.

Tasnim et al. [2011c] extended Mahmud and Fraser’s [2009] work by incorporating Brinkman-Forchheimer momentum equation in their porous thermoacoustic system modeling. They linearized the Forchheimer inertia term using Taylor series expansion and developed analytical solution for fluctuating velocity, temperature, and energy flux inside the porous medium in terms of several dimensionless parameters (e.g., Darcy number, Swift number, and temperature gradient ratio). Their analytical solution was compared with the experimental results and a good agreement was obtained.

Tasnim et al. [2012a] investigated the thermoacoustic performance of a heat pump system in terms of heat flux, work flux, coefficient of performance, and temperature difference across the heat exchangers. They used two regular and one random porous media as the stack for their heat pump system. The authors concluded that for the random porous medium stack system the temperature difference across the stack increased with increasing porosity (form 20 to 80 PPI).

2.3 MAGNETIC THERMOACOUSTIC SYSTEMS

Time phasing is an important aspect in operating the thermoacoustic systems efficiently. Time phasing can be defined as a time lag of heat flow between the stack wall and the oscillating fluid (Haddad et al. [2014]). Time phasing affects the fluctuating velocity, temperature, and pressure of a thermoacoustic system which eventually influences the thermoacoustic system’s performance (Swift [1988]). Such time phasing can be compared to the piston-valve time phasing relationship of an automotive engine. Swift et al. [1985] used electrically conductive liquid sodium as a working fluid for the theoretical analysis of a thermoacoustic engine under the influence of magnetic force. The external magnetic force is used in Swift et al. [1985] analysis to control such time lag. Subsequently, Wheatley et al. [1986] constructed a thermoacoustic engine by using liquid sodium as working fluid. They also demonstrated that an external magnetic field can be
used to control such time lag. The external magnetic force can also be used to achieve the thermoacoustic oscillation stability (Artamonov et al. [1979]). Ramos et al. [2002] and Ovando et al. [2005] also studied the influence of magnetic force on the stability of thermoacoustic oscillation of water with sodium chloride. Ovando et al. [2005] identified the limiting range of magnetic force to obtain a stable oscillation.

Wu et al. [2001] analyzed the heat transfer rate of a ferromagnetic fluid in the presence of acoustic and magnetic field using experimental and analytical approaches. They derived an analytical model for non-dimensional effective thermal diffusivity as a function of acoustic wave, frequency and magnetic induction. They found that the effective thermal diffusivity increased with the increasing acoustic wave frequency or the increasing magnetic induction. A fair agreement with a tolerance of less than 10% was found between their analytical and experimental results.

Mahmud and Fraser [2005c] considered single plate stack where magnetic force was applied to the transverse direction of the oscillating fluid. They identified three operating modes: heat pump, prime mover, and useless mode by observing the variation in the sign of the total heat flux and work flux with dimensionless Hartmann number (the ratio of electromagnetic force to the viscous force). Two critical Hartmann numbers were identified for which the time average in total heat flux and work flux were zero. All three operating modes were observed at a relatively higher temperature gradient. However, the prime mover mode was absent at a relatively lower temperature gradient.

Mahmud and Fraser [2006a] proposed analytical models for wave equation, fluctuating velocity, temperature, and complex Nusselt number as a function of Hartmann number. They observed that the boundary layer thickness became zero in the limit of a very large Hartmann number. At a lower temperature gradient, the variation in the Hartmann number has an insignificant effect on the dimensionless temperature profile. However, a significant effect was observed at a high temperature gradient. The Nusselt number was expressed as a function of mean temperature gradient. The authors identified that the magnitude of the Nusselt number was maximum when the mean temperature gradient
was equal to the critical temperature gradient. The variation in the magnitude of the Nusselt number decreased with mean temperature gradient’s variation when the Hartmann number increased. No variations were observed in the magnitude of Nusselt number when Hartmann number was greater than one.

Mahmud and Fraser [2006b] extended Mahmud and Fraser’s [2006a] work by analyzing irreversibility in terms of entropy generation rate associated with transverse magnetic field. They divided global entropy generation into three parts: global entropy generation due to magnetic dissipation, fluid friction, and heat transfer. The authors observed that the global entropy generation for each of the parts for two limiting cases of Hartmann number (zero and infinite). Entropy generation due to magnetic dissipation approached zero for both the limiting cases. However, entropy generation was at a maximum when Hartmann number was close to one. Entropy generation for fluid friction was zero for a very large Hartmann number. A non-zero entropy generation for heat transfer was observed for both the Hartmann number limits. They also observed that entropy generation decreased with increasing frequency when Hartmann number was less than one.

2.4 Research Gaps and Objectives on Porous Magnetic Thermoacoustic Systems

The limited choice on stack material leads to the investigation of the suitability of a porous medium. Although the porous medium increases heat transfer surface area, it also increases thermoacoustic irreversibility (Tasnim et al. [2011b]). However, thermoacoustic irreversibility can be minimized by increasing external magnetic force (Mahmud and Fraser [2006b]). Therefore, the following objectives are included in this research to improve the thermoacoustic system’s performance.

1. Modeling and analysis of a thermoacoustic system considering stack as a porous medium attached to a thick solid plate and a transverse magnetic field is applied to the system.
   - Finding analytical solutions for fluctuating velocity, temperature, Nusselt number, entropy generation, heat flux, work flux and efficiency.
Analyzing the effect of magnetic field on fluctuating velocity, temperature, Nusselt number, entropy generation, heat flux, work flux, and efficiency.

Validating the analytical solutions with the results available in the literature

2.5 HEAT TRANSFER INSIDE THE THERMOCOUSTIC SYSTEMS

The dimensionless Nusselt number is used to calculate the convective heat transfer rate. The study on Nusselt number subjected to a pulsating incompressible flow has received an extensive attention over the years (Yu et al. [2004], Clamen and Minton [1977], Zohir et al. [2006], Ranjbar et al. [2010], and Mehta and Khandekar [2014]). However, only few articles are devoted to calculate the convection heat transfer coefficient when the compressible fluid undergoes compression and expansion cycles periodically due to pressure gradient. Such periodic oscillations are observed in the fluid inside the stack of a thermoacoustic system during its operation.

Besnoin and Knio [2001] presented a numerical analysis for a thermoacoustic refrigeration system where compressible fluid in the thin-plate and low Mach-number limits are considered. They observed that Nusselt number was a function of the gap between two parallel plates in the stack and the heat exchanger length: the Nusselt number decreases with the increase in the heat-exchanger length at a well-defined gap. Numerical studies for determining the heat transfer coefficient of thermoacoustic refrigerators were conducted in similar line of researches (Worlikar et al. [1998] and Worlikar and Knio [1999]). Wetzel and Herman [1999] performed experiments and reported that local heat transfer coefficient does not affect significantly with varying drive ratios (fluctuating pressure/mean pressure).

Mahmud and Fraser [2005b] developed an analytical solution for the root-mean-squared of the Nusselt number for a multi-plate thermoacoustic system. They observed that the Nusselt number was independent of Swift’s number’s variation when the ratio of mean to critical temperature gradient was zero or infinite. The Nusselt number varies with the Swift number for any finite non-zero values of this ratio.
Kornhauser and Smith [1994] presented an experimental study on complex Nusselt number for oscillating pressure and flow. They observed that the Newton’s law of convection is not enough to describe the heat transfer phenomenon when the fluid compression or expansion is out of phase with the temperature difference between the bulk fluid and wall temperature. They expressed the heat transfer phenomenon in two parts: the first part is proportional to the temperature difference between the bulk fluid and wall temperature and the second part is proportional to the rate of change of wall temperature. The Nusselt number was maximum when these two parts were in the same phase. The complex Nusselt number can provide such magnitude and phase information (Kornhauser and Smith [1994]). Thus, the heat transfer between the oscillating fluid and solid wall can be calculated more accurately using complex Nusselt number for a thermoacoustic system (Kornhauser and Smith [1994]).

In an earlier work, Gedeon [1986] developed an exact solution for the complex Nusselt number where a laminar incompressible oscillating flow was considered between two parallel plates subjected to a longitudinal temperature gradient. He observed that the real part of the Nusselt number was much larger than the imaginary part at a relatively lower value of plate spacing. However, the value of the Nusselt number sharply increased at a relatively higher value of the plate spacing when the phase difference between the real and imaginary parts of the Nusselt number was minimum.

Liu and Garrett [2006] calculated the complex Nusselt number for oscillatory flows and expressed it in terms of thermoviscous functions (i.e., Rott’s functions). The thermoviscous functions are complex hyperbolic tangent functions which depend on the ratio of hydraulic radius to the thermal and viscous penetration depths. They considered circular pores, square pores, and parallel plates to calculate the complex Nusselt number subjected to an oscillating pressure. The authors observed that the real part of the complex Nusselt number is much larger than the imaginary part when the pore size is much smaller than the thermal penetration depth. They also compared their result with Gedeon’s [1986] and found an excellent agreement. Guoqiang and Ping [2000] studied the Nusselt number for thermoacoustic transport phenomena in a tube. They observed
that the Nusselt number is a complex number, and there was a phase shift between the heat flux and the temperature difference between the wall and oscillating fluid. The maximum Nusselt was observed when heat flux and the temperature difference between the wall were in phase.

Mahmud and Fraser [2009] investigated complex Nusselt numbers at different Swift and Darcy numbers for a multi-plate thermoacoustic system filled with porous medium. They observed that the real part of the complex Nusselt number was constant for any Darcy number when Swift number was less than unity. They also found a very good agreement with Liu and Garrett’s [2006] analytical model in terms of the complex Nusselt number when the Darcy number approaches infinity.

2.6 ENERGY, HEAT, AND WORK FLUXES

Energy flux density is an important parameter to measure the thermoacoustic system’s performance. The total amount of energy passing through a unit area perpendicular to the direction of the fluid velocity in unit time is defined as the energy flux density (Landau and Lifshitz [1982]). The thermal efficiency of a thermoacoustic device is the ratio of work and heat fluxes, and the COP is the ratio of heat and work fluxes. The heat flux is defined by the hydrodynamic transport of entropy that carried by the oscillatory velocity of the fluid (Swift [1988]). On the other hand, the work flux is the product of oscillating pressure and fluctuating volume of the fluid, which is the plate area times thermal penetration depth which is a measure of lateral thermal diffusion in a characteristics time interval (Swift [1988]). The same fluid volume is consider to calculate the heat and work flux.

Analytical models of heat, work, and energy fluxes of a thermoacoustic device have been developed by Swift [1988] to understand and predict the thermoacoustic systems’ performance. Cao et al. [1996] used SOLA–ICE method to solve full 2D Navier–Stokes equations to investigate time-averaged energy flux density and heat flux of an isothermal multi-plate thermoacoustic system. Authors identified that the magnitude of the time-
averaged energy flux density is maximum near the ends of the stack while it is nearly zero elsewhere on the stack.

Mozurkewich [1998] developed an analytical model to investigate the heat flux density considering an isothermal and isolated parallel plate stake where the stake pore is of constant cross-sectional area. He observed that the heat-flux density is maximum to the vicinity of the pore ends. The energy flux approaches to zero near the end of the thermally isolated pore. The author compared his simulation results with Cao et al. [1996] and found good agreement.

Worlikar et al. [1998] used the finite-difference technique to measure the mean energy and heat flux of a multi-plate thermoacoustic system. They observed that the intensity of mean energy fluxes is maximum at the corner of the plates, which agree with Cao et al.’s [1996] and Mozurkewich’s [1998] numerical and analytical results respectively. The intensity of energy flux is high at a relatively higher drive ratio. Nearly one-dimensional mean energy flux was observed that is directed from cold end to hot end of the stack at a relatively lower drive ratio. At a higher drive ratio, higher percentages of total heat flux leaves by the stack side. The mean heat exchange is negligible in the middle portion of the stack.

Ishikawa and Mee [2002] used the commercial code PHOENICS (a 2D full Navier–Stokes solver) to solve the governing continuity, momentum, and energy equations of a multi-plate thermoacoustic system for a 2D compressible ideal gas at low Mach number. They observed two types of time-averaged energy flux at the stack surface: positive (heat fluxes leaving the domain through stack) and negative (heat fluxes entering the domain through stack). Similar results were obtained by Worlikar et al. [1998] and Mozurkewich [1998]. The energy dissipation increases quadratically when the plate spacing approaches to the thermal penetration depth. The energy flux decreases when the gap between two adjacent plates is less than the thermal penetration depth.

David et al. [2003] performed experiment to analyze the performance (heat and work fluxes) of a multi-plate thermoacoustic refrigerator. They observed that thermal and
viscous losses increase with increasing input electric power resulting in a reduction of the thermal efficiency. The experimental results were compared with the DELTAE [references] thermoacoustic modeling predictions.

Mahmud and Fraser [2005a] developed an analytical model to investigate heat and work fluxes of a single plate thermoacoustic system. They considered non-zero plate thickness and the entire problem is solved as a conjugate heat transfer problem. The authors observed that the heat and work fluxes depend on the thermophysical properties of the fluid and stack, stack geometry, and the difference between mean and critical temperature gradient.

Mahmud and Fraser [2005b] developed an analytical model to investigate heat, work, and energy fluxes of a parallel plate thermoacoustic system. The governing continuity, momentum, and energy equations are simplified using a first-order perturbation expansion. They observed that the heat flux increases with increasing the gap between the stack plates in the refrigeration mode. The variation in the heat flux is insignificant at a lower Swift number with the different operational modes (i.e., heat pump, prime mover).

Piccolo and Pistone [2006] performed numerical investigation using finite difference technique to solve the linearized thermoacoustic equations to analyze the temperature and heat flux density distributions inside a thermally isolated parallel plate stake. They observed that the net heat exchange between the working fluid and the plate wall takes place only within a limited distance from the stack edges. The authors also identified the optimal length of heat exchangers for different plate spacing to maximize the heat transfer between the working fluid and the plate wall. They ignored the non-linear effects such as the turbulent oscillatory flow, presence of harmonics greater than the fundamental, or vortex generation.

Piccolo and Pistone [2007] used a finite difference technique to solve an energy balance calculus-scheme of a multi-plate thermally isolated thermoacoustic stack. They observed that time-averaged heat flux density is maximum near the edges of the stack plate, which was also observed by Cao et al. [1996], Ishikawa and Mee [2002]. The maximum time-
averaged enthalpy flux density is observed when the plate spacing approaches to the thermal penetration depth.

Mahmud and Fraser [2009] developed an analytical model to investigate the energy flux density at different Swift and Darcy numbers for a multi-plate thermoacoustic system filled with a porous medium. They observed that the Darcy number has a greater influence on the energy flux density at a relatively higher swift number. Tasnim et al. [2009] also obtained the similar results.

Kang et al. [2009] developed an analytical model to investigate the heat flux, cooling power, temperature gradient and coefficient of performance of thermoacoustic refrigerator with different combination of hydraulic radiiuses and acoustic fields for both standing and travelling wave thermoacoustic systems. They observed that the optimal hydraulic radius for the traveling wave device is much smaller than that for the standing wave device. For the case of standing wave device, heat is transferred towards the pressure antinodes.

Zoontjens et al. [2009] used the commercial CFD software FLUENT to study of flow and energy fields on a parallel plate thermoacoustic system considering non-zero plate thickness. They found that an increase in plate thickness increases the total heat transfer rate and heat flux but entropy generation increases. Shimizu and Sugimoto [2010] analyzed numerically to calculate mean acoustic energy flux and mean heat flux using quantitative analysis of a thermoacoustic field by Taconis oscillations [Taconis et al. [1949]].

Wang et al. [2011] developed an analytical model to calculate time-averaged enthalpy flux, acoustic power, entropy flux, and energy flux using an additional heat source named as mid-heater. They observed that the use of mid-heater results in higher acoustic power. The authors compared the analytical solution results with the DELTAEC generated simulation results.
Mahmud and Pop [2011] developed an analytical model to study the effect of the Darcy number on the energy flow of a thermoacoustic system filled with porous-medium. The dimensionless parameter Darcy number is the ratio of permeability and the square of viscous penetration depth. Darcy-Brinkman momentum and energy equations are simplified. The simplified equations were linearized by using a first order perturbation analysis. They found that heat flux increases with increasing Darcy number. Thermoacoustic modes of operation can also be adjusted by increasing or decreasing the Darcy number. Mahmud et al. [2011] extended Mahmud and Pop’s work [2011] by considering the stack as a porous medium lying over a thick solid plate.

Tasnim et al. [2011c] considered the stack as a porous medium to model a thermoacoustic system. The inertia term of the governing Brinkman-Forchheimer momentum equation was linearized using Taylor series expansion. The analytical solutions for fluctuating velocity, temperature, heat, work, and energy flux inside the porous medium as functions were expressed in terms of Darcy number, Swift number, and temperature gradient ratio. The authors observed a pattern of increasing heat flux at a relatively lower swift number with increasing Darcy number. They performed experiments to validate the analytical results.

Piccolo [2011] used the finite-difference technique to solve the governing equation of a multi-plate thermoacoustic refrigerator to analyze the effect of geometrical parameters on heat exchanger performance (heat and work fluxes). He observed that heat flux/cooling load increases with increasing heat exchanger fin length up to a certain length for a selected drive ratio. The shorter fin length reduces the viscous loss.

Tasnim et al. [2013] developed analytical models to analyze a thermoacoustic system filled with porous medium and observed that the heat flux is the maximum at a distance equivalent to thermal penetration depth from the plate surface.

Li and Zhao [2013] used finite volume method to solve the unsteady Navier–Stokes equations on a T-shaped standing-wave thermoacoustic system. They observed that the heat flux increases with increasing heater temperature as long as the heater temperature is
lower than a specific temperature. The transient behavior of heat flux is nonlinear, and
the thermoacoustic system’s efficiency increases with increasing the inlet flow velocity.
Li and Zhao [2013] performed experiments to validate the numerical results.

2.7 RESEARCH GAPS AND OBJECTIVES ON MULTIPLATE MAGNETIC
THERMOACOUSTIC SYSTEMS

The complex Nusselt number is a function of wall temperature gradient (Kornhauser and
Smith [1994]) which depends on the thermal boundary layer thickness of the fluid. The
thermal boundary layer thickness can be controlled using an external magnetic field
(Mahmud and Fraser [2004], Islam et al. [2016]). Therefore, it is important to conduct a
thorough investigation in order to determine the influence of magnetic field on the
complex Nusselt number.

Although a considerable number of articles have been published in which the complex
Nusselt number for thermoacoustic systems have been calculated (Kornhauser and Smith
[1994], Gedeon [1986], Liu and Garrett [2006], Guoqiang and Ping [2000], and Mahmud
and Fraser [2009]) a very limited study existed in the literature that calculates the
complex Nusselt number in the presence of magnetic field. The influence of magnetic
field on the complex Nusselt number was studied in (Mahmud and Fraser [2006a]);
however, only a single plate thermoacoustic system was considered in this work.
Therefore, the following objectives are included in this research to increase the energy
conversion efficiency of the multi-plate thermoacoustic system.

1. Modelling and analysis of a multi-plate thermoacoustic system and a transverse
magnetic field is applied to the system.

2. Determining the effect of magnetic field on the complex Nusselt number for a
multi-plate thermoacoustic system.
   - Finding analytical solutions for complex Nusselt number using both bulk mean
     and average temperature as a reference temperature.
   - Analyzing the effect of magnetic field on the complex Nusselt number.
   - Validating the analytical solutions with the results available in the literature.
3. Modelling and analysis of a multi-plate thermoacoustic refrigerator where transverse magnetic field is applied across the stack.
   - Finding analytical expression for the energy, work, and heat fluxes.
   - Analyzing the effect of magnetic field on the energy, work, and heat fluxes.
   - Validating the analytical solutions with the results available in the literature

2.8 EXPERIMENTAL WORK

The experimental work is important to validate the analytical or numerical models. Several experimental works have been reported in the literature to improve understanding of the thermoacoustic phenomena. Adeff et al. [1998] reported experimentally that using reticulated vitreous carbon (RVC) as stack material instead of a traditional plastic roll stack increased the performance of the thermoacoustic prime mover and refrigerator. RVC is poorly conductive, relatively inexpensive, and easy to machine material. It is a highly porous material and has a higher specific heat. However, the major disadvantage of RVC is its higher brittleness. Adeff et al. [1998] did not develop a theoretical model for RVC as stack material. However, they verified their experimental results with Swift’s [1988] general theories of thermoacoustic and found good agreements.

Tijani [2001] developed a loudspeaker-driven thermoacoustic refrigerator and reported their experimental investigations. He used a Mylar sheet with a thermal conductivity of 0.16 W/m K and a thickness of 0.06 mm as stack material. Helium at 10 bars was used as the working fluid, and the drive ratio was 1.4% for all measurements. He also used parallel and spiral plate stacks. The performance of the refrigerator was measured by varying the space (the gap in two successive plates) between 0.15 and 0.7 mm. and reported a temperature of -67.3 °C with a COPR of 11% for the parallel-plate stack, with a spacing of 0.38 mm. For the spiral plate stack, the temperature reached -58 °C with a COPR of 9%. The author concluded that the performance of the thermoacoustic refrigerator increases using parallel plate stack than the spiral ones.

Tijani et al. [2002a] described the design procedure of a thermoacoustic refrigerator and calculated the optimal normalized stack center position from the acoustic driver’s end and normalized stack length by using the approximate short-stack and boundary-layer
expressions for acoustic power and heat flow. They also calculated the performance of the thermoacoustic refrigerator as a function of the normalized stack length and normalized stack center position for different heat loads at the cold heat exchanger using Design Environment for Low-Amplitude Thermoacoustic Engines (DeltaE) thermoacoustic simulation software [Ward and Swift [2001]], achieving a low temperature of -65 °C for the optimal stack length and position.

Akhavanbazaz et al. [2007] find the optimim stack length and position using approximate short-stack and boundary-layer expressions [Tijani et al. [2002a]]. They performed experiments using the optimized stack to investigate the impact of the gas blockage on the performance of a thermoacoustic refrigerator. In doing this, they analyzed three different cases: no heat exchanger, a heat exchanger with a large thermal contact area, and a heat exchanger with a small thermal contact area. Their observations indicated that the gas blockage fraction is proportional to the temperature difference across the two ends of the stack. The heat transfer between the heat exchanger fluid and the stack increases with increasing thermal contact area but reduces cooling power due to increased gas blockage.

Hariharan et al. [2013a] used photographic film and Mylar sheet as stack material to analyze the performance of a thermoacoustic refrigerator driven by a standing wave twin thermoacoustic prime mover. The spacing between the two successive layers was kept at 0.4 mm and 0.8 mm, and helium at 1 MPa pressure was used as a working fluid. The heat input of the thermoacoustic prime mover was 360W. They found that the Mylar sheet stack performed better than the photographic film stack and obtained a maximum temperature difference of 16 °C across the two ends of the mylar stack when the gap was 0.4 mm, which supports Tijani’s [2001] results.

Putra and Agustina [2013] performed experiments to investigate the influence of stack plate thickness (0.15, 0.5, and 1mm) and voltage input (4-9) on the performance of a loudspeaker-driven thermoacoustic refrigerator. They observed that a decrease in plate thickness leads to significant increase in cooling rate. Although the largest temperature
difference (14.8 °C) was observed for a plate thickness of 0.15 mm at 9 Volt but for a plate thickness of 0.5 mm, cooling load is consistent.

Guédra et al. [2015] used different stack materials such as pile of stainless steel wire meshes, ceramic catalyst, carbon and metallic foams with common geometrical properties (porosity and average pore's radius) to investigate and characterize the thermoacoustic system. They observed that the carbon foam achieved the highest temperature gradients.

Zolpakanar et al. [2016] used a multi-objective genetic algorithm (MOGA) for the numerical investigation, as well as different stack materials (celcor ceramic and mylar), geometries (square and spiral), lengths (3.5 cm–4.5 cm), and positions from the driver’s end (3 cm to 5 cm). They found that the Mylar stack demonstrated maximum performance in terms of generating temperature difference between the hot and cold ends of the stack, with the optimized stack center position of 4 cm from the pressure antinode for an optimized stack length of 4 cm.

The working fluid plays a vital role in the performance of a thermoacoustic refrigerator. Belcher et al. [1999] performed experiments to find the best working fluid for a thermoacoustic refrigerator and observed that the best fluid has high ratios of specific heats and low Prandtl numbers. These properties can be achieved by mixing heavy polyatomic gas and light noble gas, but the mixing ratio needs to be optimized based on the particular thermoacoustic application. Using a helium-argon mixture as a working fluid, Reid and Swift [2000] performed an experimental study on a loudspeaker-driven thermoacoustic refrigerator and achieved temperatures from 35°C to 27 °C.

Campo et al. [2011] estimated experimentally the minimum Prandtl number for the binary gas mixtures formed with a light gas such as helium and a heavier gasses such as oxygen, carbon dioxide, nitrogen, xenon, methane, tetrafluoromethane or carbon tetrafluoride and sulfur hexafluoride for a thermoacoustic refrigerator application. They observed that the helium and xenon mixtures had the minimum Prandtl number out of the seven binary gas mixtures.
Tasnim et al. [2012b] performed experimental and numerical investigations to study the effects of the Prandtl numbers of the working fluid on the performance of a thermoacoustic refrigerator. They varied the Prandtl numbers of the working fluid between 0.7 and 0.28 for two different spacing of the stack at an atmospheric pressure and DR of 1.7%. They observed that when the Prandtl number was reduced from 0.7 to 0.28, the COP increased by 78% for the larger stack spacing. However, the COP decreased when the Prandtl number was reduced for the other stack. They concluded that reducing the Prandtl number does not ensure the improved performance of a thermoacoustic refrigerator for all operating conditions.

Nsofor et al. [2007] performed experiments to study the heat transfer characteristics at the heat exchanger of thermoacoustic refrigeration systems. They developed correlations of the Nusselt number, Prandtl number and Reynolds number for the heat transfer at the heat exchangers from the experimental results. They observed significant difference between the results that are obtained by using their correlations and laminar straight-flow heat transfer correlations developed by Shah et al. [1987].

Tijani et al. [2002c] developed a gas-spring system to shift the mechanical resonance frequency of the driver for a loudspeaker-driven thermoacoustic refrigerator. By performing experiments they observed that electroacoustic efficiency increases by 35% when the acoustic resonance frequency and the mechanical resonance frequency of the driver are equal.

Wantha and Assawamartbunlue [2013] performed an experimental investigation to study the effects of driver housing and resonance tube length on temperature differences generated across stack ends. They observed that the size of the back volume of the acoustic driver and the length of the resonance tube affected both the temperature differences across the stack and optimal frequency. The size of the back volume can increase/decrease the frequency of the driver and the acoustic resonance frequency. They
also concluded that the resonance frequency of the system depends on the resonance tube length.

Jebali et al. [2004] investigated the performance of a thermoacoustic refrigerator using both experimental and numerical analyses for different cooling loads and frequencies. In the numerical model, the one-dimensional cross-section-averaged equations [Swift, 2002] were discretized using the network analogy. They maintained the hot end at an ambient temperature, while the cold end temperature was varied to achieve three different temperature differences along the stack. They also varied the frequency for each of the temperature differences while calculating the cooling load and observed that the maximum cooling load occurs at the resonance frequency when the temperature differences across the stack are maximized.

Paek et al. [2007] performed numerical analyses by developing a simulation model using DELTAE thermoacoustic simulation software. They applied the developed program to two standing-wave thermoacoustic coolers to find the best possible COPRs for various temperature differences across the stack-end temperatures for mean pressures of 1, 2, and 3 MPa. They observed that the standing wave thermoacoustic cooler is less efficient for low or high temperature applications (with operating temperatures from -20 °C to 35 °C), such as cryogenic cooling or air conditioning. They concluded that the best use of thermoacoustic cooling system can be a refrigerator with operating temperatures from 10 °C to 35 °C.

Nsofor and Ali [2009] performed experiments to evaluate the influence of the mean pressure of the working fluid and operating frequencies on the performance of a thermoacoustic refrigerator. Four different mean pressures: 3, 4, 5, and 6 Bars and frequencies from 250 to 500Hz were considered and it was observed that the cooling loads increase with increasing temperature differences across the stack end. They also observed that the maximum cooling load can be achieved by operating the thermoacoustic refrigerator at an optimam frequency and pressure.
Tijani et al. [2002b] designed and developed an experimental setup of a thermoacoustic refrigerator by evaluating its performance in terms of COP, COPR, and temperature differences as a function of cooling load for four different drive ratios (0.7%, 1.4%, and 2.1%). They observed maximum COPR at a drive ratio 1.4%. The COPR does not increase with further increasing drive ratio. But the maximum cooling load cross ponding to maximum temperature increases with increasing drive ratio. The lowest temperature they achieved is -65 °C in their experiment at a drive ratio 2.1%.

Tijani and Spoelstra [2008] performed experiments to study a thermoacoustic-Stirling cooler. Three different drive ratios of 2.2%, 3.3%, and 4.3% were used to measure the performance of the cooler. They obtained the maximum COPR of 25% at a cooling load of 25W and -11 °C for a drive ratio of 4.3%. The minimum temperature obtained was -54 °C without any cooling load. They also observed that using a quarter-wavelength resonator instead of a half-wavelength resonator resulted in increased COPR of about 35%.

Bao et al. [2006] experimentally investigated the resonant tube shape of a thermoacoustic engine and observed that a tapered tube performed better than a cylindrical tube. Tijani [2001] obtained similar results for a thermoacoustic refrigerator.

Tasnim et al. [2011a] measured the temperature field at different locations on a stack plate and in the surrounding working fluid of a thermoacoustic heat pump. They observed that the temperature difference between the hot end of the stack and the corresponding gas is relatively higher than the temperature difference between the cold end of the stack and the corresponding gas. Thus, a revealed a high convective heat transfer occours at the hot end of the stack compared to the cold end.

Tasnim et al. [2011c] performed experimental and analytical studies on a thermoacoustic system filled with a porous medium. They modelled the thermoacoustic system using the Brinkman-Forchheimer momentum equation and linearized the Forchheimer inertia term using Taylor expansion series expansion. They also obtained analytical solutions for the
fluctuating temperature, velocity, heat, work, and energy flux inside the porous medium as a function of the Darcy number, Swift number, and temperature gradient ratio and observed that heat flux increases as the Darcy number increases. Overall, they obtained good agreement between the analytical and experimental results.

Li and Zhao [2013] studied numerical and experimental analyses to calculate the heat flux, energy flux and efficiency of a T-shaped standing-wave thermoacoustic system. They observed that the heat flux increases with increasing heater temperature of below a certain value. By increasing the inlet flow velocity, the energy conversion efficiency can be increased. Zink et al. [2010] studied the benefits of using a thermoacoustic refrigerator compared to conventional cooling in vehicles in terms of Total Equivalent Warming Impact (TEWI).

To increase the efficiency of a thermoacoustic refrigerator, Slaton and Zeegers [2006] coupled a thermoelectric device to a thermoacoustic heat pump to produce electrical power. Dhuley and Atrey [2013] studied the transient phenomena in a low-cooling thermoacoustic refrigerator, and Hasegawa et al. [2013] conducted numerical investigations to measure the performance of a thermoacoustic refrigerator driven by a multistage thermoacoustic engine. They also developed numerical models based on Rott’s [1969, 1975] first-order differential equation. They used helium as a working fluid at 1 MPa. They obtained the Carnot efficiency over 21% for the combined apparatus. Jaworski and Mao [2013] performed an experimental study on a thermoacoustic refrigerator coupled with a two-stage thermoacoustic engine and reported a maximum cooling load of 133W and a COP of 2.06.

Muzet et al. [2014] performed both numerical and experimental investigations to measure the performance of a thermoacoustic refrigerator coupled to a solar power-driven thermoacoustic engine. As a result, a cooling power of 400 Watts was obtained at a temperature equal to or less than -20 °C, with an average coefficient of performance of 21%.
2.9  **Research Gaps and Objectives on the Experimental Works**

Although for improving the performance of thermoacoustic refrigeration systems several prototypes have been developed over the years, the COP of these systems is still relatively low [Garrett *et al.* [1993], Tijani *et al.* [2002a], Wetzel and Herman [1997], Poese *et al.* [2004], Tasnim *et al.* [2012]]. Thus, it is important that more investigations need to be carried to increase the COP of this system. Tijani *et al.* [2002a] used thermoacoustic simulation software DeltaE to predict the optimal stack length and position. The stack length is selected from the DeltaE result and the position was selected based on their cooling requirements. But they did not determine the stack position based on the temperature difference across the end of the stack. Therefore, the following objectives are included in this research to improve the thermoacoustic system’s performance.

1. Designing, developing, and measuring the performance of a thermoacoustic refrigerator.
   - Finding the optimal stack length and position using analytical calculation, numerical simulation (DeltaE), and experimental results.
   - Analyzing the effect of drive ratio and the stack length on the performance of the thermoacoustic refrigerator.

2.10  **Summary**

The literature is reviewed on the following aspects: porous thermoacoustic systems; magnetic thermoacoustic systems; heat transfer inside the thermoacoustic systems; energy, heat, and work fluxes of the thermoacoustic refrigerators; and experimental work. The research gaps and objective show how this current work will contribute to the existing literature. In the next chapter, the modeling and analysis of a thermoacoustic system considering stack as a porous medium attached to a thick solid plate in the presence of transverse magnetic field will be described.
Chapter 3

INFLUENCE OF MAGNETIC FIELD ON THE PERIODICALLY OSCILLATING FLUID INSIDE A POROUS MEDIUM ATTACHED TO A THICK SOLID PLATE

3.1. INTRODUCTION

This chapter presents a rigorous mathematical investigation of the influence of a magnetic field on the periodically oscillating fluid inside a porous medium. A porous medium coupled with a thick solid plate and the magnetic field is considered to be perpendicular to the direction of fluid oscillations. The hydrodynamic and thermal interactions of the oscillating fluid with the porous medium and the thick solid plate are modeled analytically as a thermoacoustic system under the influence of a transverse magnetic field. The velocity and temperature expressions of the oscillating fluid are derived using the perturbation technique after simplifying the governing Darcy momentum and energy equations. From the flow and thermal fields’ results, Nusselt number, heat flux, and work flux are calculated and presented graphically. Consequently, the entropy generation rate for the overall system is investigated to assess the irreversibility associated with the proposed system enabling one to improve the efficiency of the system. Finally, the efficiency of the proposed thermoacoustic system is determined using the expressions of heat and work fluxes. It is observed that the thermoacoustic irreversibility can be minimized by increasing the applied magnetic force resulting in increased efficiency of the proposed system.

3.2. PROBLEM FORMULATION

The schematic diagrams of the problem under consideration with a CAD drawing are presented in Figs. 1(a) and (b). The length of the porous stack is $\Delta x$ which is aligned with the $x$-axis. $x$-axis also represents the fluctuating flow direction of the fluid. The width of the plate ($\Pi /2$) is aligned with the $z$-axis. The transverse co-ordinate directions in the porous medium and solid walls are represented by $y$-axis and $\bar{y}$-axis, respectively. Note that the thermal penetration depths in the porous medium and solid
walls are aligned with \( y \)-axis and \( \bar{y} \)-axis, respectively. Thermal penetration depths are important parameters for any thermoacoustic analysis and details on the penetration depths will be introduced later in this paper. The applied magnetic force is assumed parallel to the positive \( y \)-axis. The hydrodynamic and thermal interactions of periodically oscillating fluid with the solid matrix of porous medium, solid wall, and magnetic force produce several thermoacoustic effects (e.g., heat flux, work flux, complex Nusselt number). The flow and thermal fields’ characteristics will be identified first to evaluate important thermoacoustic effects as described above. Sections 3.3 to 3.6 below summarize the detailed modeling effort for this chapter.

Figure 3.1: (a) Schematic diagram of the problem, (b) Schematic diagram of the stack.
3.3. FLOW FIELD ANALYSIS

For simplicity of analysis, the Darcy momentum equation (Nield and Bejan [2006]) is utilized to model the flow field inside the porous stack. The Darcy momentum equation established a relationship between the flow velocity and the pressure gradient (Bejan [1984]). The Darcy momentum equation with electromagnetic volume force can be written as (Mahmud and Fraser [2006a]):

\[
\frac{\mu}{K} \mathbf{V} = -\nabla p + \mathbf{F}_{em},
\]

where \( \mathbf{V}, \ K, \ \mu, \ \nabla p, \) and \( \mathbf{F}_{em} \) represent the velocity vector, permeability of the porous medium, viscosity of the fluid, pressure gradient, and electromagnetic volume force, respectively. The electromagnetic volume force \( \mathbf{F}_{em} \) can be estimated from the following equation (Mahmud and Fraser [2006a]):

\[
\mathbf{F}_{em} = \mathbf{J} \times \mathbf{B} \text{ with } \mathbf{J} = \sigma_k (\mathbf{E} + \mathbf{V} \times \mathbf{B}),
\]

where \( \mathbf{J}, \ \mathbf{B}, \ \sigma_k, \) and \( \mathbf{E} \) represent the volume current density, magnetic induction, electrical conductivity of the fluid, and electrical field intensity, respectively. The electromagnetic volume force \( \mathbf{F}_{em} \) has two parts: the first part, \( \sigma_k (\mathbf{E} \times \mathbf{B}) \), accelerates or decelerates the flow depending on the relative direction of \( \mathbf{E} \) and \( \mathbf{B} \). The second part, \( \sigma_k (\mathbf{V} \times \mathbf{B}) \times \mathbf{B} \), always decelerates the fluid flow. As an unidirectional magnetic field (i.e., \( \mathbf{B} \approx B_y \hat{\mathbf{j}} \)) is considered which acts parallel to the \( y \)-axis (see Fig. 1b) the \( x \)-momentum equation for the current problem can be simplified to

\[
u = -\frac{K}{\mu} \left( \frac{\partial p}{\partial x} + \sigma_k u B_y^2 \right).
\]

Variables \( u \) and \( p \) in Eq. (3.3) are both time and space dependent. According to Rott’s [1980] linearized thermoacoustic theory, the first order term of the fluctuating part is sufficient enough to describe the thermoacoustic phenomenon. Equation (3.3) is linearized based on some assumptions which are summarized below:

a) The solid wall and the porous medium are perfectly rigid and stationary;

b) Nonlinear effects (i.e. streaming effect and turbulence) are neglected;
c) The thermal and viscous penetration depths ($\delta_k$ and $\delta_v$) are much smaller compared to the acoustic wave length ($\lambda$);

d) Fluid pressure is a function of $x$-axis only;

e) The mean fluid velocities ($u_m$ and $v_m$) are zero.

According to Rott’s [1980] linear thermoacoustic approximation, all physical quantities such as pressure, density, temperature, and velocity can be expanded into time averaged and time varying quantities as shown below:

\[
\begin{align*}
\rho &= \rho_m + \rho_1(x) e^{j\omega t}, \\
\rho &\equiv \rho, \\
\rho_m &\equiv \rho_m, \\
\rho_1 &\equiv \rho_1(x), \\
\rho_1 &\equiv \rho_1, \\
\omega &\equiv \omega, \\
u &= u_m + u_1(x,y) e^{j\omega t}, \\
v &\equiv v, \\
u_m &\equiv \nu_m, \\
u_1 &\equiv \nu_1(x,y), \\
\omega &\equiv \omega, \\
\lambda &\equiv \lambda, \\
\end{align*}
\]

(3.4)

where $p$, $\rho$, $\rho_1$, $\omega$, $u$, and $v$ represent pressure, density, temperature, angular frequency, velocity in the $x$-direction, and velocity in the $y$-direction, respectively. The mean and fluctuating part of any physical quantities are described with the subscripts ‘$m$’ and ‘1’, respectively.

Considering all of the assumptions and substituting variable expansions as shown in Eq. (3.4) into Eq. (3.3), the Darcy momentum equation can be further simplified to

\[
\begin{align*}
\frac{\partial u_1}{\partial x} &= -\frac{\rho_1 K}{\mu} \frac{1}{1 + \frac{K}{\delta_v^2} \left( B, \frac{\mu}{a} \right)^2}
\]

(3.5)

where $u_1$ is the fluctuating velocity in the $x$-direction, $p_1 = p_0 \left[ \sin(2\pi x / \lambda) + i \cos(2\pi x / \lambda) \right]$ is the fluctuating pressure of the standing wave (Mahmud et al. [2011a]), $p_0$ is the fluctuating pressure amplitude and $\partial p_1 / \partial x$ is the pressure gradient along $x$-axis. Further simplification of Eq. (3.5) results in

\[
\begin{align*}
\frac{\partial u_1}{\partial x} &= -\frac{\rho_1 K}{\mu} \left( \frac{1}{1 + Da Ha^2} \right)
\]

(3.6)

where $Da = (K / \delta_v^2)$ and $Ha$ are the Darcy number and Hartmann number, respectively. The dimensionless parameter Darcy number ($Da$) is the ratio of permeability and the square of viscous penetration depth. The second dimensionless parameter
Hartmann number \(( Ha_\delta = B_\delta \delta \sqrt{\sigma / \mu} )\) is the ratio of Lorentz force to viscous force. If an external magnetic force is applied perpendicular to the flow direction of an electrically conductive fluid, it experiences an electric field and produces current perpendicular to both magnetic field and flow direction. The product of electric current and magnetic field creates a force which is known as Lorentz force (Davidson [2001]). The direction of the Lorentz force is always opposite to the direction of fluid flow in the absence of an applied electric field. Thus, the free stream velocity can be controlled by controlling \( Ha_\delta \).

3.4. THERMAL FIELD ANALYSIS

The energy equation inside the porous medium can be written as (Mahmud and Fraser [2006a]):

\[
\rho C_p \left[ \sigma \frac{\partial T}{\partial t} + \mathbf{V} \cdot \nabla T \right] = k \nabla^2 T + \beta T \frac{\partial P}{\partial t} + \frac{|\mathbf{J}|^2}{\sigma_c} + \mu \Psi
\]  

(3.7)

where \( \sigma, k, C_p, \) and \( \beta \) are the porous medium heat capacity ratio, effective thermal conductivity, specific heat of the fluid, and thermal expansion coefficient, respectively. The expressions of \( \sigma \) and \( k \) can be written as (Bejan [2003])

\[
\sigma = \phi + (1-\phi) \rho_{sm} C_{sm}/(\rho C_p)
\]  

(3.8a)

\[
k = (1-\phi) k_{sm} + \phi k_f
\]  

(3.8b)

where \( \phi \) is the porosity (=void volume/total volume) of the porous medium. The properties that correspond to the solid matrix and fluid are shown in Eq. (3.8a) and Eq. (3.8b) by the subscripts ‘sm’ and ‘f’, respectively. Joule heating and viscous dissipation expression in Eq. (3.7) contain velocity squared terms; therefore, these expressions are second order and can be neglected using the linear thermoacoustic (i.e., first order) approximation. Then the general form of the energy equation reduces to

\[
\rho C_p \left[ \sigma \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right] = k \left[ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right] + \beta T \left[ \frac{\partial P}{\partial t} + u \frac{\partial P}{\partial x} + v \frac{\partial P}{\partial y} \right].
\]  

(3.9)

At steady state, the following scale is considered based on our previous assumptions: \( x \sim \tilde{\lambda}, \ y \sim \delta_v \) where \( \tilde{\lambda} = \lambda / 2\pi \). Therefore, from mass conservation equation (Bejan [2003]), \( u/\tilde{\lambda} \sim v/\delta_v \), therefore, \( v \sim (\delta_v / \tilde{\lambda}) u \). Since, it is assumed that \( \tilde{\lambda} >> \delta_v \) thus,
\( \delta_y / \tilde{\lambda} << 1 \) results in \( u >> v \), it is also assumed that pressure is not a function of \( y \)-axis, thus, \( \partial p / \partial y \approx 0 \) and the temperature gradient along \( x \)-axis is constant, therefore, \( \partial^2 T / \partial x^2 \approx 0 \). Eq. (3.9) becomes

\[
\rho C_p \left[ \sigma \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} \right] = k \left[ \frac{\partial^2 T}{\partial y^2} \right] + \beta T \left[ \frac{\partial P}{\partial t} + u \frac{\partial p}{\partial x} \right].
\] (3.10)

Using the linear expansions, already described in Eq. (3.4), one can further simplify Eq. (3.10) to its linear form as shown in the following equation:

\[
\rho_m C_p \sigma (i \omega T_1) + \rho_m C_p u_1 \frac{\partial T_m}{\partial x} = k \frac{\partial^2 T_m}{\partial y^2} + \beta T_m (i \omega p_1).
\] (3.11)

After rearrangement, Eq. (3.11) can be further simplified to

\[
\frac{\partial^2 T_m}{\partial y^2} - \left( \frac{i \omega \rho_m C_p \sigma}{k} \right) T_1 = \frac{C_p \rho_m}{k} \frac{\partial T_m}{\partial x} u_1 - \left( \frac{i \omega T_m p_1}{k} \right).
\] (3.12)

The general solution to Eq. (3.12) is

\[
T_1 = C_1 e^{\left( \frac{i \omega \sqrt{\sigma} y}{2 \delta_i} \right)} + C_2 e^{\left( -\frac{i \omega \sqrt{\sigma} y}{2 \delta_i} \right)} - \left[ \frac{u_1}{i \omega \sigma} \frac{\partial T_m}{\partial x} \frac{\beta T_m p_1}{\rho_m C_p \sigma} \right].
\] (3.13)

where \( \delta_i = \sqrt{2 \alpha / \omega} \) is the thermal penetration depth inside the porous medium and \( C_1 \) and \( C_2 \) are two integration constants. The first integration constant, \( C_1 \), becomes 0 if one applies the following boundary condition: \( y \rightarrow \infty, \partial T_1 / \partial y = 0 \). The second integration constant, \( C_2 \), can be calculated after using \( y = 0, T_1 = T_w \) in the following form

\[
C_2 = T_w + \left[ \frac{u_1}{i \omega \sigma} \frac{\partial T_m}{\partial x} \frac{\beta T_m p_1}{\rho_m C_p \sigma} \right].
\] (3.14)

In Eq. (3.14), \( T_w \) is the temperature of the interface between the porous medium and thick solid wall which is still unknown and needs to be determined. The energy equation inside the thick solid plate is required to estimate \( T_w \) in Eq. (3.14) to have the final expression of \( C_2 \). The following energy equation (Mahmud and Fraser [2005a]) is used to model the temperature fluctuation inside the solid plate:

\[
\rho_s C_s \frac{\partial T_s}{\partial t} = k_s \frac{\partial^2 T_s}{\partial y^2}.
\] (3.15)
where \( \rho_s, C_s, \) and \( k_s \) are the density, specific heat, and solid wall thermal conductivity, respectively. In Eq. (3.15), \( \bar{y} \) represents the transverse coordinate in the solid region. Equation (3.15) can be linearized by applying the same linearization principles used in the previous sections. The final linearized form of Eq. (3.15) is

\[
 i \omega \rho_{ms} C_s T_{s1} = k_s \frac{\partial^2 T_{s1}}{\partial \bar{y}^2}
\]

where \( \rho_{ms} \) is the mean density of the solid. The general solution to Eq. (16) is

\[
 T_{s1} = C_3 \exp \left( \frac{i \omega \rho_{ms} C_s}{k_s} \bar{y} \right) + C_4 \exp \left( -\frac{i \omega \rho_{ms} C_s}{k_s} \bar{y} \right).
\]

The first integration constant \( C_3 \) in Eq. (17), can be set equal to 0 by approximating a finite temperature inside the solid wall far away from the interface of the porous medium and the solid wall (i.e., \( T_{s1}(\bar{y} \to \infty) = \text{finite} \)). One can apply the interface temperature boundary condition, i.e., \( T_1(y = 0) = T_w = T_{s1}(\bar{y} = 0) \) to obtain an expression of the temperature distribution inside the thick solid wall

\[
 T_{s1} = T_w \exp \left( -\frac{i \omega \rho_{ms} C_s}{k_s} \bar{y} \right).
\]

In Eq. (18), \( T_w \) is still an unknown quantity which can finally be obtained by using the continuity heat flow at the interface, i.e.,

\[
 k \frac{\partial T_1}{\partial \bar{y}} \bigg|_{1=0} = -k_s \frac{\partial T_{s1}}{\partial \bar{y}} \bigg|_{\bar{y}=0}.
\]

The final expression of \( C_2 \) is

\[
 C_2 = -\left[ \frac{T_m \beta p_t}{\sigma \rho_s C_p} - \frac{u_1}{i \sigma \omega} \frac{\partial T_m}{\partial x} \right] \left[ \frac{1}{1 + \sqrt{\sigma \varepsilon}} \right]
\]

where \( \varepsilon = \sqrt{\rho_s C_p k_s / \rho_s C_k} \) is the heat capacity ratio of solid wall to porous medium.

Now, substituting the expression of \( C_2 \) from Eq. (20) into Eq. (13) one can obtain the final expression of the temperature inside the porous medium in the following form.
\[ T_i = \left[ \frac{T_m \beta p_l}{\sigma \rho_n C_p} - \frac{u_i}{i \sigma \omega} \frac{\partial T_m}{\partial x} \right] \left[ 1 - \frac{1}{1 + i \sqrt{\sigma} \varepsilon} \exp \left( - \frac{1+i}{\delta_k} \sqrt{\sigma} y \right) \right] \]  \hspace{1cm} (3.21)

where \( \delta_k \left( = \sqrt{\frac{2\alpha}{\omega}} \right) \) is the thermal penetration depth in the porous medium. Equation (3.21) is the simplified form of analytical solution for the fluctuating temperature inside the porous medium. Equation (3.21) consists of two components: (i) a \( y \)-independent temperature (terms inside the first square bracket) and (ii) a \( y \)-dependent temperature (terms inside the second square bracket). The \( y \)-dependent temperature term contains a negative exponential term and it vanishes when \( y \) approaches to a large value (i.e., \( y \to \infty \)). For this limiting case, the fluctuating temperature \( (T_i) \) approaches the free stream temperature (i.e, \( T_{i,\infty} \)) which can be obtained by taking limit as shown in the following equation

\[ T_{i,\infty} = \lim_{y \to \infty} (T_i) = \frac{T_m \beta p_l}{\sigma \rho_n C_p} - \frac{u_i}{i \sigma \omega} \frac{\partial T_m}{\partial x} . \]  \hspace{1cm} (3.22)

In Eq. (3.22), the fluid properties, temperature gradient, flow properties, geometric parameters, and porous medium properties can be set in such a way that results in \( T_{i,\infty} = 0 \). For this special case, temperature gradient can be termed as a critical temperature gradient (\( \nabla T_{cr} \)). The expression of the critical temperature can be derived from Eq. (3.22) in the following form:

\[ \nabla T_{cr} = \frac{T_m \beta \mu \lambda f \left( 1 + DaHa_{\delta}^2 \right)}{K\rho_n C_p} . \]  \hspace{1cm} (3.23)

The solid wall temperature can be obtained by substituting the value \( T_i (y = 0) \) in Eq. (3.21) and can be written as follows:

\[ T_w = \left[ \frac{T_m \beta p_l}{\sigma \rho_n C_p} + \frac{1}{i \sigma \omega} \frac{\partial T_m}{\partial x} \left\{ \frac{\Phi \partial p_l}{\partial x} \right\} \right] \left[ 1 - \frac{1}{1 + \sqrt{\sigma} \varepsilon} \right] . \]  \hspace{1cm} (3.24)

Finally, the temperature distribution inside the solid wall, \( T_{sl} \), can be found by using Eqs. (3.18), (3.20) and (3.14) in the following form:

\[ T_{sl} = \left[ \frac{T_m \beta p_l}{\sigma \rho_n C_p} + \frac{1}{i \sigma \omega} \frac{\partial T_m}{\partial x} \left\{ \frac{\Phi \partial p_l}{\partial x} \right\} \right] \frac{\sqrt{\sigma} \varepsilon}{1 + \sqrt{\sigma} \varepsilon} \exp \left( - \frac{1+i}{\delta_k} \sqrt{\sigma} \right) . \]  \hspace{1cm} (3.25)
where $\delta_s = \sqrt{2\alpha_s / \omega}$ is the thermal penetration depth in the solid wall.

### 3.5. Heat Transfer and the Nusselt Number

In this chapter, Nusselt number ($Nu$) is used to calculate the heat transfer rate from the wall. The following definition of Nusselt number is used (Mahmud and Fraser [2006a]):

$$Nu = \left( \frac{\delta_k}{T_w - T_{i,\infty}} \right) \frac{\partial T_1}{\partial y} \bigg|_{y=0}$$  \hspace{1cm} (3.26)

After substituting Eqs. (3.21), (3.22), and (3.24) into Eq. (3.26) one can obtain the following expression for the Nusselt number

$$Nu = (1 + i)\sqrt{\sigma}.$$  \hspace{1cm} (3.27)

### 3.6. Heat Transfer Irreversibility

The volumetric entropy generation rate for a thermoacoustic system inside a porous medium in the presence of a magnetic field can be expressed by the following equation (Mahmud and Fraser [2006b]):

$$s_{gen} = \left[ \frac{k}{T_m^2} \left( \frac{\partial T_1}{\partial y} \right)^2 \right] + \left[ \frac{k_i}{T_m^2} \left( \frac{\partial T_{1,\infty}}{\partial y} \right)^2 \right] + \left[ \frac{\mu}{KT_m} u_{i,\eta}^2 \right] + \left[ \frac{\sigma B_y^2}{KT_m} u_{i,\eta}^2 \right]$$  \hspace{1cm} (3.28)

where $T_m$ is the mean temperature. The time dependency of entropy generation is defined by the subscript “1” with any variable on the right hand side of Eq. (3.28). The right hand side of Eq. (3.28) contains three parts: the first part refers to the heat transfer entropy generation rate (inside the porous medium and solid wall), the second part refers to the fluid friction irreversibility, and the third part refers to the entropy generation rate due to the presence of magnetic field. The time averaged (Swift [1988]) heat transfer entropy generation for the porous medium can be calculated as:

$$\bar{s}_{HT, porous} = \frac{k}{T_m^2} \left( \frac{\partial T_1}{\partial y} \right) \frac{\partial T_1}{\partial y} \left[ \frac{2\sigma Pr}{\delta_v^2 \left( 1 + \sqrt{\sigma} \varepsilon \right)} \right] \left[ -2\sqrt{\sigma} Pr \left( \frac{y}{\delta_v} \right) \right]$$  \hspace{1cm} (3.29)
where \( \Re[ ] \) and \( \sim \) refer to the real part and complex conjugate of a complex expression, respectively. After substituting the expression of \( T_i \) (Eq. (3.21)) into Eq. (3.29) one can obtain the time averaged heat transfer entropy generation rate in the following form:

\[
\bar{s}_{HT, \text{porous}}^m = \frac{1}{2T_m^2} \left[ \frac{1}{\sigma} \frac{T_m \beta}{\rho_m C_p} P_0 \right]^2 \left[ 1 - \frac{\nabla T_m}{\nabla T_{cr}} \right]^2 \left\{ \frac{2 \sigma Pr}{\delta_y^2 \left[ 1 + \sqrt{\sigma} \varepsilon \right]^2} \right\} \exp \left\{ -2 \sqrt{\sigma} Pr \left( \frac{\varepsilon}{\delta_y} \right) \right\}, \quad (3.29a)
\]

Similarly, the time averaged heat transfer entropy generation for solid wall can be expressed as follows:

\[
\bar{s}_{HT, \text{solid}}^m = \frac{k}{2T_m} \Re \left[ \frac{\partial T_{sl}}{\partial y} \frac{\partial \tilde{T}_{sl}}{\partial y} \right] = \frac{1}{2T_m^2} \Re \left[ T_{1s} \tilde{T}_{1s} \right] \left\{ \frac{2 \sigma \varepsilon^2}{\left( 1 + \sqrt{\sigma} \varepsilon \right)^2 \delta_y^2} \right\} \exp \left\{ -2 \frac{\varepsilon}{\delta_y} \right\}, \quad (3.30)
\]

After substituting the expression of \( T_{sl} \) (Eq. (3.25)) into Eq. (3.30) one can obtain the time averaged heat transfer entropy generation rate inside the solid wall in the following form:

\[
\bar{s}_{HT, \text{solid}}^m = \frac{1}{2T_m^2} \left[ \frac{1}{\sigma} \frac{T_m \beta}{\rho_m C_p} P_0 \right]^2 \left[ 1 - \frac{\nabla T_m}{\nabla T_{cr}} \right]^2 \left\{ \frac{2 \sigma \varepsilon^2}{\left( 1 + \sqrt{\sigma} \varepsilon \right)^2 \delta_y^2} \right\} \exp \left\{ -2 \frac{\varepsilon}{\delta_y} \right\}, \quad (3.30a)
\]

The total time averaged heat transfer entropy generation can be obtained by adding Eq. (3.29a) and Eq. (3.30a) as shown below:

\[
\bar{s}_{HT}^m = \left( \frac{1}{T_m} \frac{T_m \beta}{\sigma \rho_m C_p} P_0 \right)^2 \left[ 1 - \frac{\nabla T_m}{\nabla T_{cr}} \right]^2 \sigma \left[ \frac{k}{\delta_k^2} \exp \left\{ -2 \frac{\sqrt{\sigma} y}{\delta_k} \right\} + \varepsilon^2 k_s \exp \left\{ -2 \frac{\varepsilon y}{\delta_s} \right\} \right] \quad (3.31)
\]

where \( \nabla T_{cr} \) is the critical temperature gradient that is expressed in Eq. (3.23). The detailed discussion about the critical temperature gradient is given in the results and discussions section. The time averaged entropy generation due to the fluid friction inside the porous medium can be calculated as follows:

\[
\bar{s}_{FF}^m = \frac{\mu}{K T_m} \frac{1}{2} \Re [\mu \tilde{u}] = \frac{1}{2} \frac{\mu}{K T_m} \left( \frac{2 \pi K p_0}{\mu \lambda} \right)^2 \left( \frac{1}{1 + DaHa_b} \right)^2 = 2 \pi^2 \frac{K}{\mu T_m} \left( \frac{p_0}{\lambda (1 + DaHa_b)} \right)^2 \quad (3.32)
\]

The time averaged entropy generation due to the magnetic effect inside the porous medium can be calculated as follows:

38
The fluctuating pressure equation

To calculate the pressure fluctuation, the wave equation must be solved. The general governing continuity equation can be written as follows:

\[
\frac{\partial (\phi \rho)}{\partial t} + \frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} = 0.
\]  (3.34)

Using linear expansions, as described in Eq. (3.4), one can further simplify Eq. (3.34) to its linear form as shown in the following equation:

\[
i \omega \phi = \frac{\partial (\rho u)}{\partial x} = 0.
\]  (3.35)

By substituting Eq. (3.6) into Eq. (3.35) and taking derivative with respect to \( x \), one obtains

\[
\rho_1 = -i \omega \phi \frac{\partial (\rho u)}{\partial x} = -i \omega \phi \frac{\partial}{\partial x} \left( -\rho_\phi \frac{\partial \phi}{\partial x} \right) = \frac{\rho_\phi \frac{\partial^2 \phi}{\partial x^2}}{i \omega \phi}.
\]  (3.36)

Using the thermodynamic relation, \( \rho_1 = \rho_\phi \beta T_1 + \left( \frac{\gamma}{c^2} \right) p_1 \), \( \rho_1 \) can be eliminated from Eq. (3.36) and can be written as

\[
-\rho_\phi \beta T_1 + \frac{\gamma}{c^2} p_1 = \frac{\rho_\phi \frac{\partial^2 \phi}{\partial x^2}}{i \omega \phi}.
\]  (3.37)

Simplifying Eq. (3.37) results in

\[
\frac{\partial^2 p_1}{\partial x^2} - \frac{\gamma}{c^2} \rho_\phi \frac{\partial^2 \phi}{\partial x^2} + \rho_\phi \beta T_1 = 0.
\]  (3.38)

By substituting Eq. (2.21) into Eq. (3.38) and after for the limit \( \gamma \to \infty \) one can achieve:

\[
\frac{\partial^2 p_1}{\partial x^2} + \frac{\beta_\phi \frac{\partial T_\phi}{\partial x}}{\sigma} \frac{\partial p_1}{\partial x} + \frac{i \omega \phi}{\rho_\phi \frac{\partial^2 \phi}{\partial x^2}} \left( \frac{T_\phi^2}{\sigma C_p} - \frac{\gamma}{c^2} \right) p_1 = 0.
\]  (3.39)

Using \( c = \sqrt{\gamma RT_m} \) and \( C_p = \gamma R / (\gamma - 1) \), Eq. (3.39) can be further simplified to

\[
\frac{\partial^2 p_1}{\partial x^2} + \frac{\beta_\phi \frac{\partial T_\phi}{\partial x}}{\sigma} \frac{\partial p_1}{\partial x} + \frac{i \omega \phi}{\rho_\phi} \left( \frac{\frac{T_\phi^2}{\sigma} \beta^2 (\gamma - 1) - \sigma \gamma}{c^2} \right) p_1 = 0.
\]  (3.40)
Finally, the wave equation is derived after substituting the expression of $\Phi$ into Eq. (3.40)
\[
\frac{\partial^2 p_1}{\partial x^2} + \frac{\phi}{\sigma} \frac{\partial \ln T_m}{\partial x} \frac{\partial p_1}{\partial x} + \left(1 + DaHa_b^2\right) \frac{\bar{\sigma}^2}{c^2} \left[\frac{(1-\sigma)\gamma-1}{\sigma}\right] p_1 = 0,
\]  
(3.41)
where $\bar{\sigma} = i\omega\mu / (\rho_m K)$. The general solution to Eq. (3.41) can be written as
\[
p_1 = C_5 \exp(\varphi_1 x) + C_6 \exp(\varphi_2 x)
\]  
(3.42)
where the $\varphi_1$ and $\varphi_2$ can be expressed as
\[
\varphi_1 = -\frac{1}{2} \frac{\phi}{\sigma} \frac{\partial \ln T_m}{\partial x} - \frac{1}{2} \left\{ \frac{\phi}{\sigma} \frac{\partial \ln T_m}{\partial x} \right\}^2 - 4\left(1 + DaHa_b^2\right) \frac{\bar{\sigma}^2}{c^2} \left[\frac{(1-\sigma)\gamma-1}{\sigma}\right],
\]  
(3.43)
\[
\varphi_2 = -\frac{1}{2} \frac{\phi}{\sigma} \frac{\partial \ln T_m}{\partial x} + \frac{1}{2} \left\{ \frac{\phi}{\sigma} \frac{\partial \ln T_m}{\partial x} \right\}^2 - 4\left(1 + DaHa_b^2\right) \frac{\bar{\sigma}^2}{c^2} \left[\frac{(1-\sigma)\gamma-1}{\sigma}\right].
\]  
(3.44)
Considering the standing wave pressure $p^s(x) = p_0 \left[ \sin(x_0 / \lambda) + i \cos(x_0 / \lambda) \right]$ at the starting point of the stack (i.e., $x = x_s$) and $p^s(x_e) = p_0 \left[ \sin(x_e / \lambda) + i \cos(x_e / \lambda) \right]$ at the stack exit (i.e., $x = x_e$). The constants, $C_5$ and $C_6$ of Eq. (3.42), can be calculated and are given as:
\[
C_5 = \frac{p^s(x_s) e^{\Theta_{x_s}} - p^s(x_e) e^{\Theta_{x_e}}}{e^{\Theta_{x_s}} - e^{\Theta_{x_e}}} , \quad C_6 = -\frac{p^s(x_s) e^{\Theta_{x_s}} - p^s(x_e) e^{\Theta_{x_e}}}{e^{\Theta_{x_s}} - e^{\Theta_{x_e}}} .
\]  
(3.45)
Finally, the expression of $p_1$ becomes
\[
p_1 = \left[ \frac{p^s(x_s) e^{\Theta_{x_s}} - p^s(x_e) e^{\Theta_{x_e}}}{e^{\Theta_{x_s}} - e^{\Theta_{x_e}}} \right] e^{\Theta_{x_s}} - \left[ \frac{p^s(x_s) e^{\Theta_{x_s}} - p^s(x_e) e^{\Theta_{x_e}}}{e^{\Theta_{x_s}} - e^{\Theta_{x_e}}} \right] e^{\Theta_{x_e}} .
\]  
(3.46)
In Figs. 3.2(a), 3.2(b), and 3.2(c), $p_1$ is plotted as a function of frequency for different values of $Ha_b$ using Eq. (46). $p_0$ is calculated using $p_m \times DR$, where $p_m$ is the mean pressure and DR is the drive ratio. The value of DR is considered to be 0.01 for Figs. 3.2(a), 3.2(b), and 3.2(c). $p_m$ is considered to be 1.0 and 10.0 bar for Figs. 3.2(a) and 3.2(b), respectively. The same values of $Ha_b$ (0, 5, 10, 15, and 20) are considered for both Figs. 3.2(a) and 3.2(b). Fig. 3.2(a) shows that the resonance frequency (where the pressure amplitude is maximum) decreases with increasing $Ha_b$. However, Fig. 3.2(b) shows that the resonance frequency does not change significantly by changing the value.
of $Ha_\delta$ because of the use of relatively higher value of $p_m$ in Fig. 3.2(b) compared to Fig. 3.2(a).

As Fig. 3.2(b), Fig. 3.2(c) is plotted considering $p_m$ of 10.0 bar. Compared to Fig. 3.2(b), higher values of $Ha_\delta$ (=0, 50, 100, 150, and 200) are considered in Fig. 3.2(c). Fig. 3.2(c) shows that the resonance frequency decreases significantly with increasing $Ha_\delta$. Since, lower values of $Ha_\delta$ (0-20) have an insignificant effect on resonance frequency for a pressurized thermoacoustic system ($p_m$ =10.0 bar), a constant frequency (=350 Hz) has considered in the simulation.
Figure 3.2: (a) Fluctuating pressure as a function of frequency at different Hartman numbers when \( p_m = 1.0 \) bar (b) \( p_m = 10.0 \) bars (c) \( p_m = 10.0 \) bars for the values of Hartman numbers higher than (b).
3.8. RESULTS AND DISCUSSIONS

3.8.1 CRITICAL TEMPERATURE GRADIENT

In a typical thermoacoustic system, the relationship between mean temperature gradient across the stack ($\nabla T_m$) and critical temperature gradient ($\nabla T_{cr}$) (described in Eq. (3.23)), plays an important role in determining modes of thermoacoustic system’s operation. Swift [1988] observed two modes of operation based on the relative magnitude of $\nabla T_m$ and $\nabla T_{cr}$. A heat pump mode is observed when $\nabla T_m < \nabla T_{cr}$ and a prime mover mode when $\nabla T_m > \nabla T_{cr}$. Thus, for a constant $\nabla T_m$, the critical temperature gradient can be defined as a parameter for determining the transition between the heat pump and prime mover mode of operations [Swift [1988]]. However, calculation of $\nabla T_{cr}$ becomes complicated in the presence of viscosity, frequency of oscillating fluid, longitudinal thermal conductivity, porous medium, and magnetic field.

In Fig. 3.3, $\nabla T_{cr}$ is plotted as a function of $Ha_\delta$ at different $Da$ using Eq. (3.23). The range of $Ha_\delta$ is considered between 0 and 5 and $Da$ between 0.01 and 0.10. Helium is considered as a working fluid and the properties of Helium at 300 K ($T_m$) is used in the calculation in Fig. 3.3. It can be observed from Eq. (3.23) that for a given value of $T_m, \beta, \mu, \lambda, f, K, \rho_m,$ and $C_p$ the $\nabla T_{cr}$ directly depends on the product $Da Ha_\delta^2$. For a constant $Ha_\delta$, the magnitude of the $\nabla T_{cr}$ is higher for the higher $Da$ value as observed in Fig.3.3. Higher $Da$ value increases porosity which results in the reduction of conductive surface area, as well as power density, which leads to an increase in $\nabla T_{cr}$. The magnitude of the $\nabla T_{cr}$ increases with increasing $Ha_\delta$ for a given $Da$ value. Eq. (3.23) also shows that $\nabla T_{cr}$ is proportional to the square of $Ha_\delta$. Therefore, the magnitude of $\nabla T_{cr}$ is higher at higher $Ha_\delta$. It is further observed from Fig. 3.3 that the $Ha_\delta$ has less effect on $\nabla T_{cr}$ for a relatively lower $Da (=0.01)$ value, while $Ha_\delta$ has significant effect on $\nabla T_{cr}$ for a relatively higher $Da (=0.1)$ value.
Figure 3.3: Critical temperature gradient as a function of Hartman number at different values of $Da$.

### 3.8.2 TEMPERATURE DISTRIBUTIONS INSIDE THE POROUS MEDIUM

The right hand side of the free stream fluctuating temperature (Eqs. (3.22) and (3.47)) has two parts. The first part represents the contribution of the adiabatic compression or expansion of the fluid and the second part is the contribution of the mean temperature gradient of the stack and fluid fluctuation velocity [Swift [1988]].

$$T_{1,\infty} = \lim_{\nu \to \infty} (T_1) = \frac{T_a \beta \rho_i}{\sigma \rho_m C_p} - \frac{u_i}{i \sigma} \frac{\partial T_m}{\partial x} = T_a - T_{sw}$$  \hspace{1cm} (3.47)

where $T_a$ is due to the adiabatic compression or expansion of the fluid and $T_{sw}$ is due to the mean temperature gradient of the stack and fluid fluctuation velocity. A dimensionless form of the fluctuating temperature can be obtained, after rearranging Eq. (3.21), as follows:

$$\frac{T_1}{T_a} = \left[ 1 - \frac{F^*}{1 + Da Ha_s} \right] \left[ 1 - \frac{1}{1 + \sqrt{\sigma} \varepsilon} \exp \left( - \frac{1 + i}{\delta_k} \sqrt{\sigma} y \right) \right],$$  \hspace{1cm} (3.48)
where $\Gamma^* = \nabla T_m K p_m C_p / T_m \beta \mu \lambda f$, which is the ratio of the mean temperature gradient to the critical temperature gradient in the absence of magnetic field. Fig. 3.4 presents the dimensionless temperature profile ($T_i / T_a$) as a function of dimensionless distance ($y / \delta_k$) inside the porous medium for different values of $Ha_{\delta}$. Six different values of $Ha_{\delta} (=0, 5, 10, 20, 40, \text{ and } \infty)$ are considered for a constant $\Gamma^* (=0.4)$ and $Da (=0.01)$. Applied magnetic field is higher on the system for a higher $Ha_{\delta}$. Fig. 3.4 shows that $Ha_{\delta}$ has significant effect on $T_i / T_a$. The magnitude of $T_{sw}$ in Eq. (3.47) depends on the magnitude of $u_i$. According to Eq. (3.6), $u_i$ has an inverse relationship with the square of $Ha_{\delta}$. Therefore, larger value of $Ha_{\delta}$ reduces the value of $u_i$ and increases the value of $T_{i,\infty}$. The resulting effect is the increase of $T_i / T_a$ in Eq. (3.48) which is also observed from Fig. 3.4. Fig. 3.4 also shows that the $y$-independent part of $T_i / T_a$ approaches a value of 1 with increasing $Ha_{\delta}$. In the limit of a very large $Ha_{\delta}$ (i.e., $Ha_{\delta} \to \infty$) the magnitude of $T_i / T_a$ approaches 1. For this limiting case (i.e., $Ha_{\delta} \to \infty$), the $T_i / T_a$ fluctuation inside the porous medium is only due to the adiabatic compression or expansion.

Figure 3.4: Dimensionless temperature profile as a function of Hartman numbers
Fig. 3.4 also shows that the thermal boundary layer thickness decreases with increasing \( Ha_\delta \). A closed observation reveals that the magnitude of \( T_i/T_a \) at the wall is maximum when \( Ha_\delta \) approaches infinity. The \( y \)-dependent part, represented by the term with the negative exponential function in Eq. (3.48), will vanish when \( y \rightarrow \infty \).

3.8.3 TEMPERATURE DISTRIBUTIONS INSIDE THE SOLID WALL

The temperature distribution equation inside the thick solid plate is expressed in Eq. (3.25). The dimensionless form of the temperature distribution equation inside the solid wall can be written as follows:

\[
\frac{T_{si}}{T_a} = \left[ 1 - \frac{\Gamma^*}{(1 + DaHa_\delta^2)} \right] \frac{\sqrt{\sigma \varepsilon}}{1 + \sqrt{\sigma \varepsilon}} \exp \left( -\frac{1+i}{\delta} \frac{y}{\delta_s} \right)
\] (3.49)

The dimensionless temperature distribution equation (Eq. 3.49) inside the thick solid plate is plotted in Fig. 3.5.

![Figure 3.5: Dimensionless temperature distribution inside the thick solid plate as a function of Hartman numbers](image)
The $T_s / T_a$ profiles are different for different $Ha_\delta$ at the wall ($\bar{v} = 0$) due to the imposed boundary condition at the interface of solid wall and porous medium. The magnitude of $T_s / T_a$ at the wall is maximum when $Ha_\delta$ approaches infinity. The dimensionless fluctuating temperature $T_s / T_a$ approaches zero at a distance approximately four times the thermal penetration depth ($=4\delta_s$). However, this distance reduces significantly with increasing $Ha_\delta$.

### 3.8.4 Heat Flux

The hydrodynamic transport of entropy carried by the oscillating fluid along the $x$-direction is defined as heat flux of a thermoacoustic system [Swift [1988]]. The average heat flux generated per unit area by the oscillating fluid in the period of oscillation $\tau = 2\pi / \omega$ is known as time average heat flux [Swift [1988]]. The time average heat flux is a product of two first order quantities and usually expressed as a second order time average of heat flux. The total heat flux along the plate is known as global heat flux [Swift [1988]]. The general equation for the second order global heat flux [Swift [1988]] is shown below:

$$
\dot{Q}_2 = \Pi \int_0^{\infty} \tilde{q}_d dy = \Pi \int_0^{\infty} \left\{ \frac{1}{2} \rho_m C_p \Re [T_i \tilde{u}_t] - \frac{1}{2} T_m \beta \Re [p_i \tilde{u}_t] \right\} dy
$$

(3.50)

where $\dot{Q}_2$, $\tilde{q}_d$ and $\Pi$ are the second order local heat flux, second order time averaged heat flux, and width of the plate, respectively. After substituting the expression of $T_i$, $u_t$, and $p_i$ into Eq. (3.50) one can obtain an expression for global heat flux as

$$
\dot{Q}_2 = -\frac{\Pi \delta_k}{2} \frac{T_m \beta}{(1 + \sqrt{\sigma} \varepsilon)\sqrt{\sigma}} \frac{\pi \rho_0^2}{\lambda \sigma} \frac{K}{\mu} \frac{1}{1 + Da Ha_\delta^2} \left[ \frac{\nabla T_m}{\nabla T_{cr}} - 1 \right]
$$

(3.51)

In the absence of magnetic field and porous media (i.e. $Ha_\delta = 0$ and $\sigma = 1$) and in the thin plate limit (i.e., $\varepsilon = 0$), Eq. (3.51) reduces to the heat flux equation obtained by Swift [1988] for an inviscid single plate thermoacoustic system. Now, a reference global heat flux ($\dot{Q}_0$) is proposed using the following substitution in Eq. (3.51):
\( \varepsilon = 0, \nabla T_m = 0, \sigma = 1, \) and \( Ha_\delta = 0 \). The expression of \( \dot{Q}_0 \) is used to make the heat flux expression in Eq. (3.51) dimensionless. The magnitude of \( \dot{Q}_0 \) can be written as

\[
\dot{Q}_0 = \pi \frac{\ln B}{2} \frac{T_m B}{\ell} \frac{p_0^2}{\lambda} \frac{K}{\mu} \left[ \frac{\nabla T_m}{\nabla T_{cr}} - 1 \right].
\] (3.52)

The resultant dimensionless heat flux can be written, after dividing Eq. (3.51) by Eq. (3.52), as

\[
Q = \frac{\dot{Q}_0}{\dot{Q}_0} = \frac{1}{1 + \sqrt{\sigma \varepsilon}} \frac{1}{\sigma} \frac{1}{1 + Da Ha^2_\delta} \left[ \frac{\nabla T_m}{\nabla T_{cr}} - 1 \right].
\] (3.53)

Eq. (3.53) is an important modeling result of this paper. It is observed that the term \((\nabla T_m / \nabla T_{cr} - 1)\) on the right hand side of Eq. (3.53) determines the sign of the heat flux \(Q\) and can be treated as a controlling term of the operating mode of the proposed thermoacoustic system. A special case with \( \nabla T_m = \nabla T_{cr} \) results in \((\nabla T_m / \nabla T_{cr} - 1) = 0\) which produces a zero heat flux. A case with \( \nabla T_m > \nabla T_{cr} \) results in \((\nabla T_m / \nabla T_{cr} - 1) > 0\) which produces a negative heat flux. A negative heat flux can be interpreted as heat flowing towards the pressure node. The opposite case with \( \nabla T_m < \nabla T_{cr} \) results in \((\nabla T_m / \nabla T_{cr} - 1) < 0\) which produces a positive heat flux. This case can be interpreted as heat flowing towards the pressure anti-node. Note that the value of \( \nabla T_{cr} \) can be varied by varying \( Ha_\delta \) which is already observed from Eq. (3.23). Therefore, the operating mode of the thermoacoustic system can be controlled by controlling \( Ha_\delta \).

The dimensionless heat flux Eq.(3.53) is further simplified to achieve an expression which is independent of material and fluid properties. The simplified expression can be written as follows:

\[
\frac{Q}{f_1(\sigma, \varepsilon)} = \frac{1}{1 + Da Ha^2_\delta} \left[ \frac{\Gamma^*}{1 + Da Ha^2_\delta} - 1 \right]
\] (3.54)

where \( f_1(\sigma, \varepsilon) = 1/(1 + \sqrt{\sigma \varepsilon})/\sigma^{3/2} \). The values of \( \varepsilon \) and \( \sigma \) depends on the properties of materials and working fluids.

Fig.3.6 presents surface and contour plot of dimensionless heat flux \((Q / f_1(\sigma, \varepsilon))\) as a function of \( \Gamma^* \) and \( Ha_\delta \) according to Eq. (3.54). The surface plot helps to understand the
variation in the magnitude of $Q/ f_i(\sigma, \varepsilon)$ with $\Gamma^*$ and $Ha_\delta$. The contour plot, which is plotted at the bottom of the surface plot, is very useful in selecting the combination of two input parameters ($\Gamma^*$ and $Ha_\delta$) for which heat flux is constant. As shown in Fig. 3.6, to generate the plot, the values of $\Gamma^*$ and $Ha_\delta$ are varied from 0 to 2.0 and 0 to 20, respectively. The value of $Q/ f_i(\sigma, \varepsilon)$ decreases with increasing $Ha_\delta$ when $\Gamma^* \leq 1$. For $\Gamma^* \geq 1$, $Q/ f_i(\sigma, \varepsilon)$ increases significantly when $Ha_\delta < 4.5$. A negative value of $Q/ f_i(\sigma, \varepsilon)$ is observed when $Ha_\delta < 10$. However, a positive values of $Q/ f_i(\sigma, \varepsilon)$ is observed when $Ha_\delta > 10$. Thus, the mode of operations can be changed by changing $Ha_\delta$. The value of $Q/ f_i(\sigma, \varepsilon)$ is zero when $Ha_\delta$ is close to 10.

3.8.5 Work Flux

Work flux is the measure of acoustic power input/output of a given thermoacoustic system. A fluid parcel, approximately at a thermal penetration depth away from the plate, undergoes thermal expansion and compression due to the pressure and temperature
gradient in the plate along the \( x \)-axis in order to produce or absorb work. The time average work flux is calculated as acoustic power per unit volume that is produced or absorbed in a period of \( \tau = 2\pi / \omega \). The general equation for the global work flux is derived in [Swift [1988]] and can be written as follows:

\[
\dot{W}_2 = \Pi \Delta x \overline{\dot{w}}_2 d y = \Pi \Delta x \left\{ \frac{\alpha \beta}{2} \Re \left[ -ip_i T_i \right] \right\} d y
\]  

(3.55)

where \( \dot{W}_2 \), \( \overline{\dot{w}}_2 \), \( \Pi \), and \( \Delta x \) are the global work flux, second order time averaged work flux, width of the plate, and length of the plate, respectively. After substituting expressions of \( T_i \) and \( \overline{p}_i \) into Eq. (3.55), one obtains a simplified expression of global heat flux as follows:

\[
\dot{W}_2 = \frac{\Pi \delta_k}{4} \frac{\omega \Delta x}{1 + \varepsilon \sqrt{\sigma}} \frac{1}{\sigma} \frac{T_m \beta^2}{\rho_m C_p} p_0^2 \left[ \frac{\nabla T_m}{\nabla T_{cr}} - 1 \right].
\]  

(3.56)

In the absence of magnetic field and porous media (i.e. \( Ha_\delta = 0 \) and \( \sigma = 1 \)) and for the case of thin plate (\( \varepsilon = 0 \)), Eq. (3.56) reduces to an expression developed by Swift [1988] for a single plate inviscid thermoacoustic system. Now, a reference global work flux \( \dot{W}_0 \) is proposed using the following substitution in Eq. (3.56): \( \varepsilon = 0, \nabla T_m = 0, \) and \( \sigma = 1 \). The expression of \( \dot{W}_0 \) is used to make the work flux expression in Eq. (3.56) dimensionless. \( \dot{W}_0 \) can be written as

\[
\dot{W}_0 = \frac{\Pi \delta_k}{4} \frac{\omega \Delta x}{1 + \varepsilon \sqrt{\sigma}} \frac{T_m \beta^2}{\rho_m C_p} p_0^2
\]  

(3.57)

The resultant dimensionless work flux can be written, after dividing Eq. (3.56) by Eq. (3.57)

\[
W = \frac{\dot{W}_2}{\dot{W}_0} = \frac{1}{1 + \varepsilon \sqrt{\sigma}} \frac{1}{\sigma} \left[ \frac{\nabla T_m}{\nabla T_{cr}} - 1 \right].
\]  

(3.58)

Equation (3.58) represents the dimensionless global work flux which is proportional to \( (\nabla T_m / \nabla T_{cr} - 1) \). A special case with \( \nabla T_m = \nabla T_{cr} \) results in \( (\nabla T_m / \nabla T_{cr} - 1) = 0 \) in Eq. (3.58) which produces a zero work flux. This special case results in a useless thermoacoustic state. The value of \( \nabla T_{cr} \) can be adjusted by controlling the value of \( Ha_\delta \).
according to Eq. (3.23). The specific value of $H_{a\delta}$ that causes $\dot{W}_2/\dot{W}_0 = 0$ can be treated as a critical Hartman number for work flux. For $\nabla T_m > \nabla T_{cr}$ (in Eq. (3.58)), results in $(\nabla T_m/\nabla T_{cr} - 1) > 0$ which produces a positive work flux. A positive work flux can be interpreted as work produced by the system. The opposite case with $\nabla T_m < \nabla T_{cr}$ results in $(\nabla T_m/\nabla T_{cr} - 1) < 0$ which produces a negative flux. A negative work flux can be interpreted as work consumed by the system. Eq. (3.58) can be simplified further as

$$\frac{W}{f_1(\sigma, \varepsilon)} = \left[\frac{\Gamma^*}{1 + DaH_{a\delta}^2} - 1\right]. \tag{3.59}$$

Fig. 3.7 presents the surface and contour plots of dimensionless work flux $(W/f_1(\sigma, \varepsilon))$ as a function of $\Gamma^*$ and $H_{a\delta}$ using Eq. (3.59). As shown in Fig. 3.7, $\Gamma^*$ and $H_{a\delta}$ are varied from 0 to 2.0 and 0 to 20, respectively. The maximum value of $W/f_1(\sigma, \varepsilon)$ is observed when $\Gamma^* = 2$ and $H_{a\delta} = 0$.

![Figure 3.7: Dimensionless work flux as functions of $\Gamma^*$ and $H_{a\delta}$](image)
The value of $W / f_1(\sigma, \varepsilon)$ decreases with increasing $Ha_\delta$ regardless of the mode of operation. The mode of operation can be changed by increasing $Ha_\delta$. $W / f_1(\sigma, \varepsilon)$ is observed to be zero when $Ha_\delta$ is close to 10

### 3.8.6 Entropy Generation

Entropy generation for the heat transfer problem is well studied in the heat transfer literature [Bejan [1996]]. The thermodynamic irreversibility can be measured by calculating global entropy generation rate. The inherent irreversibility of a thermoacoustic system is the major cause of its poor efficiency [Ishikawa and Mee [2002]]. In this problem, the time averaged entropy generation rate is subdivided into four parts in order to make an easier interpretation: (i) the time averaged heat transfer entropy generation for the porous medium (Eq. 3.29a), (ii) the time averaged heat transfer entropy generation for solid wall (Eq. 3.30a), (iii) the time averaged entropy generation due to the fluid friction inside the porous medium (Eq. 3.32), and (iv) the time averaged entropy generation due to the magnetic effect inside the porous medium (Eq. 3.33). The global entropy generation can be computed using the following equation

$$
\dot{S}_{gen} = (\Pi \Delta x) \int_0^\infty \dot{s}_{HT,\text{porous}} d y + (\Pi \Delta x) \int_0^\infty \dot{s}_{HT,\text{solid}} d y + (\Pi \Delta x) \int_0^\infty \dot{s}_{FF} d y + (\Pi \Delta x) \int_0^\infty \dot{s}_{Mag} d y.
$$

(3.60)

The last two terms of Eq. (3.60) contain the total entropy generation rates due to fluid friction and magnetic force. The time average entropy generation rates for these two terms do not contain $y$-dependent parts (see Eq. (3.32) and Eq. (3.33)). Also, an inviscid fluid has considered for the simulation, the entropy generation due to the fluid friction is zero [Bejan [1996]]. Therefore, the magnitude of $\dot{s}_{FF}$ and $\dot{s}_{Mag}$ are constant at any transverse location inside the porous medium provided that all other parameters (e.g., $\sigma, K, T_m, \mu, \lambda, Da, B_y, p_0$, and $Ha_\delta$) remains unchanged.

By neglecting those two terms ($\dot{s}_{FF}$ and $\dot{s}_{Mag}$), the global entropy generation can be calculated using the first two terms on the right hand side of Eq. (3.60) as follows

$$
\dot{S}_{gen} = \left( \frac{\Pi \Delta x}{2} \right) \left( \frac{1}{T_m} \frac{T_m \beta}{\alpha \rho_m C_p} p_0 \right)^2 \left[ \nabla T_m - \frac{1}{\nabla T_{cr}} \right]^2 \frac{\sqrt{\sigma}}{(1 + \sqrt{\sigma} \varepsilon) \delta_k}.
$$

(3.61)
For a special case of absence of porous media ($\sigma = 1$), zero mean temperature gradient ($\nabla T_m = 0$), and thin plate ($\varepsilon = 0$), Eq. (61) is reduced to

$$\dot{S}_0 = \left( \frac{\Pi \Delta x}{2} \right) \left( \frac{1}{T_m} \frac{T_m \beta}{\rho_m C_p} p_0 \right)^2 \frac{k}{\delta_k}$$  \hspace{1cm} (3.62)

A dimensionless entropy generation rate can be obtained after dividing Eq. (3.61) by Eq. (3.62) and can be expressed as

$$S = \frac{\dot{S}_{\text{gen}}}{\dot{S}_0} = \left[ \frac{\nabla T_m}{\nabla T_{cr}} - 1 \right]^2 \frac{\sqrt{\sigma}}{(1 + \sqrt{\sigma} \varepsilon)}.$$  \hspace{1cm} (3.63)

Equation (3.63) can be expressed further in terms of $Ha_\delta$ as

$$\frac{S}{f_2(\sigma, \varepsilon)} = \left[ \frac{I^*}{(1 + DaHa_\delta^2)} - 1 \right]^2,$$  \hspace{1cm} (3.64)

where $f_2(\sigma, \varepsilon) = \sqrt{\sigma} / (1 + \sqrt{\sigma} \varepsilon)$. The dimensionless global entropy generation ($S / f_2(\sigma, \varepsilon)$) is plotted in Fig. 3.8 as a function of $I^*$ and $Ha_\delta$ using Eq. (3.64). The ranges of $I^*$ and $Ha_\delta$ are considered from 0 to 2.0 and 0 to 20, respectively.
For \( T^* > 1 \) (i.e., engine mode), \( S / f_2(\sigma, \delta) \) is maximum when \( Ha_\delta = 0 \). \( S / f_2(\sigma, \delta) \) decreases with the increase of \( Ha_\delta \) as long as \( Ha_\delta < 10 \). However, \( S / f_2(\sigma, \delta) \) increases with increasing \( Ha_\delta \) when \( Ha_\delta > 10 \). \( S / f_2(\sigma, \delta) \) is minimum when \( Ha_\delta \) is closed to 10.

For \( T^* < 1 \) (i.e., Heat pump mode), \( S / f_2(\sigma, \delta) \) increases with increasing \( Ha_\delta \) when \( 10 < Ha_\delta < 100 \). However, \( S / f_2(\sigma, \delta) \) decreases with the increase of \( Ha_\delta \) as long as \( 10 > Ha_\delta > 10 \).

3.8.7 EFFICIENCY

The efficiency (\( \eta \)) of a thermoacoustic prime mover can be calculated by taking the ratio of absolute value of Eqs. (3.56) and (3.51):

\[
\eta = \frac{\dot{W}_2}{\dot{Q}_2} = \frac{\beta \mu f \Delta x}{K \rho_m C_p} \left( 1 + Da Ha_\delta^2 \right)
\]

Equation (3.65) can be expressed in terms of \( \nabla T_{cr} \) (see Eq. (3.23)) as shown below

\[
\eta = \frac{\nabla T_m \Delta x}{T_m} = \frac{\Delta T}{\Gamma T_m} = \frac{\eta_c}{\Gamma}
\]

where \( \eta_c \) is the Carnot efficiency and \( \Gamma \) is the ratio of \( \nabla T_m \) and \( \nabla T_{cr} \). Section 8.1 already described that a thermoacoustic system will work as a prime mover when \( \nabla T_m > \nabla T_{cr} \). Thus, the value of \( \Gamma \) must be greater than 1 for the prime mover mode of operation. Therefore, the efficiency of a prime mover using the described stack configuration is less than \( \eta_c \) and it depends on the value of \( \Gamma \). Now for a fixed value of \( \nabla T_m \), \( \Gamma \) can be varied by varying \( \nabla T_{cr} \). The larger value of \( \Gamma \) reduces the value of \( \eta \) and the smaller value of \( \Gamma \) increases the efficiency. According to Eq. (3.23), the magnitude of \( \nabla T_{cr} \) is higher as \( Ha_\delta \) increases. Therefore, larger value of \( Ha_\delta \) reduces the value of \( \Gamma \) and increases the value of \( \eta \). The value of \( Ha_\delta \) can be increased until \( \nabla T_{cr} = \nabla T_m \). For this limiting case (i.e., \( \nabla T_{cr} = \nabla T_m \)), the work flux is zero which is seen in Eq. (3.65).
3.9. Summary

In this paper, the stack of a thermoacoustic device as a porous medium with a thick solid plate attached to it have been modelled. The stack is exposed to an oscillating, compressible and inviscid fluid. A magnetic field is applied to the oscillating flow transversally. The aim of the current research effort is to incorporate the influence of magnetic force on the thermoacoustic effect for such stack. The governing equations for the fluid flow and temperature were simplified using a first order linear perturbation technique. In the limit of a large transverse distance \((y \to \infty)\), the fluctuating temperature approaches \(y\)-independent free stream temperature inside the porous medium. It was found that \(H a_{\delta}\) has a significant effect on \(\nabla T_{cr}\) which plays an important role in determining the system’s mode of operation, for a relatively higher \(Da(=0.1)\) value. \(H a_{\delta}\) does not have any effect on Nusselt number. It is observed that the dimensionless heat flux increases significantly with increasing \(H a_{\delta}\) when \(H a_{\delta} < 4.5\), while dimensionless work flux decreases with increasing \(H a_{\delta}\). Entropy generation is minimum when \(H a_{\delta}\) is close to 10. The efficiency was calculated using the heat and work flux equation. The efficiency of a thermoacoustic prime mover is maximum when \(\nabla T_{cr} \approx \nabla T_m\). It was shown that the \(\nabla T_{cr}\) can be controlled by \(H a_{\delta}\). Therefore, \(H a_{\delta}\) can be a control mechanism to maximize the thermoacoustic prime mover’s performance. The effect of magnetic field on the complex Nusselt number of a multi-plate thermoacoustic system and how the magnetic field effect on the heat transfer rate in a thermoacoustic system are shown in the next chapter.
Chapter 4

**EFFECT OF MAGNETIC FIELD ON NUSSELT NUMBER OF A MULTI-PLATE THERMOACOUSTIC SYSTEM**

4.1. **INTRODUCTION**

This chapter presents the results of heat transfer of a multi-plate thermoacoustic system using complex Nusselt number in the presence of magnetic field. Assuming the applied magnetic field is perpendicular to the direction of the oscillating fluid flow, the expressions for the fluctuating velocity and temperature from the governing unsteady-compressible-viscous forms of the continuity, momentum, and energy equations are derived. These equations are simplified assuming small amplitude oscillations, a long wave, and a short stack. The hydrodynamic and thermal boundary layers are considered to be very small compared to the acoustic wavelength, and the longitudinal conduction heat transfer inside the boundary layers is assumed to be negligible. Both bulk mean and space averaged temperatures are considered as reference temperatures in the analytical solution. The complex Nusselt number equations are simplified and expressed as a function of the Hartmann number ($Ha_\delta$), the Swift number ($S_w$), the modified Swift number ($\bar{S}_w$) and Rott’s functions ($f_v$ and $f_k$). The effect of $Ha_\delta$, $S_w$, and $\bar{S}_w$ on the Nusselt number is analyzed and presented graphically for both viscous and inviscid fluids. The value of Nusselt number in the boundary layer limit for the inviscid fluid is also analyzed. In the absence of a magnetic field, the simplified complex Nusselt number expression that is obtained by using the space averaged temperature as a reference temperature is compared with the data available in the literature and an excellent agreement is observed. This study will offer insight into ways to increase convection heat transfer rate, consequently help the thermoacoustic system designer to design a more power dense thermoacoustic system.

4.2. **PROBLEM FORMULATION**

Fig. 4.1(a) presents the schematic diagram of the multi-plate stack system with heat exchanges inside a resonant chamber of the proposed thermoacoustic system. The acoustic
driver, placed at one end of the resonant chamber, creates a fluid oscillation inside the resonant chamber as well as in the gap between the parallel plate stack. Fig. 4.1(b) presents the calculation domain and the corresponding coordinate frame to model and analyze the flow and thermal fields for the proposed thermoacoustic system. Cartesian coordinate system has been considered for this analysis and locate the origin at the left midpoint. The calculation domain consists of two parallel plates each having a length $L$ and are spaced $2y_0$ from each other. The plate length is aligned with the $x$-axis which is the direction of the fluctuating fluid as well. The $y$-axis is considered to be perpendicular to the plate, and the $z$-axis is aligned with the plate width. The applied magnetic field is assumed to be parallel and directed to the positive $y$-axis. Since two-dimensional analyses are carried out in this paper, the variables in the $z$-direction are assumed to be constant. The hydrodynamic and thermal interactions of oscillating compressible fluid with the stack walls and the presence of magnetic field produce several thermoacoustic effects, e.g., heat flux and work flux. The details of the modeling are presented in Sub-sections 4.2.1 to 4.2.4.

Figure 4.1: (a) A multi-plate thermoacoustic system, (b) coordinate frame associated with this system.
For a thermoacoustic system, heat transfer rate from the stack wall can be calculated using the complex Nusselt number. Heat transfer from the stack wall to the pulsating fluid is mainly governed by convection heat transfer, which can be calculated using [White [2008]]

\[
\dot{Q} = h (T_w - T_{ref}),
\]

where \(\dot{Q}, A, h, T_w, \) and \(T_{ref}\) are the rate of the convective heat transfer, total heat transfer area, the convective heat transfer coefficient, wall temperature, and a reference temperature. By establishing a balance between the conduction and convection at a particular location of the wall exposed to the fluid, one can express the convective heat transfer coefficient as

\[
h = \frac{k}{(T_w - T_{ref})} \left. \frac{\partial T_i}{\partial y} \right|_{y=y_0}.
\]

The following definition for the dimensionless Nusselt number is used in this paper:

\[
Nu = \frac{h D_h}{k},
\]

where \(D_h\) is the hydraulic diameter which is the ratio of the cross-sectional area to wetted perimeter of the stack channel. After substituting the expressions of \(D_h\) and \(h\) into Eq. (4.3), Eq. (4.3) can be re-expressed as

\[
Nu = \frac{4 y_0}{(T_w - T_{ref})} \left. \frac{\partial T_i}{\partial y} \right|_{y=y_0}.
\]

As can be seen from Eq. (4.4), an expression of fluctuating temperature \((T_i)\) is required prior to determine \(Nu\). However, \(T_i\) depends on the fluctuating velocity \((u_i)\). Therefore, the flow and thermal field’s analyses need to be carried out first before identifying the expression of \(Nu\).

### 4.2.1 Fluid Flow Analysis

The mass conservation equation for the current problem can be written as [Bejan [2003]]
\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0,
\] (4.5)

where \( \rho \), \( \mathbf{V} \), and \( t \) are the density, velocity vector (i.e., \( \mathbf{V} = u \hat{i} + v \hat{j} + w \hat{k} \)) and time, respectively. The momentum conservation can be written as

\[
\rho \frac{D \mathbf{V}}{D t} = -\nabla p + \mu \nabla^2 \mathbf{V} + F_{em},
\] (4.6)

where \( p \), \( \mu \), and \( F_{em} \) are the pressure, viscosity of the fluid, and electromagnetic volume force, respectively. The term \( F_{em} \) is given by

\[
F_{em} = \mathbf{J} \times \mathbf{B},
\] (4.7)

where \( \mathbf{J} \) is the volume current density, and \( \mathbf{B} \) is the magnetic induction. The term \( \mathbf{J} \) can be expressed as \( \sigma (\mathbf{E} + \mathbf{V} \times \mathbf{B}) \) [Mahmud and Fraser [2006a]] where \( \sigma \) is the electrical conductivity of the fluid and \( \mathbf{E} \) is the electrical field intensity. The term \( F_{em} \) is the sum of two forces: \( \sigma (\mathbf{E} \times \mathbf{B}) \) and \( \sigma (\mathbf{V} \times \mathbf{B}) \times \mathbf{B} \). The relative directions of \( \mathbf{E} \) and \( \mathbf{B} \) determine the sign of the force \( \sigma (\mathbf{E} \times \mathbf{B}) \) and consequently this force may accelerates or decelerates the fluid flow. The force \( \sigma (\mathbf{V} \times \mathbf{B}) \times \mathbf{B} \), which is a cross product of the back emf (\( \sigma (\mathbf{V} \times \mathbf{B}) \)) and magnetic induction (\( \mathbf{B} \)), always decelerates the fluid flow. As the direction of the applied magnetic field is assumed to be parallel to the \( y \)-axis, the vector \( \mathbf{B} \) is reduced to \( B_y \hat{j} \). Thus, the simplified version of the momentum equation in the \( x \)-direction for our problem can be written as

\[
\rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \sigma u B_y^2.
\] (4.8)

The velocity, pressure, and density terms in Eq. (4.8) are periodic and functions of time. To capture certain thermoacoustic physical quantities such as density, temperature, pressure, and velocity, only the first order terms of the fluctuating parts of these periodic functions can be considered according to the Rott’s linearized thermoacoustic theory [Rott [1980]]. For linearization, the following assumptions are made:

a) parallel plates are perfectly rigid and stationary;

b) nonlinear effects (e.g., turbulence and streaming) are neglected;
c) only the first order terms of all variables are considered

d) the acoustic wavelength ($\lambda$) is much larger than the penetration depths ($\delta_x$ and $\delta_y$)

e) the acoustic wavelength ($\lambda$) is much larger than the space between the plates ($2y_o$)

f) the acoustic wavelength ($\lambda$) is much longer than the stack length ($L$)

g) the pressure of the fluctuating fluid is a function of $x$-axis only

h) the mean fluid velocities are zero ($u_m$ and $v_m$).

The physical quantities can be expanded into their mean values (terms with the subscripts 'm') and time varying quantities (term with the subscripts '1') as shown below [Rott [1980]]:

$$
\rho = \rho_m + \rho_1(x)e^{i\omega t}, \quad T = T_m + T_1(x,y)e^{i\omega t}, \quad p = p_m + p_1(x)e^{i\omega t},$

$$
u = u_m + u_1(x,y)e^{i\omega t}, \quad v = v_m + v_1(x,y)e^{i\omega t},$

(4.9)

The term $\exp(i\omega t)$ represents the time dependency of any physical quantity in Eq. (4.9) where the angular frequency ($\omega$) can be expressed as $2\pi f$ with $f$ being the frequency of the fluid oscillation.

After substituting Eq. (4.9) into Eq. (4.8) and keeping only the first order terms, thus, Eq. (4.8) can be further simplified to the following form:

$$
\frac{\partial^2 u_1}{\partial y^2} - \left( \frac{i\omega}{v} + \frac{\sigma B^2}{\mu} \right) u_1 = \frac{1}{\mu} \frac{\partial p_1}{\partial x}.
$$

(4.10)

In order to obtain Eq. (4.10) from Eq. (4.8) the following scales are considered: $x \sim \tilde{\lambda}$ where $\tilde{\lambda} = \lambda / 2\pi$, $u \sim u_1$, $v \sim v_1$, and $y \sim \delta_y$. Therefore, at steady state, one obtains $u_1 / \tilde{\lambda} \sim v_1 / \delta_y$ from Eq. (4.5) which results in $v_1 \sim (\delta_y / \tilde{\lambda})u_1$. One obtains $u_1 \gg v_1$ because of the $\delta_y << \tilde{\lambda}$ assumption.

A closed form solution to Eq. (4.10) can be obtained by applying the following boundary conditions: (a) symmetry condition at the channel centerline, i.e., at $y = 0$, $\partial u_1 / \partial y = 0$.
and (b) no-slip condition at the channel wall, i.e., at \( y = y_0, u_i = 0 \). After solving the Eq. (4.10) and further simplifying, the following expression for \( u_i \) is thus obtained

\[
u_i = \frac{i}{\rho_m \omega(1 + Ha_s^2 / 2i)} \frac{\partial p_i}{\partial x} \left\{ 1 - \frac{\cosh[(1+i) \sqrt{1+Ha_s^2 / 2i S_w Y}]}{\cosh[(1+i) \sqrt{1+Ha_s^2 / 2i S_w}]} \right\},
\]

(4.11)

where \( Ha_s, S_w, \) and \( Y \) are the Hartmann number \( (= B_s \delta_v \sqrt{\sigma / \mu}) \), Swift number \( (= y_0 / \delta_v) \), and dimensionless transverse distance \( (= y / \delta_v) \), respectively. The dimensionless number \( Ha_s \) is the ratio of the Lorentz force to the viscous force. For a given flow, larger values of \( Ha_s \) indicate that larger magnetic fields are applied to a system. The parameter \( \delta_v \) \( (= \sqrt{2v / \omega}) \) is the viscous penetration depth which is a measure of the lateral momentum diffusion in a characteristic time interval \( (= 2 / \omega) \). The characteristic time interval can be calculated from the period of oscillation \( (\tau = 2\pi / \omega) \) divided by \( \pi \). The parameter \( S_w \) is a measure of the narrowness or the wideness of the channel. An expression of the average velocity \( (u_{i,av}) \) can be obtained from Eq. (4.11) by using the following integration

\[
u_{i,av} = (y_0)^{-1} \int_0^{y_0} u_i dy = \frac{i}{\rho_m \omega(1 + Ha_s^2 / 2i)} \frac{\partial p_i}{\partial x} \left\{ 1 - \frac{\tanh[(1+i) \sqrt{1+Ha_s^2 / 2i S_w}]}{(1+i) \sqrt{1+Ha_s^2 / 2i S_w}} \right\}.
\]

(4.12)

Eqs. (4.11) and (4.12) represent the final analytical solutions for the fluctuating and average velocities for fluid inside a multi-plate thermoacoustic system in the presence of a transverse magnetic field. These complex velocity expressions will be utilized next to derive analytical expressions for the fluctuating temperature and the complex Nusselt number.

### 4.2.2 Thermal Field Analysis

The energy equation for the proposed thermoacoustic system can be written as [Mahmud and Fraser [2006a]]

\[
\rho C_p \left[ \frac{\partial T}{\partial t} + \mathbf{V} \cdot \nabla T \right] = k \nabla^2 T + \beta T \frac{Dp}{Dt} + \left| \frac{\mathbf{J}}{\sigma} \right|^2 + \mu \Phi,
\]

(4.13)
where \( C_p, k, \beta \), and \( \Phi \) are the specific heat of the fluid, thermal conductivity, thermal expansion coefficient, and viscous dissipation function, respectively. Considering the assumptions stated in Sec 4.2.2 and substituting the expansions given in Eq. (4.9) into Eq. (4.13), the energy equation can further be simplified to the following form:

\[
\frac{\partial^2 T_1}{\partial y^2} - \left( \frac{i \omega}{\alpha_f} \right) T_1 = \frac{\nabla T_m}{\alpha_f} u_i - \frac{i \omega \beta T_m}{\rho_m C_p \alpha_f} p_1, \tag{4.14}
\]

where \( \alpha_f = k / \rho_f C_p \) is the thermal diffusivity of the fluid. After substituting \( u_i \) from Eq. (4.11) into Eq. (4.14) and obtained

\[
\frac{\partial^2 T_1}{\partial y^2} - \left( \frac{i \omega}{\alpha_f} \right) T_1 = \frac{\nabla T_m}{\alpha_f \rho_m \omega(1 + \frac{H a_s^2}{2i})} \frac{\partial p_1}{\partial x} \left\{ 1 - \frac{\cosh[(1 + i)\sqrt{1 + \frac{H a_s^2}{2i}S_y}]}{\cosh[(1 + i)\sqrt{1 + \frac{H a_s^2}{2i}S_y}]} \right\}
\]

\[
- \frac{i \omega \beta T_m}{\rho_m C_p \alpha_f} p_1. \tag{4.15}
\]

The following boundary conditions are applied to solve Eq. (4.15): (a) symmetry condition at the channel centerline, i.e., at \( y = 0 \), \( \partial T_1 / \partial y = 0 \) and (b) no fluctuating temperature at the channel wall (thin plate approximation [Swift [1988]]), i.e., at \( y = y_0, T_i = 0 \). After solving the Eq. (4.15) and further simplifying it, the following expression for \( T_i \) is obtained:

\[
T_i = \frac{\beta T_m p_1}{\rho_m C_p} - \frac{\nabla T_m \nabla p_1}{\rho_m \omega(1 + \frac{H a_s^2}{2i})} \left\{ 1 - \frac{\Pr}{\Pr - \frac{H a_s^2}{2i} - 1} \frac{\cosh[(1 + i)\sqrt{1 + \frac{H a_s^2}{2i}S_y}]}{\cosh[(1 + i)\sqrt{1 + \frac{H a_s^2}{2i}S_y}]} \right\}
\]

\[
- \left\{ \frac{\beta T_m p_1}{\rho_m C_p} + \frac{1}{\Pr - \frac{H a_s^2}{2i} - 1} \frac{\nabla T_m \nabla p_1}{\rho_m \omega^2} \right\} \frac{\cosh[(1 + i)\sqrt{\Pr S_y}]}{\cosh[(1 + i)\sqrt{\Pr S_y}]} , \tag{4.16}
\]

where \( \Pr = (\delta_v^2 / \delta_k^2) \) is the Prandtl number of the fluid. The space averaged temperature \((T_{i,ave})\) is calculated following the similar procedure of calculating \( u_{i,ave} \) and is given by the following equation:

\[
T_{i,ave} = \frac{\beta T_m p_1}{\rho_m C_p} (1 - f_k) - \frac{\nabla T_m \nabla p_1}{\rho_m \omega^2} \left\{ \frac{\left( \Pr - \frac{H a_s^2}{2i} - 1 \right)}{\Pr - \frac{H a_s^2}{2i} - 1} \frac{1 + \frac{H a_s^2}{2i}}{\left(1 + \frac{H a_s^2}{2i}\right)} \right\} . \tag{4.17}
\]

The parameters \( f_v \) and \( f_k \) in Eq. (4.17) can be expressed as
\[
    f_v = \frac{\tanh[(1 + i) \sqrt{1 + \frac{Ha_\beta^2}{2i} S_w}]}{(1 + i) \sqrt{1 + \frac{Ha_\beta^2}{2i} S_w}}, \tag{4.18a}
\]

\[
    f_k = \frac{\tanh[(1 + i) \sqrt{\frac{Pr S_w}{S_w}}]}{(1 + i) \sqrt{\frac{Pr S_w}{S_w}}}. \tag{4.18b}
\]

In the absence of a magnetic field, Eqs. (4.18a) and (4.18b) are reduced to Rott’s thermoacoustic functions [Rott [1980]- Swift [2002]].

The bulk mean temperature \(T_b\) can be calculated using Eq. (4.19) [Burmeister [1993]] as follows:
\[
    T_b = \frac{1}{u_{1,av} A} \int_A u_1 T_1 dA = \frac{1}{u_{1,av} y_0} \int_0^{y_0} u_1 T_1 dy. \tag{4.19}
\]

After substituting Eqs. (4.11), (4.12), and (4.16) into Eq. (4.19), an expression of \(T_b\) can be calculated in the following form
\[
    T_b = \left[ \frac{\beta T_m p_i}{\rho_m C_p} - \frac{\nabla T_m \nabla p_i}{\omega^2 \rho_m (1 + Ha_\beta^2 / 2i)} \right] - \frac{\nabla T_m \nabla p_i}{\omega^2 \rho_m (1 + Ha_\beta^2 / 2i)} \left( \frac{Pr}{Pr - Ha_\beta^2 / 2i - 1} \right) \left( 1 - f_v^2 \frac{f_v}{1 - f_v} \Phi_v \right) \\
- \left[ \frac{\beta T_m p_i}{\rho_m C_p} - \frac{\nabla T_m \nabla p_i}{\omega^2 \rho_m (Pr - Ha_\beta^2 / 2i - 1)} \left( \frac{1 + Ha_\beta^2 / 2i}{Pr - Ha_\beta^2 / 2i - 1} \right) \left( f_v - f_k \right) \right], \tag{4.20}
\]

where \( \Phi_v = 2i (1 + Ha_\beta^2 / 2i) S_w^2 \).

### 4.2.3 Calculation of the Nusselt Number

An analytical expression for the complex Nusselt number \((Nu)\) can be obtained after substituting Eq. (4.16) into Eq. (4.4) and is given by
\[
    Nu = \left[ \frac{4}{T_w - T_{ref}} \right] \left[ \frac{\nabla T_m \nabla p_i}{\omega^2 \rho_m (Pr - Ha_\beta^2 / 2i - 1)} \left( \frac{Pr}{1 + Ha_\beta^2 / 2i} \Phi_v f_v - \Phi_k f_k \right) - \frac{\beta T_m p_i}{\rho_m C_p} \Phi_k f_k \right], \tag{4.21}
\]

where \( \Phi_k = 2i \Pr S_w^2 \). A suitable \( T_{ref} \) is required to find the final expression of \( Nu \) from Eq. (4.21). In the heat transfer literature, the bulk mean or the mixing-cup temperature \((T_b)\) [Burmeister [1993]] is used as \( T_{ref} \) to calculate \( Nu \) for the steady-state channel flow problem. Mahmud and Fraser [2005b] considered \( T_b = T_{ref} \) for a multi-plate thermoacoustic system. In contrast, Liu and Garrett [2006] used \( T_{ref} = T_{1,av} \) for calculating
with the justification of relatively easy experimental measurement of $T_{i,av}$ [Wilen [1998]]. The complex Nusselt number is further discussed in Section 4.3.3.

4.3. **RESULTS AND DISCUSSIONS**

In this section, graphical results for the derived thermoacoustic parameters in Sec. 4.2.2 to Sec. 4.2.4 (e.g., fluctuating velocity, temperature, and the complex Nusselt number) are presented and interpreted. For comparison with the existing literature and better interpretation purposes, these derived expressions are also simplified using appropriate assumptions (e.g., no magnetic field, boundary layer, narrow channel, etc.). Thermoacoustic stacks are also assumed to be placed inside the quarter acoustic wavelength ($0 \leq x \leq \lambda/4$) to avoid the sign confusion (+ or -) in the calculation. Most of the results are presented in dimensionless forms for the derived thermoacoustic parameters. However, in case if it is required, the thermophysical properties of the helium at the mean temperature (=298 K) is considered. However, the mean density is calculated from the mean pressure using ideal gas law.

4.3.1 **DISCUSSION ON THE FLOW FIELD**

The expression of $u_1$ (given in Eq. (4.11)) is a product of a $y$-independent component (outside the curly bracket) as well as a $y$-dependent component (inside the curly bracket). The hyperbolic cosine functions in the $y$-dependent part, which include $S_w$ and $Y$, approach to zero when $S_w$ approaches to a very large value (i.e., wide channel limit). For this specific case of $S_w \rightarrow \infty$, $u_1$ reduces to its $y$-independent part as shown in the following equation

$$u_{1,S_w \rightarrow \infty} = \frac{i}{\rho_m \omega (1 + \frac{Ha_s^2}{2i})} \frac{\partial p_1}{\partial x}. \quad (4.22)$$

The $y$-independent part of the velocity can also be considered as the boundary layer limit of the velocity expression because it is obtained by setting a very large value of $S_w$. In the limit of a very large $Ha_s$ (i.e., $Ha_s \rightarrow \infty$), the magnitude of $u_1$ approaches to zero.
On the other hand, the standing wave velocity expression \( u_i^* \) [Swift [1988]] can be recovered from Eq. (4.22) by setting \( Ha_\delta \to 0 \) and is given by

\[
\Delta u_i = \frac{\Delta p_1}{\rho_m \omega} \frac{\partial p_1}{\partial x}.
\]

The dimensionless fluctuating velocity \( U \) expression can be obtained by dividing Eq. (4.11) by Eq. (4.12). Thus,

\[
U = \frac{(1+i)\sqrt{1+Ha_\delta^2/2i S_w}}{(1+i)\sqrt{1+Ha_\delta^2/2i S_w} - \tanh[(1+i)\sqrt{1+Ha_\delta^2/2i S_w}] \cosh[(1+i)\sqrt{1+Ha_\delta^2/2i S_w}]}.
\]

In the absence of a magnetic field (i.e., \( Ha_\delta = 0 \)), Eq. (4.24) reduces to a velocity profile that is reported in the literature [Swift [1988], Mahmud and Fraser [2005b – Swift [2002]] for a multi-plate thermoacoustic system.

Fig. 4.2(a) presents \( U \) as a function of \( Y \) for different values of \( S_w (= 0.1, 0.5, 1, 1.5, 2, 2.5, 3, 5, 10, 20, and \infty) \) where the applied magnetic field is zero (i.e., \( Ha_\delta = 0 \)). The velocity boundary layer thickness decreases with increasing \( S_w \). For a relatively larger \( S_w \) (i.e., \( S_w \geq 5 \)), \( U \) becomes independent of \( Y \) within a few \( \delta_{\lambda} \); a thin shear layer is formed and a large portion of the fluid is unaffected by the influence of viscosity and boundary wall. As \( S_w \) approaches to infinity, the boundary layer thickness becomes zero. In contrast to larger \( S_w \), the velocity boundary layer thickness increases with decreasing \( S_w \). For a relatively smaller \( S_w \) (i.e., \( S_w \leq 3 \)), the profile of \( U \) shows similar profile as Poiseuille flow velocity profile [Landau and Lifshitz [1982]].

However, in order to show the impact of Swift number on the velocity profile in the presence of magnetic field, Fig. 4.2(b) has been added. Fig. 4.2(b) presents the effect of the applied magnetic field on \( U \) as a function of \( Y \) for different \( S_w (=0.1, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 5.0, and \infty) \). The magnitude of \( Ha_\delta \) is set to 3.0. It is observed that the variation in boundary layer thickness with increasing or decreasing \( S_w \) is similar to Fig. 4.2(a).
Figure 4.2: (a) Dimensionless velocity as functions of Swift numbers in the absence of magnetic field (b) Dimensionless velocity as functions of Swift numbers for $Ha_\delta=3.0$
In order to explain the influence of magnetic field on $U$, Fig. 4.3 shows the variation in $U$ as a function of $Y$ for different $Ha_\delta (=0, 1, 3, 5, 10, 30, \text{ and } \infty)$ at constant $S_w (=1)$. The boundary layer thickness decreases with increasing $Ha_\delta$, and it approaches zero when $Ha_\delta \rightarrow \infty$. Therefore, the trend observed in Fig. 4.3 implies that the external magnetic field imposes a resistance to the fluid flow. A closed observation of Eq. (4.11) reveals that the $y$-independent part of $u_1$ decreases with increasing $Ha_\delta$. The $y$-dependent part of $u_1$ contains the hyperbolic cosine functions and approaches to zero for a large value of $Ha_\delta$. Therefore, the combine effect of $y$-independent and dependent parts causes the velocity boundary layer thickness decreases with increasing $Ha_\delta$.

### 4.3.2 Discussion on the Thermal Field

The expression of $T_1$, given by Eq. (4.16), consists of three parts. The $y$-independent first part is the contribution from the adiabatic compression or expansion of the fluid. The $y$-dependent second and third parts contain hyperbolic cosine functions with $S_w$ and $Y$. 

![Figure 4.3: Dimensionless velocity as functions of Hartmann numbers for $S_w = 1.0$](image)
which represent the temperature amplitude. The \( y \)-dependent parts approach to zero when \( S_w \) approaches to large values. For this special case, Eq. (4.16) can be simplified further to

\[
T_0 = \frac{\beta T_m p_1}{\rho_m C_p} - \frac{\nabla T_m \nabla p_1}{\omega^2 \rho_m (1 + Ha_S^2 / 2i)} = T_{ad} - \frac{T_{sw}}{(1 + Ha_S^2 / 2i)}. \tag{4.25}
\]

In Eq. (4.25), \( T_{ad} = \beta T_m p_1 / \rho_m C_p \) represents the fluctuating temperature generated due to an adiabatic compression or expansion of the oscillating fluid and \( T_{sw} = \nabla T_m \nabla p_1 / \omega^2 \rho_m \) is similar to a standing wave temperature amplitude [Swift [1988]]. The appearance of \((1 + Ha_S^2 / 2i)\) terms signifies the influence of the magnetic field on the temperature amplitude.

In Eq. (4.25), the values of the parameters \( \beta, \rho_m, p_1, \nabla p_1, \omega, \) and \( C_p \) can be set in such a way that results in \( T_0 = 0 \). The corresponding temperature gradient can be termed as a critical temperature gradient \((\nabla T_{cr})\) and is given by

\[
\nabla T_{cr} = \frac{p_1}{\nabla p_1} \frac{\beta T_m \omega^2}{C_p} (1 + Ha_S^2 / 2i).
\tag{4.26}
\]

The ratio of \( \nabla T_{cr} \) to \( \nabla T_m \) is known as the temperature gradient ratio and is used to determine the mode of operation of a thermoacoustic system [Swift [1988]]. Typically, a heat pump mode is observed when \( \nabla T_m < \nabla T_{cr} \), while a prime mover mode is observed when \( \nabla T_m > \nabla T_{cr} \) [Swift [1988]]. Thus, for a given \( \nabla T_m \) value, \( \nabla T_{cr} \) plays an important role defining the mode of thermoacoustic system’s operation. It is observed in Eq. (4.26) that \( \nabla T_{cr} \) is proportional to the square of \( Ha_S \). Therefore, the magnitude of the \( \nabla T_{cr} \) increases when \( Ha_S \) increases. In the limit of a small Hartmann number (i.e., \( Ha_S \to 0 \)), the expression of the fluctuating temperature reduces to

\[
T_i = \frac{\beta T_m p_1}{\rho_m C_p} - \frac{\nabla T_m \nabla p_1}{\rho_m \omega^2} \left[ \frac{1}{Pr} \cosh[(1 + i) S_w] \cosh[(1 + i) S_Y] \right] - \frac{1}{Pr} \frac{\nabla T_m \nabla p_1}{\rho_m \omega^2} \cosh[(1 + i) \sqrt{Pr} S_w] \cosh[(1 + i) \sqrt{Pr} S_Y]. \tag{4.27}
\]
A similar expression is available in the literature [Swift [1988], Mahmud and Fraser [2005b]] for a thermoacoustic system within the thin plate stack limit.

The dimensionless temperature ($\Theta$) is presented in Fig. 4.4 and Fig. 4.5. An expression of $\Theta$ can be obtained after dividing Eq. (4.16) by Eq. (4.17) and is given by

$$
\Theta = \frac{(\Gamma_0 - 1)+ \left[ \frac{\Pr}{\Pr- Ha_\delta^2 / 2i - 1} \right] \cosh[(1+i)\sqrt{1+ Ha_\delta^2 / 2i S_w Y}] - \left[ \frac{1+ Ha_\delta^2 / 2i}{Pr- Ha_\delta^2 / 2i - 1} + \Gamma_0 \right] \cosh[(1+i)\sqrt{Pr S_w Y}]}{\cosh[(1+i)\sqrt{Pr S_w Y}]},
$$

where $\Gamma_0$ is the temperature gradient ratio ($=\nabla T_r / \nabla T_m$), where $\Gamma_0$ is the inverse of an ordinary temperature gradient ratio[Swift [1988]]. $\Gamma_0$ can be expressed as follows:

$$
\Gamma_0 = \frac{p_i}{\nabla p_i} \frac{\beta T_m \omega^2}{C_p \nabla T_m} (1+ Ha_\delta^2 / 2i).
$$

The term $\Gamma_0$ in Eq. (4.29) defines the mode of operation of a thermoacoustic system. The values of $\Gamma_0$ that are greater than 1 represents heat pump mode and smaller than 1 represents prime mover mode of thermoacoustic operation.

Fig. 4.4 presents $\Theta$ as a function of $Y$ for different $S_w (=0.1, 0.5, 1, 1.5, 2, 2.5, 3, 5, 10, 20, \text{ and } \infty)$ in the absence of magnetic field (i.e. $Ha_\delta = 0$). The magnitude of $\Gamma_0$ is set to 2.0 based on Swift’s [1988] recommendation of having a value of $\Gamma_0$ close to unity for an efficient thermoacoustic device operation. The magnitude of $\Theta$ is zero at the wall due to the imposed thermal boundary condition. At a relatively larger $S_w$ (i.e., $S_w \geq 5$), the imposed wall thermal boundary condition does not have any effect on the fluctuating fluid temperature beyond a few thermal penetration depths during a period of oscillation; a thinner thermal boundary layer is observed. As $S_w$ approaches to infinity, the thermal boundary layer thickness becomes zero (i.e., $y$-independent). In contrast to a larger $S_w$,
Figure 4.4: Dimensionless temperature as a function of Swift Number

Figure 4.5: Dimensionless temperature as a function of Swift and Hartmann Number
for a value of $S_w \leq 2$, the thermal boundary layer increases with decreasing the value of $S_w$, and the profile looks parabolic from the wall to the centerline of the channel.

Fig. 4.5 presents the effect of the applied magnetic field on $\Theta$ as a function of $Y$ for different $S_w$ ($=0.5, 3.5, \text{and} 10$) at a constant $\Gamma_0 (=2)$. Four different values of $Ha_\delta (=0, 0.1, 1, \text{and} 10)$ are considered for each $S_w$. It is observed that the centerline temperature decreases with increasing $Ha_\delta$ at relatively lower $S_w (=0.5)$. In contrast, for $S_w = 3.5$, the centerline temperature increases with an increase in $Ha_\delta$. Thermal boundary layer thickness increases when $Ha_\delta$ increases. At relatively higher $S_w (=10)$, the shape of the temperature profile is not changed significantly with increasing $Ha_\delta$. $Ha_\delta$ does not have significant effect on thermal boundary layer. A closed observation of Eq. (4.16) shows that $T_1$ has three parts. The first part is independent from $y$ and the 2nd and 3rd parts dependent on $y$. For a given value of $S_w$, the $y$-dependent parts of $T_1$ approaches to zero for a large value of $Ha_\delta$ (see Eq. (4.25)) and the generated $T_1$ is due to an adiabatic compression or expansion of the oscillating fluid become independent of boundary layer thickened.

4.3.3 DISCUSSION ON THE COMPLEX NUSSELT NUMBER

In this section, a discussion on the complex Nusselt number for the proposed thermoacoustic system is presented. The working fluid inside the thermoacoustic system undergoes a periodic oscillatory motion with simultaneous compression and expansion while exchanging heat with the stack wall. The temperature gradient, $\partial T_1 / \partial y\big|_{y=y_0}$, at the stack wall is not always in phase with the wall to reference temperature difference $(T_w - T_{ref})$ which is a typical characteristic of the steady-state channel flow [Liu and Garrett [2006]]. Moreover, the value of $\partial T_1 / \partial y\big|_{y=y_0}$ increases with decreasing boundary layer thickness as observed in Figs. 4.4 and 4.5. Therefore, it is necessary to investigate the complex Nusselt number further to better understand of the heat transfer phenomenon in a multi-plate magnetic thermoacoustic system. The complex Nusselt number considering both $T_{1,av}$ and $T_b$ as $T_{ref}$ is discussed in the subsequent sections.
4.3.3.1 Complex Nusselt Number Considering the $T_{1,av}$ as $T_{ref}$

In this section, the space averaged temperature ($T_{1,av}$) is selected as a reference temperature in the $Nu$ expression in Eq. (4.21). After substituting $T_{1,av}$ from Eq. (4.17) into Eq. (4.21) as $T_{ref}$ and considering negligible $T_w$ compared to the $T_{ref}$, the simplified expression for the $Nu_{av}$ can be written as follows:

$$Nu_{av} = 4 \times \frac{Pr \Phi_v f_v - \Phi_k f_k (1 + Ha_0^2 / 2i)}{Pr - Ha_0^2 / 2i - 1} - \Gamma_0 \Phi_k f_k \frac{Pr - Ha_0^2 / 2i - 1 - Pr f_v + f_k (1 + Ha_0^2 / 2i)}{(Pr - Ha_0^2 / 2i - 1) - \Gamma_0 (1 - f_k)}.$$  \hspace{1cm} (4.30)

The two limiting values of $\Gamma_0$ (i.e., $\Gamma_0 \to 0$ and $\Gamma_0 \to \infty$) are considered to interpret $Nu_{av}$ in Eq. (4.30) for the two operational modes.

For the case of $\Gamma_0 \to 0$, $\nabla T_m >> \nabla T_{cr}$, which is a case of a thermoacoustic prime mover. For this special case, Eq. (4.30) can be modified to

$$Nu_{av}' = 8i(1 + Ha_0^2 / 2i) Pr S_w^2 \frac{f_v - f_k}{Pr - Ha_0^2 / 2i - 1 - Pr f_v + f_k (1 + Ha_0^2 / 2i)}.$$  \hspace{1cm} (4.31)

The real part of $Nu_{av}'$ obtained from Eq. (4.31) is plotted in Fig. 4.6 as a function of $S_w$ for five different values of $Ha_0$ ($=0, 2, 3, 5, \text{ and } \infty$). The value of $Nu_{av}'$, is observed 10 for $Ha_0 \to 0$ and 12 for $Ha_0 \to \infty$ when $S_w < 1$. The value of $Nu_{av}'$ increases to a great extent when $S_w > 1$. For a given $Ha_0$, the magnitude of $Nu_{av}'$ does not increase significantly in the narrow channel limit (i.e., $S_w << 1$). In the boundary layer limit, the magnitude of $Nu_{av}'$ increases significantly for all values of $Ha_0$. 


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In the absence of a magnetic field Eq. (4.31) can be written as:

\[
Nu'' = 4 \times 2i \cdot Lc_k \left\{ \frac{(1 - f_v)(1 - Pr)}{(1 - f_k) - Pr(1 - f_v)} - 1 \right\}, \tag{4.32}
\]

where \( Lc_k \) (\( = \sqrt{Pr \cdot \delta_w} \)) is the Lautrec number used by Liu and Garrett [2006]. Eq. (4.32) can be compared to Eq. (37) of Liu’s and Garrett’s [2006] published article on the single pore geometry. It should be noted that Liu and Garrett [2006] used \( r_h = y_0 \); whereas, in this research \( r_h = 4y_0 \) is used.

For the special case of an inviscid fluid without the presence of magnetic field, \( Ha_d, \delta_v, \) Pr, and \( f_v \) approach to zero and Eq. (4.31) can be reduced further to

\[
Nu'_{av-inv} = \frac{8iS_w^2 f_k}{1 - f_k}, \tag{4.33}
\]
where $\tilde{S}_w (= y_0 / \delta_k)$ is the modified Swift number. In case of a very large $\tilde{S}_w (\rightarrow \infty)$, the parameter $f_k$ in Eq. (4.33) can be expressed as

$$f_k\big|_{\tilde{S}_w \rightarrow \infty} = \frac{\tanh[(1+i)\tilde{S}_w]}{(1+i)\tilde{S}_w} \approx \frac{1}{(1+i)\tilde{S}_w}.$$  

Eq. (4.34) can be used to obtain the boundary layer limit of $Nu'_{\text{av, inv}}$. Substituting Eq. (4.34) into Eq. (4.33) and using the Taylor series expansion while keeping the first order term only, the boundary layer limit of $Nu'_{\text{av, inv}}$ can be expressed as

$$Nu'_{\text{av, inv, \infty}} = \frac{8i\tilde{S}_w^3}{(1+i)\tilde{S}_w - 1} \approx 4(1+i)\tilde{S}_w + 4.$$  

Fig. 4.7 presents the real and imaginary parts of Eq. (4.33). The real part of the $Nu'_{\text{av, inv}}$ is much larger than the imaginary part at $\tilde{S}_w << 1$. It implies that $\partial T_1 / \partial y|_{y=y_0}$ is in phase with $T_w - T_{\text{av}}$ when $\tilde{S}_w << 1$. Both real and imaginary parts of $Nu'_{\text{av, inv}}$ increase with increasing $\tilde{S}_w$ when $\tilde{S}_w > 2$. The nearly equal magnitude of the real and imaginary parts of $Nu'_{\text{av, inv}}$ at $\tilde{S}_w > 2$ implies that the phase difference between $\partial T_1 / \partial y|_{y=y_0}$ and $T_w - T_{\text{av}}$ is $\pi/4$ in the boundary layer limit. The real part of Eq. (4.35) is also plotted in Fig. 4.7 for comparison purpose. The agreement between $Nu'_{\text{av, inv}}$ and $Nu'_{\text{av, inv, \infty}}$ is excellent when $\tilde{S}_w > 2$. A decrease in the value of $\delta_k$ increases the wall temperature gradient which results in increasing magnitude of the Nusselt number. Liu and Garrett [2006] also observed a similar trend for the inviscid fluid.

The phase information for the complex Nusselt number where the $T_{T,av}$ is considered as $T_{\text{req}}$ is plotted in Figs. 4.8 (a) and (b). Fig. 4.8 (a) shows that $\partial T_1 / \partial y|_{y=y_0}$ is in phase with $T_w - T_{\text{av}}$ when $S_w = 0.5$ and $Ha_\delta = 0$. Fig. 4.8(b) shows that the phase difference between $\partial T_1 / \partial y|_{y=y_0}$ and $T_w - T_{\text{av}}$ is $\pi/4$ when $S_w = 30$. 

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Figure 4.7: The profiles of complex Nusselt number for the inviscid fluid when space averaged temperature is considered as reference temperature at different modified Swift numbers.
Figure 4.8: Temperature gradient at the stack plate surface \( (\partial T_y / \partial y)_{y=y_0} \) and the difference between the stack wall and the space averaged temperature \( (T_w - T_{1,av}) \) as a function of time for (a) \( S_w = 0.5 \) and (b) \( S_w = 30 \) when \( Ha_\delta = 0 \).

For the case of \( \Gamma_0 \to \infty, \nabla T_m \ll \nabla T_{cr} \), which represents the heat pump mode of a thermoacoustic system. For this special case, Eq. (4.30) can be modified to

\[
Nu''_{av} = \frac{8iS^2_w f_k}{1 - f_k}.
\]  

Eq. (4.36) is identical to Eq. (4.33). The real and imaginary part of Eq. (4.36) is shown in Fig. (4.7). It can be concluded from Eq. (4.36) that the magnetic field has no influence on \( Nu''_{av} \) if space averaged temperature is considered as reference temperature while the thermoacoustic system is operating as a heat pump.

However, to determine the range of \( \Gamma_0 \) over which \( Ha_\delta \) has an influence on the complex Nusselt number, Fig. 4.9 has been plotted using Eq. (4.30). Fig. 4.9 shows the surface and contour plots of \( Nu''_{av} \) as functions of \( \Gamma_0 \) and \( Ha_\delta \) for \( S_w = 0.5 \). The surface plot shows the variation in the magnitude of \( Nu''_{av} \) with \( \Gamma_0 \) and \( Ha_\delta \).
Figure 4.9: The real part of complex Nusselt number in Eq. (4.30) as functions of $\Gamma_0$ and $Ha_\delta$ when $S_w = 0.5$

The contour plot provides information of the combination of $\Gamma_0$ and $Ha_\delta$ for which $Nu''_{av}$ is constant. The values of $\Gamma_0$ and $Ha_\delta$ are varied from 1.5 to 10 and 0 to 100, respectively. The value of $Nu''_{av}$ increases when $Ha_\delta$ increases for $Ha_\delta \leq 10$. It is observed that the value of $Nu''_{av}$ decreases when $Ha_\delta$ increases for $Ha_\delta > 10$. The maximum value of $Nu''_{av}$ is observed when $\Gamma_0 = 1.5$ and $Ha_\delta = 10$. A negligible increase in the value of $Nu''_{av}$ is observed when $\Gamma_0 = 10$ and $Ha_\delta = 10$. At a constant value of $Ha_\delta (=10)$, $Nu''_{av}$ decreases with increasing $\Gamma_0$. Thus, the effect of $Ha_\delta$ on the value of $Nu''_{av}$ is minimal when $\Gamma_0 > 10$.

**4.3.3.2 Complex Nusselt Number Considering the $T_b$ as $T_{ref}$**

In this section, $T_b$ is selected as $T_{ref}$ to calculate $Nu$ expression. After substituting Eq. (4.20) into Eq. (4.21) the expression of the Nusselt number becomes
The $Nu_b$ expression in Eq. (4.37) is also simplified and interpreted for the two limiting values of $\Gamma_0$; i.e., $\Gamma_0 \to 0$ and $\Gamma_0 \to \infty$.

For the case of $\Gamma_0 \to 0$, Eq. (4.37) can be modified to

$$Nu_b' = 4 \frac{2i(1 + Ha_b^2/2i)S_w^2 Pr f_v - 2i Pr S_w^2 f_i(1 + Ha_b^2/2i)}{Pr - Ha_b^2/2i - 1} \frac{Pr - Ha_b^2/2i - 1}{1 + \frac{Pr}{Pr - Ha_b^2/2i}} \left(1 - \frac{f_v}{1 - f_v} \right)^2 \left(1 - \frac{f_i}{1 - f_i} \right).$$

Fig. 4.10 presents the real part of Eq. (4.38) as a function of $S_w$ for different $Ha_b (=0, 1, 2, 3, 5, \text{ and } \infty)$. It is observed that the magnitude of $Nu_b'$ increases with increasing $S_w$.

For a given $S_w$, the magnitude of $Nu_b'$ is higher at higher value of $Ha_b$. 

$Nu_b = -4 \frac{Pr \phi f_v - \phi_i f_i \left(1 + Ha_b^2/2i\right)}{Pr - Ha_b^2/2i - 1} - \Gamma_0 \phi_i f_i.$

(4.37)
Figure 4.10: The real part of the complex Nusselt number in Eq. (4.38) at different Swift and Hartmann numbers when $\Gamma_0 \rightarrow 0$.

For the special case of an inviscid fluid without the presence of magnetic field, $Ha_\delta$, $\delta_v$, $Pr$, and $f_v$ approach to zero and Eq. (4.38) can be reduced further to

$$Nu_{b,\text{inv}}' = \frac{8i\bar{S}^2 f_k}{1+f_k},$$  \hspace{1cm} (4.39)

The boundary layer limit of $Nu_{b,\text{inv}}'$ can be expressed by substituting Eq. (4.34) into Eq. (4.39).

$$Nu_{b,\text{inv},w}' = \frac{8i\bar{S}^2}{(1+i)\bar{S}_w + 1} \approx 4(1+i)\bar{S}_w - 4.$$  \hspace{1cm} (4.40)

The real and imaginary parts of Eq. (4.39) are plotted in Fig. 4.11. The imaginary part of the $Nu_{b,\text{inv}}'$ shows higher values than the real part at $\bar{S}_w \ll 1$. It implies that $\partial T_i/\partial y|_{y=y_0}$ and $T_w - T_k$ are nearly 90 degree out of phase and there is no heat transfer between the stack wall and the working fluid when the value of $\delta_k$ is much larger compared to the channel width ($y_0$). A larger thermal penetration depth is observed
compared to \( y_0 \) in Figs. 4.4 and 4.5 at a relatively lower \( S_w \) which reduces the value of \( \frac{\partial T_i}{\partial y} \bigg|_{y=y_0} \).

In contrast, \( \delta_k \) is much smaller than \( y_0 \) for a higher \( S_w (\geq 2) \) and in such a case \( Nu'_{b, inv} \) approaches to its boundary layer limit. The phase difference between the \( \frac{\partial T_i}{\partial y} \bigg|_{y=y_0} \) and \( T_w - T_b \) is \( \pi/4 \) in the boundary layer limit. In this case value of the real and imaginary parts are same. Fig. 4.11 also shows the real part of \( Nu'_{b, inv, x} \) in Eq. (4.40) has the similar trend with the real part of Eq. (4.39).

Figs. 4.12 (a) and (b) have been plotted considering \( T_b \) as \( T_{ref} \) when \( Ha_\delta = 0 \). Fig. 4.12 (a) shows that \( \frac{\partial T_i}{\partial y} \bigg|_{y=y_0} \) and \( T_w - T_b \) are nearly 90 degree out of phase when \( S_w = 0.5 \).
Fig. 4.12 (b) shows that the phase difference between the $\partial T_t/\partial y|_{y=y_0}$ and $T_w-T_b$ is $\pi/4$ when $S_w = 30$.

The maximum phase difference between the $\partial T_t/\partial y|_{y=y_0}$ and $T_w-T_{ref}$ is observed in Fig. 4.12(a) when $S_w = 0.5$ and $Ha_\delta=0$. $Ha_\delta$ can be used to reduce the phase difference. The influence of $Ha_\delta$ on these phase is shown in Fig. 4.13. It is observed from Fig. 4.13 that $\partial T_t/\partial y|_{y=y_0}$ is in phase with $T_w-T_b$ when $S_w = 0.5$ for $Ha_\delta=50$. This implies that the real part of the complex Nusselt number is much larger than the imaginary part and results in higher rate of heat transfer.

For the case of $\Gamma_0 \to \infty$, Eq. (4.37) can be simplified to

$$Nu''_b = \frac{8i Pr S^2 f_k}{1-\left(\frac{1+Ha_\delta^2/2i}{Pr-Ha_\delta^2/2i-1}\right)\left(f_v-f_k\right)}.$$  \hspace{1cm} (4.41)

Eq. (4.41) shows that $Nu''_b$ is proportional to the square of $S_w$. Therefore, the magnitude of $Nu''_b$ is higher at higher $S_w$. To understand the functional relationships of $Nu''_b$ with $S_w$ and $Ha_\delta$, the real part of Eq. (4.41) is plotted in Fig. 4.14 as a function of $S_w$ for five different values of $Ha_\delta$ ($=0, 2, 3, 5, 50, \text{and } \infty$). A variation in $Nu''_b$ values is observed for different $Ha_\delta$ when $S_w \leq 1$. The value of $Nu''_b=10$ for $Ha_\delta \to 0$ and $Nu''_b=12$ for $Ha_\delta \to \infty$ when $S_w \leq 1$. A change in the value of $Ha_\delta$ has an insignificant impact on the magnitude of $Nu''_b$ when $S_w > 1$. However, the magnitude of $Nu''_b$ increases significantly in the boundary layer limit regardless the value of $Ha_\delta$.

In the case of an inviscid fluid without a magnetic field, Eq. (4.41) can be further reduced to

$$Nu''_{b\_inv} = \frac{8iS^2 f_k}{1-f_k}.$$  \hspace{1cm} (4.42)

Eq. (4.42) is similar as Eq. (4.33). The real and imaginary parts of Eq. (4.42) are presented in Fig. 4.7. The trend of Nusselt number is described for both narrow channel and boundary layer limits in the previous section.
Figure 4.12: Temperature gradient at the stack plate surface \( \frac{\partial T_i}{\partial y} |_{y=y_0} \) and the difference between the stack wall and the bulk mean temperature \( T_w - T_b \) as a function of time for (a) \( S_w = 0.5 \) and (b) \( S_w = 30 \) when \( H\alpha = 0 \).
Figure 4.13: Temperature gradient at the stack plate surface \( \left( \frac{\partial T_i}{\partial y}\big|_{y=y_0} \right) \) and the difference between the stack wall and the bulk mean temperature \( (T_w - T_b) \) as a function of time for \( S_w = 0.5 \) when \( H_a = 50 \).

Figure 4.14: The real part of complex Nusselt number in Eq. (4.41) at different Swift and Hartmann numbers when \( \Gamma_0 \to \infty \).
4.4. **Summary**

In this chapter, an analytical solution for the complex Nusselt number was derived for a magnetic thermoacoustic system. The complex Nusselt number was calculated using both space averaged and bulk mean temperatures as reference temperatures. The influence of the transverse magnetic field and Swift number on the complex Nusselt number was analyzed and presented graphically.

The complex Nusselt number expression that was derived using the space averaged temperature as a reference temperature is analyzed for two distinct modes of operation: prime mover and heat pump. For the case of a prime mover mode, the real part of the complex Nusselt number increases with increasing magnetic force. For a specific value of the magnetic field, the real part of the complex Nusselt number is constant when the value of the Swift or modified Swift number is less than one. However, a sharp increase in the real part of the complex Nusselt number was observed when the value of the Swift or modified Swift number was greater than one. The complex Nusselt number is also calculated for the boundary layer limit for the inviscid fluid. The complex Nusselt number expression was simplified by neglecting magnetic force to compare our results with Liu and Garrett’s [2006] results an excellent agreement was obtained. In contrast, for the case of heat pump mode, a magnetic field has insignificant influence on the complex Nusselt number but a similar trend was observed for different modified swift number. Two different trends were observed for two distinct modes of operation when the the bulk mean temperature is used as a reference temperature. For both of the modes of operation, the real part of the complex Nusselt number increases with increasing Hartmann number. However, for the heat pump mode, the real part of the complex Nusselt number is constant when the value of the Swift or modified Swift number is less than one. It was also observed that magnetic force has a relatively minor effect on the value of the real part of the complex Nusselt number, whereas the plate spacing has a significant effect on the real part of the complex Nusselt number regardless the mode of operations and the way of calculations (i.e., \( T_{w,av} \) or \( T_{ref} \) ). It was also observed that magnetic force can reduce the phase difference between \( \partial T_1/\partial y \bigg|_{y=y_0} \) and \( T_w - T_h \) resulting in higher heat transfer.
Mahmud and Fraser [2006a] considered a single plate magnetic system thermoacoustic system. They considered free steam temperature as a reference temperature and studied the Nusselt number distribution (one time period) as a function of time. Using proposed approach, the heat transfer behavior of a magnetic thermoacoustic system can be predicted with good accuracy using this study when the magnetic field and the space between the plates are known. The results of this paper can shed light on designing an efficient thermoacoustic system. As a future work, this work can be extended by developing an experimental setup to validate the developed analytical solution. The effect of magnetic field on the energy, work, and heat fluxes are described in the next chapter.
Chapter 5

INFLUENCE OF A MAGNETIC FIELD ON THE ENERGY, HEAT, AND WORK FLUXES OF A MULTI-PLATE THERMOACOUSTIC REFRIGERATOR

5.1 INTRODUCTION

This chapter describes research on clean and efficient energy conversion is extremely important to mitigate the high price of fossil fuel and its adverse effects on the environment. Thermoacoustic is a clean energy conversion technology that uses the conversion of acoustic to thermal energy and vice versa. However, the efficient conversion of acoustic to thermal energy using thermoacoustic systems (e.g., engine, refrigerator, or heat pump) demands research on working fluids, operational, and geometric parameters. The present study is a contribution to improve the efficiency of a thermoacoustic heat pump by introducing a magnetic field perpendicular to the direction of the oscillating fluid. The major focus of this study is to examine the effect of a magnetic field on three important performance parameters; energy, heat, and work fluxes of a multi-plate thermoacoustic heat pump. Initially, analytical expressions for the fluctuating velocity and temperature are derived from the governing continuity, momentum, and energy equations by applying the first order perturbation technique and solving the equations. The derived first order analytical equations for the fluctuating velocity and temperature enable us to calculate the energy, heat, and work flux and are expressed in terms of dimensionless Hartmann number ($Ha$), temperature gradient ratio ($\Gamma_0$), Swift number ($S_w$), Prandlt number ($Pr$), and modified Rott’s and Swift’s parameters ($f_c$ and $f_k$). It is observed that the normalized energy flux density increases with increasing $Ha$ and $\Gamma_0$ when $S_w < 1.5$. The heat flux and work flux densities also increase with increasing $Ha$ and $\Gamma_0$ when $S_w < 1.5$ and decrease when $Ha > 1.5$. The findings of this research will provide useful information to thermoacoustic system’s designer for the development of efficient magnetic thermoacoustic heat pumps.
5.2 Problem Formulation

In the current study, a two-dimensional, unsteady-state, and compressible fluid is considered flowing through a parallel-plate channel. Fig. 5.1(a) presents the schematic diagram of the proposed thermoacoustic system. The acoustic driver is placed at the open end of the resonant tube and a transverse magnetic field is applied parallel to the $y$-axis. Fig. 5.1(b) shows the calculation domain. Cartesian coordinate system has been considered for this analysis. The origin is considered at the middle point of the left side as shown in Fig. 5.1 (b). The stack of parallel plate channel is placed inside one end of the closed resonant tube. The stack length, parallel to the $x$-axis, is to be $L$, and width, parallel to the $z$-axis, is to be $\Pi/2$, and the gap between two parallel plates, parallel to the $y$-axis, is to be $2y_0$. In the presence of a magnetic field, the hydrodynamic and thermal interactions between the stack wall and the oscillating compressible fluid produce several thermoacoustic effects (i.e., energy, heat flux, and work flux). The physics behind such effects can be modelled by the continuity, momentum, and energy equations.

![Figure 5.1](image_url)

Figure 5.1: (a) A multi-plate thermoacoustic system, (b) coordinate frame associated with this system.
The continuity equation for the current problem can be written as [White [2008]]:

\[
\frac{\partial \rho}{\partial t} + \rho \frac{\partial u}{\partial x} + \rho \frac{\partial v}{\partial y} = 0. \tag{5.1}
\]

The momentum equations in the time domain can be written as:

\[
\rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + (J \times B), \quad \text{and} \tag{5.2}
\]

\[
\rho \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + (J \times B). \tag{5.3}
\]

The energy equation in the time domain can be written as [Mahmud and Fraser [2006a]]:

\[
\rho C_p \left[ \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right] = k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \beta T \left( \frac{\partial p}{\partial x} + u \frac{\partial p}{\partial x} + v \frac{\partial p}{\partial y} \right) + \frac{|J|^2}{\sigma} + \mu \Phi, \tag{5.4}
\]

where \( \Phi = 2 \left[ \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 + \left( \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right)^2 - \frac{2}{3} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)^2 \right]. \]

In Eqs. (5.1)-(5.4), \( \rho \), \( u \), \( v \), \( t \), \( T \), \( p \), \( \mu \), \( J \), \( B \), \( C_p \), \( k \), \( \sigma \), \( \beta \), and \( \Phi \) are the density of the fluid, axial velocity, transverse velocity, time, temperature, pressure, viscosity of the fluid, volume current density, magnetic induction, fluid specific heat, thermal conductivity, electrical conductivity, thermal expansion coefficient, and the viscous dissipation function, respectively. The last two terms (i.e., Joule heating and viscous dissipation terms) on the right hand side of Eq. (5.4) are of second order and hence are neglected in the further calculations.

### 5.3 Flow and Thermal Field Analysis

Equations (5.1) - (5.4) are linearized to model the oscillating fluid and temperature inside the stack. The analytical solutions of fluctuating velocity, temperature, and pressure are calculated using the following assumptions: a). Stack, composed of parallel plates, are stationary and perfectly rigid; b). stack length \( (L) \ll \) acoustic wave length \( (\lambda) \); c). the thermal penetration depth \( \delta_t \ll \lambda \); d). the viscous penetration depth \( \delta_v \ll \lambda \); e). the gap between two adjacent parallel plates \( 2y_0 \ll \lambda \); f). the mean fluid velocities \( u_m \) and \( v_m = 0 \); g). turbulence and streaming effects are neglected; and h). only the first order
terms of all variables are considered. In Eqs. (5.1) - (5.4), all the dependent variables and the physical quantities can be expanded using Rott’s [1980] thermoacoustic approximations as follows:

\[ \rho = \rho_m + \rho_i(x)e^{i\omega t}, \quad T = T_m + T_i(x,y)e^{i\omega t}, \quad p = p_m + p_i(x)e^{i\omega t}, \]

\[ u = u_m + u_i(x,y)e^{i\omega t}, \quad v = v_m + v_i(x,y)e^{i\omega t} \]

(5.5)

where the first and second part of the right hand side of each equation are the mean and time varying quantities. \( e^{i\omega t} \) expresses the time dependency of any physical quantity, \( \omega \) (= \( 2\pi f \)) is the angular frequency, and \( f \) is the frequency of the oscillating fluid.

A scale analysis is carried out to derive the simplified form of analytical solutions. For the scale analysis, the following scales are considered: \( x \sim \tilde{\lambda} \) where \( \tilde{\lambda} = \lambda / 2\pi \), \( y \sim \delta_v \), \( u \sim u_i \), and \( v \sim v_i \). Considering the above assumptions, at steady state, from the continuity equation one can obtain \( u_i / \tilde{\lambda} \sim v_i / \delta_v \). Since \( \delta_v \ll \tilde{\lambda} \), it is obvious that the axial velocity \( (u_i) \) is much larger than the transverse velocity \( (v_i) \) of the oscillating fluid. By substituting Eq. (5.5) into the momentum equations, Eqs. (5.2) and (5.3) and the energy equation, Eq. (5.4) and using above scale analysis, the \( x \) momentum (Eq. (5.2)) and energy (Eq. (5.4)) equations can further be simplified to the following forms:

\[ \frac{\partial^2 u_i}{\partial y^2} - \left( \frac{i\omega}{\nu} + \frac{\sigma B_i^2}{\mu} \right) u_i = \frac{1}{\mu} \frac{\partial p_i}{\partial x} \quad \text{and} \]

\[ \frac{\partial^2 T_i}{\partial y^2} - \left( \frac{i\omega}{\alpha_f} \right) T_i = \frac{\nabla T_m}{\alpha_f} u_i - \frac{i\omega \delta T_m}{\rho \alpha_f C_p} p_i. \]

(5.6)

(5.7)

The solution to Eq. (5.6) can be obtained after applying the boundary conditions: (a) at \( y = 0 \), \( \partial u_i / \partial y = 0 \) and (b) at \( y = y_o \), \( u_i = 0 \). After solving and simplifying Eq. (5.6), the following expression for \( u_i \) is obtained:

\[ u_i = \frac{i}{\rho \omega (1 + Ha_\delta^2 / 2i)} \frac{\partial p_i}{\partial x} \left[ 1 - \frac{\cosh[(1 + i) \sqrt{1 + Ha_\delta^2 / 2i} S_Y]}{\cosh[(1 + i) \sqrt{1 + Ha_\delta^2 / 2i} S_Y]} \right], \]

(5.8)

where \( S_Y \), \( Y \), and \( Ha_\delta \) are Swift number (= \( y_o / \delta_v \)), dimensionless transverse distance (= \( y / y_o \)), and the Hartmann number (= \( B_s \delta_v \sqrt{\sigma / \mu} \)), respectively. The viscous penetration depth, \( \delta_v (= \sqrt{2\nu / \omega}) \), is a measure of the lateral momentum diffusion in a
characteristic time interval \(= 2/\omega\). The parameter \(S_w\) is a measure of the wideness or narrowness of the channel between two successive parallel plates of the stack, and the parameter \(Ha_\delta\) is the ratio of the Lorentz force to the viscous force. As \(Ha_\delta\) is proportional to \(B_y\), higher the \(Ha_\delta\) higher the magnetic fields are applied to a system.

The solution to Eq. (5.7) can be obtained by substituting \(u_t\) from Eq. (5.8) into Eq. (5.7) and using the boundary conditions: (c) at \(y = 0\), \(\partial T_1 / \partial y = 0\) and (d) at \(y = y_0\), \(T_1 = 0\) (thin plate approximation [Swift [1988]]). After solving and simplifying, the following expression for \(T_1\) is obtained

\[
T_1 = \frac{\beta T_m p_t}{\rho_mC_p} - \frac{\nabla T_m \nabla p_t}{\rho_m \omega^2 (1 + Ha_\delta^2 / 2i)} \left\{ 1 - \frac{Pr}{Pr - (Ha_\delta^2 / 2i) - 1} \frac{\cosh[(1 + i) \sqrt{1 + Ha_\delta^2 / 2i} \ S_w]}{\cosh[(1 + i) \sqrt{1 + Ha_\delta^2 / 2i} \ S_w]} \right\}
\]

\[ \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad (5.9) \]

where \(Pr = (\delta_y^2 / \delta_k^2)\) is the Prandtl number and \(\delta_k = (2\alpha / \omega)\) is the fluid and thermal penetration depth. The parameter \(\delta_k\) is the lateral thermal diffusion that is measured in a characteristic time interval.

The temperature gradient in the stack is the thermoacoustic effect of a thermoacoustic refrigerator. The temperature gradient creates temperature fluctuation in the working fluid. Such temperature fluctuation becomes zero at a distance from the plate for a certain temperature gradient which is known as critical temperature gradient \((\nabla T_{cr})\). The equation for the \(\nabla T_{cr}\) can be derived from the expression of \(T_1\), given by Eq. (5.9). The \(y\)-dependent parts of Eq. (5.9) approach to zero for a large value of \(S_w\).

Thus, Eq. (5.9) is reduced to

\[
T_\infty = \frac{\beta T_m p_t}{\rho_mC_p} - \frac{\nabla T_m \nabla p_t}{\omega^2 \rho_m (1 + Ha_\delta^2 / 2i)} = T_{ad} - \frac{T_{\infty w}}{(1 + Ha_\delta^2 / 2i)}.
\]

\[ \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad (5.10) \]

The right hand side of Eq. (5.10) has two parts. The first part is to calculate the adiabatic compression or expansion of the oscillating fluid \((\beta T_m p_t / \rho_mC_p)\), and the second part is to calculate the temperature amplitude. In the absence of magnetic field, the expression for \(T_\infty\) (Eq. (5.10)) reduce to Swift’s [1988] equation. In Eq. (5.10), the stack mean
temperature gradient can be expressed in terms of other parameters after setting \( T_\infty = 0 \). Such temperature gradient is known as a critical temperature gradient \( \nabla T_{cr} \)

\[
\nabla T_{cr} = \frac{P_1}{\nabla P_1} \frac{\beta T_m \omega^2}{C_p} \left(1 + Ha_\delta^2 / 2i\right). \tag{5.11}
\]

In Eq. (5.11), the \( \nabla T_{cr} \) is a function of \( Ha_\delta \). \( \nabla T_{cr} \) is proportional to the square of \( Ha_\delta \). Therefore, the magnitude of the \( \nabla T_{cr} \) increases/decreases when \( Ha_\delta \) increases/decreases.

An interesting relationship is observed between \( \nabla T_{cr} \) and \( \nabla T_m \). For a thermoacoustics system operating as a prime mover, \( \nabla T_{cr} < \nabla T_m \). On the other hand, for a thermoacoustic heat pump/refrigerator \( \nabla T_{cr} > \nabla T_m \) [Swift’s [1988]]. The ratio of \( \nabla T_{cr} \) to \( \nabla T_m \) is termed as the temperature gradient ratio \( \Gamma_0 \), which can be expressed as follows:

\[
\Gamma_0 = \frac{P_1}{\nabla P_1} \frac{\beta T_m \omega^2}{C_p \nabla T_m} \left(1 + Ha_\delta^2 / 2i\right). \tag{5.12}
\]

It is obvious from the above discussion that the thermoacoustic mode of operation can be determined by calculating the value of \( \Gamma_0 \). Such as for a heat pump/refrigerator mode, \( \Gamma_0 > 1 \), prime mover mode, \( \Gamma_0 < 1 \), and useless mode, \( \Gamma_0 < 0 \). In the useless mode, the system itself absorbs the produced heat and work flux. The current study further shows that the thermoacoustic mode of operation can be changed by changing the value of \( Ha_\delta \).

As this study is on a thermoacoustic refrigerator/heat pump, the values of \( \Gamma_0 > 1 \) are used to calculate the energy flux densities.

### 5.4 Wave Equation

The presence of the stack in a thermoacoustic system modified the standing wave. Such modification creates two important thermoacoustic effects: a time-averaged heat flux and work flux. Both the thermoacoustic effects generate near the surface of the stack [Swift’s [1988]]. The longitudinal directional fluctuating pressure can be calculated by constructing a wave equation. The wave equation can be obtained from the continuity, momentum, and state equation. The general governing continuity equation can be written as follows:
\[
\frac{\partial (\rho)}{\partial t} + \rho \left[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right] + \left[ u \frac{\partial \rho}{\partial x} + v \frac{\partial \rho}{\partial y} \right] = 0
\] 
(5.13)

By substituting Eq. (5.5) into Eq. (5.13) and considering the assumptions of Section 5.3, Eq. (5.13) can further simplify into the following form:

\[
i\omega \rho_i + \frac{\partial}{\partial x} (\rho_m u_i) + \rho_m \frac{\partial v_i}{\partial y} = 0,
\]
(5.14)

Equation (5.6) is further simplified to solve Eq. (5.14) to the following form

\[
u_i = -\frac{1}{i\omega \rho_m (1 + Ha_s^2 / 2i)} \frac{\partial p_1}{\partial x} + \frac{\mu}{i\omega \rho_m (1 + Ha_s^2 / 2i)} \frac{\partial^2 u_i}{\partial y^2}
\]
(5.15)

After substituting Eq. (5.15) into Eq. (5.14), one can obtain the following expression:

\[
\omega^2 \left(1 + Ha_s^2 / 2i\right) \rho_1 - i \omega \rho_m \left(1 + Ha_s^2 / 2i\right) \frac{\partial v_i}{\partial y} = \frac{\partial}{\partial x} \left( \mu \frac{\partial^2 u_i}{\partial y^2} \right) - \frac{\partial^2 p_1}{\partial x^2}
\]
(5.16)

Using the thermodynamic relation, \( \rho_1 = \rho_m \beta T_1 + (\gamma / c^2) p_1 \), \( \rho_1 \) can be eliminated from Eq. (5.16) and can be written as follows:

\[
\frac{\partial}{\partial x} \left( \mu \frac{\partial^2 u_i}{\partial y^2} \right) - \frac{\partial^2 p_1}{\partial x^2} + \left(1 + Ha_s^2 / 2i\right) \left[ \omega^2 \rho_m \beta T_1 - \frac{\gamma \omega^2}{c^2} p_1 + i \omega \rho_m \frac{\partial v_i}{\partial y} \right] = 0
\]
(5.17)

By integrating Eq. (5.17) with respect to \( y \) and substituting the boundary condition (e) at \( y = 0, v_i = 0 \) and (f) at \( y = y_o, v_i = 0 \), the differential equation for \( p_1 \) can be expressed as a function of \( x \). Finally, the wave equation becomes

\[
\frac{\partial^2 p_1}{\partial x^2} + \frac{\beta \nabla T_m}{(1 - f_v)} \left\{1 - \frac{\Pr f_v}{\Pr - Ha_s^2 / 2i - 1} + \frac{\left(1 + Ha_s^2 / 2i\right) f_k}{\Pr - Ha_s^2 / 2i - 1} \right\} \frac{\partial p_1}{\partial x}
\]

\[
- \frac{\left(1 + Ha_s^2 / 2i\right)}{k_o^2 (1 - f_v)} (\gamma - 1)(1 - f_k - \gamma) p_1 = 0
\]
(5.18)

In Eq. (5.18), \( k_o \) is the ratio of sound velocity and angular frequency \( (c / \omega) \). The parameters \( f_v \) and \( f_k \) in Eq. (5.18) can be expressed as

\[
f_v = \frac{\tanh[(1 + i)\sqrt{1 + Ha_s^2 / 2i} S_v]}{(1 + i)\sqrt{1 + Ha_s^2 / 2i} S_v},
\]
(5.19a)

\[
f_k = \frac{\tanh[(1 + i)\sqrt{\Pr S_w}]}{(1 + i)\sqrt{\Pr S_w}}.
\]
(5.19b)
In the absence of magnetic field, Eqs. (5.19a) and (5.19b) are reduced to Rott’s thermoacoustic functions [Rott [1980]]. For an inviscid fluid (Pr → 0, f, → 0) without any imposed magnetic field (i.e., Ha = 0) Eq. (5.18) reduces to the following expression

\[ \frac{\partial^2 p_1}{\partial x^2} + \left( \frac{\omega}{c} \right)^2 p_1 = 0 \]  

which is the well known Helmholtz equation [Kinsler et al. [2000]], which is a time independent form of wave equation.

The general solution to Eq. (5.18) can be obtained by assuming the co-efficients of \( \partial p_1 / \partial x \) and \( p_1 \) are constant and written as:

\[ p_1 = C_5 \exp(\phi_1 x) + C_6 \exp(\phi_2 x) \]  

where The \( \phi_1 \) and \( \phi_2 \) can be expressed as

\[ \phi_1 = - \frac{1}{2} \frac{\beta \sqrt{T_m}}{1 - f_\nu} \left\{ 1 - \frac{\text{Pr} f_\nu}{\text{Pr} - \text{Ha}^2 / 2i - 1} + \left( 1 + \frac{\text{Ha}^2 / 2i}{\text{Pr} - \text{Ha}^2 / 2i - 1} \right) f_k \right\} \]

\[ - \frac{1}{2} \frac{\beta^2 (\sqrt{T_m})^2}{(1 - f_\nu)^2} \left\{ 1 - \frac{\text{Pr} f_\nu}{\text{Pr} - \text{Ha}^2 / 2i - 1} + \left( 1 + \frac{\text{Ha}^2 / 2i}{\text{Pr} - \text{Ha}^2 / 2i - 1} \right) f_k \right\}^2 \]  

\[ - \frac{1}{2} \frac{\beta^2 (\sqrt{T_m})^2}{(1 - f_\nu)^2} \left\{ 1 - \frac{\text{Pr} f_\nu}{\text{Pr} - \text{Ha}^2 / 2i - 1} + \left( 1 + \frac{\text{Ha}^2 / 2i}{\text{Pr} - \text{Ha}^2 / 2i - 1} \right) f_k \right\}^2 \]  

\[ - \frac{1}{2} \frac{\beta^2 (\sqrt{T_m})^2}{(1 - f_\nu)^2} \left\{ 1 - \frac{\text{Pr} f_\nu}{\text{Pr} - \text{Ha}^2 / 2i - 1} + \left( 1 + \frac{\text{Ha}^2 / 2i}{\text{Pr} - \text{Ha}^2 / 2i - 1} \right) f_k \right\}^2 \]  

\[ - \frac{1}{2} \frac{\beta^2 (\sqrt{T_m})^2}{(1 - f_\nu)^2} \left\{ 1 - \frac{\text{Pr} f_\nu}{\text{Pr} - \text{Ha}^2 / 2i - 1} + \left( 1 + \frac{\text{Ha}^2 / 2i}{\text{Pr} - \text{Ha}^2 / 2i - 1} \right) f_k \right\}^2 \]  

The value of \( C_5 \) and \( C_6 \) of Eq. (5.21) can be calculated by using the boundary conditions: (a) at the starting point of the stack, i.e., \( x = x_s \), \( p'(x_s) = p_0 \left[ \sin(x_s / \tilde{\lambda}) + i \cos(x_s / \tilde{\lambda}) \right] \) and (b) at the stack exit, i.e., \( x = x_e \), \( p'(x_e) = p_0 \left[ \sin(x_e / \tilde{\lambda}) + i \cos(x_e / \tilde{\lambda}) \right] \). Thus, one can obtain
\[ C_s = \frac{p'(x_1)e^{p_1x} - p'(x_0)e^{p_2x}}{e^{p_1x} - e^{p_2x}}, \quad C_o = \frac{p'(x_1)e^{p_1x} - p'(x_0)e^{p_2x}}{e^{p_1x} - e^{p_2x}}. \]  

(5.24)

Finally, the expression of \( p_1 \) becomes

\[ p_1 = \left[ \frac{p'(x_1)e^{p_1x} - p'(x_0)e^{p_2x}}{e^{p_1x} - e^{p_2x}} \right] e^{p_1x} = \left[ \frac{p'(x_1)e^{p_1x} - p'(x_0)e^{p_2x}}{e^{p_1x} - e^{p_2x}} \right] e^{p_1x}. \]  

(5.25)

5.5 ENERGY FLUX DENSITY

Energy flux density (\( \dot{E} \)) is an important parameter to measure the thermoacoustic system’s performance. In concept, \( \dot{E} \) is the total amount of energy passing through a unit area perpendicular to the direction of the fluid velocity in unit time [Landau and Lifshitz [1982]]. The thermal efficiency of a thermoacoustic device, working in an engine mode, is the ratio of the work flux to the heat flux. Similarly, COP of a thermoacoustic system, working in a refrigerator/heat pump mode, is the ratio of the heat flux to the work flux. The heat flux is defined by the hydrodynamic transport of entropy that carried by the oscillatory velocity of the fluid [Swift’s [1988]]. On the other hand, the work flux is the product of oscillating pressure and the volume of the fluid. The fluid volume is considered to be the plate area times thermal penetration depth which is a measure of lateral thermal diffusion in a characteristics time interval [Swift’s [1988]]. The same fluid volume is considered to calculate the heat and work fluxes. Present study will examine the effect of magnetic field on heat, work, and energy flux densities of a multi-plate thermoacoustic system.

\( \dot{E} \) is a conserved quantity for a thermoacoustic system which can be compared to the mass flux density in an incompressible fluid. In the steady state, the time-averaged energy flux along the \( x \)-direction is independent of \( x \) [Swift’s [1988]] if there is no lateral heat flows to the surroundings. Landau and Lifshitz [1982] introduced the concept of \( \dot{E} \) and developed a general vector form expression for \( \dot{E} \). The general expression of \( \dot{E} \) including a magnetic field is available in Mahmud and Fraser [2007] and is given below:

\[ \dot{E} = \rho v \left( \frac{1}{2} |v|^2 + h \right) - \mathbf{v} \cdot \mathbf{\sigma} - k \nabla T + \frac{1}{\sigma \mu_0} (\mathbf{J} \times \mathbf{B}) + \frac{1}{\mu_0} [\mathbf{B} \times (\mathbf{v} \times \mathbf{B})], \]  

(5.26)
where \( \sigma, v, h, \) and \( \mu_0 \) are the viscous stress tensor, velocity vector, enthalpy, and permeability of the free space, respectively. The right hand side of Eq. (5.26) consists of five terms representing kinetic energy, enthalpy, viscous dissipation, heat conduction, and magnetic energy. Equation (5.26) can be further simplified by neglecting viscous contributions \((v \cdot \sigma)\) and kinetic energy \( (1/2 \rho |v|^2)\) terms since they are third order in velocity [Cao et al. [1996]] and can be written as follows:

\[
\dot{E} \approx \rho u h - \frac{2}{\mu_0} u B_y^2 - k \frac{\partial T}{\partial x}.
\]

In Eq. (5.27), \( h \) is a function of entropy and pressure, i.e., \( h = f(s, p) \), and can be written according to Bejan [2006] as follows:

\[
\frac{dh}{ds}_p = \frac{\partial h}{\partial s} ds + \frac{\partial h}{\partial p} dp = T ds + \frac{dp}{\rho} = C_p dT + (1 - T \beta) \frac{dp}{\rho}.
\]

The value of \( h \) is obtained by integrating Eq. (5.28) with respect to \( s \) and \( p \). Substituting Eq.(5.5) and the thermodynamic relation \( s = (C_p / T_m)T_i - (\beta / \rho_m) p_i \) into Eq. (5.28), Eq. (5.27) can be simplified further to

\[
\dot{E}_2 \approx \rho_m C_p (T_i u_i) + (1 - T_m \beta)(p_i u_i) - \frac{2}{\mu_0} u_i B_y^2 - k \frac{\partial T_m}{\partial x}.
\]

As the applied magnetic field is considered time independent in this paper, the time averaged value of the third term on the right hand side of Eq. (5.29) will be zero. After time and space averaging, the energy flux density Eq. (5.29) can be expressed as follows:

\[
\frac{\dot{E}_i}{\Pi} = \frac{1}{2} \rho_m C_p \Re \left[ \frac{\gamma_0}{\partial y} T_i \tilde{u}_i dy \right] + \frac{1}{2} (1 - \beta T_m) \Re \left[ \frac{\gamma_0}{\partial y} p_i \tilde{u}_i dy \right] - \gamma_0 k \nabla T_m.
\]

where tilde \((\sim)\) denotes the complex conjugation and \( \Re[\ ] \) signifies the real part. After substituting Eq. (5.8) and Eq. (5.9) into Eq. (5.30) and simplifying, the following expression can be obtained
\[
\dot{E}_2 = \frac{v_o \Pi}{2} \left[ -\frac{i \rho_o \omega}{\rho_o \omega} \left( 1 - \frac{H a^2}{2i} \right) \left( 1 - \frac{f_v}{f_k} \right) + \beta T_m \left( 1 - \frac{H a^2}{2i} \right) \left( \frac{f_v}{f_k} - \frac{f_v}{f_k} \right) \right] + \frac{i C_p \nabla T_m \nabla p_v \nabla \tilde{p}}{\rho_o \omega (1 + H a^2 / 2i)(1 - H a^2 / 2i)} \left( 1 - \frac{f_v}{f_k} \right) - \frac{Pr}{Pr - H a^2 / 2i - 1} \left( f_v - \frac{f_v}{f_k} \right)
\]

(5.31)

By using \( \Gamma_0 \), the normalize global energy flux equation can be expressed as follows:

\[
\frac{\dot{E}_2}{E_0} = \Re \left[ -i \Gamma_0 \left( 1 - \frac{f_v}{f_k} \right) + \beta T_m \left( 1 - \frac{H a^2}{2i} \right) \left( \frac{f_v}{f_k} - \frac{f_v}{f_k} \right) \right] + \frac{i}{\Gamma_0} \left( 1 - \frac{f_v}{f_k} \right) - \frac{Pr}{Pr - H a^2 / 2i - 1} \left( f_v - \frac{f_v}{f_k} \right) - \frac{Pr - H a^2 / 2i - 1}{Pr - H a^2 / 2i - 1} \left( 1 - \frac{H a^2}{2i} \right) \left( \frac{f_v}{f_k} - \frac{f_v}{f_k} \right) - \Gamma_{\text{cond}}
\]

(5.32)

where \( E_0 \) is a reference global energy flux and can be calculated from Eq. (5.31) as follows:

\[
\dot{E}_0 = \frac{v_o \Pi}{2} \frac{C_p \nabla T_m \nabla p_v \nabla \tilde{p}}{\rho_o \omega (1 + H a^2 / 2i)(1 - H a^2 / 2i)}
\]

(5.33)

In Eq. (5.32), \( \Gamma_{\text{cond}} \) is the ratio of axial conduction to \( E_0 \). In the absence of a magnetic field in an inviscid fluid (\( \text{Pr}=0, \delta_v=0 \)), Eq. (5.32) reduces to

\[
\dot{E}_{2,\text{Swift}} = \frac{\Pi \delta_k}{4} \beta T_m \left( u_t \cdot u_t \right) \left[ \frac{1}{\Gamma_0} - 1 \right] - \left[ \Pi v_o k \frac{\partial T_m}{\partial x} \right]_{\text{cond}}.
\]

(5.34)

Equation (5.34) is identical to the energy flux density equation reported in Swift’s [1988].

Work flux is the measure of acoustic power input of a given thermoacoustic refrigerator/heat pump. In order to produce a temperature gradient along the \( x \)-axis, the fluid parcel undergoes a thermal expansion and compression cycle by absorbing work flux. The time average work flux is calculated as acoustic power per unit volume (area×thermal penetration depth) that is absorbed in a period of \( \tau = 2\pi / \omega \) [Swift [1988]].
The time-averaged product of the fluctuating velocity and pressure is integrated with respect to \( y \) from the channel centerline to the wall to calculate global work flux \( \dot{W}_z \) in the following form:

\[
\dot{W}_z = \Pi \int_0^{y_0} \frac{\dot{p}_1 \mu_1 dy}{\rho_m \omega} = -\frac{y_0 \Pi}{2 \rho_m \omega} \Re \left[ \frac{i \dot{p}_1 \Delta \dot{p}_1}{(1 - Ha^2 / 2i)} \left(1 - \bar{f}_v \right) \right]
\]  \hspace{1cm} \text{(5.35)}

In the absence of a magnetic field, the global work flux density expression (Eq. (5.35)) reduces to

\[
\dot{W}_z = -\frac{y_0 \Pi}{2 \rho_m \omega} \Re \left[ i \dot{p}_1 \Delta \dot{p}_1 \left(1 - \bar{f}_v \right) \right]
\]  \hspace{1cm} \text{(5.36)}

which is reported in the thermoacoustic literature [Mahmud and Fraser [2005b]].

The heat flux of a thermoacoustic system can be defined as the hydrodynamic entropy transport by the oscillating fluid along the \( x \)-axis [Swift [1988]]. In this paper the global heat flux is calculated by subtracting the global work flux from the global energy flux. Therefore, the expression of the global heat flux \( \dot{Q}_z \) can be obtained by using Eq. (5.35) from Eq. (5.31) in the following form:

\[
\dot{Q}_z = \frac{y_0 \Pi}{2} \Re \left[ -\frac{i \dot{p}_1 \Delta \dot{p}_1}{\rho_m \omega (1 - Ha^2 / 2i)} \left( \frac{\beta}{Pr} \left( 1 - Ha^2 / 2i \right) \left( \bar{f}_v - f_k \right) \right) \right]
\]  \hspace{1cm} \text{(5.37)}

In the absence of a magnetic field Eq. (5.37) reduces to

\[
\dot{Q}_z = -\frac{y_0 \Pi}{2} \Re \left[ i \dot{p}_1 \Delta \dot{p}_1 \left( \frac{\beta}{Pr + 1} \right) \left( \bar{f}_v - f_k \right) \right] + \frac{y_0 \Pi}{2} \frac{\nabla T_m \nabla \rho_1 \nabla \dot{p}_1}{\rho_m \omega^3} \left[ \frac{Pr}{Pr - 1} \left( \bar{f}_v - f_k \right) \right]
\]  \hspace{1cm} \text{(5.38)}

which is reported in the literature [Mahmud and Fraser [2005b]].
5.6 RESULTS AND DISCUSSION

In this section, a discussion on the energy flux, work flux, and heat flux are presented for the proposed multi-plate thermoacoustic system under the influence of an external magnetic field. The analytical solutions, obtained in Section 5.5, are analyzed graphically. As a reference working fluid, helium is considered in this paper. The thermophysical properties of the helium are calculated at the mean temperature (=298 K). The mean pressure of the fluid inside the system is kept equal to 1 bar and the mean density of the working fluid is calculated at the mean pressure by using the ideal gas law.

5.6.1 DISCUSSION ON THE ENERGY FLUX DENSITY

Figure 5.2 presents the real part of Eq. (5.32) as a function of $S_w$ for different values of $\Gamma_0$ (=3.5, 4.0, 4.5, and 5.0) and $Hd_0 = 0$. Note that the conduction term ($\Gamma_{cond}$) in Eq. (5.32) is neglected while calculating $\dot{E}_2/\dot{E}_0$ in Fig. 5.2. Conduction represents an inherent irreversibility characteristic of a thermoacoustic system and, therefore, poses a negative effect on the energy flux density.

Therefore, minimizing $\Gamma_{cond}$ will increase the performance of a thermoacoustic refrigerator. The narrowness or wideness of the channel between the stacks is measured by the parameter $S_w$. Therefore, relatively smaller values of $S_w$ represent narrower chanled widths, while relatively larger values of $S_w$ represent the opposite. It is observed from Fig. 5.2 that $\dot{E}_2/\dot{E}_0$ is negligible for the narrow channel ($S_w < 0.55$) for all values of $\Gamma_0$. The value of $\dot{E}_2/\dot{E}_0$ increases when $0.55 < S_w < 1.5$ and then decreases when $S_w > 1.5$. For $\Gamma_0 > 1$, the imposed $\nabla T_m$ is small and the value of $\dot{E}_2/\dot{E}_0$ is positive over the range of $S_w$, implies thermoacoustic refrigerator/heat pump effect. A relatively higher value of $\dot{E}_2/\dot{E}_0$ is observed for a relatively higher value of $\Gamma_0$. 

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Figure 5.2: Normalize energy flux density as a function of $S_w$ for different values of $\Gamma_0$ at $Ha_\delta = 0$.

The influence of $Ha_\delta$ on the energy flux density is shown in Fig. 5.3. The real part of Eq. (5.32) is plotted in Fig. 5.3 as a function of $S_w$ for different values of $Ha_\delta (=0, 2, 3, 5, 10, \text{ and } \infty)$ selecting a thermoacoustic system’s functioning in the heat pump/refrigerator mode ($\Gamma_0 = 5$). It is observed that the effects of imposed magnetic field on $\dot{E}_2/\dot{E}_0$ are negligible for the narrow channel ($S_w < 0.25$). It is further observed that the magnitude of $\dot{E}_2/\dot{E}_0$ increases until $S_w \approx 1.5$ and then decreases when $S_w > 1.5$ for all considered values of $Ha_\delta$. In the absence of magnetic field ($Ha_\delta = 0$), the value of $\dot{E}_2/\dot{E}_0$ is zero for $S_w < 0.55$. The maximum variations in $\dot{E}_2/\dot{E}_0$ are observed for the value of $Ha_\delta$ between 0 and 2 for a given $S_w$. 
Figure 5.3: Normalize energy flux density as a function of $S_w$ for a heat pump/refrigerator ($\Gamma_0 = 5$) at different values of $Ha_\delta$.

5.6.2 DISCUSSION ON THE WORK FLUX DENSITY

Figure 5.4 shows the distribution of the real part of $\dot{V}_z$ as a function of $S_w$ for different values of $DR$ (=0.01, 0.02, 0.03, 0.04, 0.05, and 0.06) at $Ha_\delta = 0$. Fig. 5.4 shows a negative $\dot{W}_2$ which implies the stack is absorbing the work flux and generating a temperature gradient across the two ends of the stack. A close observation of Eq. (5.35) reveals that $\dot{W}_2$ is proportional to $y_0\Pi$ which is the cross-sectional area of the fluid in the $y-z$ plane. Therefore, an increasing cross-sectional area increases the magnitude of $\dot{W}_2$. Thus, $\dot{W}_2$ becomes negligible for the narrow channel ($S_w < 0.25$) which is also observed from Fig. 5.4. It is also observed that the magnitude of $\dot{W}_2$ increases when $S_w$ increases until $S_w \approx 1.5$, then $\dot{W}_2$ decreases with further increases in $S_w$ for all values of drive ratio. Eq. (5.35) also shows that $\dot{W}_2$ is proportional to the square of the $DR$. Therefore, an increasing $DR$ increases the value of $\dot{W}_2$. Note that the parameter $DR$ can...
be interpreted as a produce of Mach number \((Ma)\) and specific heat ratio \((\gamma = C_p / C_v)\) of the fluid where \(Ma\) is the ratio of the amplitude of the fluctuating velocity \((u_i)\) to the velocity of sound \((c_m)\) calculated at the mean fluid temperature \((T_m)\). For the narrow channel \((S_w < 0.4)\), \(\dot{W}_2\) approaches zero for a relatively lower value of \(DR\).

Figure 5.5 shows the variation in \(\dot{W}_2\) as a function of \(Ha_{\delta}\) for different values of \(DR\) \((=0.01, 0.02, 0.03, 0.04, 0.05, \text{ and } 0.06)\) at \(S_w = 2\). The magnitude of \(\dot{W}_2\) increases with \(Ha_{\delta}\) when \(Ha_{\delta} < 1.5\) and reaches its maximum when \(Ha_{\delta} = 1.5\) for all \(DR\). The influence of \(Ha_{\delta}\) on \(\dot{W}_2\) is observed significant for \(Ha_{\delta} < 1.5\). \(\dot{W}_2\) shows a trend to approach very small value \(Ha_{\delta}\) approaches a large value.

Figure 5.6 presents \(\dot{V}_z\) distribution as a function of \(Ha_{\delta}\) for different values of \(S_w\) \((=0.25, 0.5, 1.0, 1.25, 1.50, 2.0)\) at \(DR = 0.01\). Maximum \(\dot{W}_2\) is observed at \(Ha_{\delta} \approx 1.5\) for all values of \(S_w\). The higher magnitude of \(\dot{W}_2\) is observed for a relatively higher
value of \( S_w (=2) \). \( \dot{W}_2 \) approaches zero when \( Ha_\delta \rightarrow \infty \). A negligible influence of \( Ha_\delta \) on \( \dot{W}_2 \) is observed for the narrow channel ranges \(( S_w < 0.5 \).\)

Figure 5.5: Work flux density as a function of \( Ha_\delta \) for different values of \( DR \) at \( S_w = 2 \).

Figure 5.6: Work flux density as a function of \( Ha_\delta \) for different values of \( S_w \) at \( DR = 0.01 \).
5.6.3 Discussion on the heat flux density

The distribution of the real part of heat flux is presented in Fig. 5.7 as a function of $S_w$ for four different values of $\Gamma_0$ (=5, 4, 3, and 1) at $DR = 0.01$ in the absence of magnetic field ($Ha_0 = 0$). A close observation of Eq. (5.37) reveals that $\dot{Q}_2$ is proportional to $v_0 \Pi$. Therefore, a reducing cross-sectional area reduces $\dot{Q}_2$. Fig. 5.7 shows that $\dot{Q}_2$ is negligible for the narrow channel ($S_w < 0.25$). A positive value of $\dot{Q}_2$ implies the refrigeration/heat pump mode of operation of a thermoacoustic system. For the narrow channel ($0 < S_w < 0.7$), the effect of $\Gamma_0$ is insignificant on the variation of $\dot{Q}_2$. A considerable variation in $\dot{Q}_2$ for different values of $\Gamma_0$ is observed for the wider channel, i.e., $S_w > 0.75$. The value of $\dot{Q}_2$ increases sharply when $0 < S_w < 1.4$. The value of $\dot{Q}_2$ does not increase significantly when $S_w > 1.4$. For the range $0.7 < S_w < 1.4$, a relatively higher value of $\Gamma_0$ produces higher $\dot{Q}_2$.

![Figure 5.7: Heat flux density as a function of $S_w$ for different values of $\Gamma_0$ at $DR = 0.01$ and $Ha_0 = 0$.](image-url)
Figure 5.8: Heat flux density as a function of $Ha_\delta$ for different values of $DR$ at $S_w = 2$.

Figure 5.8 presents variation in the heat flux as a function of $Ha_\delta$ for six different values of $DR$ (=0.01, 0.02, 0.03, 0.04, 0.05, and 0.06) at $S_w = 2.0$. The value of $\dot{Q}_2$ increases when $Ha_\delta < 1.5$ and $\dot{Q}_2$ decreases when $Ha_\delta > 1.5$ for all values of $DR$. For a given $Ha_\delta$, $\dot{Q}_2$ increases when $DR$ increases.

Figure 5.9 presents the distribution of $\dot{Q}_2$ as a function of $Ha_\delta$ for different values of $S_w$ (=0.25, 0.5, 1.0, 1.25, 1.50, 2.0) at $DR = 0.01$. The magnitude of $\dot{Q}_2$ increases when $Ha_\delta \leq 1.2$. The magnitude of $\dot{Q}_2$ decreases when $Ha_\delta > 1.2$ and approaches zero when $Ha_\delta \rightarrow \infty$ for all values of $DR$. For a given $Ha_\delta$, the magnitude of $\dot{Q}_2$ is high at a higher $S_w$. 
Figure 5.9: Heat flux density as a function of $\text{Ha}_\delta$ for different values of $S_w$ at $DR = 0.01$.

### 5.7 Summary

In this chapter, a multi-plate thermoacoustic system under the influence of an external magnetic field is modeled and analyzed to evaluate its performance. Three performance parameters: energy, work, and heat fluxes are evaluated. In the absence of a magnetic field, the analytical solutions of energy, work, and heat fluxes are compared with the solutions available in the existing literature. The simplified analytical solutions to the energy, heat, and work fluxes are analyzed through graphical presentations. It is observed that the energy flux density increases with increasing temperature gradient ratio when $S_w < 1.5$ and decreases when $S_w > 1.5$. The energy flux density also increases with increasing $\text{Ha}_\delta$ when $S_w < 1.5$ and decreases when $S_w > 1.5$. The work flux density increases with increasing drive ratio and Swift number. The work flux also increases with increasing $\text{Ha}_\delta$ when $\text{Ha}_\delta < 1.5$ and decreases when $\text{Ha}_\delta > 1.5$. A similar pattern of work flux density is observed for heat flux density where the sign of heat flux density is opposite to the work flux density. This study will give an insight to the thermoacoustic system designer regarding the behavior of energy, heat, and work fluxes of a parallel plate thermoacoustic system when a magnetic field is applied to the
transverse direction of the stack. The effect of the spacing between two adjacent parallel plates on the energy, work, and heat fluxes is also examined in this research. The next step of this current research is to conduct an experimental investigation to validate the developed analytical modeling. The optimal design, development, and measurement of the performance of a thermoacoustic refrigerator are described in the next chapter.
Chapter 6

EXPERIMENTAL INVESTIGATION ON THE PERFORMANCE OF A POROUS MEDIUM THERMOACOUSTIC REFRIGERATOR

6.1 INTRODUCTION

This chapter describes the design, development, and the performance measurement of a porous thermoacoustic refrigerator. Numerical, analytical, and experimental analysis are carried out to optimize the parameters of the thermoacoustic refrigerator. Experiments are performed with air at atmospheric pressure using four different stack lengths (100mm, 150mm, 175mm, and 200mm) and drive ratios (0.09, 0.87, 0.78, and 0.063) at a working frequency of 50Hz. The performance of the thermoacoustic system was measured in terms of the temperature difference across the two ends of the stack. Consequently, the coefficient of performance (COP) and the COP relative to the Carnot’ coefficient of performance (COPR) are calculated for different cooling loads. It is observed that higher drive ratios produce higher temperature differences across the stack ends, and that the COPR increases with the temperature span of the standing wave thermoacoustic refrigerator. The stack length and position from the acoustic driver’s end are optimized, which in this case (a 175mm stack length at a distance of 42mm from the driver’s end) and achieved a temperature of 88.9 °C at the hot end of the stack, with a cold end temperature of -8.5 °C. The maximum cooling capacity achieved is 14.75 watt at 5.2% COPR. This study will provide meaningful insight into the design of standing wave thermoacoustic refrigerators.

6.2 EXPERIMENTAL SETUP OF A THERMOACOUSTIC REFRIGERATOR

The experimental setup and schematic diagram of our developed prototype thermoacoustic refrigerator are shown in Figs. 6.1 (a) and (b). This setup contains of four elements: A resonator, a stack, two heat exchangers, and an acoustic driver. The componenets of the thermoacoustic system are described in the following subsections:
6.2.1 ACOUSTIC DRIVER

Qdrive (model no. 1S132D-24743) manufactured by Chart Inc. (Qdrive Group) is used as an acoustic driver to build the thermoacoustic refrigerator. Fig. C.1 shows the acoustic driver used for this experiment. The area exposed to oscillating pressure is $1.807 \times 10^{-03}$ m$^2$, the electric resistance of the coil is 2.2 ohm, the electrical inductance $L$ of the coil
(when prevented from moving) is $4.66 \times 10^{-02}$ H, the moving mass is 0.7148 kg, the spring constant is $4.07 \times 1004$ N/m, and the mechanical resistance is 6.99 N.s/m.

The drive electronics power supply unit is use to drive the Q-drive. Fig. C.2 shows the Drive Electronics power supply unit. It allows changing in output voltage at a constant frequency 50 Hz.

### 6.2.2 RESONATOR

An end-flanged PVC tube is used as a resonator. One end of the resonator is attached to the acoustic driver and the other end is kept open. A rubber O-ring is placed between the flanges of the acoustic driver and the resonator for sealing. Fig. 6.2 shows the one end flanged resonator with a rubber O-ring mounted on it. The inner diameter of the resonator is 51.59 mm, which is the closest diameter to the acoustic driver’s output port (47.30 mm) available on the market. Air at one atmospheric pressure is considered as the working fluid. An optimized length of the resonator tube is $\lambda / 4$ for the open-end condition or $\lambda / 2$ for the closed-end condition [Tijani [2001]].

---

Figure 6.2: Resonator
The acoustic power loss (viscous and thermal dissipation) at the wall of the resonator for a resonator length of $\lambda/4$ is less than that for a resonator length of $\lambda/2$ [Hofler [1986]]. Considering the resonance frequency and the acoustic power losses, I have developed an experimental setup with an open-end resonator tube of length $\lambda/4$.

### 6.2.3 Stack

The stack is the most important element of the thermoacoustic refrigeration system, as its material properties and geometry have a great impact on the performance of the thermoacoustic refrigerator [Adeff et al. [1998]]. The thermoacoustic effects are produced by thermal and hydrodynamic interactions between the oscillating fluid and the stack solid wall [Swift’s [1988]]. The thermal conductivity of the stack material is to be low (reduces axial heat conduction) for designing an effective thermoacoustic system. I chose porous Corning celcor ceramic (parallel plate type) manufactured by Corning Inc. for the stack material because of its low thermal conductivity (1.46 W/mK) and availability.

![Figure 6.3: Corning celcor ceramic stack](image_url)
Corning celcor ceramic stack’s other material properties are heat capacity (1000 J/kg K), temperature capability (up to 1400°C), density (2500 Kg/m³). It also can carry a good compressive load [Corning [2010]]. Fig. 6.3 shows the hot end and cold end views of the stack with the parallel plate arrangement. The plate spacing and thickness are measured as 1.00 mm and 0.36 mm, respectively.

### 6.2.4 Heat Exchangers

Within a limited distance from the stack ends, the net heat exchange between the working fluid and the plate wall takes place [Piccolo and Pistone [2006]]. It means that the heat-flux density is maximum at the two stack extremities [Cao *et al.* [1996], Worlikar *et al.* [1998], Mozurkewich [1998]]. The stack end closer to the acoustic driver becomes hotter and the other end becomes colder due to the acoustic effect. Two heat exchangers can be used to extract heat from the hot side and supply heat to the cold side. Ni-Cr heater wire is used to supply heat to the cold side of the stack. In the experiment, I have made shallow grooves on the cold side of the stack, as shown in Fig.6.3 to accommodate the Ni-Cr heater wire. Ni-Cr heater wire is a resistive load that is easy to measure the amount of heat that can be pumped using the thermoacoustic refrigerator. I used BK precision (Fig. C.3) DC power supply unit to supply power to the Ni-Cr heater wire. I did not extract the heat from the hot side; rather, two thermocouple wires were simply attached to both ends of the stack.

### 6.3 Measurements

The thermoacoustic refrigerator’s performance is measured using several measuring devices, such as a thermocouple thermometer, a dynamic pressure sensor, and a data acquisition card. A description of each measuring device is given below.

#### 6.3.1 Thermometer

An Omega-HH374, 4-channel type K data logger thermometer was used to measure the temperature in Celsius. The accuracy of the thermocouple is ± 0.1% (°C), and the measurement range is -200 to +1372°C. The Omega-HH374 measured the temperature in every second and recorded the measurements in the built-in memory. The recorded
data were then downloaded using SE374 software. Fig. C.4 shows the thermometer that used to measure the temperature in the experiment.

6.3.2 Dynamic Pressure Sensors and Data Acquisition

Two high frequency ICP© dynamic pressure sensors (model 113B28), manufactured by PCB Piezotronics Inc., are used to measure the fluctuating pressure \( p_1 \) and operating frequency. Fig. C.5 shows the dynamic sensors that used to measure the fluctuating pressure inside the resonant tube. These sensors produce electric potential in response to pressure change. The sensitivity of the sensor is 101.8 mV/PSI or 14.77 mV/kPa, with an accuracy ±1%. The sensors’ analog signals are processed, recorded, and analyzed using NI-DAQ (Model NI USB 6008) shown in Fig. C.6 with an NI LabVIEW 82015 interface. The sampling rate is 1 data/sec. Fig. C.7 shows the ICP signal conditioner (model 482C05) that provides an adjustable current source to drive the ICP dynamic pressure sensors is used to integrate the dynamic pressure sensors with the DAQ.

6.3.3 Input Acoustic Power

The input acoustic power is measured using the following equation [Swift [2001]]:

\[
\dot{W} = \frac{A}{2\omega \rho_m \Delta x} |p_A||p_B|\sin \theta, \quad (6.1)
\]

where \( A \), \( p_A \), \( p_B \), \( \Delta x \), and \( \theta \) are the area of the duct, peak pressure by the first pressure transducer, peak pressure by the second pressure transducer, the gap between the 1\(^{\text{st}}\) and 2\(^{\text{nd}}\) transducer, and the phase angle by which \( p_A \) lead by \( p_B \). The fluctuating pressure is calculated as \( p_i = (p_A + p_B)/2 \).

6.4 Refrigerator Performance

The performance of a refrigerator is measured by the coefficient of performance (COP). The COP is the ratio of extracted heat from the cold heat exchanger and the supplied acoustic power. COP can be calculated as:

\[
COP = \frac{\dot{Q}_c}{\dot{W}}, \quad (6.2)
\]

where \( \dot{Q}_c \) is the applied load in the system and \( \dot{W} \) is computed using Eq. (6.1).
The temperature differences across the stack ends are measured to calculate the Carnot coefficient of performance (COPC). COPC is a measure of the maximum performance for all refrigerators. The COPC is calculated using the following equation:

\[
COPC = \frac{T_c}{T_h - T_c},
\]

where \(T_c\) and \(T_h\) are the cold and hot end temperatures, respectively. The COP relative to Carnot’s coefficient of performance is calculated as:

\[
COPR = \frac{COP}{COPC}.
\]

### 6.5 Experimental Results and Discussions

In this section, the experimental results are presented in three different parts: design optimization of a thermoacoustic refrigerator, effect of drive ratios on the performance of a thermoacoustic refrigerator, and effect of stack length on the performance of a thermoacoustic refrigerator.

#### 6.5.1 Design Optimization of a Thermoacoustic Refrigerator

The acoustic driver generates fluctuating pressure \(p_1\) in the working fluid (i.e. air) inside the regenerator. The operating frequency of the acoustic driver is 50Hz. A higher fluctuating pressure produces a higher temperature gradient across the stack ends. The dynamic pressure sensors are used to measure the magnitude of the fluctuating pressure. It produces relatively a higher voltage in response to a relatively higher pressure. Fig. 6.4 shows the dynamic pressure sensors reading of fluctuating voltage as a function of time. The sensors \(p_A\) and \(p_B\) are mounted on the resonator 96.3 mm and 312 mm away from the driver’s end respectively. The values of \(p_A\) is 0.14 volt and \(p_B\) is 0.13 volt which crossponds to 9.47 kpa and 8.6kpa respectively. The value of \(p_1\) is calculated as follows:

\[
p_1 = \frac{p_A + p_B}{2} = 9.03 \text{kpa}
\]
The stack length, and position are needed to be optimized for obtaining the maximum temperature difference across the stack ends at a certain frequency [Swift [2001], Tijani [2001], Hariharan et al. [2013]]. Tijani et al. [2002a] developed an analytical solution for calculating the COP of the thermoacoustic refrigerator as a function of Normalized stack length \((= 2\pi f L_s / a)\) and Normalized stack position \((= 2\pi f X_s / a)\), where \(L_s\) is the length of the stack, \(f\) is the frequency of the acoustic driver, \(a\) is the velocity of sound, and \(X_s\) is the distance from the acoustic driver’s end to the mid length of stack.

The normalized cooling power, \(Q_{cn} \left(= \dot{Q}_c / p_m a A\right)\) is calculated using the following equation [Tijani et al. [2002a]]:

\[
Q_{cn} = -\frac{\delta_{kn} DR^2 \sin(2X_{sn})}{8\gamma (1 + Pr) A} \left( \frac{\Delta T_{mn} \tan(X_{sn})}{1 + \sqrt{Pr} + Pr} - \frac{1 + \sqrt{Pr} - \sqrt{Pr} \delta_{kn}}{1 + \sqrt{Pr}} \right),
\]

where \(\delta_{kn}\), \(DR\), \(X_{sn}\), \(\gamma\), \(\Delta T_{mn}\), \(L_{sn}\), \(Pr\), and \(B\) are Normalized thermal penetration depth \((= \delta_k / y_0)\), Drive ratio \((= p_1 / p_m)\), Normalized stack position, heat capacity ratio of air, Normalized temperature difference \((= \Delta T_m / T_m)\), Normalized stack length, Prandtl...
number, and Blockage ratio \((= y_0 /(y_0 + l))\) respectively. \(2y_0\) is the gap between the two parallel plate, and \(2l\) is the thickness of the plate.

The normalized acoustic power, \(W_n = \dot{W} / (p_m a A)\) is calculated using the following expression [Tijani et al. [2002a]]:

\[
W_n = \frac{\delta_{kn} L_{sn} DR^2}{4\gamma} (\gamma - 1) B \cos^2(X_{sn}) \times \left( \frac{\Delta T_{mn} \tan(X_{sn})}{BL_{sn} (\gamma - 1)(1 + \sqrt{\Pr})\Lambda} - 1 \right),
\]

\[
- \frac{\delta_{kn} L_{sn} DR^2}{4\gamma} \sqrt{\Pr \sin^2(X_{sn})} \frac{BL_{sn} (\gamma - 1)(1 + \sqrt{\Pr})\Lambda}{(\gamma - 1) B \Lambda},
\]

(6.7)

where

\[
\Lambda = 1 - \sqrt{\Pr \delta_{kn}} + \frac{1}{2} \Pr \delta_{kn}^2.
\]

(6.8)

The coefficient of performance (COP) of the stack can be obtained by dividing Eq. (6.6) by Eq. (6.7). Thus,

\[
COP = \frac{\dot{Q}_c}{W} = \frac{Q_m}{W_n}
\]

(6.9)

6.5.2 RESULTS AND DISCUSSIONS

The COP obtained from Eq. (6.9) is plotted in Fig. 6.5 as a function of \(L_{sn}\) for different values of \(X_{sn}\) (=0.06, 0.08, 0.12, 0.22, and 0.32). In Eq. (6.9), the thermophysical properties of air at one atmospheric pressure are considered. Other parameters such as the acoustic driver’s operating frequency are measured as 50Hz, the Blockage ratio is calculated as \(B = 0.735\), the Sound velocity \((a)\) is 345 m/s, the drive ratio is measured as \(DR=0.09\), the normalized temperature difference is calculated as \(\Delta T_{mn} = 0.43\), the mean temperature is considered as \(T_m = 300\ \text{K}\), and the normalized thermal penetration depth is calculated as \(\delta_{kn} = 0.768\). Fig. 6.5 indicates either that a relatively smaller stack length at a relatively closer position to the acoustic driver’s end produces higher COP or that a
fixed stack length at a relatively closer position to the driver’s end produces higher COP. In both cases, an estimation on stack length and position that would achieve maximum COP is obtained. As the analytical solution does not provide exact optimal stack length and position, DeltaE thermoacoustic simulation software is used to predict the optimal stack length and position as described next.

Design Environment for Low-Amplitude Thermoacoustic Engines (DeltaE) simulation software was developed by the Los Alamos group [Ward and Swift [2001]] to predict thermoacoustic system response. In this section, I will describe the simulation results and the modelling of a thermoacoustic refrigerator using DeltaE thermoacoustic simulation software. Fig. 6.1b shows the geometry of the developed thermoacoustic refrigerator, which can be simulated using DeltaE as a sequence of different acoustic segments. To simulate this system, six of these segments are used. Specifically, BEGIN is used to set the initial condition, VSPEAKER is used for modelling the acoustic driver, DUCT is used for modelling the pipe, HX is used for modelling the heat exchanger, STKRECT is used for modelling the stack, and COMPLIANCE is used to create the open-end
condition. The BEGIN segment takes the global parameters, such as frequency, mean pressure, and fluctuating pressure. The other segments take the local parameters, such as area, perimeter, porosity, etc. Air at one atmospheric pressure with a drive ratio of 0.09 is used for simulations, while the hot end temperature is kept constant at 300K. I calculated the coefficient of the performance relative to Carnot coefficient of performance (COPR) for different stack lengths, which is a function of the heat load. The heat load is applied to the cold heat exchanger, which was varied from 0 to 40 Watts at increments of 0.25 Watts.

Fig. 6.6 presents the COPR as a function of the cooling load for three different stack lengths (=150mm, 175mm, and 200mm) and initial position (=20mm, 30mm, 42mm, and 50mm) obtained by DeltaE. The stack’s hot end is placed 42 mm away from the acoustic driver’s end the drive ratio is chosen as 0.09. The DeltaE simulation result (Fig. 6.6) provides similar information as analytical solution (Fig. 6.5) that a larger COPR is produced for a fixed stack length at a relatively closer position to the driver’s end. It is observed that the maximum COPR is achieved for a stack length of 175mm for all four stack initial positions.

Experiments were performed with a stack length of 175mm by varying four different initial stack positions from the acoustic driver’s end (=20, 30, 42, 50mm) to obtain maximum cooling effect. Fig. 6.7 shows the experimental results of the measured temperature (°C) at the cold end of the stack (measured immediately after the acoustic power is supplied) as a function of time at DR=0.09. I observed that the temperature at the far end of the stack from the acoustic driver started to decrease with time and that the system became steady state after approximately 4.0 minutes. The cooling temperature decreased with the increase of the distance between the acoustic
Figure 6.6: COPR as a function of cooling load for different stack lengths and positions (obtained by DeltaE).

Figure 6.7: The cold end temperatures of the stack versus time for the different stack initial positions (from the driver’s end).
driver’s end and the stack’s initial position when this distance is \( \leq 42 \text{mm} \). Furthermore, increasing this distance to \( >42 \text{mm} \) raises the temperature. Finally, a temperature of \(-8.5^\circ\text{C}\) at the cold side and \(88.9^\circ\text{C}\) at the hot side are observed for a stack position of \(42 \text{mm}\) away from the driver’s end.

### 6.5.3 Drive Ratios Effect on the Performance of a Thermoacoustic Refrigerator

The drive ratio \( (DR = \frac{p_1}{p_m}) \) is defined as the ratio of fluctuating pressure to mean pressure. The acoustic driver generates fluctuating pressure \( (p_1) \) in the working fluid (i.e. air) inside the regenerator tube. The acoustic driver Q-drive used to produce acoustic power in this experiment produces higher fluctuating pressure when the input voltage is relatively higher. A larger fluctuating pressure produces a larger drive ratio at a constant mean pressure and the cooling power is proportional to the square of the drive ratio [Swift 1988]. In this section, the influence of the drive ratios on the performance of a thermoacoustic refrigerator have been discussed. The performances are measured in terms of temperature difference across the stack ends, COP and COPR as a function of cooling load for different drive ratios.

The dynamic pressure sensors are used to measure the magnitude of the fluctuating pressure. The dynamic pressure sensors produce voltages proportional to the pressure generated by the acoustic driver, i.e., higher voltage is produced by the pressure sensors in response to a higher fluid pressure. Two pressure sensors \( p_A \) and \( p_B \) (Fig. C.5) are placed on the resonator \(96.3 \text{ mm}\) and \(312 \text{ mm}\) away from the driver’s end respectively. The dynamic pressure is measured for four different driving voltage (=40V, 35V, 30V, and 25V). Fig. 6.8 shows the dynamic pressure sensors output (i.e., \( p_A \) and \( p_B \)) as a function of time. The higher the input voltage into the acoustic driver the higher the fluctuating voltage is. For an input voltage of 40 V, the values of \( p_A \) is 0.14 volt and \( p_B \)
is 0.13 volt which corresponds to 9.47 kpa and 8.6 kpa respectively. The value of \( p_1 \) is calculated as \((p_A + p_B)/2 = 9.03 \text{ kpa}\) and the \( DR = 0.090 \). For an input voltage of 35 V, the values of \( p_A \) is 0.13 volt and \( p_B \) is 0.12 volt which corresponds to 8.79 kpa and 8.60 kpa, respectively. The value of \( p_1 \) is calculated as \((p_A + p_B)/2 = 8.69 \text{ kpa}\) and the \( DR = 0.087 \). For an input voltage of 30 V, the values of \( p_A \) is 0.11 volt and \( p_B \) is 0.10 volt which crossponds to 7.43 kpa and 6.91 kpa, respectively. The value of \( p_1 \) is calculated as \((p_A + p_B)/2 = 7.17 \text{ kpa}\) and the \( DR = 0.072 \). For an input voltage of 25 V, the values of \( p_A \) is 0.096 volt and \( p_B \) is 0.093 volt which crossponds to 6.42 kpa and 6.23 kpa, respectively. The value of \( p_1 \) is calculated as \((p_A + p_B)/2 = 6.33 \text{ kpa}\) and the \( DR = 0.063 \).

The experiments have been performed to study the effect of drive ratio on the thermoacoustic refrigerator for four different drive ratios (=0.09, 0.087, 0.072, and 0.063),with a stack length of 175 mm placed 42mm away from the acoustic driver’s end.
to produce optimal cooling effects (as described in Subsection 6.5.2). Fig. 6.9 shows the measured temperature (°C) at the hot and cold ends of the stack as a function of time. Both hot and cold end temperatures were measured immediately after the acoustic power is supplied. Results show that the temperature of the stack end that is closed to the acoustic driver started to increase and the other end started to decrease. At the initial stage of operation when there is no temperature gradient in the stack, the diffusive heat from the hot end to the cold end is not significant, thus a rapid increase in temperature difference across stack is observed. The diffusive heat flow increases with increasing temperature gradient in the stack material and become steady state after 4.00 minutes. Once the diffusive heat flow is constant, the temperature difference across two ends becomes time independent.

The maximum temperature observed at the hot and cold side of the stack are 88.9 °C and -8.5 °C for a drive ratio of 0.09. As can be seen from Fig. 6.9, the temperatures across the stack end decreases with decreasing drive ratio (<0.09).

![Figure 6.9: Hot and cold end temperatures of the stack versus time for different drive ratios.](image-url)
For drive ratios of 0.087, 0.72, and 0.063, the hot end temperatures are 81.1 °C, 75.1 °C, and 72.6 °C and the cold end temperatures are -7.4 °C, -4.3 °C, and -1.5 °C, respectively.

The higher rate of temperature increase at the hot side relative to the temperature decrease at the cold side is due to the dissipative heat because of viscous and thermal processes.

Fig. 6.10 presents the experimental results of the measured temperature difference between the two sides of the stack (\(VT = T_h - T_c\)) as a function of heat load for four different values of \(DR\) (=0.09, 0.087, 0.072, and 0.063). It shows that \(\dot{Q}\) is a linear function of \(VT\) with a negative slope at a relatively lower value of \(DR\). The temperature difference across the stack ends reduces with the higher cooling load. For a given \(VT\), the magnitude of \(\dot{Q}\) is higher at higher value of \(DR\).

The COP of the thermoacoustic refrigerator is calculated using Eq. (6.9). Ni-Cr heater wire is used as a resistive load on the cold end of the stack as shown in Fig. 6.3. Electric power is supplied to the Ni-Cr heater wire from a DC voltage source (Fig. C.3). The applied voltage was varied from 0 to 6.4 Volt and the current was varied from 0 to 4.3 amp. Thus, The supplied heat to the cold end of the thermoacoustic refrigerator was calculated in watt that vary from 0-27.52 watt. The supplied heat is a measure of heat that can be pumped using the thermoacoustic refrigerator. The acoustic power produced by the Q-drive is measured using Eq. (6.1). Fig. 6.11 presents the experimental results of COP as functions of cooling load (\(\dot{Q}\)) for four different values of \(DR\) (=0.09, 0.087, 0.072, and 0.063). The COP increases with increasing \(\dot{Q}\) for all \(DR\). For a given \(\dot{Q}\), the magnitude of COP is higher for lower values of \(DR\).

Fig. 6.12 presents experimental results of COPR as a function of \(\dot{Q}\) for four different values of \(DR\) (=0.09, 0.087, 0.072, and 0.063). As can be seen, the magnitude of COPR increases with the increase of \(\dot{Q}\) up to a certain value of \(\dot{Q}\) after which it starts decreasing. Therefore, the COPR shows a parabolic pattern for all four DRs, but the magnitude of the maximum COPR is increased as the DR increases. The cooling load corresponding to the maximum COPR increases with increasing DR. The maximum COPR is observed at \(DR\) (=0.090) is 5.42% for a cooling load 17.85 watt.
Figure 6.10: Hot and cold end temperatures difference of the stack as functions of total heat load at different drive ratio.

Figure 6.11: COP as a function of total heat load at different drive ratios.
6.5.4 THE EFFECT OF STACK LENGTH ON THE PERFORMANCE OF A THERMOACOUSTIC REFRIGERATOR

This section described the effect of stack length on the performance of a thermoacoustic refrigerator. The fluctuating pressure is measured using the dynamic pressure sensors and consequently the drive ratio is calculated. Fig. 6.13 shows the dynamic pressure sensors reading of fluctuating voltage due to the fluctuating pressure as a function of time. The values of $p_A$ is 0.14 volt and $p_B$ is 0.13 volt which corresponds to 9.47 kpa and 8.6 kpa respectively. The value of $p_1$ is calculated as $(p_A + p_B)/2 = 9.03$ kpa and the $DR = 0.090$.

DeltaE simulation is carried out to predict COPR for different stack length with varying cooling load. Fig. 6.14 presents the DeltaE simulation results for COPR as a function of the cooling load for five different stack lengths (= 100mm, 125mm, 150mm, 175mm, and 200mm). The stack’s hot end is placed 42 mm away from the acoustic driver’s end. It is observed that the maximum COPR increases with increasing stack length when the stack
Figure 6.13: Fluctuating voltage due to the fluctuating pressure as a function of time

Figure 6.14: COPR as a function of cooling load for different stack lengths (results obtained by DeltaE).
length \( \leq 175\text{mm} \). Further increases in stack length (>175mm), however, results in decreased COPR. The maximum COPR is 5.6% for a stack length of 175mm.

Experiments are also conducted to study the effect of the stack length on the thermoacoustic refrigerator for four different stack lengths (= 100mm, 150mm, 175mm, and 200mm). The hot end of the stack is placed 42 mm away from the acoustic driver’s end. Fig. 15 shows the experimental results of the measured temperature (°C) at the hot and cold ends of the stack as a function of time for stack length (= 100mm, 150mm, 175mm, and 200mm). Both hot and cold end temperatures were measured right after the acoustic power is supplied. It is observed that the temperature difference between the two sides of the stack increases with increasing stack length up to a certain length (<175mm) and start decreasing when this length is further increased (>175mm). The maximum temperature observed at the hot is 88.9 °C and the cold side of the stack is -8.5 °C for a stack length of 175mm.

Figure 6.15: Hot and cold end temperatures of the stack as functions of time for different stack lengths.
The coefficient of performance relative to Carnot (COPR) has been calculated as a function of $\dot{Q}$ for four different stack lengths (100mm, 150mm, 175mm, and 200mm) shown in Fig. 6.16. The magnitude of COPR increases with the increase of $\dot{Q}$ up to a certain value of $\dot{Q}$ after which a decrease is observed. The maximum COPR is 5.42\% for a cooling load of 17.85 watt for a stack length of 175mm. The experimental results support the DeltaEc simulation results.

6.6 SUMMARY

A prototype thermoacoustic refrigerator is developed based on the performance prediction of DeltaE thermoacoustic simulation software. To obtain the maximum cooling effect, the stack length and position were optimized using analytical analysis and numerical simulation (DeltaE) as the first step in designing the thermoacoustic refrigerator. DeltaE was used to obtain the requires stack length generating the maximum COPR. Using that length, experiments were performed for different initial stack positions. The minimum temperature achieved at the cold end was -8.5 °C for a stack length of 175mm at a distance of 42mm from the driver’s end.
Then the effects of drive ratio on the performance of the thermoacoustic refrigerator is investigated experimentally. Celcore ceramic material of 175mm length is used as a stack in the experiment and placed at a distance 42mm from the driver’s end. performed the experiment for four different drive ratios (=0.09, 0.087, 0.072, and 0.063). It is seen that a higher drive ratio produces higher temperature difference across the stack ends. The cooling load increases with increasing drive ratio. The maximum temperature difference was observed 97.14 °C for drive ratio 0.09. A close observation also shows that the cooling load is proportional to the temperature difference between the stack ends which support the result of the study by Jebali et al. [2004], and Nsofor and Ali [2009].

Then the effect of stack lengths on the performance of the thermoacoustic refrigerator are discussed from the experimental investigation. Celcore ceramic material of four different lengths (= 100mm, 150mm, 175mm, and 200mm) are used as a stack in the experiment. Every stack is placed at a distance 42mm from the driver’s end. A resistance heating on the cold side of the stack is applied as the cooling load. The maximum temperature observes at the hot is 88.9 °C and the cold side of the stack is -8.5 °C for a stack length of 175mm. The maximum COPR is observed is 5.42% for a cooling load 17.85 watt for a stack length of 175mm. Therefore, there exit an optimal stack length for a given frequency and drive ratio to get the optimal temperature difference and cooling load from a thermoacoustic refrigerator.
Chapter 7
CONCLUSIONS AND FUTURE WORK

7.1 CONCLUSIONS

This thesis describes the analytical modelling of the thermoacoustic system where a transverse magnetic field is applied across the stack perpendicular to the oscillating fluid flow. The objective of the this research effort is to investigate the magnetic and porous media thermoacoustic systems by developing simplified analytical models and subsequent experiments. Therefore, I used two different types of stacks for the analytical modelling: a porous medium with a thick solid plate attached to it and a multi-plate stack with thin wall approximation, and porous medium for the experiments.

For the stack contains of porous medium with a thick solid plate attached to it exposed to an oscillating, compressible and inviscid fluid. A magnetic field is applied to the oscillating flow transversally. The governing equations for the fluid flow and temperature were simplified using a first order linear perturbation technique. The major conclusions are:

- $Ha_\delta$ has a significant effect on $\nabla T_{cr}$ which plays an important role in determining the system’s mode of operation, for a relatively higher $Da (=0.1)$ value.
- $Ha_\delta$ does not have any effect on Nusselt number.
- The dimensionless heat flux increases significantly with increasing $Ha_\delta$ when $Ha_\delta < 4.5$, while dimensionless work flux decreases with increasing $Ha_\delta$.
- Entropy generation is minimum when $Ha_\delta$ is close to 10.
- The efficiency of a thermoacoustic prime mover is maximum when $\nabla T_{cr} \approx \nabla T_m$.

It was shown that the $\nabla T_{cr}$ can be controlled by $Ha_\delta$. Therefore, $Ha_\delta$ can be a control mechanism to maximize the thermoacoustic prime mover's performance.

For the multi-plate stack with thin wall approximation, the analytical expressions for the complex Nusselt number was derived for the magnetic thermoacoustic system. The
complex Nusselt number was calculated using both space averaged and bulk mean temperatures as reference temperatures. The influence of the transverse magnetic field and Swift number on the complex Nusselt number was analyzed and presented graphically. The complex Nusselt number expression that was derived using the space averaged temperature as a reference temperature is analyzed for two distinct modes of operation: prime mover and heat pump. The major conclusions are:

- The real part of the complex Nusselt number increases with increasing magnetic force.
- For a specific value of the magnetic field, the real part of the complex Nusselt number is constant when the value of the Swift or modified Swift number is less than one and increases sharply when the value of the Swift or modified Swift number was greater than one.
- The magnetic force has a relatively minor effect on the value of the real part of the complex Nusselt number, whereas the plate spacing has a significant effect on the real part of the complex Nusselt number regardless the mode of operations and the way of calculations (i.e., \( T_{av} \), \( T_{ref} \), or \( T_{av} \) as \( T_{ref} \)).
- It was also observed that magnetic force can reduce the phase difference between \( \partial T_1 / \partial y \bigg|_{y=y_0} \) and \( T_w - T_b \) resulting in higher heat transfer.

The analytical solutions for the energy, work, and heat flux of the multi-plate stack are also calculated. The major conclusions are:

- The energy flux density increases with increasing temperature gradient ratio when \( S_w < 1.5 \) and decreases when \( S_w > 1.5 \).
- The energy flux density also increases with increasing \( Ha_s \) when \( S_w < 1.5 \) and decreases when \( S_w > 1.5 \).
- The work flux density increases with increasing drive ratio and Swift number. The work flux also increases with increasing \( Ha_s \) when \( Ha_s < 1.5 \) and decreases when \( Ha_s > 1.5 \).
From the experimental investigation the major conclusions are:

- Higher drive ratio produces higher temperature difference across the stack ends and the cooling load increases with increasing drive ratio for a certain stack length and position.
- The maximum temperature achieved at the hot side of the stack is 88.9 °C and the minimum temperature achieved at the cold side of the stack is -8.5 °C for a stack length of 175mm at a distance of 42mm from the driver’s end.
- The maximum COPR is observed is 5.42% for a cooling load 17.85 watt.

7.2 Future Work

One of the most important conclusion in this research is that the magnetic field reduces the entropy generation rate and that lead to increase the efficiency of the thermoacoustic system. The velocity and thermal boundary layers thickness decreases with increasing of the magnetic fields. This novel findings allows to decrease the gap between the two successieve parallel plates of the stack. It is obvious that applying a transverse magnetic field and increasing the number of parallel plates in stack will increase the power density. This research is just a start to wards achieving an efficient and improved thermoacoustic system. The recommendations for the next steps are as follows:

- The parallal plate stack field with porous medium with the transversed magnetic fields can be used to increase the thermoacoustic system’s power density and efficiency. An indepth investigation is required on the stack materials.
- The analytical and numerical solution can be developed to see the effect of magnetic field on parallel plate stack filled with porous media.
- The design of heat exchanger needs more attention for effective thermal energy removal. The high thermal conductivity foam with metal tube can be used as an heat exchanger.
- A small scale thermoacoustic system can be developed to validate the developed theory. It will require high electrical conductive fluid, proper strength magnetic field, and a special arrangement in the experimental setup.
REFERENCES


Hariharan, N.M., P. Sivashanmugam, S. Kasthirirengan, Experimental investigation of a thermoacoustic refrigerator driven by a standing wave twin thermoacoustic prime mover, international journal of refrigeration 36 (2013a) 2420-2425.


APPENDIX A

A.1. EXPERIMENTAL MEASUREMENT UNCERTAINTIES

The experimental measurement uncertainties can be defined as the true and measured value of the measurand. There is always an uncertainty in every measurement. The measurement uncertainties can arise from the measuring device, environment, item being measured, or from other sources. The error in the measured value occurs because of two major reasons: the systematic error of the measuring device known as bias error/uncertainties and the precision error due to the lack of repeatability in the measurement output known as random uncertainty. The bias and random uncertainties that need to be considered in the experimental results of this thesis as estimated as follows:

A.1.1. BIAS UNCERTAINTY

The systematic or bias uncertainties do not vary with the repeating of the measurement. The bias uncertainties can be calculated by combining using root-sum squares method.

\[ B_x = \left[ \sum_{i=1}^{K} B_i^2 \right]^{1/2}, \]  

(A.1)

where \( k \) is the device bias uncertainties regarding to the measuring variable \( x \).

A.1.2. RANDOM UNCERTAINTY

The random uncertainties vary with the repeating of the measurement. The standard deviation can be calculated using the following equations:

\[ S_x = \left[ \sum_{i=1}^{n} \frac{(x_i - \bar{x})^2}{(n-1)} \right]^{1/2}, \]  

(A.2)

where \( x_i \) is the measured value of \( i^{th} \) measurement. \( \bar{x} \) is the average value, and \( n \) is the number of samples.
The random uncertainty, assuming that the output of the experimental result follow normal distribution and that the mean corresponds to a 95% confidence interval, can be estimated as

\[ P_x = \pm t \frac{S_x}{\sqrt{n}}, \quad (A.3) \]

where \( t \) is to be 2.262 for 10 independent measurements (\( n \)).

**A.1.3. Total Uncertainty**

The total measurement uncertainties which is come from the bias and random uncertainties can be calculated as:

\[ U_{\text{total}} = \sqrt{B_x^2 + P_x^2}, \quad (A.4) \]

**A.1.4. Experimental Measurement Uncertainties**

The uncertainties in the experimental results are estimated in the following two subsections:

**A.1.4.1. Temperature Measurement**

In the experiments, the temperatures of the hot end and cold end side of the stack were measured using an Omega-HH374, 4-channel type K data logger thermometer. The bias uncertainty of the Omega-HH374 thermometer and thermocouple wire are

\[ B_x = \sqrt{0.01^2 + 1.2^2} = 1.2 \, ^\circ\text{C} \]

For the cold end temperature, the standard deviation (Eq. (A.2)) of 10 independent measurements is calculated as 0.62. Thus the random uncertainties at the cold end is

\[ P_x = \pm t \frac{S_x}{\sqrt{n}} = \pm 2.262 \frac{0.62}{\sqrt{10}} = 0.44 \, ^\circ\text{C}. \]

For the hot end temperature, the standard deviation (Eq. (A.2)) of 10 independent measurements is calculated as 3.51 Thus the random uncertainties at the hot end is

\[ P_x = \pm t \frac{S_x}{\sqrt{n}} = \pm 2.262 \frac{3.51}{\sqrt{10}} = 0.51 \, ^\circ\text{C}. \]

The total uncertainties at the cold end measurement \( U_{C_{\text{total}}} = \sqrt{1.2^2 + 0.44^2} = 1.28 \, ^\circ\text{C}. \)
The total uncertainties at the hot end measurement \( U_{H_{\text{total}}} = \sqrt{1.2^2 + 2.51^2} = 2.78^\circ C \).

which is 4.08 % of the mean measured value at the cold end of the stack and 4.27% of the mean temperature at the hot end of the stack at a 95% confidence interval.

A.1.4.2. **Fluctuating Pressure Measurement**

The fluctuating pressure was measured using ICP© dynamic pressure sensors (model 113B28). The sensors accuracy is ±1% (mVolt) and the accuracy of NI USB 6008 is ±1.43 mVolt

The bias uncertainty is \( B_x = 1.43 \text{ mVol} \)

The random uncertainties is \( P_x = \pm t \frac{S_x}{\sqrt{n}} = \pm 2.262 \frac{0.21}{\sqrt{10}} = 0.15 \text{ mVol} \).

The total uncertainties \( U_{\text{total}} = \sqrt{1.43^2 + 0.15^2} = 1.43 \text{ mVol} \)

which is 1.0% of the measured value at a 95% confidence interval
APPENDIX B

B.1. SAMPLE MAPLE SCRIPT

In this section a sample MAPLE script is presented. The MAPLE script shows the calculation of Equation (3.46) in Chapter 3 which is plotted in Fig. 3.2 (a) Fluctuating pressure as a function of frequency at different Hartman numbers when mean pressure is 1 bar.

```maple
restart:
`\\varpi;` := (I*2)*Pi*f*phi0*mu/(rho0*K);

\[ \omega := \frac{2\Pi f \phi_0 \mu}{\rho_0 K} \]

Phi1 := -.5*phi0*dTm/(sigma*Tm) - .5*sqrt((phi0*dTm/(sigma*Tm))^2 - (4*(Da*Ha^2+1))*`\\varpi;`*((1-sigma)*gamma1 - 1)/(C^2*sigma));

Phi2 := -.5*phi0*dTm/(sigma*Tm) + .5*sqrt((phi0*dTm/(sigma*Tm))^2 - (4*(Da*Ha^2+1))*`\\varpi;`*((1-sigma)*gamma1 - 1)/(C^2*sigma));

denominator := exp(Phi1*xs)*exp(Phi2*xe) - exp(Phi1*xe)*exp(Phi2*xs);
```
Definition of different variables

\[ P_{\text{int}} := Pa \left( \sin\left(\frac{xs}{\lambda_0}\right) + i \cos\left(\frac{xs}{\lambda_0}\right) \right) \]

\[ P_{\text{exit}} := Pa \left( \sin\left(\frac{xe}{\lambda_0}\right) + i \cos\left(\frac{xe}{\lambda_0}\right) \right) \]

\[ \lambda_0 := \frac{1}{2} \frac{\lambda}{\pi} \]

\[ \lambda := \frac{C}{f} \]

\[ C := \sqrt{\frac{\gamma l R T_m}{M}} \]

\[ Pa := \frac{Pm}{DR} \]

\[ \rho_0 := \frac{Pm M}{R T_m} \]

\[ \beta := \frac{1}{T_m} \]

\[ \sigma := \phi_0 + (1 - \phi_0) \sigma_0 \]
Properties of Helium at 298.15 K

\[ Tm := 300; \quad dTm := 700; \]
\[ Pm := 100000; \quad M := 0.0040026; \quad R := 8.3145; \quad Cp := 5197.61; \quad \rho_AL := 2702; \]
\[ Cp_AL := 903; \quad \sigma_0 := (\rho_AL \cdot Cp_AL) / (\rho_0 \cdot Cp); \quad \gamma_1 := 5/3; \]
\[ Da := 0.01; \quad \phi_0 := 0.33; \quad \mu := 1.9938E-05; \quad \chi := 0.127; \quad \chi_e := 0.381; \quad \chi := 0.254; \quad K := 0.209e-7; \quad Pa := Pm \ast DR; \]
\[ DR := 0.01; \quad \pi := 3.14; \quad \rho_0 := Pm \ast M / (R \ast Tm); \]

\[ \gamma_1 := \frac{5}{3} \]
\[ Da := 0.01 \]
\[ \phi_0 := 0.33 \]
\[ \mu := 0.000019938 \]
\[ \chi := 0.127 \]
\[ \chi_e := 0.381 \]
\[ \chi := 0.254 \]
\[ K := 2.09 \ast 10^{-8} \]
\[ Pa := 100000 \ast DR \]
\[ DR := 0.01 \]
\[ \pi := 3.14 \]
\[ \rho_0 := 0.1604666546 \]

\[ p1 := (P_{int} \ast \exp(\Phi_2 \ast \chi_e) - P_{exit} \ast \exp(\Phi_2 \ast \chi_s) \ast \exp(\Phi_1 \ast \chi)) / \text{denominator} - \]
\((P_{\text{int}} \exp(\Phi_1 x e) - P_{\text{exit}} \exp(\Phi_1 x s)) \exp(\Phi_2 x)/\text{denominator};\)

\[ p_l := \left\{ \begin{aligned} &\left(1000.00 \sin(0.0002492308170 f/\pi) \\
&+ 1000.00 \cos(0.0002492308170 f/\pi)\right) \\
&\exp(-0.00007482611225 + 0.1905 \sqrt{1.542824057 \times 10^{-7} + 0.02517970328 \left(0.01 H a^2 + 1\right) \pi f} \\
&- \left(1000.00 \sin(0.0007476924511 f/\pi) \\
&+ 1000.00 \cos(0.0007476924511 f/\pi)\right) \\
&\exp(-0.00002494203742 + 0.0635 \sqrt{1.542824057 \times 10^{-7} + 0.02517970328 \left(0.01 H a^2 + 1\right) \pi f} \\
&- 0.000004988407483 - 0.1270 \sqrt{1.542824057 \times 10^{-7} + 0.02517970328 \left(0.01 H a^2 + 1\right) \pi f} \\
&\exp(-0.000002494203742 - 0.0635 \sqrt{1.542824057 \times 10^{-7} + 0.02517970328 \left(0.01 H a^2 + 1\right) \pi f} \\
&- 0.000007482611225 - 0.1905 \sqrt{1.542824057 \times 10^{-7} + 0.02517970328 \left(0.01 H a^2 + 1\right) \pi f} \\
&\exp(-0.000002494203742 + 0.0635 \sqrt{1.542824057 \times 10^{-7} + 0.02517970328 \left(0.01 H a^2 + 1\right) \pi f} \right) \\
&\exp\left(0.0002492308170 f/\pi \right) + 1000.00 \cos\left(0.0002492308170 f/\pi \right)\right) e^{-0.00C} \\
&+ 1000.00 \cos\left(0.0007476924511 f/\pi \right)\right) e^{-0.00C} \\
&\exp\left(0.00002494203742 - 0.0635 \sqrt{1.542824057 \times 10^{-7} + 0.02517970328 \left(0.01 H a^2 + 1\right) \pi f} \\
&- 0.000004988407483 + 0.1270 \sqrt{1.542824057 \times 10^{-7} + 0.02517970328 \left(0.01 H a^2 + 1\right) \pi f} \\
&\exp(-0.000002494203742 - 0.0635 \sqrt{1.542824057 \times 10^{-7} + 0.02517970328 \left(0.01 H a^2 + 1\right) \pi f} \\
&- 0.000007482611225 + 0.1905 \sqrt{1.542824057 \times 10^{-7} + 0.02517970328 \left(0.01 H a^2 + 1\right) \pi f} \\
&\exp(-0.000002494203742 + 0.0635 \sqrt{1.542824057 \times 10^{-7} + 0.02517970328 \left(0.01 H a^2 + 1\right) \pi f} \right) \\
&\exp\left(0.0002492308170 f/\pi \right) + 1000.00 \cos\left(0.0002492308170 f/\pi \right)\right) \\
\end{aligned} \right\}

> pp1:=subs(Ha=0, Re(p1));
> pp2:=subs(Ha=5, Re(p1));
> pp3:=subs(Ha=10, Re(p1));
> pp4:=subs(Ha=15, Re(p1));
> pp5:=subs(Ha=20, Re(p1));
> plot([pp1, pp2, pp3, pp4, pp5], f = 0 .. 2000, color = [red, blue, black, green]);
APPENDIX C

C. 1. DIFFERENT COMPONENTS OF THE THERMOACOUSTIC REFRIGERATOR

Figure C.1: Acoustic driver (Qdrive)

Figure C.2: Acoustic drivers's power supply unit (Drive Electronics)
Figure C.3: DC power supply unit

Figure C.4: Thermocouple thermometer (Omega-HH374)
Figure C.5: Dynamic pressure sensors

Figure C.6: NI- DAC card
Figure C.7: IPC signal conditioner