Production and Incentives in Teams

by

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ABSTRACT

PRODUCTION AND INCENTIVES IN TEAMS

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The first chapter looks at the impact of uncertainty about the types of teammates and about the production process on effort decisions of agents. The model involves solving a moral hazard problem with generation of agents that overlap. The economy contains two types of agents, pyramid organizational structures, (heterogeneous or homogeneous) combination of manager-worker teams, and promotion and retirement reward structure that affects incentives. When cost of effort is low, “bad luck” in the production process is low, and proportion of low efficiency types are high, then this reduces the incentives for individuals to free ride on the efforts of the team.

The second chapter looks at the impact of cost of effort and price of workers on a firm’s choice toward team composition when worker types differ in efficiency levels. The production technology the firm employs require a team of two workers. The analysis tackles the idea of team composition under two scenarios; when a firm can hire the team from a pool of applicants and when a firm must organize an existing workforce of employees. Under the first scenario, if costs of effort for workers and hiring costs are sufficiently low, hiring only high efficiency types is optimal (homogeneous). As both these costs increase, the firm uses its resources to incentivize one (more expensive) high efficiency type to exert effort and fill the other position with cheap, low efficiency, labour (heterogeneous). Instead, if there is an existing workforce, most often heterogeneous teams are selected.

The third chapter analyzes a specific incentive scheme of firms to induce effort, overtime participation, that is not directly related to an individual’s contract; namely promotions. To find the causal effect of expected monetary incentives of a promotion on an individual’s overtime participation, the Arellano-Bond estimator is employed. The results indicate that there is a positive and significant impact of financial incentives from a promotion on
overtime participation for low hourly wage individuals. An increase in the expected monetary incentives of a promotion increases the likelihood an individual works more than the minimum regular work hours.
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Chapter 1

1 Incentives for Effort with Team Production

1.1 Introduction

This article provides a further understanding of the interactions between individuals working in a firm with joint production. The focus is on the effort decisions made by agents of differing types with uncertainty and asymmetric information in regards to the agent’s teammate and production process. The results provide an explanation, based on cost of effort of agents, luck in the production process, and proportion of types, for the teamwork that can exist in work environments. The analysis also includes several important features that reflects some aspects of a professional work environment such as a pyramid organizational structure and team rewards.

An important assumption of the analysis is joint production in firms. Joint production involves a group of agents who jointly produce some observable output. Each agent’s input towards the production of output is unobservable. The observable output determines the rewards of the team members.\(^1\) Moral hazard may exist when each agent’s input is unobservable. A teammate may have an incentive to contribute less input (i.e., high cost of effort) and free ride on the inputs of others.

The framework is related to the theoretical literature regarding teams in the presence of asymmetric information. Many models focus on the actions and incentive scheme of the principle to induce efficient effort from its workers in teams.\(^2\) This article adds to

\(^1\)Suppose a team of workers produced a single unit of output that a firm considered good quality. The firm gives the “team” a good reputation instead of reputation per individual because firms cannot observe a workers type or effort. Each member of the team, therefore, has a good reputation regardless of the effort each member exerted in producing the good quality output. Each member is given the individual reward that accompanies a good reputation. Since the reputation is given to the entire team regardless of type and effort, there exists incentives of some members to free ride on the efforts of others to achieve a good reputation.

\(^2\)Alchian and Demsetz (1972) states that residual from production should be given to an individual whose
the literature regarding incentives in teams in the presence of asymmetric information by including the impact of uncertainty created by the production process and unknown type of teammate on the shirking decision of the agent.

Most theoretical work is inadequate for many applications. The model in this article incorporates some characteristics of professional work environments to better explain the teamwork that exists. First, there is pyramid organizational structure in which a manager is responsible for multiple agents. Second, are two levels of uncertainty in the form of the type of a teammate and in the production process (“bad luck”). Lastly, firms use a promotion scheme to 1) incentivize its workers to exert effort and 2) act as a imperfect screening mechanism to only promote high ability individuals.

It is of more interest to investigate and analyze a scenario that uses a more commonly used or observed incentive scheme to induce effort in teams (i.e., promotions). Jeon (1995) and Auriol et al. (2002) considered an incentive scheme in which wages and payoffs are dependent on the team outcome (incentives tied/linked together). By exerting effort, the team performance is better which leads to higher payoffs for the worker. Breton et al. (2000) and Breton et al. (2002) analyzed incentive schemes for teamwork if teams are composed by workers of different ages. By pairing young workers that are unestablished with old workers with an established reputation, the team output is a better signal for the young workers type. Therefore, concern for his own reputation for the future, the young worker exerts effort. Huck et al. (2001) use a common incentive scheme in bonuses for team output but assume a social norm, such as disutility of exerting less effort than his team, exists in the work environment. The bonus increases individual effort but the existence of the social norm increases the effort exerted much more. The incentive scheme this analysis uses is

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promotions. If a team of workers perform successfully for a firm, all low level workers in the
team are offered a promotion to manager. If they do not perform well, they are not offered
a promotion. As shown later, the promotion can be an incentive to exert effort and it acts
as a imperfect screening mechanism for the firm to promote high ability type individuals to
be managers.

This article is closest to the analysis of Bar-Isaac (2007). He introduces a framework
with overlapping generations of agents with three period lifespans. Agents can start their
own firms and hire junior workers that produce output with them. The junior worker
can eventually have the option to purchase the firm from his manager. It proceeds to
characterize an equilibrium where agents exert costly and efficient effort throughout their
life by comparing the expected payoff from sticking to the equilibrium strategy to all possible
deviations. He concludes that by working alone, an agent with an established reputation
cannot credibly commit to exerting costly effort. However, if the agent hires a junior
of unknown reputation, the uncertainty acts as an incentive for the agent to exert effort
because of concerns for his own reputation. This article builds upon Bar-Isaac (2007)
by incorporating another level of uncertainty in the production process and finds that the
introduction of uncertainty in the production process creates the existence of an equilibrium
of no shirking. Also, a different incentive scheme (promotions) is used to incentive the
juniors to exert effort which provides a different result. Although the methodology is similar,
this analysis provides a solution using a different incentive scheme and additional level of
uncertainty that is more widely observed in reality.

This article is best suited towards illustrating work environments where production of
output is by teams of workers and the individual effort is unverifiable. Examples of such
work environments are in industries such as consulting, law, or manufacturing. Within
these organizations, the productivity of a single individual is hard to verify when output is
produced in teams. For example, consider a service based organization such as a law firm.
When a team of lawyers defend a case in court, it is hard to know the effort each lawyer put
into the case. If efforts of each lawyer is unobserved, incentives cannot be given according
to effort. The reward may be given to the entire team with each member having an equal
share (promoting all members of the team). When rewards are given in this manner, there
exists the possibility of incentives for individuals to “free ride” on the efforts of others and reap the benefits of success without exerting any costly effort of their own.\textsuperscript{3}

Consider an economy that contains many firms that produce output. Firms employ a technology that requires a team of two individuals to produce a single unit of output. One member of the team is the manager; an individual with experience in production. The other member is the worker; an individual with or without experience in production. The technology also allows the manager to manage two workers separately but simultaneously. The manager’s action is applied to both teams. This creates two teams producing two units of output for the firm.\textsuperscript{4} The output produced can be of two qualities, good or bad, and the probability that determines the quality is dependent on the effort levels the individual team members choose to exert. The firm values good quality (success) greater than bad quality (failure).

The firm’s role is to select individuals to fill the positions for production and organize them into teams for production. The firm offers a contract with wages that are dependent on the position and work history of the individual. The contract also states the amount of retirement income the individual will receive depending on the individual’s work history. The wages are equal to an individual’s expected worth for his age and position in the firm.

There are two types of individuals that a firm can hire to fill the positions for production (managers and workers). The types differ by their efficiency levels. A high type individual is someone that can choose to exert effort but it is costless (cost of effort is equal to zero). The other type, low types, can exert the same amount of effort as a high type but incurs a positive cost of effort. Since a high type and low type can exert the same amount of effort but low types incur a cost, a high type is considered more efficient compared to a low type.

Firms provide incentives for individuals to exert effort in the form of promotions and retirement income. If a worker’s team performs successfully in production (produce good

\textsuperscript{3}A relatable example is group assignments. A professor issues a group assignment to his class. Students hand in the assignment and the professor grades it. If the professor knows the amount of work each student put into the assignment, the professor can give a grade according to the percentage of work each member gave. A practice observed in some courses requires students to hand in evaluations of their group members and the professor gives grades to individual students according to those evaluations. If the professor does not know the work put in by each group member, each member receives the same grade (team reward).

\textsuperscript{4}This creates a vertical hierarchy structure within firms. This is often referred to as the pyramid organizational structure where a manager supervises/manages more than one worker.
quality output) for a firm, he is promoted to manager. If a worker’s team does not perform successfully, he remains a worker but is now experienced (one period older). Managers that perform successfully (produce at least one unit of good quality output) is rewarded with more in retirement. Promoting individuals with good work histories (team success) is an imperfect screening mechanism for high efficiency types. Firms would prefer to promote high type individuals to managers. Since a high type individual incurs zero cost of effort, exerting effort is a weakly dominant strategy. Also, the effort he exerts is applied to both teams. This makes hiring a high type individual as a manager a more cost effective way to increase the probability of all teams producing good quality output. Since type and effort are unobservable, promoting a worker because of his work history provides the highest probability of obtaining a high efficiency type as a manager.

Individuals live for three periods; the first two are working periods and the final is a retirement period. A period ends after production has taken place and the quality of output is observed. They first apply for a job as a worker in a firm. They are considered “young” or inexperienced. The firm pairs him with a manager in the firm for production. Types of individuals are unobservable to firms and other individuals although age is observable. The “young” worker chooses an effort level to exert in team production. After production, the quality of the output is observed. If the quality is good, the worker is offered a promotion to a manager position by a firm. As a manager, the individual is paired with two workers. He manages then separately but simultaneously. The effort he chooses to exert is applied to both teams. The production process occurs again. When the quality of team output is realized, the firm gives the manager a retirement bonus if at least one of his teams was able to produce good quality output. The individual retires after the production process as a manager. If the team produced bad quality output, a “young” worker is not offered a promotion. He remains a worker but he is “old” and experienced. He is paired with a manager and the production process occurs again. He retires after his second round of production as a worker. In neither of the manager’s teams produces at least one unit of output that is good quality, then he receives no retirement income. The wages and retirement for all the positions, young and old worker and manager, were agreed upon when the firm offered a contract to the individuals.
The analysis focuses on 3 possible equilibrium strategies of an individual. It proposes a specific equilibrium strategy and finds the exogenous parameters that supports the strategy. The three equilibrium strategies that this analysis focuses on focus on the effort decisions of a low efficiency type individual. The individuals always work for the firm and the individual chooses whether to exert effort only as a “young” worker, only as a manager, or in both positions. Other equilibrium strategies, such as not working for the firm or rejecting promotions if offered, are not considered in this analysis.

A simulation of the model illustrates that these three alternative possible pure strategy equilibria depend on the parameters of the model; $c$ (cost of effort), $\beta$ (“bad luck” in the production process), and $\gamma$ (proportion of low efficiency types in each generation). High efficiency types have a weakly dominant strategy to exert effort in all positions, worker and manager, as the effort they exert is costless. Old and low efficiency type workers never exert effort because there are no incentives to do so. The analysis focuses on the actions of the low efficiency types. Low efficiency types can choose to exert effort in three possible scenarios; in all positions (young worker and manager), only exert effort as young (first period) workers, or only exert effort if promoted to manager. When cost of effort, $c$, is sufficiently low, $\beta$ is sufficiently low, and $\gamma$ is sufficiently high, effort is exerted in all positions and no shirking occurs. As cost of effort increases, exerting effort becomes less attractive. The agent chooses only one position to exert effort. When $\beta$ is low or $\gamma$ is high, the individual only exerts effort as a manager. If $\beta$ is high or $\gamma$ is low, the individual only exerts effort as a young worker.

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5Only exerting effort in one of the positions or in only one period of life means the individual free rides on the efforts of the team to obtain a reward. Exerting effort in all positions or both periods means he does not free ride. An agent never exerts effort as a “old” worker because there are no incentives in the model for him to exert effort (no retirement income for “old” workers). Also, a high efficiency type has zero cost of effort and therefore, always exerts effort as a weakly dominant strategy.

6It is believed that these are the most interesting equilibria and trivial equilibrium strategies, such as no individuals work for the firm, are not considered.

7The design of the model does not offer a reward to workers in their second working period if they produced good quality output. It can certainly be included into the model and possibly several other equilibrium spaces are created to account for the parameters that satisfy those conditions. But the qualitative results of the current equilibria should not change significantly. The addition of a retirement package for old workers to incentivize effort increases the overall expectation for a manager to produce good quality output. This impacts a manager’s decision regarding his effort choice.
1.2 The Model

This section presents a model to illustrate the actions of an agent throughout his lifetime while working for a firm that uses teams in the production process. It demonstrates the impact of uncertainty and asymmetric information, in regards to an agent’s teammate and the production process, on an agent’s contribution to the team.

There are important features that help the model reflect and provide a better understanding of professional work environments. There is a probability function that determines quality of production which obscures the actions of agents in the team. The quality of the output does not provide information on the effort choices of individual team members. A team of agents produce a single unit of output. The quality of output is dependent on the total amount of effort exerted by team members (joint production) and the public cannot tell which team member exerted effort.\(^8\) Also, the vertical hierarchy of firms reflect an environment where an individual manages a team of workers, which is believed to be more observed in reality (i.e., pyramid organizational structure). Finally, the rewards for production are given to the entire team regardless of effort because it is unobservable.

The analysis focuses on three possible equilibrium strategies of agents while working for a firm. Agents always choose to work in a firm, if possible, rather than choosing their reservation payoffs. The \(H\) type agent always chooses to exert effort while working for a firm. The \(L\) type agent can choose to exert effort as a young worker and manager, only as a young worker, or only as a manager. The analysis constructs conditions for the \(H\) and \(L\) type agents for sticking to an equilibrium strategy and compares them to any deviations. The equilibrium strategy presented in this article is where the \(L\) type agent exerts effort as a young worker and manager. The construction of the conditions for other equilibriums are similar and the differences are explained in footnotes of the article. To illustrate the existence of the equilibrium, a simulation is performed and the conditions are illustrated in

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\(^8\) "Two men jointly lift heavy cargo into trucks. Solely by observing the total weight loaded per day, it is impossible to determine each person’s marginal productivity. With team production it is difficult, solely by observing total output, to either define or determine each individual’s contribution to this output of the cooperating input." - Alchian and Demsetz (1972), pg. 779. Assuming a supervisor (in the case of this article, the firm) saw the heavy cargo on the truck, they would assume that half the weight was lifted by one man and the other half lifted by the other. Therefore, half the weight is attributed to one man and the other half to the other (assuming the supervisor was not observing the process of loading the truck).
a parameter space. Certain combinations of values of parameters will satisfy the conditions of different equilibria creating areas with multiple equilibria. The equilibrium strategy that provides an agent with the highest lifetime expected payoff is considered the dominant strategy. Other equilibrium strategies are possible but this analysis focuses on, what is believed to be, the more interesting equilibrium strategies.

The model introduces a framework with overlapping generations of agents that live for three periods and work for a firm; the first two periods are working periods and the final period is a retirement period where agents only consume. It characterizes equilibria where agents choose to exert costly effort in one or both working periods while working for a firm. The proof of equilibria results begin by constructing the expected payoff of the agent along his decision path and then checking if there are incentives to deviate. The expected payoffs of an agent sticking to the equilibrium strategy is first constructed and then compared to the expected payoff from a deviation. If an agent behaves rationally and sticks to the equilibrium strategy, then there is a pure strategy equilibrium. A simulation is used to illustrate that any deviation from the equilibrium strategy, given certain parameters, leads to a lower expected payoff for the agent.

1.2.1 Technology

The technology firms employ to produce a single unit of output for production requires a team of two agents. One of the agents is a manager. A manager is an agent that has worked in a firm in the previous period and was part of a team that produced good quality output. Due to his good work history (good quality output), he was offered a promotion to a manager position in a firm. The other member of the team is a worker. A worker is an agent that can be young or old. A young worker is an agent that has no experience with the production technology (an agent in their first working period). An old worker is an agent, similar to a manager, with experience in production (worked in a firm in the previous period) but did not produce good quality output and obtained a bad work history. He is not promoted and remained a worker. More details regarding how agents become managers and workers is explained in a later section.

The technology allows a manager to work with two workers simultaneously but
separately. Therefore, for every manager in a firm, there are two workers hired. Since a team is composed of one manager and one worker, this creates two teams that produce a total of two units of output for the firm. The effort that a manager exerts is applied to both teams. This technology also creates a pyramid organizational structure where a higher level employee (manager) is in charge of one or more lower level employees (workers) but the lower level employees work separately.

The output produced by the team of agents can be of two qualities: good or bad. The probability that the output is of a certain quality is dependent on the amount of effort that each member of the team exerts. An explanation of the probability is provided in a later section.

1.2.2 Agents

Agents live for three periods. The first two periods are working periods where the agent chooses to work for the firm and the final period is a retirement period where he only consumes. A new generation of agents with a mass of 1 is born each period creating overlapping generations of agents.

An agent may be born high (H) or low (L) type which is unobservable to firms and other agents. There is a probability \( \gamma \) that the worker is born L type and \( 1 - \gamma \) is born H type where \( 0 < \gamma < 1 \). At birth, an agent knows his own type.

Agents have an effort choice while working denoted by \( e \in \{0, \bar{e}\} \) where \( \bar{e} > 0 \). A H type agent always chooses to exert an effort level of \( \bar{e} \). Exerting this level of effort is costless \( (c = 0) \) to the agent. Since a H type agent does not incur a cost \( (c = 0) \) while choosing to exert effort \( (\bar{e}) \), he always chooses to exert effort. Exerting effort is a weakly dominant strategy. A L type agent chooses an effort level to exert. If he chooses \( \bar{e} \), he incurs a cost of \( c > 0 \). Choosing not to exert effort incurs no cost \( (c = 0) \). The difference between the two types of agents is the efficiency level. A H type agent can exert the same amount of

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There exists a possibility where workers from the same group are paired together next period in a manager-worker relationship. The successful worker is promoted to manager and is randomly paired with the failed worker from the other team the manager supervises. The model in this paper does not model the aspect of interpersonal relationships and externalities between group/team members. It is assumed that everyone does not have information regarding their teammates except for the common information such as \( \gamma \).
effort as a $L$ type agent but at a lower cost.

All agents are risk neutral, maximize their expected lifetime earnings, and have a discount factor of 1 (discount rate of 0).

1.2.3 Firms

There exists many identical risk neutral firms. An agent’s type and effort is unobservable to the firm but age of agents is observable. Firms hire one agent that was in his first working period in his previous period and produced good quality output in a team to be a manager.\footnote{In this article, agents in their first working period are referred to as “young”. Agents in their second working period are referred to as “old”.} They also hire two agents to be workers managed by the manager.

The joint production process of the firm requires two agents; one manager and one worker. As mentioned previously, the manager manages two workers simultaneously creating two teams with a team comprising of the manager and one of the two workers.\footnote{The probability that each team produces good quality output is statistically independent but the variables determining the probabilities are common. The probability that one of the manager’s teams produces good quality output does not depend on the quality produced from the other team.} Therefore, each period, two units of output are produced by each manager for the firm.

Each unit of output can be of two qualities: Good ($G$) or Bad ($B$). The firm values good quality output at 1 and bad quality output at zero.

Firms cannot observe the type and effort of agents. Due to the nature of the joint production process, firms are unable to identify from the quality produced, which agent exerted effort. Therefore, firms arbitrarily attribute half ($\frac{1}{2}$) the value from production to each member of the team.\footnote{The agents in the model are risk neutral. The $H$ type agent exerts effort but it is costless. The $L$ type agent can exert effort but incurs a cost of $c > 0$. Therefore, if the $L$ type agent exerts effort, they obtain the same expected payoff as the $H$ type agent minus cost of effort, $c$. Suppose a firm changed the share of value from production between workers and managers where it increases the expected payoff for $H$ type agents. They will move to the new firm offering the different contract. Since the expected payoffs are the same between the two types, the new share also increases the expected payoff of $L$ type agents. The new contract has attracted all types of workers but at a different share. Therefore, the share does not affect the qualitative results of the analysis.}

Reputation is earned from production in a firm. Quality of output produced in a team determines an agent’s reputation. Since effort is indistinguishable between agents in teams, the reputation earned from production by a team is given to all team members regardless of effort. This means the team member’s reputation is equal to the team’s reputation.
the quality of output is $G$, agents earn good reputations, otherwise, $B$ quality output earns no reputations.

Rewards accompany good reputations. A young worker with good reputation is offered a promotion for the subsequent period to manager by a firm. Firms benefit from promoting $H$ types to manager because they have a weakly dominant strategy to exert effort for zero cost. By promoting $H$ types to manager, this increases the probability of producing good quality output of all teams at the lowest cost to induce effort. Since type and effort is unobservable, promoting workers that produced good quality output gives the highest probability of a $H$ type manager. An old worker with a good reputation is rewarded with nothing and retires next period. A manager earns a good reputation as long as one of the teams he manages produces good quality output. A manager with good reputation is given $R$ in retirement. Agents with bad reputations are given nothing next period. In this case, a worker re-applies for a position as worker when he is old and a manager retires with nothing in the retirement period.

For services of the agents, firms pay them a wage according to their position (worker or manager) and age. They pay a wage equal to the expected value they bring to the firm. Therefore, firms make an expected profit of zero. Employment contracts are in the form of $(w_{YW}, w_{OW}, w_M, R)$ where $w_{YW}$ is the wage that a young worker is paid, $w_{OW}$ is the wage an old worker is paid, $w_M$ is the wage a manager is paid, and $R$ is the retirement consumption given to managers.

1.2.4 Timing

In the beginning of an agent’s first working period, the agent is born and the type is revealed to him. The agent chooses to apply for a job with a firm in a worker position. A worker is considered as the low level employee in the vertical hierarchal structure. If he does not apply or is not hired by a firm, he obtains a reservation payoff of $\bar{P}$ equivalent to

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It is true that providing different $R$’s dependent on whether the outcome is one or two units of good quality output for the manager can potentially provide a lower expected $R$ required to induce effort. For simplicity of the results, only one $R$ is used.

Despite the possibility that the employment contract can be in the form of $(0, 0, 0, R)$ with $R$ sufficiently large and achieve similar results, for the purpose of realism, this analysis assume a cap on $R$ to achieve positive wage levels. The potential of including risk aversion for agents in the future provides a reason for the analysis performed here.
the expected value of producing a single unit of output by himself.

If the agent is hired to be a worker in the first working period, he is paired with another agent in a manager position. The manager is considered to be a high level position on the vertical hierarchal structure. The two agents work together to produce a single unit of output for the firm. The worker decides whether or not to exert effort. If he exerts effort, the probability of producing good quality output increases. The probability of producing good quality output is dependent on the combined efforts of the worker and manager.\textsuperscript{15} Likewise, the effort choice of the worker impacts the probability of producing good quality output of the manager.

In this joint production process, the effort decision of an agent can influence the outcomes of his teammate and \textit{vice versa}. By exerting effort, an agent increases the team’s probability of producing good quality output. Not exerting effort hurts both agents by not increasing the probability. Effort is unobservable and the quality of output does not provide information about which member of the team exerted effort. Firms and other agents cannot tell which agent exerted effort. This production process allows an agent to shirk on his teammate without incurring the true total cost of the team because costs are incurred by both team members.\textsuperscript{16}

In the second working period, the quality of output from production in the first working period is observed (given he was hired by a firm in the first working period). If the team produced good quality output, the team members are all given their corresponding rewards. The worker is given a promotion to manager from a firm. The worker decides to accept the promotion and becomes a manager. The firm hires two workers for the manager to manage simultaneously. This creates two teams with a team comprising of the manager plus one of the two workers. The manager has an effort decision. The effort decision made by the manager is applied to both teams. The output produced from each team is independent from the other.

\textsuperscript{15}Each team of workers produce one unit of output that can be of two qualities: good or bad. The firm values each quality differently. The section on firms, presented later in the article, provides details on the role of the firm and its valuation on quality.

\textsuperscript{16}If efforts are observable, it is possible to only penalize the agent that chose not to exert effort. Therefore, the agent incurs the full cost of shirking. If effort is unobservable, the team is punished and therefore some costs are incurred by the agent that chooses to exert effort.
If he rejects the promotion, he chooses to work alone and earns a reservation payoff of $P$ equivalent to the expected value of producing a single unit of output by himself given that he produced good quality output in a firm in the previous period.\footnote{An individual that does not have a history (bad) of production from a firm has a reservation payoff of $\bar{P}$ (P “bar”). This is given to young or old agents that are not hired by firms. An individual with a (good) history of production from a firm has a different reservation payoff given by $P$. The reservation payoff for an agent with a production history is updated to reflect the “good reputation” he earned from a firm. (i.e., Using a successful history at a firm to advertise his own business.)}

If bad quality was produced in the first working period, the worker is not rewarded with a promotion to manager. The agent applies for a worker position in his second working period. The reputation earned from producing bad quality is unobservable to the public.

Similarly, if the agent was not hired in the first working period as a worker, he applies for a job in his second working period. Producing good quality output while working alone does not get any rewards from a firm.

The output quality from an individual’s second working period is observed. If, at least one of the teams, good quality output is produced, then the manager is given a retirement consumption of $R > 0$ in the retirement period. If neither of the teams produced good quality output, the manager is given 0. Workers in their second working period also retire but receive no retirement benefits. Therefore, the only way to obtain $R$ is to work for a firm as a manager and produce at least one unit of good quality output in each working period.

### 1.2.5 Probability of Quality

In general, when members of a team all choose to exert effort, the probability of producing high quality output is usually greater compared to if they choose not to exert effort. Even though the team members exert effort, sometimes people make mistakes and therefore the quality of work suffers or the task was completed not to standard. There are no guarantees in the outcome of a task unless all team members choose to not exert effort.

This article uses a probability function to determine the quality of output produced by a team in a firm that incorporates characteristics of teamwork explained above. The function adopted, for $N$ team members, is the following,

\[
p(e_1, \ldots, e_N) = \frac{\sum_{i=1}^{N} e_i}{\sum_{i=1}^{N} e_i + \beta} \quad \text{where} \quad \beta > 0 \quad \text{and} \quad e_i \in \{0, \bar{e}\} \quad (1)
\]
The equation is a modification of the popular Tullock contest success function. Instead of the players exerting effort to defeat the other, they exert effort to help the team succeed.18

First, \( p(e_1, \ldots, e_N) \) is increasing and concave in all \( e_i \)’s. This equation satisfies the property that the probability of good quality is increasing in each individual’s effort.

Second, the function includes a measurement of error involved in production. The error can arise from team member accidents, mistakes, bad luck, or any other uncontrollable/unobservable factors. The magnitude of these factors’ effect on the production process is captured in \( \beta \). The \( \beta \) captures the extent to which luck as opposed to effort determines the success in a team environment. As \( \beta \) increases, the probability of good quality decreases (i.e., the likelihood of a good quality product depends more on luck).

When \( \beta \) is strictly positive, the probability of good quality is never 1. If the entire team exerts effort, that is the highest probability of producing good quality output the team can achieve. Therefore, if they produced a bad quality output while all team members exert effort, it is caused strictly by uncontrollable factors and not by the efforts of team members. On the other hand, if all members of the team exert zero effort, then the team produces bad quality output with certainty.

In this article, a team working for a firm is composed of two agents; a worker and a manager. Since types are unobservable, agents form expectations of the probability that good quality output will be produced. Therefore, the probability conditional on the worker’s type for a worker and manager position are denoted by \( p^W(e_W; e_M^H, e_M^L) \) and \( p^M(e_M; e_W^H, e_W^L) \), respectively. The worker’s probability is a function of the effort choice of the worker, \( e_W \), and the effort choice of the manager he is paired with in equilibrium depending on whether he is \( H \), \( e_M^H \), or \( L \) type, \( e_M^L \). The manager’s probability is a function of the effort choice of the manager, \( e_M \), and the effort choice of the workers in equilibrium depending on whether they are \( H \), \( e_W^H \), or \( L \) types, \( e_W^L \). The probabilities, \( p^W(e_W; e_M^H, e_M^L) \) and \( p^M(e_M; e_W^H, e_W^L) \), must all be consistent with the on and off equilibrium actions of the agents.

\(^{18}\)Amegashie (2006) proposed a similar contest success function in which he examined the degree to which luck as opposed to effort affects behaviour in different contest settings. His paper presents and discusses the properties of the contest success function which is very similar to the probability function used in this article.
1.2.6 Wages

As mentioned in a previous section, firms compete for agents and pay a wage equal to the expected value they bring to the firm.

The wage given to a young worker is,

\[
w_{YW} = \frac{1}{2} \left[ p(H^{YW})p_H^W(e_W; e_M^H, e_L^M) + p(L^{YW})p_L^W(e_W; e_M^H, e_L^M) \right]
\] (2)

The wage is constructed as follows. If the team produces good quality output, the firm values it at one and attributes half \((\frac{1}{2})\) the value produced to the young worker. To the firm, the worker can be of two types; \(H\) and \(L\). If the young worker is \(H\) type, \(p(H^{YW})\), he is expected to produce good quality with probability \(p_H^W(e_W; e_M^H, e_L^M)\). If he is \(L\) type, \(p(L^{YW})\), he is expected to produce good quality output with probability \(p_L^W(e_W; e_M^H, e_L^M)\). If bad quality output is produced, the firm values it at zero.

The wage given to an old worker is,

\[
w_{OW} = \frac{1}{2} \left( p(U) + p(F) \right) \left[ p(H^{OW})p^W(\bar{e}; e_M^H, e_M^L) + p(L^{OW})p^W(0; e_M^H, e_M^L) \right]
\] (3)

The wage is constructed as follows. If the team produces good quality output, the firm values it at one and attributes half \((\frac{1}{2})\) the value produced to the old worker. For an agent to be an old worker, he must have been unemployed in the previous period, \(p(U)\), or part of a team that produced bad quality output in the previous period, \(p(F)\). Similarly to the young worker’s wage, the probability of an old worker producing good quality output is given by the probability the worker is of either type, \(p(H^{OW})\) and \(p(L^{OW})\), multiplied by the probability of producing good quality output, \(p^W(\bar{e}; e_M^H, e_M^L)\) and \(p^W(0; e_M^H, e_M^L)\), depending on whether he is \(H\) or \(L\) type respectively.\(^{19}\)

\(^{19}\)An \(L\) type old worker chooses to never exert effort. He gets paid an old workers wage for being hired but there are no incentives for him to exert effort because there are no rewards in his retirement period.
The wage given to a manager is,

\[
w_M = \left[ 2(p(H^M)p^M(e_H^M; e_H^W, e_L^W) + p(L^M)p^M(e_L^M; e_H^W, e_L^W)) \right.

\left. + (p(H^M)p^{M1}(e_H^M; e_H^W, e_L^W) + p(L^M)p^{M1}(e_L^M; e_H^W, e_L^W)) \right]

- 2(p(N)w_{YW} + (p(U) + p(F)w_{OW}) - Rp^M(e_M; e_H^W, e_L^W) \right]

(4)

The wage given to a manager is constructed as follows. The manager is paid the residual of the expected value earned in production. A manager supervises two teams simultaneously. If both teams produce good quality output, the firm values the output at 2. The probability of both teams producing good quality output is given by the probability the manager is H or L type, \(p(H^M)\) and \(p(L^M)\), respectively, multiplied by the corresponding probability both his teams produce good quality output given the managers type, \(p^M(e_H^M; e_H^W, e_L^W)\) and \(p^M(e_L^M; e_H^W, e_L^W)\). Similarly, if only one of the manager’s teams produces good quality output, the firm receives a value of 1 and is multiplied by the probability of only one team producing good quality output, \(p(H^M)p^{M1}(e_H^M; e_H^W, e_L^W) + p(L^M)p^{M1}(e_L^M; e_H^W, e_L^W)\). If no teams produce good quality output, the value is zero. The expected value from production for a firm is the first two terms in the managerial wage. The firm must pay wages to the two workers hired for production. The workers hired by a firm can be young or old. Finally, the firm pays the manager a retirement consumption of \(R\) next period if at least one of his teams produces good quality output, \(p^M(e_M; e_H^W, e_L^W)\). Subtracting the expected workers wages and retirement consumption of the manager from the expected value from production is the manager’s wage.

If an agent does not work for a firm, he receives a reservation payoff equal to the expected value from production if he was working alone instead of in a team. There are two potential reservation payoffs for the worker; a reservation payoff if he does not have a history of production from a firm or if he produced bad quality output with a firm in the previous working period (\(P\)) or a reservation payoff if he produced good quality output in a firm in the previous working period (\(\bar{P}\)).

Suppose an agent was in their first working period and was not hired by a firm or in their second period but produced bad quality output in the previous period, he has no
production history. Therefore, his reservation payoff is given by,

\[ P = p(H) \frac{\bar{e}}{\bar{e} + \beta} \]  

(5)

The reservation payoff is the expected payoff from producing output by himself. Good quality output is valued at 1 and bad quality valued at zero. The expected payoff is the probability the agent is \( H \) type provided his age is observable multiplied by the probability of producing good quality output while working alone. \( L \) type agents working alone do not exert effort and produce bad quality with certainty because there are no rewards next period for producing output alone. Since there are no incentives next period, an agent chooses no effort.

If an agent produced good quality output in a firm the previous period, the agent accepts a promotion to manager in equilibrium. But if he rejects the promotion, the agent produces alone. Given a history of good quality output last period in a firm, the agent has a different reservation payoff.

\[ \bar{P} = p(H|G) \frac{\bar{e}}{\bar{e} + \beta} \]  

(6)

Similar to \( P \), the reservation payoff is the probability the agent is \( H \) type given he produced good quality output in a firm in the previous period multiplied by the probability of producing good quality output while working by himself.

### 1.2.7 Probability Agents Hired as Workers

In each working period, there are agents, young and old, applying for jobs as a worker. Applying for a job with a firm as a worker does not guarantee the agent a job. The agent has an probability of being hired by the firm in each working period. The probability that an agent is hired as a worker in a working period is constructed as follows.

The number of workers hired by firms is directly related to the number of managers hired by firms. For each manager hired by a firm, two workers are hired. The expected number of managers in a period is given by \( N \),

\[ N = \alpha p^W(e_W; e^H_M, e^L_M) \]  

(7)
The above equation is constructed as follows. To become a manager in the second working period, the agent must have been a worker in the first working period. Therefore, the agent was a young worker. Each period, a new generation of agents is born with a size of 1. Of the agents born, a proportion of them apply to be workers and are hired. This is represented by the variable $\alpha$. To be a manager, the worker must have been part of a team that produced good quality output. The probability that a worker produces good quality output is given by $p^W(e_W; e^H_M, e^L_M)$. Therefore, the expected number of managers in a period is $N$.

Each firm hires two workers for each manager hired. Therefore, the number of worker vacancies ($V$) each period is,

$$V = 2\alpha p^W(e_W; e^H_M, e^L_M)$$

(8)

The number of applicants ($A$) for worker positions in a period is given by,

$$A = 1 + (1 - \alpha) + \alpha \left(1 - p^W(e_W; e^H_M, e^L_M)\right)$$

(9)

The above equation is constructed as follows. There are three groups of agents that apply for worker positions in a given period. Each period, there is a new generation of agents born with a size 1. There is a proportion of young agents last period that were not hired by firms last period $(1 - \alpha)$. Lastly, there are agents that were hired in their first working periods as workers, $\alpha$, that were part of a team that produced bad quality output, $(1 - p^W(e_W; e^H_M, e^L_M))$. They re-apply for worker position in their second working period.

The probability of being hired as a worker in equilibrium ($\alpha$) is the number of vacancies ($V$) divided by the number of applicants ($A$).

$$\alpha = \frac{V}{A} = \frac{2\alpha p^W(e_W; e^H_M, e^L_M)}{1 + (1 - \alpha) + \alpha \left(1 - p^W(e_W; e^H_M, e^L_M)\right)} \implies \alpha = \frac{2(1 - p^W(e_W; e^H_M, e^L_M))}{p^W(e_W; e^H_M, e^L_M)}$$

(10)

1.3 Equilibrium Strategies

This article focuses on the three possible equilibrium strategies that are believed to be the most interesting. Other equilibrium strategies and even multiple equilibria are possible.
but are not analyzed in this article but is considered in future iterations. The analysis focuses on the four possible equilibrium strategies of effort for an $L$ type agent; exerting effort as a young worker and manager, only as a young worker, or only as a manager. All agents choose to work for a firm if possible. Other possible equilibrium strategies such as deviations in their career decisions (i.e., not working for the firm or rejecting promotions to manager) are not looked at.

The analysis begins by hypothesizing the existence of one of the equilibrium strategies of the $L$ type individual and finding the exogenous parameter values that support its existence. The proof of the results starts by outlining the equilibrium strategy of agents in each period of life and checking possible deviations from the equilibrium strategy to verify no agent has an incentive to deviate. The expected payoff of each action in the equilibrium strategy is constructed and compared to the expected payoff of a deviation. Conditions are formed to show that, given a set of exogenous parameters, any deviations from the equilibrium strategy leads to a lower expected payoff. The analysis starts with the last decision an agent makes and works backwards in his decision path towards the first decision of his life.

The conditions constructed in this model are similar to incentive compatibility and participation constraints. They illustrate that, if the expected payoff is greater when performing an action compared to another, the action with the higher expected payoff is taken. Assuming agents in this model are rational, the following inequalities can be considered as rationality conditions as agents choose the action that provides the higher expected payoff.\footnote{In the standard view, rational choice is defined to mean the process of determining what options are available and then choosing the most preferred one according to some consistent criterion. In a certain sense, this rational choice model is already an optimization-based approach.” - Jonathan Levin and Paul Milgrom. Introduction to Choice Theory. Jonathan Levin: Teaching and Lecture Notes. http://web.stanford.edu/~jdlevin/Econ%202020/Choice%20Theory.pdf. September 2004.}

The equilibrium strategy of both types is to apply for jobs when possible and accept any employment contract offered by a firm so long as such contract offers him in equilibrium at least as high an expected payoff as his reservation payoff of working alone.\footnote{The $L$ type agent never exerts effort as an old worker because they retire next period and is guaranteed zero regardless of the effort exerted. Also, the equilibrium strategy is described in terms of the $L$ types actions}

The conditions constructed in this section illustrate an equilibrium strategy where the $L$ type agent chooses to exert effort as a young worker and manager.\footnote{The $L$ type agent never exerts effort as a old worker because they retire next period and is guaranteed zero regardless of the effort exerted. Also, the equilibrium strategy is described in terms of the $L$ types actions}
strategies with the $L$ type agent exerting effort in a different combination of positions (i.e., only young worker, only manager, or never exert effort) is constructed in a similar approach. The expected payoff of sticking to the equilibrium strategy is constructed and compared to the expected payoff of a deviation. The changes in wages and $\alpha$ reflect the beliefs and on and off equilibrium actions of the agents and firms.

The $H$ type agent always choose to exert effort and it is costless ($c = 0$). As long as the expected payoff from working for a firm is greater than his reservation payoff, $H$ type agents always work for a firm. Suppose a $H$ type agent is offered a promotion in his second working period because he produced good quality output in the previous period in his team, the $H$ type worker accepts the promotion if,

$$w_M + p^M(\bar{e}; e_H^W, e_L^W)R \geq \bar{P}$$

(11)

The above inequality is constructed as follows. The left hand side (LHS) of the inequality is the expected payoff of an $H$ type agent accepting the promotion. The manager is given a wage of $w_M$. If one of the teams that he is part of produces good quality output, $p^M(e_M; e_H^W, e_L^W)$, the firm pays him a retirement package of $R$. The right hand side (RHS) of the inequality is the expected payoff of a worker that rejects the promotion and chooses to work alone. He makes $\bar{P}$, the income he obtains by selling output produced alone given he produced good quality output in a firm last period.

Inequality (11) can be re-written as,

$$w_M + p^M(\bar{e}; e_H^W, e_L^W)R - \bar{P} \geq 0$$

(11*)

If a $H$ type agent produces bad quality output in the previous period in a team, he re-applies for a worker position with a firm in his second working period.

$$\alpha w_{OW} + (1 - \alpha)P \geq P \implies w_{OW} \geq P \implies w_{OW} - P \geq 0$$

(12)

because the $H$ types do not have conditions to determine their effort choice. Since all types’ equilibrium strategy is to always work for the firm, the focus of the analysis is on the actions of the $L$ type agent while working for a firm.
The above inequality is constructed as follows. The LHS of the equation is the expected payoff of a $H$ type agent if he re-applies for a worker position in a firm in his second working period. If he is hired as a worker, $\alpha$, he is given a wage of $w_{OW}$. If the agent is not hired by the firm, $1 - \alpha$, the agent works alone and earns $P$, a reservation payoff for an agent with no history of production from a firm. If he chooses not to apply for a job, he obtains $P$ with certainty (RHS). Since cost of applying for a job is zero, if the wage earned in a firm is greater than the reservation payoff, $P$, the $H$ type agent applies.

Moving backwards on the decision path, the $H$ type agent applies for a worker position in a firm in his first working period.

\[
\begin{align*}
\text{LHS: } & w_{YW} + p^W(\bar{e}; e^H_M, e^L_M) (w_M + p^M(\bar{e}; e^H_W, e^L_W)R) + (1 - p^W(\bar{e}; e^H_M, e^L_M)) (\alpha w_{OW} + (1 - \alpha)P) \\
\text{RHS: } & P + \alpha w_{OW} + (1 - \alpha)P
\end{align*}
\]

(13)

Again, since cost of applying is zero, if the expected payoff from working in a firm is greater than his expected reservation payoff, the agent applies. If the $H$ type agent is hired, he earns the young worker’s wage, $w_{YW}$. In production, if his team produces good quality output, $p^W(\bar{e}; e^H_M, e^L_M)$, the agent has an expected payoff of accepting a promotion (Inequality (11)). If the team produces bad quality output, $1 - p^W(\bar{e}; e^H_M, e^L_M)$, the agent re-applies for a job in the next period and has an expected payoff of re-applying for a worker position (Inequality (12)). If the $H$ type agent does not apply for a job as a worker (RHS), the agent receives $P$ with certainty in the first working period for producing alone and next period applies for a job as a worker and has an expected payoff of applying in the second working period (Inequality (12)).

Equation (13) can be re-written as,

\[
\begin{align*}
\text{RHS: } & w_{YW} - P - p^W(\bar{e}; e^H_M, e^L_M) [\alpha w_{OW} + (1 - \alpha)P - w_M - p^M(\bar{e}; e^H_W, e^L_W)R] \\
\geq & 0 \quad (13^*)
\end{align*}
\]

\footnotetext[22]{If an agent was unemployed in the first working period, they are equivalent to an agent that produced bad quality output in a firm. In the second working period, the condition for an agent to re-apply as a worker in his second working period after producing bad quality output in the previous period is the same for an agent that was working alone in the previous period.}
The results of the analysis focus on the \( L \) type agent because of their effort decisions. Similar to the methodology for the \( H \) type agent, the analysis begins with the final decision an \( L \) type agent makes in his lifetime and works backwards towards the first decision. The equilibrium strategy of a \( L \) type agent is to exert effort as a young worker and manager and the conditions presented compare the expected payoffs of sticking to the equilibrium strategy to a deviation. The last decision an \( L \) type agent makes is an effort choice as a manager.\(^{23}\)

\[
-c + p^M(\bar{e}; e^H_W, e^L_W)R \geq p^M(0; e^H_W, e^L_W)R
\]

(14)

The above inequality is constructed as follows. The LHS of the inequality is the expected payoff of a \( L \) type manager that chooses to exert effort. If he exerts effort, he incurs a cost of \( c \). If the manager produces at least one unit of good quality output with his teams, \( p^M(\bar{e}; e^H_W, e^L_W) \), he receives a retirement package of \( R \). The RHS is the expected payoff of a manager that chooses not to exert effort. The manager does not incur the cost of \( c \) because he does not exert effort. The manager receives a retirement package of \( R \) if at least one of his teams produces good quality output, \( p^M(0; e^H_W, e^L_W) \). The \( H \) type agent exerts effort as a manager if the LHS is greater than or equal to the RHS.\(^{24}\)

Inequality (14) can be re-written as,

\[
[p^M(\bar{e}; e^H_W, e^L_W) - p^M(0; e^H_W, e^L_W)]R \geq c
\]

(14*)

The action prior to exerting effort as a manager is accepting a promotion offered by a firm. If a \( L \) type agent exerts effort as a manager in equilibrium, the condition for the agent to accept a promotion from a firm is,

\[
w_M - c + p^M(\bar{e}; e^H_W, e^L_W)R \geq \bar{P}
\]

(15)

The above inequality is constructed as follows. The LHS is the expected payoff of a

\(^{23}\)This assumes the \( L \) type manager produced good quality output last period, a firm offered a promotion, and the agent accepted the promotion.

\(^{24}\)If the equilibrium strategy is for the \( L \) type agent to not exert effort as a manager, inequality (14) is reversed with the RHS greater than or equal to the LHS with consistent beliefs.
$L$ type agent accepting the promotion to manager. By accepting, the manager receives a wage $w_M$. If the manager exerts effort in equilibrium, the expected payoff from production is given by the LHS of inequality (14). The RHS is the reservation payoff from rejecting the promotion and choosing to work alone given he produced good quality output in the previous period in a firm, $P$.

Inequality (15) can be re-written as,

$$w_M - \bar{P} + p^M(\bar{e}; e_H^M, e_L^M)R \geq c$$  \hspace{1cm} (15*)

The decisions made by a $L$ type agent so far relate if the agent produced good quality output in the previous period. If the agent produced bad quality output, the agent does not receive an offer of promotion. He re-applies for a position with a firm as a worker. This condition was explained previously and is the same for a $H$ type worker. This condition is inequality (12).

Moving backwards, the $L$ type agent makes an effort decision in his first working period while working for a firm as a worker.

$$-c + p^W(\bar{e}; e_H^L, e_L^L)(w_M - c + p^M(\bar{e}; e_H^L, e_L^L)R) + (1 - p^W(\bar{e}; e_H^L, e_L^L))(\alpha w_{OW} + (1 - \alpha)P)$$ \geq 

$$p^W(0, e_H^L, e_L^L)(w_M - c + p^M(\bar{e}; e_H^L, e_L^L)R) + (1 - p^W(0, e_H^L, e_L^L))(\alpha w_{OW} + (1 - \alpha)P)$$  \hspace{1cm} (16)

Inequality (16) is constructed as follows. The LHS is the expected payoff of a $L$ type worker choosing to exert effort in the first working period while working for a firm. He incurs a cost of $c$ for exerting effort. If his team produces good quality output for the firm, $p^W(\bar{e}; e_H^L, e_L^L)$, he has an expected payoff given by accepting a promotion and exerting effort as a manager (LHS of inequality (15)). If the worker’s team produces bad quality output, $1 - p^W(\bar{e}; e_H^L, e_L^L)$, the worker has an expected payoff given by re-applying for a worker position with a firm next period (LHS of inequality (12)). The RHS is the expected payoff

\footnote{If the equilibrium strategy for $L$ type agents is not to exert effort as managers, the inequality sign in condition (14) is reversed with the RHS greater than or equal to the LHS. The expected payoff from production on the LHS of inequality (15), $-c + p^M(\bar{e}; e_H^L, e_L^L)R$, becomes the RHS of inequality (14) because not exerting effort is the rational decision, $w_M + p^M(0; e_H^L, e_L^L)R$.}
of a $L$ type worker choosing not to exert effort in the first working period. The worker does not incur a cost of $c$. If his team produces good quality output, $p^W(0, e^H_M, e^L_M)$, he obtains an expected payoff given by accepting a promotion to manager and exerting effort. If the team produces bad quality output, his expected payoff is given by re-applying for a worker position next period. The expected payoffs for the second working period are the same if he exerts effort or not in the first working period. A $L$ type agent sticks to the equilibrium strategy of exerting effort as a young worker if the LHS is greater than or equal to the RHS.\footnote{If the equilibrium strategy for $L$ type agents is not to exert effort as young workers, the RHS is greater than or equal to the LHS. Furthermore, if the equilibrium strategy is for $L$ type agents to never exert effort as young workers and managers, first, the RHS is greater than or equal to the LHS and, in addition, the second working period expected payoff for producing good quality output changes to the expected payoff for not exerting effort as a manager.}

Inequality (16) can be re-written as,

$$\frac{p^W(\bar{e}; e^H_M, e^L_M) - p^W(0, e^H_M, e^L_M)}{1 + p^W(\bar{e}; e^H_M, e^L_M) - p^W(0, e^H_M, e^L_M)} \left(w_M + p^M(\bar{e}; e^H_W, e^L_W)R - \alpha w_{OW} - (1 - \alpha)P\right) \geq c$$

(16*)

The first decision the $L$ type agent makes is to apply for a job as a worker in a firm in his first working period. Since cost of applying for a job is zero, if expected payoff of being hired is greater than the reservation payoff of producing alone, the $L$ type agents apply for jobs in the first working period.

$$w_{YW} - c + p^W(\bar{e}, e^H_M, e^L_M)(w_M - c + p^M(\bar{e}, e^H_W, e^L_W)R) + (1 - p^W(\bar{e}, e^H_M, e^L_M))(\alpha w_{OW} + (1 - \alpha)P) \geq P + \alpha w_{OW} + (1 - \alpha)P$$

(17)

The above inequality is constructed as follows beginning with the LHS. If the agent is hired, he obtains a wage of $w_{YW}$ and an expected payoff given by exerting effort as a young worker and manager and accepting a promotion if offered one by a firm (LHS of inequality (16)). For an agent to apply for a job in his first working period, this must be greater than the expected payoff of the RHS. If the agent chooses to not apply, he receives $P$ for working alone and an expected payoff in the second working period given by re-applying for a worker.
Inequality (17) can be re-written as,

\[
\frac{1}{1 + p^W(\bar{e}, e_{M}, e_{M})} \left( w_{YW} - P + p^W(\bar{e}, e_{M}, e_{L}) (w_{M} + p^M(\bar{e}, e_{W}, e_{L}) R - \alpha w_{OW} - (1 - \alpha) P) \right) \geq c
\]  

(17*)

If the equilibrium strategy for the \( L \) type agent is to exert effort as a young worker and manager, the sufficient conditions are (11*) to (17*). The cost of effort must be sufficiently low for a \( L \) type agent to choose to exert effort in all working periods.

1.4 Simulation

To provide a comprehensible solution to the problem presented in the previous section, a simulation is performed. The exogenous variables in the model are maximum effort levels \((\bar{e})\), cost of effort \((c)\), luck \((\beta)\), and proportion of types \((\gamma)\). Values are selected for certain parameters to show the impact of a change in other exogenous variable on effort choices of agents in teams.

The simulation begins by presenting the conditions for each equilibrium strategy. Values previously selected are substituted into the conditions and plotted in a parameter space of exogenous variables (i.e., values are selected for \( \bar{e} \) and \( \gamma \) and the conditions are illustrated in a \((c, \beta)\) parameter space). The area in which the parameter values satisfy all conditions is the equilibrium space where the equilibrium strategy exists given selected parameter values. After all equilibrium strategies are shown, the parameter spaces are combined into a single graph. Any overlapping areas, where parameter values satisfy conditions for multiple equilibrium strategies, are resolved by determining the strategy that provides the agent with the highest lifetime expected payoff. The strategy that provides the highest lifetime expected payoff is considered the preferred strategy.

The maximum value for effort is set to 1 \((\bar{e} = 1)\). There are parameter values of \( c \) (cost of effort), \( \beta \) (“bad luck”), and \( \gamma \) (probability agent is born \( L \) type) that exist to satisfy all

\(27\) Similar changes occur to inequality (17) if the equilibrium strategy for \( L \) type agents were to not exert effort in either position of young worker or manager as inequality (16). If the equilibrium strategy is \( L \) type agents do not exert effort as young workers, the LHS does not include \( c \) and the probability for production changes. If no effort is exert as a manager, the expected payoff given good quality output is produced changes. A combination of the two changes can also occur depending on the equilibrium strategy.
conditions that characterize an equilibrium. The following graphs illustrate areas in a \((\beta, c)\)
and \((\gamma, c)\) space where the conditions are satisfied for each equilibrium strategy (exerting
effort in one working period, both, or none).

The graphs presented in this section are only of the \(L\) type agents. If the parameter
values satisfy a rationality condition for a certain equilibrium strategy for \(L\) types, by
transitivity, \(H\) types’ conditions are satisfied. Agents are risk neutral. A \(H\) type agent
always chooses to exert effort and exerting effort is costless \((c = 0)\). Therefore, a \(H\) type
agent has a weakly dominant strategy to exert effort. The contract only needs to satisfy a
\(H\) type agent’s participation constraints to work for a firm. Suppose a \(L\) type agent chooses
to work for the firm and exerts effort. The \(L\) type agent receives the same expected payoffs
as the \(H\) type agent and has the same probability of receiving a reward from production.
The difference is the \(L\) type agent incurs a cost of effort lowering his expected payoff. If the
contract satisfies the \(L\) type agent to work for a firm and exert effort, a \(H\) type agent also
chooses to work for a firm. On the other hand, an equilibrium strategy of a \(L\) type agent
can be to work for a firm but exert no effort. The similarity between the two types of agents
is they do not incur a cost of effort but the \(H\) type has a higher probability of obtaining a
reward, increasing his expected payoff. Therefore, if a contract satisfies the conditions for
a \(L\) type to work for a firm, a \(H\) type also works for the firm.

Figure 1: The graphs illustrate the combination of parameters that satisfy the conditions for the equilibrium
strategy where the \(L\) type agent exerts effort in both working periods as a young worker and manager. The
graph on the left illustrates the combinations of \(\beta\) and \(c\) given that \(\gamma = \frac{2}{3}\). The graph on the right illustrates
the combinations of \(\gamma\) and \(c\) given that \(\beta = \frac{1}{4}\).
Figure 1 illustrates the parameter values that satisfy the conditions for the equilibrium strategy where $L$ type agents choose to exert effort in all working periods as a young worker and manager.\footnote{The scale of the axes are chosen to highlight and clearly illustrate the area where the parameter values satisfy the equilibrium conditions. Some lines representing actions in the equilibrium strategy lie beyond the maximum values of the axes. Each line represents the parameter values where the agent is indifferent between sticking to the equilibrium strategy and deviating. Therefore, parameter values that satisfy a specific equilibrium strategy must lie on, below, or above a line depending on the equilibrium strategy considered.} The graph on the left illustrates the combinations of $\beta$ and $c$ for the case that $\gamma = \frac{2}{3}$. For all conditions to be satisfied for this equilibrium, parameter values of $\beta$ and $c$ must lie below all the lines; cost of effort and $\beta$ must be sufficiently low.

Line (I) is the condition for the $L$ type agent to apply for a job with a firm as a worker in the first working period. All parameter values that satisfy the condition is below the curve. As bad luck increases in the production process, the probability of obtaining a reward decreases. Also, wages fall due to lower expectations of producing good quality output. Therefore, the maximum cost of effort must fall as $\beta$ increases for a $L$ type agent to work for a firm as a worker. Line (II) is the condition for a $L$ type agent to exert effort as a worker in the first working period. Cost of effort and $\beta$ must be sufficiently low for the $L$ type agent to exert effort. As $\beta$ increases past a sufficient level, the expected payoff of producing bad quality output becomes greater than the expected payoff from producing good quality. Therefore, to increase the probability of obtaining the higher expected payoff, no effort is exerted even if effort is costless. Line (III) is the condition for the $L$ type to exert effort as a manager. The parameter values that satisfy the condition for the $L$ type agent to exert effort as a manager must lie below the line. As the level of bad luck increases in the production process, the decrease in expected payoff is greater when less team effort is exerted. Therefore, a higher cost can be incurred because exerting effort decreases the agent’s expected payoff by a smaller amount.

The graph on the right illustrates the combinations of $\gamma$ and $c$ for the case that $\beta = \frac{1}{4}$. All conditions are satisfied for the equilibrium strategy below the lines; the cost of effort must be sufficiently low and $\gamma$ must be sufficiently high.

Line (I) is the condition where the $L$ type agent chooses to apply for a job with a firm as a worker in the first working period. When $\gamma$ is sufficiently low, the reservation payoff becomes more attractive than working for a firm. Line (II) is the condition where the $L$ type agent chooses to apply for a job with a firm as a worker in the second working period.
agent exerts effort as a young worker. Parameter values must fall beneath the line for the condition to be satisfied. As $\gamma$ decreases, the reservation payoff increases which increases the expected payoff of producing bad quality output. Similar to $\beta$ increasing, when $\gamma$ decreases beyond a sufficient value, the agent prefers to not exert effort to increase his probability of obtaining the higher expected payoff. Line (III) is the condition for a $L$ type agent to exert effort as a manager. The parameter values that satisfy the agent’s condition to exert effort fall beneath the line. As the proportion of $L$ type agents increase (as $\gamma$ increases), a manager is more likely to be paired with an old, $L$ type worker that does not exert effort lowering his chances of receiving a reward. As $\gamma$ increases, the decrease in the probability of obtaining a reward is greater when less total team effort is exerted (similar to increasing $\beta$). Therefore, a higher cost can be incurred because exerting effort decreases the agent’s expected payoff by a smaller amount. Line (IV) is the condition where the $L$ type agent accepts a promotion. Parameter values must fall beneath the line for the condition to be satisfied. As more $H$ types are born into each generation, $\gamma$ decreases, the expected payoff of working alone (reservation payoff) increases making working alone more attractive below a sufficient level of $\gamma$.

Figure 2: The graphs illustrate the combination of parameters that satisfy the conditions for the equilibrium strategy where the $L$ type agent exerts effort only as a young worker. The graph on the left illustrates the combinations of $\beta$ and $c$ given that $\gamma = \frac{2}{3}$. The graph on the right illustrates the combinations of $\gamma$ and $c$ given that $\beta = \frac{1}{4}$.

Figure 2 illustrates the parameter values that satisfy the conditions for the equilibrium
strategy where $L$ type agents choose to exert effort only as a young worker while working for a firm. The graph on the left illustrates the combinations of $\beta$ and $c$ for the case that $\gamma = \frac{2}{3}$. The parameter values that satisfy all the conditions for this equilibrium strategy for the $L$ type agent falls between the downward sloping line and the $\beta$-axis.

Line (I) is the condition where the $L$ type agent chooses to exert effort as a young worker. As $\beta$ increases, the probability of obtaining a reward decreases. Therefore, cost of effort must decrease to maintain the equilibrium strategy. Parameter values that satisfy this equilibrium condition falls below the curve. Line (II), which lies on top of the $\beta$-axis, is the condition where the $L$ type agent exerts effort as a manager. In this equilibrium strategy, the manager does not exert effort. Therefore, $R = 0$ because there is no need for an incentive to exert effort. Any $c$ greater than zero, the agent chooses not to exert effort.

The graph on the right illustrates the combination of parameters for $\gamma$ and $c$ that satisfy the conditions for this equilibrium strategy if $\beta = \frac{1}{4}$. The combination of parameters must fall below the two curves and above the $\gamma$-axis.

Line (I) is the condition where the $L$ type agent chooses to exert effort as a young worker. Decreasing $\gamma$ increases the expected payoff from producing bad quality output above the expected payoff of producing good quality output. To increase the probability of producing bad quality output and obtain the higher expected payoff, the agent chooses to not exert effort. Line (II), lies on top of the $\gamma$-axis, is the condition where the $L$ type agent chooses to exert effort as a manager. Since $R = 0$, if $c$ is greater than zero, he chooses not to exert effort. Line (III) is the condition where the $L$ type agent chooses apply for a job as a worker in the first working period. As $\gamma$ decreases, the expected payoff from producing good quality output falls and the reservation payoff increases. Below a sufficient level, the outside option of the agent is more attractive. Parameter values to satisfy this condition lie below the curve.

Figure 3 illustrates the combination of parameters that satisfy the conditions for the equilibrium strategy where $L$ type agents choose to exert effort only as managers while working for a firm. The graph on the left illustrates the combination of parameter values of $\beta$ and $c$ for $\gamma = \frac{2}{3}$ that satisfy all the conditions. The parameter values that satisfy all the conditions for this equilibrium strategy fall below the two top curves and above the bottom
The graphs illustrate the combination of parameters that satisfy the conditions for the equilibrium strategy where the $L$ type agent exerts effort only as a manager. The graph on the left illustrates the combinations of $\beta$ and $c$ given that $\gamma = \frac{2}{3}$. The graph on the right illustrates the combinations of $\gamma$ and $c$ given that $\beta = \frac{1}{4}$.

Curve. The value of $\beta$ must be sufficiently low and the value of $c$ must be sufficiently low but above a certain value.

Line (I) is the condition for the $L$ type agent to apply for a job as a worker in the first working period. As $\beta$ increases, the wages and the probability of receiving rewards fall. Therefore, cost of effort must fall for the agent to work for the firm. If $\beta$ is sufficiently high, even if $c = 0$, the outside option provides a higher expected payoff. Line (II) is the condition where the agent chooses to exert effort as a young worker. Similar to the curves from the other equilibriums, the increase in $\beta$ increases the expected payoff from producing bad quality output beyond the expected payoff of producing good quality output. Exerting no effort increases the probability of obtaining a higher expected payoff. Line (III) is the condition where the $L$ type agent chooses to accept a promotion to manager. As $\beta$ increases, the expected payoff from working alone decreases. From equation (4) for the managerial wage, the expected payoff of a manager is a constant. Therefore, if the expected payoff from accepting a promotion is constant and the expected payoff of rejecting is decreasing as $\beta$ increases, the maximum value of $c$ can increase and the agent sticks to the equilibrium strategy.

The graph on the right illustrates the combination of parameter values of $\gamma$ and $c$ that
satisfy all the conditions for this equilibrium strategy where $\beta = \frac{1}{4}$. The combination of parameter values for $\gamma$ and $c$ that satisfy all conditions for this equilibrium strategy falls in the area on the right above the small triangular area on the bottom right. The values of $\gamma$ must be sufficiently high and $c$ must be high above a certain value.

Line (I) is the condition for a $L$ type agent to apply for a job as a worker in the first working period. Line (II) is the condition where the $L$ type agent chooses to exert effort as a young worker. Parameter values must be above the curve for the agent to stick to the equilibrium strategy. As $\gamma$ increases, the expected payoff from producing bad quality output increases making not exerting effort more attractive. Line (III) is the condition for a $L$ type agent to accept the promotion to manager. Parameter values that satisfy the conditions for an agent to always work for a firm must fall below both curves. As $\gamma$ decreases, the reservation payoff increases making working alone more attractive than working for a firm. Line (IV) is the condition for a $L$ type agent to exert effort as a manager. As $\gamma$ increases, the probability of a manager being paired with an agent that does not exert effort increases. By exerting effort, his expected payoff increases greatly allowing for higher levels of cost ot be incurred.

The final equilibrium strategy to consider is $L$ type agents choosing to never exert effort. This equilibrium is shown to not exist because $L$ type agents always have a beneficial deviation to not applying for a position as a worker in the first working period for all values of $\gamma$. Condition (17) is violated for all values of $\gamma$. The equilibrium strategy of the $L$ type agent is to not exert effort while working for a firm. Therefore, the probability of producing good quality output in a team as a worker is low. If the probability of producing good quality output is low, the likelihood of being promoted to manager is low which lowers an agent’s expectations of being hired, $\alpha$ falls. Also, wages reflect the beliefs of the firm and since $L$ types are expected to never exert effort, wages are low. The low probability of obtaining a reward plus the low wages makes the reservation payoff of producing alone more profitable. Therefore, there are only three possible equilibriums; the $L$ type agent chooses to exert effort as a young worker and manager, only as a young worker, or only as a manager.

Overlaying the graphs from Figures 1 to 3 illustrates the parameter values that satisfy
conditions of each equilibrium strategy. From combining the graphs from Figures 1 to 3, there exists areas of multiple equilibria. These are areas with parameter values that satisfy conditions from multiple equilibria. For instance, some parameter values of $\beta$ and $c$ that satisfy sufficient equilibrium conditions for a $L$ type agent to exert effort only as a young worker also satisfy the conditions to only exert effort as a manager. To determine the dominant equilibrium in this overlapping area, the expected lifetime payoff of the $L$ type worker is compared. The dominant equilibrium is the one that provides the $L$ type agent the higher lifetime expected payoff.\textsuperscript{29} Figure 5 in the Appendix shows that, for all values of $\beta$ and $\gamma$, the equilibrium strategy where the $L$ type agent only exerts effort as a manager provides the highest lifetime expected payoff, followed by exerting effort as a young worker and manager, and followed lastly by only exerting effort as a young worker.\textsuperscript{30}

\textsuperscript{29}The existence of multiple equilibria given a set of parameters is possible in reality. Given a set of parameter values, two firms can display different work cultures among the employees. In the context of this paper, the equilibrium strategy that provides the $L$ type with the highest expected payoff is chosen because the strategy depends on the $L$ type’s decision. It is not rational for the $L$ type to choose a strategy with a lower lifetime expected payoff.

\textsuperscript{30}The reason that only exerting effort as a manager provides the highest lifetime expected payoff lies in the wage for old workers. Exerting effort in all periods incurs the highest cost. Therefore, potentially, it could not be the most profitable. Between exerting effort as a young worker or manager, the probability of success in a single team is the same because there is only one person exerting effort. Therefore, other variables such as $\alpha$ and probability of obtaining a reward as a worker is relatively the same. If the equilibrium strategy was to only exert effort as a young worker, there exist more teams that will not produce good quality output because there are teams that exert no effort (i.e., managers with old workers). Therefore, the old worker wage is much lower compared to the equilibrium strategy where agents only exert effort as managers.
Applying the criteria to determine the dominant equilibrium, the resulting graphs are illustrated in Figure 4. The graph on the left highlights the parameter values of $\beta$ and $c$ that satisfy the conditions for each equilibrium strategy of the $L$ type agent ($\gamma = \frac{2}{3}$). When cost of effort and $\beta$ are sufficiently low, effort is exerted in all periods. This is illustrated by area (I). As $\beta$ and $c$ increase, $L$ type agents choose to only exert effort as managers. This is represented by area (II). When $\beta$ gets sufficiently high, area (III), while costs remain sufficiently low, the agent only exerts effort as a young worker.\(^{31}\)

When $\beta$ is equal to zero, there is no uncertainty in the production process. If one member of the team exerts effort, good quality output is produced with certainty. Therefore, at $\beta = 0$, the equilibrium strategy for the $L$ type agent is to only exert effort as a manager as it provides a higher lifetime expected payoff. As $\beta$ increases and uncertainty is introduced into the production process, the existence of an equilibria where more effort is exerted appears. If cost of effort is sufficiently low, increasing $\beta$, uncertainty in the production process, a $L$ type agent chooses to exert more effort by also exerting effort as a young worker. Uncertainty in the production process acts as a potential incentive for $L$ type agents to exert effort under the right conditions.

The graph on the right of Figure 4 highlights the equilibrium strategies, after applying the criteria to determine the dominant equilibrium, of the $L$ type agent given parameter values for $\gamma$ and $c$ where $\beta = \frac{1}{4}$. Area (I), where $\gamma$ is sufficiently high and $c$ is sufficiently low, the parameter values satisfy the conditions for the equilibrium strategy of the $L$ type agent exerting effort as a young worker and a manager. The area above, area (II), contains the combination of parameter values that satisfy the condition for the equilibrium strategy where the $L$ type agent exerts effort only as a manager. Area (III), below the line are the parameter values of $\gamma$ and $c$ that satisfy the conditions for the equilibrium strategy where the $L$ type agent exerts effort only as a young worker.

As a $L$ type agent is more certain to be paired with a $H$ type agent that always exerts effort, a $L$ type agent deviates from his equilibrium strategy of exerting effort (or working for

\(^{31}\)The area above all lines might contain parameter values that satisfy conditions for mixed strategy equilibria. The idea of mixed strategy equilibria may have non-symmetric actions of agents of the same type. For example, a proportion of $L$ type agents may exert effort while the other does not. This is not solved in this article but might be considered in future iteration.
a firm). If $\gamma$ increases and the $L$ type agent is more likely paired with someone that might not exert effort, introducing uncertainty into the production process by way of less effort, equilibrium strategies with more effort appear. Again, uncertainty in the production process introduced by the increase in the probability of being paired with an agent that might not exert effort acts as an incentive for $L$ type agents to exert effort. Similar to increasing $\beta$, uncertainty acts as a potential incentive for effort under the right conditions.$^{32}$

1.5 Conclusion

This article illustrates the possible impact of uncertainty and asymmetric information in regards to the agent’s teammate and production process on teamwork. It included several important features of professional work environments such as promotions as an incentive, a pyramid organizational structure, team rewards, and heterogeneous agents in efficiency to better model firms seen in reality.

The model suggests, under the assumptions and framework of the model, agents tend to shirk on their teammates when cost of effort is sufficiently high. This is a obvious result. As cost of effort rises, an agent chooses to exert less effort. Since effort is binary, for an agent to exert less effort, he must choose to exert effort only in one of two periods. When the bad luck in the production process is sufficiently low or the proportion of low efficiency types are high, agents free ride as young workers to get a promotion. As bad luck increases or the proportion of low efficiency types decrease sufficiently, agents free ride as managers. The first objective behind this article is to provide firms and organization with an explanation of the teamwork that exists in their workplace. If they observe free riding among teams of workers, it could be due to the flaws of the production process or proportion of types in the workforce.

Another result is the potential positive impact of uncertainty in the production process on teamwork. When there is certainty in the production process and effort from agents provide guaranteed outcomes, agents tend to shirk. If a little uncertainty in the production process is introduced, in the form of bad luck or uncertainty in type of teammates, agents

$^{32}$Figure 6 to 8 in the Appendix further illustrates the impact of the parameter changes on the parameter areas of each equilibrium strategy.
begin to worry about the impact their teammate might create to his own probability of obtaining rewards. Therefore, agents are incentivized to exert effort to prevent major negative effects under the right conditions.
1.6 Appendix

1.6.1 Probabilities

The equation for \( p^W(e_W; e^H_M, e^L_M) \) is,

\[
p^W(e_W; e^H_M, e^L_M) = p(H^M) \left( \frac{e_W + e^H_M}{e_J + e^H_M + \beta} \right) + p(L^M) \left( \frac{e_J + e^L_M}{e_J + e^L_M + \beta} \right) \tag{18}
\]

The equation for \( p^M(e_M; e^H_W, e^L_W) \) is,

\[
p^M(e^H_W, e^L_W, e_M) = p(H^W)^2 \left[ \left( \frac{e^H_W + e_M}{e^H_W + e_M + \beta} \right)^2 + (2) \left( \frac{e^H_W + e_M}{e^H_W + e_M + \beta} \right) \left( \frac{\beta}{e^H_W + e_M + \beta} \right) \right] \\
+ (2)(p(H^W))(p(L^W)) \left[ \left( \frac{F + U}{N + F + U} \right) \left( \frac{e_M}{e_M + \beta} \right) \right] \\
+ \left( \frac{F + U}{N + F + U} \right) \left( \frac{e_M}{e_M + \beta} \right) \left( \frac{N}{N + F + U} \right) \left( \frac{e^H_W + e_M}{e^H_W + e_M + \beta} \right) \\
+ \left( \frac{F + U}{N + F + U} \right) \left( \frac{e_M}{e_M + \beta} \right) \left( \frac{N}{N + F + U} \right) \left( \frac{e^H_W + e_M}{e^H_W + e_M + \beta} \right) \\
+ \left( \frac{F + U}{N + F + U} \right) \left( \frac{e_M}{e_M + \beta} \right) \left( \frac{N}{N + F + U} \right) \left( \frac{e^H_W + e_M}{e^H_W + e_M + \beta} \right) \left( \frac{\beta}{e^H_W + e_M + \beta} \right) \tag{19}
\]

The equation for \( p^{M2}(e^H_W, e^L_W, e_M) \) is,

\[
p^{M2}(e^H_W, e^L_W, e_M) = p(H^W)^2 \left[ \left( \frac{e^H_W + e_M}{e^H_W + e_M + \beta} \right)^2 \right] \\
+ (2)(p(H^W))(p(L^W)) \left[ \left( \frac{e^H_W + e_M}{e^H_W + e_M + \beta} \right) \left( \frac{e^L_W + e_M}{e^L_W + e_M + \beta} \right) \right] \\
+ \left( \frac{F + U}{N + F + U} \right) \left( \frac{e_M}{e_M + \beta} \right) \left( \frac{N}{N + F + U} \right) \left( \frac{e^H_W + e_M}{e^H_W + e_M + \beta} \right) \left( \frac{\beta}{e^H_W + e_M + \beta} \right) \tag{20}
\]

The equation for \( p^{M1}(e^H_W, e^L_W, e_M) \) is,

\[
p^{M1}(e^H_W, e^L_W, e_M) = p^M(e^H_W, e^L_W, e_M) - p^{M2}(e^H_W, e^L_W, e_M) \tag{21}
\]
1.6.2 \textit{H Type Agent Equilibrium Strategy Conditions}

The \textit{H} type agent’s conditions for the equilibrium strategy where \textit{L} type agents choose to exert effort as a young worker and manager are inequalities (11*) to (13*). The LHS of inequalities (11*) to (13*) are greater than or equal to zero for all values of $\beta$. Substituting the values for $\gamma = \frac{2}{3}$ into the equations, the LHS of the inequalities (11*) to (13*) become, respectively,

1. $\frac{16 + 56\beta + 76\beta^2 + 50\beta^3 + 16\beta^4 + 2\beta^5}{4(\beta + 2)^3(\beta + 1)^2} \geq 0$
2. $\frac{3\beta + 4}{3(\beta + 2)(\beta + 1)} \geq 0$
3. $\frac{5.3 + 21.3\beta + 32\beta^2 + 21.3\beta^3 + 5\beta^4 - 0.33\beta^6 - 0.66\beta^5}{(\beta + 1)^3(\beta + 2)^4} \geq 0$

Referring back to inequalities (11*) to (12*), this means for all values of $\beta$, the \textit{H} type agent accepts a promotion and re-applies for a job as a worker in the second working period. The last one is downward sloping in $\beta$. The equation is equal to zero when $\beta \approx 5$. A \textit{H} type agent does not apply for a worker position in a firm in the first working period if $\beta \gtrsim 5$, which is well outside the value of $\beta$ for the equilibrium strategy of \textit{L} type agents ($\beta \lesssim 0.7$).

Similarly, the equation on the LHS for the \textit{H} type agent’s conditions (11*) to (13*) for $\beta = \frac{1}{4}$ are, respectively,

1. $0.5 \geq 0$
2. $\frac{4 + 32g}{45} \geq 0$
3. $-0.64 + 1.35g \geq 0$

The first two are positive for all values of $0 < \gamma < 1$. The \textit{H} type agent accepts a promotion and re-applies for a job as a worker in the second working period. The last equation is greater than zero if $\gamma \gtrsim 0.47$ which is consistent with the conditions for \textit{L} type agents ($g \gtrsim 0.52$).

For the equilibrium strategy where \textit{L} type agents choose to exert effort only as young
workers, conditions for $H$ type agents, (11*) to (13*), when $\gamma = \frac{2}{3}$ are, respectively,

$$\frac{162 + 918\beta + 2254.5\beta^2 + 3135\beta^3 + 2700.5\beta^4 + 1476\beta^5 + 500\beta^6 + 96\beta^7 + 8\beta^8}{(2\beta + 3)^2(\beta + 1)^2(\beta + 2)^2(4\beta^2 + 12\beta + 9)} \geq 0$$

$$\frac{3\beta + 4}{9(\beta + 2)(\beta + 1)} \geq 0$$

$$\frac{498\beta^7 + 1812.45\beta^6 + 4161.8\beta^5 + 6261.5\beta^4 + 6177\beta^3 + 3854\beta^2 + 216.0 + 78.2\beta^6 + 5.35\beta^9 + 1380\beta}{(2\beta + 3)^2(4\beta^2 + 12\beta + 9)(1 + \beta)^3(2 + \beta)^3} \geq 0$$

For all values of $\beta$, the conditions are true. The $H$ type agent always chooses to work for the firm for all values of $\beta$.

For the equilibrium strategy when $L$ type agents choose to exert effort only as young workers when $\beta = \frac{1}{4}$, the conditions for $H$ type agents (11*) to (12*), are, respectively,

$$0.5 \geq 0$$

$$\frac{4(1 - \gamma)(8\gamma + 1)}{45} \geq 0$$

For all values of $0 < \gamma < 1$, the equation on the LHS are positive. The final condition for $H$ type agents where he chooses to apply for a position as a worker in his first working period is also satisfied for all values of $\gamma \gtrsim 0.473$. The equation is very large and is not presented here. For the conditions of a $L$ type agent to hold for the equilibrium strategy where he exerts effort only as a young worker, $\gamma \gtrsim 0.476$.

For the equilibrium strategy where $L$ type agents choose to only exert effort as a manager, conditions for $H$ type agents, (11*) to (13*), when $\gamma = \frac{2}{3}$ are, respectively,

$$\frac{162 + 1476\beta^5 + 2700.5\beta^4 + 3135\beta^3 + 2254.5\beta^2 + 500\beta^6 + 8\beta^7 + 96\beta^8 + 918\beta}{(4\beta^2 + 12\beta + 9)^2(1 + \beta)^2(2 + \beta)^2} \geq 0$$

$$\frac{3\beta + 4}{3(\beta + 2)(\beta + 1)} \geq 0$$

$$\frac{648 - 11395.3\beta^5 - 9324\beta^4 - 2040\beta^3 + 3006\beta^2 - 53.3\beta^6 - 7482.7\beta^6 - 597.3\beta^8 - 2853.3\beta^7 + 2592\beta}{(4\beta^2 + 12\beta + 9)^3(2\beta + 3)(1 + \beta)^3(2 + \beta)^3} \geq 0$$

The first two conditions state illustrate that they are true for all values of $\beta$. This means that a $H$ type agent chooses to accept a promotion and re-apply as a worker in the second working period for all values of $\beta$. The last condition is true for all values of $\beta \lesssim 0.8$. Therefore, for all values of $\beta$ less than approximately 0.8, $H$ type agents choose to apply for a position in a firm as a worker in their first working period. Any $\beta \gtrsim 0.75$, the conditions for this equilibrium strategy do not hold for $L$ type agents.
For the equilibrium strategy when $L$ type agents choose to exert effort only as managers when $\beta = \frac{1}{4}$, the conditions for $H$ type agents$\left(11^{*}\right)$ to $\left(12^{*}\right)$, are, respectively,

\[
\frac{0.5\gamma^2 - 10\gamma + 50}{\gamma - 10} \geq 0
\]
\[
\frac{32\gamma + 4}{45} \geq 0
\]
\[
\frac{550.5\gamma^2 - 1483.5\gamma + 2.7\gamma^4 - 68.2\gamma^3 + 642}{(-10 + \gamma)(\gamma^2 - 20\gamma + 100)} \geq 0
\]

The first two conditions are true for all values of $0 < \gamma < 1$. $H$ type agents choose to accept a promotion or re-apply for a worker position in a firm in their second working period. The last condition is true for all values of $\gamma \gtrsim 0.53$. The minimum parameter value of $\gamma$ for all $L$ type conditions to hold in this equilibrium is $\gamma \gtrsim 0.66$.

1.6.3 Lifetime Expected Payoffs of $L$ Type Agent by Equilibrium Strategy

Figure 5: The graph on the left illustrates the expected lifetime payoff of a $L$ type agent given parameter values of $\beta$ and $c$ with $\gamma = \frac{2}{3}$. The highest curve is the expected payoff of the equilibrium strategy of only exerting effort as a manager. The second highest is the curve for the equilibrium strategy of exerting effort as a young worker and manager. The lowest curve is for the equilibrium strategy of exerting effort only as a young worker. The graph on the right is for the parameter values of $\gamma$ and $c$ with $\beta = \frac{1}{4}$. The ordering of the curves are the same as the graph on the right.
1.6.4 Equilibrium Strategy Area

Figure 6: The graphs illustrate the parameter values that satisfy the conditions for the equilibrium strategy where \( L \) type agents exert effort as a young worker and manager. The graph on the left illustrates the parameter values of \( \beta \) and \( c \). The solid line is where \( \gamma = 0.6 \), dash line is \( \gamma = 0.75 \), and dot line is \( \gamma = 0.9 \). The equilibrium area shrinks as \( \gamma \) increases. The graph in the middle illustrates the parameter values of \( \gamma \) and \( c \). The solid line is where \( \beta = 0.2 \), dash line is \( \beta = 0.5 \), and dot line is \( \beta = 0.8 \). The equilibrium area shrinks as \( \beta \) increases. The graph on the right illustrates the parameter values of \( \beta \) and \( \gamma \). The solid line is where \( c = 0.005 \), dash line is \( c = 0.01 \), and dot line is \( c = 0.015 \). As cost of effort increases, the equilibrium area shrinks. The condition shown here is for the \( L \) type agent to exert effort as a young worker. If this condition is satisfied for the equilibrium strategy, all other conditions are satisfied. As \( \gamma \) increases, a manager is more likely to be paired with a old worker that does not exert effort. This decreases the value of a reward making exerting effort less attractive unless cost of effort or \( \beta \) decreases. If \( \beta \) increases, this decreases the likelihood of obtaining a reward. To stick to the equilibrium strategy, cost of effort must fall or the reservation payoff must be less attractive (\( \gamma \) increases) for the agent to exert effort. If cost increases, the expected payoff from exerting effort decreases. If \( \beta \) falls, obtaining a reward is more likely. If \( \gamma \) increases, the reservation payoff is less attractive. Both incentivizes the agent to exert effort.
Figure 7: The graphs illustrate the parameter values that satisfy the conditions for the equilibrium strategy where $L$ type agents exert effort as a young workers. The graph on the left illustrates the parameter values of $\beta$ and $c$. The solid line is where $\gamma = 0.6$, dash line is $\gamma = 0.75$, and dot line is $\gamma = 0.9$. The equilibrium area expands as $\gamma$ increases. The graph in the middle illustrates the parameter values of $\gamma$ and $c$. The solid line is where $\beta = 0.2$, dash line is $\beta = 0.5$, and dot line is $\beta = 0.8$. The equilibrium area shrinks as $\beta$ increases. The graph on the right illustrates the parameter values of $\beta$ and $\gamma$. The solid line is where $c = 0.005$, dash line is $c = 0.01$, and dot line is $c = 0.015$. As cost of effort increases, the equilibrium area shrinks. Similar to the graphs previously, the condition illustrated is the one where an $L$ type agent chooses to exert effort as a young worker. Increasing $\gamma$ greatly decreases the value of the reservation payoff and an old worker’s wage. Therefore, a worker can incur a higher cost for exerting effort. The explanations for the other graphs are the same as the previous graphs.
Figure 8: The graphs illustrate the parameter values that satisfy the conditions for the equilibrium strategy where $L$ type agents exert effort as a manager. The graph on the left illustrates the parameter values of $\beta$ and $c$. The solid lines are where $\gamma = 0.6$, dash line is $\gamma = 0.75$, and dot line is $\gamma = 0.9$. The equilibrium area expands as $\gamma$ increases. The graph in the middle illustrates the parameter values of $\gamma$ and $c$. The solid line is where $\beta = 0.2$, dash line is $\beta = 0.5$, and dot line is $\beta = 0.8$. The equilibrium area shrinks as $\beta$ increases.

The graph on the right illustrates the parameter values of $\beta$ and $\gamma$. The solid line is where $c = 0.005$, dash line is $c = 0.01$, and dot line is $c = 0.015$. As cost of effort increases, the equilibrium area shrinks. As $\gamma$ increases, the reservation payoff greatly decreases and the likelihood of being paired with agents that do not exert effort increases. This greatly decreases the expected payoff of not exerting effort. Therefore, the agent can incur a higher cost of effort or larger $\beta$. The explanation for the other graphs are the same as previous graphs.
Chapter 2

2 Homogeneous vs. Heterogenous Teams: Optimal Team Composition

2.1 Introduction

This paper focuses on the firm’s decision regarding the composition of workers in teams when there exists a heterogeneity among workers in efficiency levels. Firms aim to maximize expected profits by choosing the optimal composition of types of workers in the team. The results indicate that the effect of both an increase in cost of effort for workers, or an increase in hiring costs of more efficient type workers, can change a firm's choice from hiring homogeneous teams to heterogeneous teams. As costs for the firm increase, less high efficiency types are hired and less effort is induced. Therefore, to complete the team, positions are filled with cheap, less efficient labour. In contrast, if a firm already has an existing workforce, heterogeneous teams are generally chosen.

This analysis aims to answer the question regarding composition of teams under two scenarios. The first is a moral hazard model that assumes a firm can hire workers from a pool of applicants to assemble a team. The pool contains two types of workers: high and low. It is assumed that high efficiency type workers are more expensive to hire compared to low efficiency types (i.e., high efficiency types have a higher reservation payoff compared to low efficiency types).\footnote{Consider a manufacturing facility that has experienced and rookie workers. The experienced workers can be considered high efficiency types. Since they have been doing the job a lot longer, they are considered more efficient compared to the rookies. The rookies are new to the job and therefore, slower and less efficient in the task. If manufacturing of goods requires a team of workers, the firm needs to decide the composition of teams in production.} The technology used by the firm requires a team of two workers.
This analysis is based on a standard moral hazard model. The results indicate that as cost of effort for workers or reservation payoffs of high efficiency types increase relative to low efficiency types, the firm changes from hiring a team of high efficiency types (homogeneous) to a heterogeneous team of high and low efficiency types. The focus is on incentivizing the high efficiency type to exert high effort while filling the second position with a low efficiency type and incentivizing low effort.

The second scenario assumes that the workers are already hired by the firm and firing workers is not allowed.\textsuperscript{34} The firm has a workforce of two high and two low efficiency types. An important difference is that this model with existing workers is not a principal-agent model. It is supposed that the owner and workers receive fixed shares of the value from production.\textsuperscript{35} As in the first model, the firm’s production process requires a team of two workers. The firm combines its workers into homogeneous (same type of workers) or heterogeneous (different types of workers) teams. The firm chooses the composition of workers that provides the highest expected value from production. Given the composition of teams and the incentives they face, workers make their effort decisions. The results indicate that a heterogeneous choice is mostly dominant for the firm. Due to the concavity of the production function, heterogeneous teams are chosen over homogeneous teams when parameters are such that alternative team composition results in the same effort choices. If parameter values are not in the latter ranges, at times homogeneous teams are the better choice.

The two models aim to analyze the problem of team composition under two scenarios. The first model may seem more realistic: a firm can hire or change its workforce into a more profitable group. The second scenario examines team formation with an existing workforce with no possibility of a change in personnel. A possible interpretation for these two scenarios could be a situation where a firm is composed of multiple levels: Upper management, middle management, low level employees, etc. as found in most firms. In this case, the moral hazard problem issues discussed in the first model would be more relevant.

\textsuperscript{34}Assume that firing costs are very high. i.e., very expensive compensation packages or a union is preventing the release of workers easily.

\textsuperscript{35}It is supposed that these shares would have been established and perpetuated through the corporate culture of the firm.
to the bottom levels of the hierarchy, where the turnover rate is generally higher. The levels of middle management or higher is more related to the second model where the turnover rate is generally lower and the cost of turnover is relatively higher. Therefore, employees are less likely to be changed and instead they are reorganized. Another explanation for the two models is the existence of unions or large severance packages. Without unions, firms are free to optimize their workforce with changes in personnel: This is the first model. Instead, if unions and high severance packages exist, firms face rigidities when firing employees and therefore must make the best of the current situation: This is reflected in the second model.

Both scenarios described above provide different perspectives into the problem of organizational structure. The analysis provided in this paper separates the hiring decision (first scenario) and allocation decision (second scenario) and analyzes them separately.36

A conventional thought regarding team composition is to form a group consisting of different types allowing for a competitive advantage where individuals perform tasks in which they specialize. Hoffman and Maier (1961) discussed that heterogeneity of characteristics allows a group to have access to many sources of information or skills to enable the group to perform optimally. Using this logic and assuming learning between members of the group, the formation of heterogeneous teams in organizations seem to be the common practice. They assume that there is heterogeneity in skills among individuals and a heterogeneous team of workers specializing in their respective field optimizes output. Hoffman and Maier (1961) discuss horizontal differentiation where workers are different according to features that can’t be ordered in an objective way. Most of the literature focuses on horizontal differentiation among workers. In contrast, this work focuses on vertical differentiation. To my knowledge, no papers are similar in nature to the analysis presented in this paper.

Most of the literature regarding team composition falls into the field of management, sociology, and psychology and the results between homogeneous and heterogeneous are mixed. Ancona and Caldwell (1992) observe that as teams increasingly get called upon to perform tasks and to cross functional boundaries within organizations, conventional wisdom suggests that teams be composed of more diverse members. Although in their

36It could be argued that that the first scenario contains elements of both while the second focuses mainly on the allocation decision.
model diversity brings more creativity to problem solving and product development, it impedes implementation because there is less capability for teamwork than there is for homogeneous teams. Hamilton et al. (2003) evaluated rationales for team participation and the effects team composition had on productivity using data from a garment plant and found heterogeneous teams, composed of more diverse workers, were more productive. Cornell and Welch (1996) and Athey et al. (2000) showed that teams tend to be composed of workers with similar backgrounds because team members “get along” easier. Mello and Ruckes (2006) present a model of team composition when agents have differing preferences for projects. A heterogeneous team provides more information in the decision making process allowing for better selection of projects but a homogeneous team exerts greater total effort because all members prefer the same project. The results of the analysis of Mello and Ruckes (2006) suggest that under most conditions, a heterogeneous composition of team members with different efficiency levels is preferred even if there is no helping/teaching involved or other factors influencing effort (i.e., reputation, social norms).

The contribution of this paper is in answering the question of composition of teams when there is a heterogeneity among workers in the levels of a single characteristic, efficiency (vertical differentiation). The model examines the impact of the team composition on the effort decision of workers and a firm’s choice for composition. The only incentive scheme is the linked payoffs between workers where the effort of one worker increases the payoff of the team which is directly related to his own payoff. In addition to the typical scenario of hiring workers and composing a team, this analysis provides another scenario in which the workers are already hired and must be optimally organized.

Examples of applications of the model include auto assembly lines and law firms. The workers mainly do similar task, have the same skills, and work in teams to produce goods. Consider a manufacturing facility that has experienced and rookie workers. The experienced

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37 The literature regarding assortative matching is similar to the ideas presented in this paper. In a marriage market, the competition for a spouse leads to sorting of mates by characteristics such as wealth, education, and other characteristics. Economists have tried to understand sorting patterns in the marriage market and other matching markets by focusing on the nature of the gain from match and the mechanism of the market force of competition. In this paper, the matching of workers in teams is done by a firm that maximizes expected profits. In assortative matching, workers of a certain type choose a partner that would maximize his own expected payoff.

38 As a tangential contribution to Chapter 1 of the thesis, the analysis provides more insight into team composition as an incentive for effort when workers produce in teams.
workers can be considered high efficiency types. Since they have been doing the job a lot longer, they are considered more efficient compared to the rookies. The rookies are new to the job and therefore, slower and less efficient in the task. If manufacturing of goods requires a team of workers, the firm needs to choose the composition of teams in production that maximizes their profits. A good example could be a manufacturing facility for automobiles or automobile parts and the manager must optimally hire or organize its workforce to maximize production.

Section 2.2 presents a moral hazard problem with hiring that assumes a firm can hire workers from a pool of applicants to assemble a team. Section 2.4 presents a model where the workers are already hired by the firm and firing workers is not allowed. The firm must optimally organize its employees. Section 2.6 concludes.

2.2 Moral Hazard Problem With Hiring

The model in this section analyzes the hiring decision and incentive schemes of a profit maximizing firm that employs a technology in production that requires a team of two workers. The firm hires two workers to form a team from a pool of applicants that contains two types of workers; high (H) and low (L) type workers. There are three possible combinations of teams the firm can compose; HH, HL, and LL. The analysis solves for the optimal wage contract under each combination of teams. Depending on the incentives of the optimal contract, one or both workers exert effort as a dominant strategy. In all scenarios the analysis looks for dominant strategy equilibria. The combination of types that provides the firm with the highest expected profit is chosen.

The types of workers in the model differ in efficiency and their reservation payoffs. A H type worker can exert a level of effort $e_H \in \{0, \bar{e}_H\}$ and a L type worker can exert a level of effort $e_L \in \{0, \bar{e}_L\}$ where $\bar{e}_H > \bar{e}_L$. When effort is exerted by either type, there is the same cost of effort to the agent, $c > 0$, and when effort is not exerted, the cost of effort is zero.\footnote{Assigning a value of 0 for low levels of effort is for simplicity.} That means, for the same amount of cost, $c$, a H type worker can exert more effort compared to a L type worker. For this reason, a H type worker is more efficient than a L
type worker.\footnote{Compared to Chapter 1 of this thesis, where workers choose the same amount of effort but different cost of efforts between types, worker efficiency in this chapter is modeled with different levels of effort between the types but with the same cost of effort. Both are assumed to provide similar qualitative results since definition of efficiency for both methods of measurement are relatively the same. The reason behind the difference/change is simplicity. The first chapter has a very complex model and so the simpler definition of worker was employed for tractability. In the second model, with a simpler model, the definition of efficiency was more elaborate to provide a richer and more informative result (i.e., Allow the \(H\) type worker to make a choice rather than have a weakly dominant strategy).}

The reservation payoffs of \(H\) type workers is \(R_H\) and \(L\) type workers is \(R_L\) where \(R_H = R_L + \Delta\) and \(\Delta > 0\). A \(H\) type worker is more expensive to hire compared to a \(L\) type worker.\footnote{A higher efficiency worker commands a higher price. i.e., Using the example regarding experienced and rookie workers, a worker with a longer job history would prefer to command a higher salary making him more expensive to hire.}

When the workers are hired by the firm, they are required to work in a team of two to produce a single product of good or bad quality. The quality of the product is stochastic and it depends on the workers’ efforts. The firm values a good quality product at 1 and a bad quality product at zero. If the outcome is good (\(G\)), the \(H\) type worker receives a wage of \(w_{HG}\) and the \(L\) type worker \(w_{LG}\). If bad quality is produced, \(H\) and \(L\) type workers receive \(w_{HB}\) and \(w_{LB}\), respectively.

The following analysis is a simple moral hazard problem. It will solve for the expected profit under three scenarios depending on the composition of teams; the firm hires two \(H\) type workers (\(HH\)), the firm hires two \(L\) type workers (\(LL\)), and the firm hires one \(H\) type and one \(L\) type worker (\(HL\)). The firm chooses the composition that provides the highest expected profit. Under each scenario, the analysis will also solve for the expected profit when both workers are incentivized to exert effort or when only one worker is incentivized.\footnote{If both workers are not incentivized to exert effort, the expected profits are zero and wages are zero.}

\subsection*{2.2.1 Quality}

The quality of the good is determined by a modified Tullock contest success function of the following form,

\[
p(e_1, e_2) = \frac{e_1 + e_2}{e_1 + e_2 + \beta} \quad \text{where} \quad e_i \in \{0, \bar{e}_i\} \quad \text{and} \quad \bar{e}_i > 0 \quad \text{and} \quad i = 1, 2 \quad (22)\]

The equation is a modification of the familiar Tullock contest success function. Instead
of the players exerting effort to defeat the other, they exert effort to help the team succeed.\footnote{Amegashie (2006) proposed a similar contest success function in which he examined the degree to which luck as opposed to effort affects behaviour in different contest settings. His paper presents and discusses the properties of the contest success function.}

First, \( p(e_1, e_2) \) is increasing in all \( e \)'s. If everyone exerts effort, the probability of good quality is greater.

Second, the function includes a measurement of error involved in production. The error can arise from team member accidents, mistakes, bad luck, or any other uncontrol-
lable/unobservable factors. The magnitude of these factors’ effect on the production process is captured in \( \beta \). The \( \beta \) captures the extent to which luck as opposed to effort determines the success in a team environment. As \( \beta \) increases, the probability of good quality will decrease (i.e., the likelihood of a good quality product will depend more on luck).

When \( \beta \) is strictly positive, the probability of success will never be 1. If the entire team exerts effort, that is the highest probability of success the team can achieve. Therefore, if they produced a bad quality product under this scenario, it is caused strictly by uncontrollable factors and not due to the effort of the team members. On the other hand, if all members of the team exert zero effort, then the team will produce a bad quality product with certainty.

### 2.2.2 Homogeneous Team (\( HH \))

If the firm chooses to hire a homogeneous team consisting of only \( H \) type workers and incentivizes both workers to exert effort, the firm’s problem is,\footnote{The superscript \( 2eH \) specifies the effort the firm will induce. The firm will incentivize both \( H \) type workers to exert effort. The subscript \( HH \) specifies the composition of the team the firm hires for production. The firm hires two \( H \) type workers.}

\[
\begin{align*}
\max_{w_{HG}, w_{HB}} \quad & E\pi_{HH}^e = \left( \frac{2\bar{e}_H}{2\bar{e}_H + \beta} \right) (1 - 2w_{HG}) + \left( \frac{\beta}{2\bar{e}_H + \beta} \right) (-2w_{HB}) \\
\text{s.t.} \quad & \left( \frac{2\bar{e}_H}{2\bar{e}_H + \beta} \right) w_{HG} + \left( \frac{\beta}{2\bar{e}_H + \beta} \right) w_{HB} - c \geq RH \quad (24) \\
& \left( \frac{2\bar{e}_H}{2\bar{e}_H + \beta} \right) w_{HG} + \left( \frac{\beta}{2\bar{e}_H + \beta} \right) w_{HB} - c \geq \left( \frac{\bar{e}_H}{\bar{e}_H + \beta} \right) w_{HG} + \left( \frac{\beta}{\bar{e}_H + \beta} \right) w_{HB} \\
& \left( \frac{\bar{e}_H}{\bar{e}_H + \beta} \right) w_{HG} + \left( \frac{\beta}{\bar{e}_H + \beta} \right) w_{HB} - c \geq w_{HB} \\
\end{align*}
\]
Equation (23) is the expected profit of the firm if it chooses to hire two $H$ type workers and incentivizes both workers to exert effort. If the team produces a good quality product, which happens with probability $\frac{2e_H}{2e_H+\beta}$, the firm values the product at 1 and pays wages of $w_{HG}$ to each of the workers in the team. If the team produces a bad quality product, which happens with probability $\frac{\beta}{2e_H+\beta}$, the firm receives a value of zero and pays a wage of $w_{HB}$ to each worker.

Inequality (24) is the participation constraint (PC) for a $H$ type worker. The expected payoff for working in the firm and exerting effort is on the left hand side (LHS) of the inequality. If the team produces a good quality product, with probability $\frac{2e_H}{2e_H+\beta}$, the worker receives a wage $w_{HG}$ and if the team produces a bad quality product, with probability $\frac{\beta}{2e_H+\beta}$, the worker receives $w_{HB}$. The $H$ type worker exerts effort and incurs a cost of $c$. For a $H$ type worker to choose and work for the firm, the expected payoff on the LHS should be greater than or equal to his reservation payoff $R_H$.

Inequality (25) is the incentive compatibility (IC) constraint for a $H$ type worker to exert effort in the team when his partner chooses to exert effort. The payoff from exerting effort, as explained above for the PC, is on the LHS of the inequality. The right hand side (RHS) of the inequality is the expected payoff of an $H$ type worker choosing to free ride on the effort of his partner. If he chooses not to exert effort, the probability of producing a good quality product decreases to $\frac{e_H}{e_H+\beta}$. If good quality is produced, the worker is paid a wage of $w_{HG}$. If a bad quality product is produced, with probability $\frac{\beta}{e_H+\beta}$, the worker receives a wage of $w_{HB}$. In this case, the worker does not exert effort and does not incur a cost of effort, $c$.

Inequality (26) is the IC constraint for a $H$ type worker to exert effort in the team when his partner chooses not to exert effort. The LHS of the inequality is the expected payoff of a $H$ type worker exerting effort when his partner chooses not to exert effort. The probability of producing a good quality product is $\frac{e_H}{e_H+\beta}$ because only one worker is exerting effort. If good quality is produced, the worker receives a wage of $w_{HG}$. If bad quality is produced, $\frac{\beta}{e_H+\beta}$, the worker receives a wage of $w_{HB}$. Since effort is exerted by the worker, he incurs a cost of $c$. The RHS is the expected payoff of a $H$ type worker not exerting effort when his partner chooses not to exert effort. Since no one exerts effort in the team, they are
guaranteed to make a bad quality product and receives a wage of $w_{HB}$.

In all scenarios, including the one above, the optimal wages are determined by assuming the incentive compatibility constraint holds with equality. The participation constraint and the iso-profit functions are parallel in the relevant region of the graph (i.e., in the section where the incentive compatibility constraint is satisfied) and therefore there is a range of optimal wages that deliver the maximized expected profits of the firm and satisfy both the incentive compatibility and participation constraint of the worker. Since all wages within the range provide the firm with the same expected profit (the firm is indifferent) and workers with the same expected payoff are indifferent, the qualitative results of the paper are not affected by this choice. The firm adopts the optimal composition of teams and induces incentives for worker effort choices to maximize its expected profits.

Before solving the problem, note that (26) is implied by (25), so (26) can be excluded.

$$
\left(\frac{2\bar{e}_H}{2\bar{e}_H + \beta}\right) w_{HG} + \left(\frac{\beta}{2\bar{e}_H + \beta}\right) w_{HB} - c \geq \left(\frac{\bar{e}_H}{e_H + \beta}\right) w_{HG} + \left(\frac{\beta}{e_H + \beta}\right) w_{HB} \\
\left(\frac{\bar{e}_H}{e_H + \beta}\right) w_{HG} + \left(\frac{\beta}{e_H + \beta}\right) w_{HB} - c \geq w_{HB}
$$

From IC constraint (25), rearranging the inequalities for $w_{HG}$,

$$w_{HG} \geq w_{HB} + \frac{c(2\bar{e}_H + \beta)(\bar{e}_H + \beta)}{e_H \beta}$$

(27)

The $H$ type’s wage given his team produced a quality product is conditional on $w_{HB}$. To maximize expected profits, $w_{HG}$ must be as small as possible while still satisfying the PC. Therefore, for a given value of $w_{HB}$, $w_{HG} = w_{HB} + \frac{c(2\bar{e}_H + \beta)(\bar{e}_H + \beta)}{e_H \beta}$.

Substituting $w_{HG}$ into the $H$ type workers PC constraint, inequality (23), and isolating $w_{HB}$,

$$w_{HB} \geq R_H - c \left(\frac{2\bar{e}_H + \beta}{\beta}\right)$$

(28)

Similarly, since wages decrease the expected profits of the firm, $w_{HB}$ should be the smallest value possible while satisfying the PC. Assuming that wages must be greater than or equal to zero, a restriction for the $H$ type’s wage given he produces bad quality output is
\[ R_H \geq c \left( \frac{2e_H + \beta}{\beta} \right). \]  

Substituting \( w_{H_B}^* \) into \( w_{H_G}^* \) to solve for the optimal wage paid to a \( H \) type worker if the team produces a good quality product,

\[ w_{H_G}^* = R_H + c \left( \frac{2e_H + \beta}{\beta} \right) \]  

(29)

Substituting the equations for \( w_{H_G}^* \) and \( w_{H_B}^* \) into the expected profit function of the firm gives,

\[ E\pi_{H_H}^{2eH} = \left( \frac{2e_H}{\bar{e}_H + \beta} \right) (1 - 2w_{H_G}^*) + \left( \frac{\beta}{\bar{e}_H + \beta} \right) (-2w_{H_B}^*) = \left( \frac{2e_H}{\bar{e}_H + \beta} \right) - 2R_H - 2c \]  

(30)

The problem for a firm that hires two \( H \) type workers but only incentivizes one worker to exert effort is,

\[ \max_{w_{H_G}, w_{H_B}} E\pi_{H_H}^{1eH} = \left( \frac{\bar{e}_H}{\bar{e}_H + \beta} \right) (1 - w_{H_G}) + \left( \frac{\beta}{\bar{e}_H + \beta} \right) (-w_{H_B}) - R_H \]  

(31)

\[ \text{s.t.} \quad \left( \frac{\bar{e}_H}{\bar{e}_H + \beta} \right) w_{H_G} + \left( \frac{\beta}{\bar{e}_H + \beta} \right) w_{H_B} - c \geq R_H \]  

(32)

\[ \left( \frac{\bar{e}_H}{\bar{e}_H + \beta} \right) w_{H_G} + \left( \frac{\beta}{\bar{e}_H + \beta} \right) w_{H_B} - c \geq w_{H_B} \]  

(33)

In this scenario, the firm offers two contracts. The first contract is for the \( H \) type worker that exerts effort. The firm offers a similar contract, as in the previous scenario, with state contingent wages. If the team produces a good quality product, the worker receives \( w_{H_G} \) and \( w_{H_B} \) if the team produces bad quality. The second contract is for the \( H \) type worker that does not exert effort. He is offered a single wage that satisfies his PC (i.e., equal to \( R_H \)) to work for the firm. \(^{46}\)

Equation (31) is the expected profit of the firm if it chooses to hire two \( H \) type workers while only incentivizing one member to exert effort. The firm offers a contract to an \( H \) type worker with wages \( w_{H_G} \) and \( w_{H_B} \) depending on the quality of output to incentive effort. If the team produces a good quality product, given by probability \( \bar{e}_H \bar{e}_H + \beta \), the firm values the

\(^{45}\) The condition ensures that wages are positive and that problem provides an interior solution.

\(^{46}\) This scenario, where the same type of worker is paid different, introduces the idea of pay inequity. Although the workers in my model are the same type, they are hired to perform different actions. Since the firm is incentivizing the same type of workers to do different actions, the workers are paid differently.
product at 1 and pays a wage of $w_{HG}$ to the worker that exerts effort in the team. If the team produces a bad quality product, with probability $\frac{\beta}{\bar{e}_H + \beta}$, the firm receives a value of zero and will pay a wage of $w_{HB}$ to the worker. Another contract worth $R_H$ is offered to another $H$ type worker to attract him to work for the firm but not exert effort.\footnote{A wage that satisfies a $H$ type’s PC constraint is offered to attract him to work for the firm. The firm only needs to offer a wage, regardless of outcome, that attracts the worker to work for the firm. This wage must satisfy his PC. The firm is also a profit maximizer and the smallest wage possible is offered. Therefore, a wage equal to a $H$ type’s reservation payoff of $R_H$ is offered.}

Inequality (32) is the PC for a $H$ type worker that chooses to exert effort. The expected payoff for working in the firm and exerting effort is on the LHS of the inequality. If the team produces a good quality product, with probability $\frac{\bar{e}_H}{\bar{e}_H + \beta}$, the worker receives a wage of $w_{HG}$ and if bad quality is produced, the worker receives $w_{HB}$. He incurs a cost of $c$ by exerting effort in production. For a $H$ type worker to choose and work for the firm, the expected payoff on the LHS should be greater than or equal to his reservation payoff $R_H$.

Inequality (33) is the IC constraint for a $H$ type worker to exert effort in the team when his partner chooses not to exert effort. The expected payoff from exerting effort, as explained above, is on the LHS of inequality (32). The RHS of the inequality is the expected payoff of an $H$ type worker choosing to not exert effort when his partner also does not exert effort. Since both workers choose not to exert effort, bad quality is produced with certainty. The worker receives a payoff of $w_{HB}$ and does not incur a cost of effort, $c$, because he did not exert effort.

From IC constraint (33), isolating $w_{HG}$,

$$ w_{HG} \geq w_{HB} + \frac{c(\bar{e}_H + \beta)}{\bar{e}_H} $$

(34)

Similar to the previous scenario, $w_{HG}$, conditional on $w_{HB}$, should be as small as possible while satisfying the worker’s PC. Therefore, $w_{HG}^* = w_{HB} + \frac{c(\bar{e}_H + \beta)}{\bar{e}_H}$.

From PC (32), substituting in $w_{HG}^*$ and isolating $w_{HB}$,

$$ w_{HB} \geq R_H $$

(35)

Minimizing wages to maximize expected profits of the firm, $w_{HB}^* = R_H$. 

53
Substituting $w^*_{HB}$ into $w^*_{HG}$,

$$w^*_{HG} = R_H + \frac{c(\bar{e}_H + \beta)}{\bar{e}_H}$$  \hspace{1cm} (36)

Substituting the equations for $w^*_{HG}$ and $w^*_{HB}$ into the expected profit of the firm gives,

$$E\pi_{HH}^* = \left(\frac{\bar{e}_H}{\bar{e}_H + \beta}\right) (1 - w^*_{HG}) + \left(\frac{\beta}{\bar{e}_H + \beta}\right) (-w^*_{HB}) - R_H = \left(\frac{\bar{e}_H}{\bar{e}_H + \beta}\right) - 2R_H - c \hspace{1cm} (37)$$

If the firm hires two $L$ type workers, the problem is similar with the same explanation for the PC and IC constraints.

### 2.2.3 Homogeneous Teams ($LL$)

If the firm chooses to hire a homogeneous team consisting of only $L$ type workers and incentivizes both workers to exert effort, the problem is,\(^{48}\)

$$\begin{align*}
\max_{w_{LG}, w_{LB}} & \quad E\pi^{2eL}_{LL} = \left(\frac{2\bar{e}_L}{2\bar{e}_L + \beta}\right) (1 - 2w_{LG}) + \left(\frac{\beta}{2\bar{e}_L + \beta}\right) (-2w_{LB}) \\
\text{s.t.} & \quad \left(\frac{2\bar{e}_L}{2\bar{e}_L + \beta}\right) w_{LG} + \left(\frac{\beta}{2\bar{e}_L + \beta}\right) w_{LB} - c \geq R_L \\
& \quad \left(\frac{2\bar{e}_L}{2\bar{e}_L + \beta}\right) w_{LG} + \left(\frac{\beta}{2\bar{e}_L + \beta}\right) w_{LB} - c \geq \left(\frac{\bar{e}_L}{\bar{e}_L + \beta}\right) w_{LG} + \left(\frac{\beta}{\bar{e}_L + \beta}\right) w_{LB} \\
& \quad \left(\frac{\bar{e}_L}{\bar{e}_L + \beta}\right) w_{LG} + \left(\frac{\beta}{\bar{e}_L + \beta}\right) w_{LB} - c \geq w_{LB}
\end{align*} \hspace{1cm} (38-41)$$

Solving the problem using the same method as above, if the firm hires two $L$ type workers and incentivizes both workers to exert effort, the optimal wages given to $L$ type workers are,

$$w^*_{LG} = R_L + c \left(\frac{2\bar{e}_L + \beta}{\beta}\right) \hspace{1cm} (42)$$

$$w^*_{LB} = R_L - c \left(\frac{2\bar{e}_L + \beta}{\beta}\right) \hspace{1cm} (43)$$

\(^{48}\)The problem for a homogeneous team of $L$ type workers is the same as the problem for a homogeneous composition of $H$ type workers. The problem has the same PC and IC constraints. The explanations for each PC and IC constraint in the section for a homogeneous composition of $H$ type workers are also applied here but instead with $L$ type workers.
The restriction $R_L \geq c \left( \frac{2\bar{e}_L + \beta}{\beta} \right)$ assumes that the wages of the $L$ type worker if his team produces bad quality output is greater than or equal to zero.

Substituting $w^*_{LG}$ and $w^*_{LB}$ into the expected profit of the firm gives,

$$E\pi_{LL}^{2eL^*} = \left( \frac{2\bar{e}_L}{2\bar{e}_L + \beta} \right) \left( 1 - 2s^*_L \right) - 2w^*_L = \frac{2\bar{e}_L}{2\bar{e}_L + \beta} - 2R_L - 2c$$ (44)

The problem for the firm if it hires a homogeneous team of $L$ type workers but only incentivizes one worker to exert effort is,

$$\max_{w_{LG},w_{LB}} E\pi_{LL}^{1eL} = \left( \frac{\bar{e}_L}{\bar{e}_L + \beta} \right) (1 - w_{LG}) + \left( \frac{\beta}{\bar{e}_L + \beta} \right) (-w_{LB}) - R_L$$ (45)

s.t.

$$\left( \frac{\bar{e}_L}{\bar{e}_L + \beta} \right) w_{LG} + \left( \frac{\beta}{\bar{e}_L + \beta} \right) w_{LB} - c \geq R_L$$ (46)

$$\left( \frac{\bar{e}_L}{\bar{e}_L + \beta} \right) w_{LG} + \left( \frac{\beta}{\bar{e}_L + \beta} \right) w_{LB} - c \geq w_L$$ (47)

Similar to the problem in the previous section for $H$ type workers, the optimal wages given to $L$ type workers are,

$$w^*_{LG} = R_L + \frac{c(\bar{e}_L + \beta)}{\bar{e}_L}$$ (48)

$$w^*_{LB} = R_L$$ (49)

Substituting the equations for $w^*_{LG}$ and $w^*_{LB}$ into the expected profit of the firm gives,

$$E\pi_{LL}^{1eL^*} = \left( \frac{\bar{e}_L}{\bar{e}_L + \beta} \right) (1 - w^*_L) + \left( \frac{\beta}{\bar{e}_L + \beta} \right) (-w^*_L) - R_L = \left( \frac{\bar{e}_L}{\bar{e}_L + \beta} \right) - 2R_L - c$$ (50)

---

*It is similar to the problem in the previous section for a homogeneous composition of $H$ type workers with the firm incentivizing only one $H$ type worker. The PC constraint, IC constraint, and the explanations are the same, respectively, but for $L$ type workers.*
2.3 Heterogeneous Teams \((H, L)\)

If the firm chooses to hire a heterogeneous team consisting of one \(H\) type worker and one \(L\) type worker and incentivizes both workers to exert effort, the problem is,

\[
\max_{w_{HG}, w_{HB}, w_{LG}, w_{LB}} E\pi_{HL} = \left( \frac{\bar{e}_H + \bar{e}_L}{\bar{e}_H + \bar{e}_L + \beta} \right) (1 - w_{HG} - w_{LG}) + \left( \frac{\beta}{\bar{e}_H + \bar{e}_L + \beta} \right) (-w_{HB} - w_{LB})
\]

(51)

s.t.

\[
\left( \frac{\bar{e}_H + \bar{e}_L}{\bar{e}_H + \bar{e}_L + \beta} \right) w_{HG} + \left( \frac{\beta}{\bar{e}_H + \bar{e}_L + \beta} \right) w_{HB} - c \geq R_H
\]

(52)

\[
\left( \frac{\bar{e}_H + \bar{e}_L}{\bar{e}_H + \bar{e}_L + \beta} \right) w_{LG} + \left( \frac{\beta}{\bar{e}_H + \bar{e}_L + \beta} \right) w_{LB} - c \geq R_L
\]

(53)

\[
\left( \frac{\bar{e}_L}{\bar{e}_L + \beta} \right) w_{HG} + \left( \frac{\beta}{\bar{e}_L + \beta} \right) w_{HB} - c \geq \left( \frac{\bar{e}_L}{\bar{e}_L + \beta} \right) w_{HG} + \left( \frac{\beta}{\bar{e}_L + \beta} \right) w_{HB}
\]

(54)

\[
\left( \frac{\bar{e}_L}{\bar{e}_L + \beta} \right) w_{LG} + \left( \frac{\beta}{\bar{e}_L + \beta} \right) w_{LB} - c \geq \left( \frac{\bar{e}_L}{\bar{e}_L + \beta} \right) w_{LG} + \left( \frac{\beta}{\bar{e}_L + \beta} \right) w_{LB}
\]

(55)

\[
\left( \frac{\bar{e}_L}{\bar{e}_L + \beta} \right) w_{LG} + \left( \frac{\beta}{\bar{e}_L + \beta} \right) w_{LB} - c \geq \left( \frac{\bar{e}_L}{\bar{e}_L + \beta} \right) w_{LG} + \left( \frac{\beta}{\bar{e}_L + \beta} \right) w_{LB}
\]

(56)

Inequalities (52) and (53) are the participation constraints for a \(H\) and \(L\) type worker, respectively. The LHS of the inequalities are the expected payoffs the \(H\) type worker and \(L\) type worker receives from working for the firm. If the team produces a good quality product, with probability \(\bar{e}_H + \bar{e}_L + \beta\), they receive their respective wages for producing a good quality product in their teams, \(w_{HG}\) and \(w_{LG}\). If the team produces a bad quality product, with probability \(\frac{\beta}{\bar{e}_H + \bar{e}_L + \beta}\), the respective wages for producing bad quality are, \(w_{HB}\) and \(w_{LB}\). Both workers incur a cost of \(c\) for exerting effort. If the LHS of inequalities (52) and (53) are greater than or equal to their respective reservation payoffs, they choose to work for the firm.

Inequalities (54) and (56) are incentive compatibility constraints for a \(H\) and \(L\) type worker, respectively, to exert effort when their partner exerts effort. The LHS of both inequalities are explained previously with the LHS of inequalities (52) and (53). The RHS is the expected payoff of the worker from not exerting effort when their partner chooses to exert effort. For the \(H\) type worker, if the team produces a good quality product with only the \(L\) type worker exerting effort, with probability \(\frac{\bar{e}_L}{\bar{e}_L + \beta}\), he receives \(w_{HG}\). If the team
produces bad quality, with probability \( \frac{\beta}{e_L + \beta} \), the worker receives \( w_{HB} \). He does not incur a cost of \( c \) because he does not exert effort.

Similarly, for the \( L \) type worker, the RHS of inequality (56) is the expected payoff from not exerting effort while his partner, \( H \) type worker, exerts effort. The construction of the expected payoff is the same except that the probability of producing a good or bad quality product changes to \( \frac{\bar{e}_H}{e_H + \beta} \) and \( \frac{\bar{e}_L}{e_L + \beta} \), respectively, because only the \( H \) type worker exerts effort instead of only the \( L \) type. Also, the wages correspond for the \( L \) type worker instead of the \( H \) type worker.

Inequalities (55) and (57) are incentive compatibility constraints for a \( H \) and \( L \) type worker, respectively, to exert effort when their partners do not exert effort. If the \( H \) or \( L \) type worker exerts effort and their partner, of the different type does not exert effort, the probability of producing a good quality product is \( \frac{\bar{e}_H}{e_H + \beta} \) and \( \frac{\bar{e}_L}{e_L + \beta} \), respectively. If the team produces a good quality product, they are given the corresponding wage to their type for producing a good quality product. If the team produces a bad quality product, with probabilities \( \frac{\beta}{e_H + \beta} \) and \( \frac{\beta}{e_L + \beta} \) respectively, the worker receives a wage corresponding to their type given the team produced a bad quality product. Since the worker exerts effort, he incurs a cost of effort, \( c \). If the worker of either type chooses to not exert effort when their partner does not, the team produces bad quality with certainty and the worker receives the bad quality wage corresponding to their type; \( w_{HB} \) for the \( H \) type and \( w_{LB} \) for the \( L \) type. If the LHS is greater than or equal to the RHS, the worker chooses to exert effort when his partner chooses not to.

Isolating \( w_{HG} \) using the IC constraints for a \( H \) type worker, inequalities (54) and (55), are respectively,

\[
w_{HG} \geq w_{HB} + \frac{c(\bar{e}_H + \bar{e}_L + \beta)(\bar{e}_L + \beta)}{\bar{e}_H \beta}
\]

\[
w_{HG} \geq w_{HB} + \frac{c(\bar{e}_H + \beta)}{\bar{e}_H}
\]

First, \( \frac{c(\bar{e}_H + \bar{e}_L + \beta)(\bar{e}_L + \beta)}{\bar{e}_H \beta} > \frac{c(\bar{e}_H + \beta)}{\bar{e}_H} \). Therefore, \( w_{HG} \) that satisfies IC constraint (54) also satisfies (55). To maximize expected profits, \( w_{HG}^* = w_{HB} + \frac{c(\bar{e}_H + \bar{e}_L + \beta)(\bar{e}_L + \beta)}{\bar{e}_H \beta} \).

Similarly, the IC constraints for a \( L \) type worker, inequalities (56) and (57), are satisfied.
with $w^*_{LG}$.

\[
w^*_{LG} = w_{LB} + \frac{c(\bar{e}_H + \bar{e}_L + \beta)(\bar{e}_H + \beta)}{\bar{e}_L \beta}\tag{58}
\]

From the $H$ type worker’s PC, inequality (52), substituting $w^*_{HG}$,

\[
w_{HB} \geq R_H - \frac{c\bar{e}_L(\bar{e}_H + \bar{e}_L + \beta)}{\bar{e}_H \beta}\tag{59}
\]

Minimizing wages, $w^*_{HB} = R_H - \frac{c\bar{e}_L(\bar{e}_H + \bar{e}_L + \beta)}{\bar{e}_H \beta}$. It is assumed that wages are greater than or equal to zero. Therefore, a restriction of $R_H \geq \frac{c\bar{e}_L(\bar{e}_H + \bar{e}_L + \beta)}{\bar{e}_H \beta}$ is applied to the $H$ type worker’s wage given his team produces bad quality output.

Similarly, from the $L$ type worker’s PC, inequality (53), substituting $w^*_{LG}$, and solving for equality give $w^*_{LB}$,

\[
w^*_{LB} = R_L - \frac{c\bar{e}_H(\bar{e}_H + \bar{e}_L + \beta)}{\bar{e}_L \beta}\tag{60}
\]

Assuming wages are greater than or equal to zero, the restriction $R_L \geq \frac{c\bar{e}_H(\bar{e}_H + \bar{e}_L + \beta)}{\bar{e}_L \beta}$ is applied to the $L$ type worker’s wage given is team produces bad quality output.

Substituting $w^*_{HG}$ and $w^*_{LG}$ into $w_{HB}$ and $w_{LB}$, respectively, gives,

\[
w^*_{HG} = R_H + \frac{c(\bar{e}_H + \bar{e}_L + \beta)}{\bar{e}_H}\tag{61}
\]

\[
w^*_{LB} = R_H + \frac{c(\bar{e}_H + \bar{e}_L + \beta)}{\bar{e}_L}\tag{62}
\]

Substituting the optimal wages, $w^*_{HG}$, $w^*_{HB}$, $w^*_{LG}$, $w^*_{LB}$, into the firm’s expected profit,

\[
E\pi^*_{HL} = \frac{\bar{e}_H + \bar{e}_L}{\bar{e}_H + \bar{e}_L + \beta} - R_H - R_L - 2c\tag{63}
\]

The problem for the firm if it hires a heterogeneous team of workers but only incentivizes
the $H$ type worker to exert effort is,

$$
\max_{w_{HG}, w_{HB}, w_{LG}, w_{LB}} \quad E \pi_{H|L} = \left( \frac{\bar{e}_H}{\bar{e}_H + \beta} \right) \left( 1 - w_{HG} - w_{LG} \right) + \left( \frac{\beta}{\bar{e}_H + \beta} \right) \left( -w_{HB} - w_{LB} \right)
$$  \hspace{1cm} (64)

subject to

$$
\left( \frac{\bar{e}_H}{\bar{e}_H + \beta} \right) w_{HG} + \left( \frac{\beta}{\bar{e}_H + \beta} \right) w_{HB} - c \geq R_H \hspace{1cm} (65)
$$

$$
\left( \frac{\bar{e}_H}{\bar{e}_H + \beta} \right) w_{LG} + \left( \frac{\beta}{\bar{e}_H + \beta} \right) w_{LB} \geq R_L \hspace{1cm} (66)
$$

$$
\left( \frac{\bar{e}_H}{\bar{e}_H + \beta} \right) w_{LG} + \left( \frac{\beta}{\bar{e}_H + \beta} \right) w_{LB} \geq \left( \frac{\bar{e}_H + \bar{e}_L}{\bar{e}_H + \bar{e}_L + \beta} \right) w_{LG} + \left( \frac{\beta}{\bar{e}_H + \bar{e}_L + \beta} \right) w_{LB} - c \hspace{1cm} (67)
$$

$$
w_{LB} \geq \left( \frac{\bar{e}_L}{\bar{e}_L + \beta} \right) w_{LG} + \left( \frac{\beta}{\bar{e}_L + \beta} \right) w_{LB} - c \hspace{1cm} (68)
$$

Inequality (65) is the PC constraint for a $H$ type worker. The LHS of the inequality is the expected payoff of the worker exerting effort in a team. He receives a wage of $w_{HG}$ if the team produces a good quality product, with probability $\frac{\bar{e}_H}{\bar{e}_H + \beta}$. He receives a wage of $w_{HB}$ if the team produces a bad quality product, with probability $\frac{\beta}{\bar{e}_H + \beta}$. He incurs a cost of $c$ for exerting effort. For the worker to work in the firm, the expected payoff should be bigger than the reservation payoff for a $H$ type worker $R_H$ on the RHS of the inequality.

Inequality (66) is the PC constraint for a $L$ type worker to work in the firm but not exert effort. The LHS is the expected payoff for not exerting effort. He receives a wage of $w_{LG}$ if the team produces a good quality product, with probability $\frac{\bar{e}_H}{\bar{e}_H + \beta}$ and $w_{LB}$ if the team produces a bad quality product. He does not incur a cost of $c$ because he chooses not to exert effort. For the worker to work in the firm, the LHS should be larger than the RHS which is the worker’s reservation payoff, $R_L$.

Inequality (67) is the IC constraint for a $H$ type worker to exert effort while his partner chooses not to exert effort.\(^{50}\) The LHS of the inequality is the expected payoff for a $H$ type worker to exert effort. The explanation was provided previously for the LHS of inequality

\(^{50}\)The current problem has been constructed where the $L$ type worker has the dominant strategy to not exert effort. The solution to the problem is simpler. It can be shown that the expected profits of the firm are the same regardless if the $L$ type worker has a dominant strategy to not exert effort or the $H$ type worker has a dominant strategy to exert effort. However, the wages in each scenario are different. If the workers do not have a dominant strategy, there exists the possibility of multiple equilibria. To incentivize one worker to have a dominant strategy, the payoff for the dominant strategy must increase to satisfy his incentive compatibility constraints (i.e., increase the payoff in the good state). But since the firm must also satisfy the worker’s participation constraint, payoffs in the bad state need to decrease. Since both workers are risk neutral in the model, the increase and decrease in payoffs are the same for both workers. Therefore, the expected profits are the regardless of which worker has a dominant strategy.
(65). If the expected payoff is greater than or equal to the reservation payoff, \( R_H \), the \( H \) type worker chooses to work for the firm.

Inequality (68) is the IC constraint for a \( L \) type worker to choose not to exert effort when his partner chooses to exert effort. If the \( L \) type worker does not exert effort, the expected payoff is given by the LHS of inequality (66). If the \( L \) type worker chooses to exert effort if his partner exerts effort, the probability of producing a good quality product is \( \frac{\bar{e}_H + \bar{e}_L}{\bar{e}_H + \bar{e}_L + \beta} \) and the probability of producing a bad quality product is \( \frac{\beta}{\bar{e}_H + \bar{e}_L + \beta} \). He receives \( w_{LG} \) for good quality and \( w_{LB} \) for bad quality. He incurs a cost of \( c \) for exerting effort.

Inequality (69) is the IC constraint for a \( L \) type worker to not exert effort when his partner does not exert effort. The LHS is the expected payoff from both team members not exerting effort. They produce bad quality with certainty and the \( L \) type worker receives \( w_{LB} \). If the \( L \) type worker chooses to exert effort, the expected payoff is on the RHS. If he exerts effort, the team produces good quality with the probability \( \frac{\bar{e}_L}{\bar{e}_L + \beta} \) and bad quality with \( \frac{\beta}{\bar{e}_L + \beta} \). The \( L \) type worker receives \( w_{LG} \) for good quality and \( w_{LB} \) for bad quality. He incurs a cost of effort, \( c \), for exerting effort.

From the PC and IC constraint for a \( H \) type worker, the optimal \( w_{HB}^* \) is,

\[
w_{HB}^* = R_H
\]  

(70)

Substituting \( w_{HB}^* \) into the PC for the \( H \) type worker (or IC constraint for the \( H \) type worker) and solving for \( w_{HG}^* \),

\[
w_{HG}^* = R_H + \frac{c(\bar{e}_H + \beta)}{\bar{e}_H}
\]  

(71)

Solving for \( w_{LG}^* \) using the IC constraints for a \( L \) type worker, inequalities (68) and (69), are respectively,

\[
w_{LG}^* = w_{LB} + \frac{c(\bar{e}_H + \bar{e}_L + \beta)(\bar{e}_H + \beta)}{\bar{e}_L \beta}
\]  

(72)

\[
w_{LG}^{**} = w_{LB} + \frac{c(\bar{e}_L + \beta)}{\bar{e}_L}
\]  

(73)
where $w_{LG}^* > w_{LG}^{**}$. Therefore, $w_{LG}^*$ is required to satisfy both IC constraints for the $L$ type worker.

Substituting $w_{LG}^*$ into the PC for the $L$ type worker and solving for $w_{LB}^*$,

$$w_{LB}^* = R_L - \frac{c\bar{e}_H(\bar{e}_H + \bar{e}_L + \beta)}{\bar{e}_L \beta}$$ (74)

Assuming that wages are greater than or equal to zero, a restriction on the $L$ type worker’s wage given his team produces bad quality output of $R_L \geq \frac{c\bar{e}_H(\bar{e}_H + \bar{e}_L + \beta)}{\bar{e}_L \beta}$ is applied.

Substituting $w_{LB}^*$ into $w_{LG}^*$,

$$w_{LG}^* = R_L - \frac{c(\bar{e}_H + \bar{e}_L + \beta)}{\bar{e}_L}$$ (75)

Assuming that wages are greater than or equal to zero for a $L$ type worker given his team produces good quality output, a restriction of $R_L \geq \frac{c(\bar{e}_H + \bar{e}_L + \beta)}{\bar{e}_L}$ is applied.

Substituting the optimal wages into the firm’s expected profits,

$$E\pi_{HL}^{1eH^*} = \frac{\bar{e}_H}{\bar{e}_H + \beta} - R_H - R_L - c$$ (76)

The problem for the firm if it hires a heterogeneous team of workers but only incentivizes the $L$ type worker to exert effort is,

$$\max_{w_{HG}, w_{HB}, w_{LG}, w_{LB}} E\pi_{HL}^{1eL} = \left(\frac{\bar{e}_L}{\bar{e}_L + \beta}\right) (1 - w_{HG} - w_{LG}) + \left(\frac{\beta}{\bar{e}_H + \beta}\right) (-w_{HB} - w_{LB})$$ (77)

s.t.

$$\left(\frac{\bar{e}_L}{\bar{e}_L + \beta}\right) w_{HG} + \left(\frac{\beta}{\bar{e}_H + \beta}\right) w_{HB} \geq R_H$$ (78)

$$\left(\frac{\bar{e}_L}{\bar{e}_L + \beta}\right) w_{LG} + \left(\frac{\beta}{\bar{e}_H + \beta}\right) w_{LB} - c \geq R_L$$ (79)

$$w_{HB} \geq \left(\frac{\bar{e}_H}{\bar{e}_H + \beta}\right) w_{HG} + \left(\frac{\beta}{\bar{e}_H + \beta}\right) w_{HB} - c$$ (80)

$$\left(\frac{\bar{e}_L}{\bar{e}_L + \beta}\right) w_{HG} + \left(\frac{\beta}{\bar{e}_H + \beta}\right) w_{HB} \geq \left(\frac{\bar{e}_H + \bar{e}_L}{\bar{e}_H + \bar{e}_L + \beta}\right) w_{HG} + \left(\frac{\beta}{\bar{e}_H + \bar{e}_L + \beta}\right) w_{HB} - c$$ (81)

$$\left(\frac{\bar{e}_L}{\bar{e}_L + \beta}\right) w_{LG} + \left(\frac{\beta}{\bar{e}_H + \beta}\right) w_{LB} - c \geq w_{LB}$$ (82)

Inequality (78) is the PC constraint for a $H$ type worker. The LHS of the inequality is the expected payoff of the worker exerting effort in a team. He receives a wage of $w_{HG}$ if the
team produces a good quality product, with probability $\frac{\bar{e}_L}{\bar{e}_L + \beta}$. He receives a wage of $w_{HB}$ if the team produces a bad quality product, with probability $\frac{\beta}{\bar{e}_L + \beta}$. He does not incur a cost of $c$ for not exerting effort. For the worker to work in the firm, the expected payoff should be bigger than the reservation payoff for a $H$ type worker $R_H$ on the RHS of the inequality.

Inequality (79) is the PC constraint for a $L$ type worker. The LHS is the expected payoff for exerting effort. He is paid a wage of $w_{LG}$ if the team produces a good quality product, with probability $\frac{\bar{e}_L}{\bar{e}_L + \beta}$ and $w_{LB}$ if the team produces a bad quality product. He incurs a cost of $c$ because he chooses to exert effort. For the worker to work in the firm, the LHS should be larger than the RHS which is the worker’s reservation payoff, $R_L$.

Inequality (80) is the IC constraint for a $H$ type worker to not exert effort while his partner chooses not to exert effort. The LHS of the inequality is the payoff for a $H$ type worker if bad quality is produced with certainty when no team members exert effort. If the $L$ type worker chooses to exert effort, the expected payoff is on the RHS. If he exerts effort, the team will produce good quality with the probability $\frac{\bar{e}_H}{\bar{e}_H + \bar{e}_L + \beta}$ and bad quality with $\frac{\beta}{\bar{e}_H + \bar{e}_L + \beta}$. The $L$ type worker receives $w_{LG}$ for good quality and $w_{LB}$ for bad quality. He incurs a cost of effort, $c$, for exerting effort.

Inequality (81) is the IC constraint for a $H$ type worker to choose not to exert effort when his partner chooses to exert effort. If the $L$ type worker does not exert effort, the expected payoff is given by the LHS of inequality (78). If the $L$ type worker chooses to exert effort if his partner exerts effort, the probability of producing a good quality product is $\frac{\bar{e}_H + \bar{e}_L}{\bar{e}_H + \bar{e}_L + \beta}$ and the probability of producing a bad quality product is $\frac{\beta}{\bar{e}_H + \bar{e}_L + \beta}$. He receives $w_{HG}$ for good quality and $w_{HB}$ for bad quality. He incurs a cost of $c$ for exerting effort.

Inequality (82) is the IC constraint for a $L$ type worker to exert effort when his partner does not exert effort. If the $L$ type worker chooses to exert effort, the expected payoff is on the LHS. If he exerts effort, the team produces good quality with the probability $\frac{\bar{e}_L}{\bar{e}_L + \beta}$ and bad quality with $\frac{\beta}{\bar{e}_L + \beta}$. The $L$ type worker receives $w_{LG}$ for good quality and $w_{LB}$ for bad quality. He incurs a cost of effort, $c$, for exerting effort. The RHS is the expected payoff.

$^{51}$The current problem has been constructed where the $H$ type worker has the dominant strategy to not exert effort. It can be shown that the expected profits of the firm are the same regardless if the $H$ type worker has a dominant strategy to not exert effort or the $L$ type worker has a dominant strategy to exert effort. I have chosen to present this version for simplicity.
from both team members not exerting effort. They produce bad quality with certainty and the $L$ type worker receives $w_{LB}$.

From the PC and IC constraints for a $L$ type worker, inequalities (79) and (82), the optimal $w_{LB}^*$ is,

$$w_{LB}^* = R_L$$ (83)

Substituting $w_{LB}^*$ into the PC for a $L$ type worker (or IC constraint for the $L$ type worker) and solving for $w_{LG}^*$,

$$w_{LG}^* = R_L + \frac{c(e_L + \beta)}{e_L}$$ (84)

Solving for $w_{HG}$ using the IC constraints for a $H$ type worker, inequalities (80) and (81), are respectively,

$$w_{HG}^* = w_{HB} + \frac{c(e_H + e_L + \beta)(e_H + e)}{e_L \beta}$$ (85)

$$w_{HG}^{**} = w_{HB} + \frac{c(e_H + \beta)}{e_H}$$ (86)

where $w_{HG}^* > w_{HG}^{**}$. Therefore, $w_{HG}^*$ is required to satisfy both IC constraints for the $H$ type worker.

Substituting $w_{HG}^*$ into the PC for the $H$ type worker and solving for $w_{HB}^*$,

$$w_{HB}^* = R_H - \frac{c(e_H + \beta)(e_H + e_L + \beta)}{(e_L + \beta)}$$ (87)

Assuming that the wage for a $H$ type worker given his team produces bad quality output is greater than or equal to zero, the restriction $R_H \geq \frac{c(e_H + e_L + e_L + \beta)}{(e_L + \beta)}$ is applied.

Substituting $w_{HB}^*$ into $w_{HG}^*$,

$$w_{HG}^* = R_H + \frac{c(e_H + \beta)(e_H + e_L + \beta)}{e_L (e_L + \beta)}$$ (88)

Substituting the optimal wages into the firm’s expected profits,

$$E\pi_{HL}^e = \frac{e_L}{e_L + \beta} - R_H - R_L - c$$ (89)
where $E_{\pi H}^{1eH} > E_{\pi HL}^{1eH}$. The firm will always choose to induce the $H$ type to exert effort rather than the $L$ type worker.

The same is true when comparing the expected payoffs from scenarios $E_{\pi H}^{1eH}$ and $E_{\pi HL}^{1eH}$. If a firm hires only one $H$ type worker to exert effort, the other position in the team should be filled with a $L$ type worker to save costs. A $L$ type worker has a lower reservation payoff compared to a $H$ type worker. Therefore $E_{\pi H}^{1eH} > E_{\pi HL}^{1eH}$.

### 2.3.1 Firm Choices

The firm chooses the composition of teams with the highest expected profits. For the firm to choose a homogeneous team consisting of only $H$ type workers, the following must be true.

52

For the firm to choose a homogeneous team consisting of only $L$ type workers, the following must be true.

52 $\Delta = R_H - R_L$. 

55

\[ E_{\pi H}^{2eH} > E_{\pi L}^{2eL} \implies \frac{\beta(\bar{e}_H - \bar{e}_L)}{(2\bar{e}_H + \beta)(2\bar{e}_L + \beta)} > \Delta \]  

(90)

\[ E_{\pi H}^{2eH} > E_{\pi L}^{1eL} \implies \frac{\beta(\bar{e}_H - \bar{e}_L)}{2(2\bar{e}_H + \beta)(\bar{e}_L + \beta)} - \frac{c}{2} > \Delta \]  

(91)

\[ E_{\pi H}^{2eH} > E_{\pi HL}^{2eH} \implies \frac{\beta(\bar{e}_H - \bar{e}_L)}{(2\bar{e}_H + \beta)(\bar{e}_L + \beta)} > \Delta \]  

(92)

\[ E_{\pi H}^{2eH} > E_{\pi HL}^{1eH} \implies \frac{\bar{e}_H}{(2\bar{e}_H + \beta)(\bar{e}_L + \beta)} - c > \Delta \]  

(93)

For the firm to choose a homogeneous team consisting of only $L$ type workers, the following must be true.

\[ E_{\pi L}^{2eL} > E_{\pi H}^{2eH} \implies \frac{\beta(\bar{e}_H - \bar{e}_L)}{(2\bar{e}_H + \beta)(2\bar{e}_L + \beta)} < \Delta \]  

(94)

\[ E_{\pi L}^{2eL} > E_{\pi L}^{1eL} \implies \frac{\bar{e}_L}{(2\bar{e}_L + \beta)(\bar{e}_L + \beta)} > c \]  

(95)

\[ E_{\pi L}^{2eL} > E_{\pi HL}^{2eL} \implies \frac{\beta(\bar{e}_H - \bar{e}_L)}{(2\bar{e}_L + \beta)(\bar{e}_H + \bar{e}_L + \beta)} < \Delta \]  

(96)

\[ E_{\pi L}^{2eL} > E_{\pi HL}^{1eH} \implies \frac{2\beta(\bar{e}_H - \bar{e}_L)}{(2\bar{e}_L + \beta)(\bar{e}_H + \beta)} + c < \Delta \]  

(97)
For the firm to choose a heterogeneous team of workers, the following must be true.

\[
E_{π_{HL}}^{2e} > E_{π_{HH}}^{2e} \implies \frac{β(\bar{e}_H - \bar{e}_L)}{(2\bar{e}_H + β)(\bar{e}_H + \bar{e}_L + β)} < \Delta \tag{98}
\]

\[
E_{π_{HL}}^{2e} > E_{π_{LL}}^{1e} \implies \frac{\bar{e}_H^β}{(2\bar{e}_L + β)(\bar{e}_H + \bar{e}_L + β)} - c > \Delta \tag{99}
\]

\[
E_{π_{HL}}^{2e} > E_{π_{LL}}^{2e} \implies \frac{β(\bar{e}_H - \bar{e}_L)}{(2\bar{e}_L + β)(\bar{e}_H + \bar{e}_L + β)} > \Delta \tag{100}
\]

\[
E_{π_{HL}}^{2e} > E_{π_{LL}}^{1e} \implies \frac{\bar{e}_L^β}{(\bar{e}_H + \bar{e}_L + β)(\bar{e}_H + β)} > c \tag{101}
\]

For the firm to choose a heterogeneous team of workers and on incentivize the \(H\) type worker to exert effort, the following must be true.

\[
E_{π_{HL}}^{1e} > E_{π_{HH}}^{2e} \implies \frac{\bar{e}_H^β}{(2\bar{e}_H + β)(\bar{e}_H + \bar{e}_L + β)} - c < \Delta \tag{102}
\]

\[
E_{π_{HL}}^{1e} > E_{π_{LL}}^{1e} \implies \frac{β(\bar{e}_H - \bar{e}_L)}{(\bar{e}_H + β)(\bar{e}_L + β)} > \Delta \tag{103}
\]

\[
E_{π_{HL}}^{1e} > E_{π_{LL}}^{2e} \implies \frac{2β(\bar{e}_H - \bar{e}_L)}{(2\bar{e}_L + β)(\bar{e}_H + β)} + c > \Delta \tag{104}
\]

\[
E_{π_{HL}}^{1e} > E_{π_{LL}}^{2e} \implies \frac{\bar{e}_L^β}{(\bar{e}_H + \bar{e}_L + β)(\bar{e}_H + β)} < c \tag{105}
\]

The conditions for the scenario where \(E_{π_{LL}}^{1e}\) has the highest expected payoff can be extracted from the above conditions (Inequalities (91), (95), (99), and (103)).

Figure 9 illustrates the firm’s choice for team formation, according to the conditions stated above, in regards to \(Δ\) and \(c\). Along the \(y\)-axis, \(Δ\) is increasing where \(Δ = R_H - R_L\) (i.e., \(H\) types are getting more expensive to hire relative to \(L\) types). Along the \(x\)-axis, the cost of effort is increasing. The areas are labeled with the optimal composition of workers that the firm hires and the effort to be induced given values of \(Δ\) and \(c\).

Figure 9 assumes that all the restrictions for positive wages are satisfied in each parameter space of the firm’s optimal choice. There is a possibility that within each parameter space, combinations of \(Δ\) and \(c\) do not satisfy the positive wage restrictions causing zero or negative wages.\(^{53}\) The restrictions are not shown in the figure because the exact lines

\(^{53}\)To approximate the wage restrictions in terms of the parameters of the figure, the analysis begins by assuming the reservation payoff of the \(L\) type worker is zero, \(R_L = 0\), and then examines the changes in the restriction with an increase in the value of \(R_L\) relative to \(R_H\).
depend on $R_H$ or $R_L$, but not necessarily the difference. The restrictions are approximated and explained in the following paragraphs.

Suppose $R_L$ is equal to zero, then $\Delta = R_H$. In the parameter space where the firm chooses to hire two $H$ type workers and incentivizes both workers to exert effort, $E\pi_{HH}^{\delta H}$, the restriction for positive values of wages, $R_H \geq \frac{c(2\delta H + \beta)}{\bar{e}^{\beta}}$, is a positively sloped straight line that passes through the origin and is steeper than the 45 degree line. The parameter values that satisfy the restriction falls to the left of the line. As $R_L$ increases, the line shifts to the right.\(^{54}\)

The parameter space where the firm chooses a heterogeneous team of workers and incentivizes both workers to exert effort contains two restrictions for positive wages, $R_H \geq \frac{c\bar{e}_L(\delta_H + \delta_L + \beta)}{\bar{e}^{\beta}}$ and $R_L \geq \frac{c\bar{e}_H(\delta_H + \delta_L + \beta)}{\bar{e}^{\beta}}$. For the former restriction that depends on the

\(^{54}\)The conditions for the firm’s optimal choice of team composition are constructed using $\Delta$ and $c$ but the restrictions for wages only contain $R_H$ or $R_L$. Therefore, the exact locations of the restrictions are difficult to determine. In this paper, the restrictions are described and approximated. A simulation can be done for specific values of reservation payoffs and cost of effort but is not shown in this paper.
reservation payoff of the $H$ type worker, it is similar to the restriction discussed above. It is a positively sloped line where the slope depends on the values of effort and $\beta$. As $R_H$ increases, the lines for the restriction shifts to the right.

There are other restrictions for positive wages for the other parameter spaces; $R_L \geq \frac{c(2\bar{e}_L+\beta)}{\beta}$ for $E_{\pi L L}^2$, $R_L \geq c\bar{e}_H(\bar{e}_H+\bar{e}_L+\beta)$ for $E_{\pi H L}^2$, and $R_L \geq \frac{c(\bar{e}_H+\bar{e}_L+\beta)}{\bar{e}_L}$ for $E_{\pi H L}^2$. These conditions all depend on the reservation payoff of the $L$ type worker. Assuming a constant value of $R_L$, the restrictions are a vertical line at a specific value of $c$ depending on values of effort and $\beta$. As $R_L$ increases, the line shifts to the right. The parameter values of $\Delta$ and $c$ that satisfy the restriction for wages lie to the left of the vertical line and inside the parameter space of the optimal choice of the firm.

Figure 9 illustrates, as $\Delta$ increases or the $H$ type becomes more expensive to hire compared to the $L$ type when cost of effort is sufficiently low, the firm’s choice changes from a homogeneous composition of $H$ type workers to a heterogeneous composition to a homogeneous composition of $L$ type workers. When cost of effort is sufficiently low, it is profitable for the firm to incentivize all its workers to exert effort. Given that the hiring costs of both types are relatively similar, $\Delta$ is sufficiently small, it is more profitable to hire two $H$ type workers. They cost relatively the same as $L$ types and, at the same cost, exert a higher level of effort.

As $\Delta$ increases, the $H$ types are more expensive to hire. Since cost of effort is still relatively low, the firm finds it profitable to incentivize all workers to exert effort. If $H$ types are more expensive, the firm hires fewer of them. At a moderate level of $\Delta$, the firm hires one less $H$ type to make a heterogeneous team of one $H$ and one $L$ type.

Further increasing $\Delta$ makes $H$ types too expensive to hire relative to $L$ types. Therefore, the firm only hires $L$ type workers and incentivizes them to exert effort if cost of effort is still sufficiently low.

The figure further illustrates the impact of cost of effort on the firms decision regarding composition of teams. Moving from left to right in the figure, all compositions when cost of effort was low changes to a heterogeneous choice while incentivizing the $H$ type worker. As cost of effort increases, it becomes more expensive for the firm to incentivize effort. To maintain a positive level of expected profits, at least one worker must exert effort.
both workers have the same cost of effort, therefore same cost to the firm to induce effort, the firm chooses to incentivize a $H$ type worker for his higher level of effort. The firm hires one $H$ type worker for one of the positions on the team. The other position is filled with a worker that does not exert effort since cost is too high. The firm fills the second position with a $L$ type worker because they are cheaper to hire.\(^{55}\) This creates a heterogeneous composition of teams.

When $\Delta$ and cost of effort is excessively high, the $H$ type is too expensive to hire. The remaining choice is to hire the cheaper worker, $L$ type and incentivize him to exert effort.\(^{56}\)

The increase in $\Delta$ means one type of worker is more expensive to hire relative to the other. Therefore, the firm chooses to hire less workers of the more expensive type as $\Delta$ increases creating a homogeneous composition of $H$ and $L$ types when $\Delta$ is low and high respectively. At the moderate range, the firm can hire some of each type creating a heterogeneous team composition. As cost of effort rises, it becomes more expensive for the firm to induce effort. Since the production technology requires the firm to hire two workers, positions with workers not exerting effort are filled with the cheapest to hire workers, $L$ types.

### 2.3.2 Conclusion

When costs to incentivize effort and to hire high efficiency relative to low efficiency types are low, homogeneous teams of high efficiency types are hired. As both costs increase, less high efficiency types are hired and heterogeneous teams are chosen.

### 2.4 Moral Hazard Problem Without Hiring

The model consists of a firm and four workers where two are high ($H$) and two are low ($L$) types. All the workers are assumed to be already hired by the firm.\(^{57}\) The role of the firm is to find the pairing (two teams, two workers per team) to optimize production.\(^{58}\) This

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\(^{55}\) The dominant strategy for the firm is to always hire $H$ types when $\Delta$ is zero. Condition (98) is violated when $\Delta$ is zero.

\(^{56}\) Conditions (91) and (99) had to be approximated in Figure 9 but the qualitative results do not change.

\(^{57}\) There are not wages to incentivize workers to join the firm but there is a profit sharing scheme to incentivize workers to exert effort.

\(^{58}\) The following model design is for simplicity and tractability of the results. The solution can be applied to scenarios with more teams, more workers, or uneven distributions of types of workers. A firm can create
creates two possible choices for the firm; homogeneous teams or heterogeneous teams. If the firm decides to make homogeneous teams, it has one team with both high type workers and the other consisting of two low type workers. If the firm decides on heterogeneous teams, it has two teams each consisting of one high type worker and one low type worker. After the teams have been formed by the firm, workers make effort decisions for the production process.

It is assumed that the share of the value of output workers receive is predetermined.\textsuperscript{59} The worker’s efforts affect the quality of the good produced, which in turn is directly related to their payoff. Many of the characteristics of the firm and workers are the same as the model in the previous section. However, one important difference is that the workers are already employed by the firm.

2.4.1 Workers

There exists two types of workers; high ($H$) and ($L$). Each type of worker can choose to exert a level of effort, $e_i \in \{0, \bar{e}_i\}$, $i = H, L$, and $\bar{e}_H > \bar{e}_L$. As in the previous model, if a worker chooses to exert an effort level of 0, it is referred to as “no effort” while exerting $\bar{e}$ is referred to as exerting “effort”,\textsuperscript{60} and the latter incurs a cost of $c > 0$. Effort is not observable.

2.4.2 Firms

Each team working for the firm produces a single product for the firm. The product can be of two qualities; good ($G$) or bad ($B$). If the product is of good quality, the value the firm receives is 1. If the quality of the product is bad, the firm receives a value of 0. When the quality of the product is realized, the firm will evenly split the value of the product produced between the firm and team with each member receiving an equal portion. If good

\textsuperscript{59}It is supposed that these shares would have been established and perpetuated through the corporate culture of the firm.

\textsuperscript{60}Equating low effort to a value of zero is for simplicity.
quality is produced, the firm will receive $\frac{1}{2}$ and each worker will receive $\frac{1}{4}$.

The firms are unable to observe the amount of effort exerted by each individual. If there was one worker exerting effort in the team, the firm is unable to observe which member exerted the effort. Therefore, it is possible to assume a fixed and equal share of the value from production between the workers.\footnote{“Two men jointly lift heavy cargo into trucks. Solely by observing the total weight loaded per day, it is impossible to determine each person’s marginal productivity. With team production it is difficult, solely by observing total output, to either define or determine each individual’s contribution to this output of the cooperating input.” - Alchian and Demsetz (1972). Assuming a supervisor (in the case of this paper, the firm) saw the heavy cargo on the truck, they would assume that half the weight was lifted by one man and the other half lifted by the other. Therefore, half the weight is attributed to one man and the other half to the other (assuming the supervisor was not observing the process of loading the truck).}

The firm chooses the composition of teams that produce the highest expected value of the product.

### 2.4.3 Quality

The quality of the good is determined by a modified Tullock contest success function in the following form,

$$p(e_1, e_2) = \frac{e_1 + e_2}{e_1 + e_2 + \beta} \quad \text{where} \quad e \in \{0, \bar{e}\}$$  \hfill (106)

The teams for production consist of two workers. If the workers choose to exert effort, the probability of success increases. If both workers choose to not exert effort, then a bad quality product is guaranteed.

The $\beta$ prevents the probability of producing a good quality product from becoming a guarantee. The term captures the magnitude of “bad luck” that can exist in the production process. If “bad luck” increases, (i.e., the $\beta$ term increases) then the probability of producing a good quality product decreases.\footnote{Amegashie (2006) proposed a similar contest success function in which he examined the degree to which luck as opposed to effort affects behaviour in different contest settings. His paper presents and discusses the properties of the contest success function.}

### 2.4.4 Homogeneous Teams

The following analysis uses backward induction to determine the optimal choice of the firm. The first step solves the best responses of the workers and then solves the choice of the
firm. The analysis aims to explain the impact of cost of effort of the worker on the actions of the firm. As cost of effort changes, this impacts the actions of the worker. Depending on the actions of its workers, the firm makes the decision regarding its workforce.

This section details the conditions for workers to exert effort to be a dominant strategy when firms choose to form homogeneous teams. There is one team consisting of only high type workers and one team consisting of only low type workers. The section will first determine the expected payoffs of the high type team and then the low type team.

For the high type team, if one worker chooses to exert effort, the condition for the other worker to choose to exert effort is,

\[
\bar{e}_H^2 (2\bar{e}_H + \beta) - c \geq \frac{\bar{e}_H}{4(\bar{e}_H + \beta)} \quad (107)
\]

The LHS of the above inequality is the expected payoff of a $H$ type worker if both workers choose to exert effort. The probability of producing a good quality product is $\frac{2\bar{e}_H}{2\bar{e}_H + \beta}$ and the good quality is valued at 1. This is divided by four since both workers receive a quarter of the value of a good quality product.\textsuperscript{63} Since the worker exerts effort, there is a cost of $c$. The LHS of condition (107) should be greater than or equal to the payoff if the worker chooses to free ride on the effort of his team member, given by the RHS. The probability of producing a good quality product if only one member of the team exerts effort is $\frac{\bar{e}_H}{\bar{e}_H + \beta}$. This is divided by four because each team member receives a quarter of the value of a good quality product. There is no cost of effort since the worker chooses not to exert effort.

Condition (107) is rearranged for $c$,

\[
c \leq \frac{\bar{e}_H \beta}{4(2\bar{e}_H + \beta)(\bar{e}_H + \beta)} \quad (108)
\]

If cost of effort is sufficiently high, the worker shirks on his partner’s effort. The critical value of cost of effort, $c^{2e_H}_{HH}$, is where a $H$ type worker decides to free ride on the effort of his $H$ type partner that exerts effort. If cost exceeds this critical value, it is not beneficial

\textsuperscript{63} Since the value of a bad quality product is zero, the payoff equation does not include the probability of producing a bad quality product.
for the $H$ type worker to exert effort.

If one of the team members chooses not to exert effort, the condition for the other worker to exert effort is,

$$\frac{\bar{e}_H}{4(\bar{e}_H + \beta)} - c \geq 0 \quad (109)$$

If the worker chooses to exert effort, only one member of the team is exerting effort. The probability of producing a good quality product is $\frac{\bar{e}_H}{\bar{e}_H + \beta}$. This is divided by four for the value received between the two members of the team. Since the worker chooses to exert effort, he will incur a cost of $c$. To exert effort, the LHS of condition (109) should be greater than or equal to zero. If both workers choose not to exert effort, the quality of the product is guaranteed to be bad and the value is zero. The worker chooses not to exert effort and therefore does not incur a cost. Therefore, the payoff of no one exerting effort is zero.

Condition (109) is rearranged to be,

$$c \leq c^{\bar{e}_H}_{HH} = \frac{\bar{e}_H}{4(\bar{e}_H + \beta)} \quad (110)$$

If cost of effort is sufficiently high, the worker chooses to not exert effort. If costs exceed the critical value, $c^{\bar{e}_H}_{HH}$, a $H$ type worker is better off not exerting effort with his $H$ type teammate.

The teams consist of two members that are of the same type. Therefore, the equations are the same for both high type members of the team. If the condition is satisfied for one member of the team, it must be satisfied for the other as well. Both team members choose the same actions.

Since the framework is symmetrical for both teams, the conditions are the same for the team consisting of only low type workers except replace $H$ with $L$. The conditions for a $L$ type worker to exert effort when his team member exerts effort and the condition for the $L$ type worker to exert effort when his team member does not exert effort are, respectively,

$$c^{\bar{e}_L}_{LL} = \frac{\bar{e}_L\beta}{4(2\bar{e}_L + \beta)(\bar{e}_L + \beta)} \geq c \quad (111)$$
Therefore, the conditions for all members of both teams to exert effort as a dominant strategy in homogeneous teams are conditions (108), (110), (111), and (112).64

\[
c_{LL}^{1e} = \frac{\bar{e}_L}{4(\bar{e}_L + \beta)} \geq c \tag{112}
\]

Therefore, the conditions for all members of both teams to exert effort as a dominant strategy in homogeneous teams are conditions (108), (110), (111), and (112).64

<table>
<thead>
<tr>
<th>i</th>
<th>(\bar{e}_i)</th>
<th>(c_{i}^{1e})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\bar{e}_i)</td>
<td>(\frac{\bar{e}_i}{2(\bar{e}_i + \beta)} - c)</td>
<td>(\frac{\bar{e}_i}{2(\bar{e}_i + \beta)} - c)</td>
</tr>
<tr>
<td>0</td>
<td>(\frac{\bar{e}_i}{4(e_i + \beta)})</td>
<td>(\frac{\bar{e}_i}{4(e_i + \beta)} - c)</td>
</tr>
</tbody>
</table>

Table 1: Game Matrix for Homogeneous formation of teams.

Table 1 illustrates the game between workers in a homogeneous composition.

The ordering of the critical values is the following except for \(c_{HH}^{2e}\),

\[
c_{HH}^{2e} > c_{LL}^{1e} > c_{LL}^{1e} \tag{113}
\]

The increase in the probability is highest for a \(H\) type worker to exert effort when his partner does not compared to a \(L\) type worker exerting effort in any scenario.\(^65\) Therefore, a \(H\) type can incur a higher cost of effort to exert effort compared to a \(L\) type if their partner does not exert effort because the marginal increase in the probability to produce a good quality product is higher compared to the \(L\) type worker’s \((c_{HH}^{2e} > c_{LL}^{1e})\).

The increase of the probability by exerting effort from zero total team effort is greater than the increase with a teammate already exerting effort. Therefore, a \(L\) type worker can incur a higher cost of effort when choosing to exert effort from zero total team effort compared to exerting effort when his partner, another \(L\) type worker, is already exerting effort. This puts \(c_{LL}^{1e}\) above \(c_{LL}^{2e}\).

If the maximum effort a \(L\) type worker can exert is sufficiently low, \(\bar{e}_L < \frac{\bar{e}_H^2}{2\bar{e}_H(e_H + \beta) + \beta^2}\), the ordering of the critical values for \(c\) is the following,

\[^64\]The dominant strategies of the workers prevent the existence of multiple equilibria and mixed equilibria that are not analyzed in this paper. The dominant strategies are for simplicity.

\[^65\]The probability function is upward sloping and concave in \(\bar{e}\).
If $\bar{e}_L$ is sufficiently low, the marginal increase in the probability of producing a good quality product of a $H$ type worker choosing to exert effort when his teammate, another $H$ type chooses to exert effort is greater than then marginal increase in the probability of producing a good quality product of a $L$ type choosing to exert effort when his teammate does not.

If $\bar{e}_L$ falls within the range 
$$
\frac{\bar{e}_H \beta^2}{2\bar{e}_H (\bar{e}_H + \beta^2)} < \bar{e}_L < \frac{\beta^2}{2\bar{e}_H},
$$
at this moderate level of maximum effort for a $L$ type worker, the marginal increase in the probability of producing a good quality product from a $L$ type worker exerting effort when his partner does not is greater than the marginal increase from a $H$ type exerting effort while his partner does exert effort. This puts $c^1_{eL} > c^2_{eH}$. But the marginal increase from a $H$ type worker is still greater than the marginal increase in the probability from a $L$ type worker when their partners exert effort. Therefore $c^2_{eH} > c^2_{eL}$.

If $L$ type workers become more efficient to the extent that, $\bar{e}_L > \frac{\beta^2}{2\bar{e}_H}$, then the marginal increase in the probability of producing a good quality product of a $L$ type is greater than the marginal increase from a $H$ type when the partner chooses to exert effort. This puts $c^2_{eH} < c^2_{eL}$.

### 2.5 Heterogeneous Teams

This section details the conditions for workers to exert effort to be a dominant strategy when firms choose to form heterogeneous teams. If heterogeneous teams are formed, there are two identical teams both consisting of a single $H$ type worker and a single $L$ type worker. Therefore, the conditions are identical for each team. The section will proceed with the
formation of the $H$ type worker’s conditions to choose effort followed by the $L$ type worker’s conditions to choose effort.

If the $L$ type worker in the team chooses to exert $\bar{e}_L$, the condition for the $H$ type worker to choose $\bar{e}_H$ is,

$$\frac{\bar{e}_H + \bar{e}_L}{4(\bar{e}_H + \bar{e}_L + \beta)} - c \geq \frac{\bar{e}_L}{4(\bar{e}_L + \beta)} \quad (114)$$

If the $H$ type worker chooses to exert effort, the probability of producing a good quality product is $\bar{e}_H + \bar{e}_L$. This probability is divided by four for receiving a quarter of the value of a good quality product. Since the worker chooses to exert effort, he incurs a cost of $c$. For the high type worker to exert effort $\bar{e}_H$, the expected payoff (RHS) must be greater than the expected payoff of not exerting effort (LHS). The probability of producing a good quality product is given by $\frac{\bar{e}_L}{\bar{e}_L + \beta}$ if he chooses not to exert effort. Again, he receives a quarter of the value from production. If the worker does not exert effort, he incurs the cost of $c$.

Condition (114) can be rearranged as,

$$c \leq c_{HL}^{2eHL} = \frac{\bar{e}_H \beta}{4(\bar{e}_H + \bar{e}_L + \beta)(\bar{e}_L + \beta)} \quad (115)$$

If cost of effort falls below the critical value, $c_{HL}^{2eHL}$, the $H$ type worker chooses to exert effort if his $L$ type teammate is also exerting effort.

If $c$, cost of effort, is less than the marginal increase in expected payoff for the $H$ type worker from exerting effort when his partner, a $L$ type worker, is also exerting effort, the $H$ type worker exerts effort.

If the $L$ type worker chooses to not exert effort, the condition for the $H$ type worker to exert $\bar{e}_H$ is,

$$\frac{\bar{e}_H}{4(\bar{e}_H + \beta)} - c \geq 0 \quad (116)$$

If only the $H$ type worker exerts effort, the probability of producing a good quality product is $\frac{\bar{e}_H}{\bar{e}_H + \beta}$. Good quality product is valued at one and each worker receives a quarter of the value. Since the $H$ type worker chooses to exert effort, he incurs a cost of $c$. This payoff (LHS) must be greater than the payoff of not exerting effort (RHS). If the $H$ type worker chooses not to exert effort, then no one within the team is exerting effort. Therefore,
the payoff is zero.

Condition (116) is rearranged to be,

\[ c \leq c_{HL}^{eH} = \frac{\bar{e}_H}{4(\bar{e}_H + \beta)} \] (117)

If cost of effort is below a the critical value, \( c_{HL}^{eH} \), a \( H \) type worker chooses to exert effort if his \( L \) type partner does not.

Conditions (115) and (117) need to be satisfied for the \( H \) type worker to choose exerting effort as his dominant strategy.

If the \( H \) type worker chooses to exert effort, the condition for a \( L \) type worker to exert \( \bar{e}_L \) in the team is,

\[ \frac{\bar{e}_H + \bar{e}_L}{4(\bar{e}_H + \bar{e}_L + \beta)} - c \geq \frac{\bar{e}_H}{4(\bar{e}_H + \beta)} \] (118)

If both team members exert effort, the probability of a good quality product is \( \frac{\bar{e}_H + \bar{e}_L}{\bar{e}_H + \bar{e}_L + \beta} \). This is divided by four because the worker receives a quarter of the value from production and since the \( L \) type worker chooses to exert effort, he incurs a cost of \( c \). This must be greater than the expected payoff of not exerting effort (RHS). If the \( L \) type worker does not exert effort, the probability of a good quality product is \( \frac{\bar{e}_H}{\bar{e}_H + \beta} \). This is divided by four because he receives a quarter of the value from production. There is not a cost of \( c \) since the worker chooses not to exert effort.

Condition (118) is rearranged to be,

\[ c \leq c_{HL}^{2eLH} = \frac{\bar{e}_L \beta}{4(\bar{e}_H + \bar{e}_L + \beta)(\bar{e}_H + \beta)} \] (119)

If cost of effort is below the critical value, \( c_{HL}^{2eLH} \), the \( L \) type worker chooses to exert effort if his \( H \) type teammate exerts effort.

If the \( H \) type worker chooses not to exert effort, the condition for a \( L \) type worker to exert effort is,

\[ \frac{\bar{e}_L}{4(\bar{e}_L + \beta)} - c \geq 0 \] (120)

If only the \( L \) type worker exerts effort in the team, the probability of producing a good
quality product is \( \frac{\bar{e}_L}{e_L + \beta} \). This value is divided by four because he receives a quarter of the value from production. Since the \( L \) type worker exerted effort, he incurs the cost of \( c \). If the \( L \) type worker does not exert effort, then with both members of the team not exerting effort, the payoff is zero.

Condition (120) is rearranged to be,

\[
c \leq c_{HL}^{LeL} = \frac{\bar{e}_L}{4(\bar{e}_L + \beta)}
\]  

(121)

If cost of effort is below the critical value, \( c_{HL}^{LeL} \), the \( L \) type worker chooses to exert effort if his \( H \) type teammate does not exert effort.

The conditions for all types of workers to exert effort as a dominant strategy are given by (115), (117), (119), and (121).

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Table 2: Game Matrix for Heterogeneous formation of teams.

Table 2 illustrates game for heterogeneous teams.

The ordering of the critical values of \( c \) except for \( c_{HL}^{2eHL} \) is,

\[
c_{HL}^{LeH} > c_{HL}^{LeL} > c_{HL}^{2eLH}
\]  

(122)

Similar to the previous ordering of critical values for homogeneous teams, the largest marginal increase in probability of producing good quality products is choosing to exert effort when the partner does not. Therefore, the worker can incur the highest costs under these scenarios with an \( H \) member incurring higher costs. This puts \( c_{HL}^{LeH} > c_{HL}^{LeL} \) at the top.

Due to the concavity of the probability function, the marginal increase in the probability when a \( L \) type worker exerts effort is greater when the partner is not exerting effort compared
to when the partner is exerting effort. Therefore, \( c_{HL}^{2eLH} \) is the smallest.

If the maximum effort levels of the \( H \) type worker is sufficiently high relative to the \( L \) type worker’s maximum effort where \( \bar{e}_H > \frac{\bar{e}_L(\bar{e}_L + \beta)}{\beta - \bar{e}_L} \), the ordering of the critical values of \( c \) is,

\[
0 < c_{HL}^{2eLH} < c_{HL}^{2eHL} < c_{HL}^{\lambda eL} = c_{HL}^{\lambda eH}.
\]

Otherwise, \( \bar{e}_H < \frac{\bar{e}_L(\bar{e}_L + \beta)}{\beta - \bar{e}_L} \), results in the following ordering of the critical values,

\[
0 < c_{HL}^{2eHL} < c_{HL}^{\lambda eL} < c_{HL}^{2eHL} < c_{HL}^{\lambda eH}.
\]

First, similar to the explanation for \( c_{HL}^{2eLH} < c_{HL}^{\lambda eL} \), the same explanation can be applied for \( c_{HL}^{2eHL} < c_{HL}^{\lambda eL} \). Second, \( c_{HL}^{2eHL} > c_{HL}^{2eLH} \) because the marginal increase in the probability from a \( H \) worker exerting effort when his partner, a \( L \) type, is also exerting effort is always greater than a marginal increase in the probability from a \( L \) type worker exerting effort when his partner, a \( H \) type worker, is also exerting effort.

If the \( H \) type worker is sufficiently efficient relative to the \( L \) type worker, or equivalently, the \( L \) type worker becomes less efficient, the increase in expected payoff from exerting effort for the \( H \) type worker is lower compared to if the \( L \) type worker is more efficient. Therefore, for a \( H \) type worker to exert effort when the \( L \) type worker is exerting effort, the cost must fall.

### 2.5.1 Firms Choice

The firm makes its decision given the actions of the workers from the previous section. There are several scenarios which involve the ordering of the critical values of \( c \) that the firm must consider.

Figures 10 to 15 in the Appendix illustrate the best response of each team member under both composition of teams. The line labeled “\( c \)” is the cost line for the values of cost of effort. The table above the cost line represents the best response of the \( H \) and \( L \) type workers in homogeneous teams at each interval of cost of effort. The notation \( H(e_i) \) or


$L(e_i)$ refers to $H$ or $L$ being the type of worker making the decision and the variable inside the bracket represents the action of his team member ($i = H, L$). The table beneath the cost line represents the best response of the $H$ and $L$ type workers in heterogeneous teams. The row labeled “Firm” show the composition of teams the firm chooses at each interval of cost. The final row labeled “$E\pi$” shows the expected value the firm receives.

The figures illustrate the firm’s choices for team composition at different intervals of cost of effort for the workers. Due to the concavity of the probability function, the firm obtains a higher expected payoff under a heterogeneous composition compared to a homogeneous composition when the same amount of total effort is exerted (i.e., everyone exerts effort in the firm). With this modified Tulloch contest success function, the idea of “do not put all your eggs in one basket” is optimal and the firm spreads out the efforts of the workers.

When cost is really high, only $H$ type workers have incentives to exert effort. When cost of effort increases, $L$ type workers begin to choose no effort because the marginal increase in expected payoff for them is lower compared to the marginal increase in expected payoff for a $H$ type worker. Therefore, at high levels of cost when $L$ types choose not to exert effort, due to the concavity of the probability function, the firm chooses a heterogeneous composition of teams. (“Don’t put all your eggs in one basket”).

At moderate levels of cost of effort, a firm chooses to form either homogeneous or heterogeneous teams depending on the relative levels of effort between the $H$ and $L$ type workers. If the maximum effort level a $L$ type worker can exert is sufficiently small relative to $H$ type worker’s maximum effort level, $\bar{e}_L < \frac{\bar{e}_H}{3\bar{e}_H + 2\bar{e}_L}$, a firm chooses a heterogeneous composition of teams as a dominant strategy. Conditional on $\bar{e}_L$, heterogeneous composition of teams is chosen at any cost of effort. At these moderate levels of cost, all $H$ type workers choose to exert effort. Under a homogeneous composition, the $H$ type team have both members

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66 If the probability function is concave (the second derivative with respect to $\bar{e}$, is negative), it must be true that $2f(\bar{e}_H + \bar{e}_L) \geq \frac{f(2\bar{e}_H + 2\bar{e}_L)}{2}$.

67 In Figures 12 and 14, there are empty spaces in Firm and $E\pi$ because in that interval of cost of effort, the firm’s choices are undetermined. In those intervals of cost, under the heterogeneous composition of teams, there is multiple equilibria where there is a potential of two $H$ types, two $L$ types, or one of each type exerting effort. Therefore, it is not possible to determine the firm’s choice unless mixed strategies is employed which is not done in the current analysis. Mixed strategies can be considered in future editions of the current paper.

68 The inequality is obtained by comparing the expected profit of the firm under a homogeneous and heterogeneous composition and then isolating for $\bar{e}_L$. 

79
exerting effort. Under a heterogeneous composition, the $H$ type workers exert effort and the best response of the $L$ type worker is to not exert effort. Due to the concavity of the probability function, it is true that, from just the $H$ type worker’s production, a heterogeneous composition is chosen by the firm. When $\bar{e}_L < \frac{\bar{e}^2_H}{3\bar{e}H+\beta}$, the efficiency of the $L$ type worker is so low that, under a homogeneous composition, the marginal increase to production with the inclusion of the $L$ type team does not exceed the expected value of production from a heterogeneous composition of teams when only $H$ type workers exert effort.

As the $L$ type worker becomes slightly more efficient, $\frac{\bar{e}^2_H}{3\bar{e}H+\beta} < \bar{e}_L < \frac{2\bar{e}^2_H}{3\bar{e}H+\beta}$, heterogeneous composition of teams is no longer a dominant strategy. The increase in expected value produced by a team of $L$ types in a homogeneous composition of teams with everyone exerting effort exceeds the expected value from production from a heterogeneous composition where only $H$ types exert effort.

As $L$ types become more efficient, $\bar{e}_L > \frac{2\bar{e}^2_H}{3\bar{e}H+\beta}$, at the moderate levels of cost, homogeneous is chosen. The $L$ types have become so efficient that a homogeneous composition of teams with the $H$ type team and only one member of the $L$ type team has a higher expected value from production then two $H$ type workers exerting effort in a heterogeneous composition of teams.

Under certain circumstances, when cost of effort is sufficiently low and the low types are not very efficient, even though the total amount of effort exerted in the firm is greater under a homogeneous composition of teams, the firm still chooses a heterogeneous composition due to the concavity assumption of the production function. Therefore, under the right circumstances, even though a homogeneous composition incentivizes more workers to exert effort, it might not provide the firm with the higher expected payoff.

### 2.5.2 Conclusion

A firm generally chooses to form heterogeneous teams and is the consequence of the concave production function. Under a specific set of parameters (i.e., efficiency of low type is sufficiently high), homogeneous teams are chosen.
2.6 Conclusion

The objective of the paper is to provide an explanation for the choices of a firm about team composition. The model assumes that the type of workers differ in their efficiency levels. A high type worker is able to exert a higher level of effort at the same level of cost as the low type worker. The worker’s expected payoff is dependent on both his partner’s characteristics and his own. This is representative of many manual labor production processes such as manufacturing.⁶⁹

The paper tackles the idea of team composition under two different scenarios. In the first model, the firm has the ability to hire any type of workers to compose a team. This scenario is similar to a firm hiring new workers. The second model illustrates a firm that has an existing work force and firing costs are high. The firm needs to optimally organize its workforce. The second scenario is similar to an existing company that has a worker’s union. The manager must get the most out of existing resources.

In the model with hiring (first model), the results indicate that, as the relative reservation payoff of high efficiency types relative to low efficiency types and costs to incentivizing workers to exert effort increases, firm’s choice changes from homogeneous to heterogeneous team compositions. When those costs are low, the firm chooses a homogeneous team composition of high efficiency workers because they are relatively cheap to hire compared to the low efficiency types and the cost to induce effort is the same. As high efficiency types become more expensive to hire, firms hire less of them. This creates a heterogeneous team of workers. Eventually, high efficiency types are so expensive relative to the low efficiency types that the firm hires only low efficiency types, creating a homogeneous composition.

If a firm hires a homogeneous team of high efficiency types and the cost of effort increases making incentivizing effort more expensive, the firm changes its choice from a homogeneous team of high efficiency types to a heterogeneous team. Since incentivizing effort is expensive, the firm only incentivizes one team member, high efficiency type, and fills the extra position with cheap low efficiency labour.

⁶⁹Consider a factory assembly line where the first worker builds a certain part of the final product and passes on the part to the next worker that completes another part of the final good. The workers are working in a team but the quality of the good produced is dependent on the effort each inputs into the production process. The first worker does not teach the second about the production process.
If a firm hires a homogeneous team of low efficiency types that both exert effort and the cost of effort increases, the firm changes its choice from a homogeneous team of low efficiency types to a heterogeneous team and as cost continue to increase move to a homogeneous team of low efficiency types. As cost of effort increases, the firm incentivizes less effort. At moderate levels of cost of effort, the firm shifts expenses to hiring one $H$ type and one $L$ type and incentivizing the $H$ type to exert effort. The cost is small enough to spend money to hire a $H$ type to replace a $L$ type in the team and incentivize effort. Further increasing costs makes hiring a $H$ type not profitable. The firm hires a homogeneous team of low efficiency types and incentivizes one to exert effort.

The model without hiring (second model) illustrates the firm’s choices depending on the cost of effort of the workers. Similar to the model with hiring, the firm in the model without hiring composes a team to maximize expected profits and worker types are different in their efficiency levels. The difference is the firms offer a fixed share of the value of output as incentive for effort. The second model focuses on the allocation problem a firm may face ignoring the hiring problem from the first model. The results indicate that a heterogeneous combination of different efficiency type workers is chosen for the most part and is the consequence of the concave production function. If the two types differ significantly in efficiency levels, a heterogeneous composition of teams in the workforce is always chosen. At moderate levels of cost of effort, if efficiency of the low efficiency type increases beyond a critical level, a homogeneous composition of teams is chosen because the marginal increase in expected value from production is increasing for low efficiency individuals.

Even if a homogeneous composition of workers in teams incentivizes more effort in total from workers, it might still be more profitable to form heterogeneous teams despite less total effort being exerted.
### 2.7 Appendix

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Figure 10: Firm choice for formation of teams #1

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83
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| Homogeneous   | $0$ | $c_{H}^{HH}e_H$ | $c_{L}^{HH}e_L$ | $c_{H}^{HL}$ | $c_{L}^{HL}$ | $c_{H}^{LL}$ | $c_{L}^{LL}$ |
| Heterogeneous | $\bar{e}_H$ | $\bar{e}_H$ | $\bar{e}_H$ | $\bar{e}_H$ | 0 | 0 |
| $0$           | 0 | 0 | 0 | 0 |

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Figure 11: Firm choice for formation of teams #2
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**Homogeneous**

* Firm: $e_L < \frac{2e_H}{\mu + \beta}$

* Ex: $\frac{2(e_H + e_L)}{\mu + e_L + \beta}$

**Heterogeneous**

* Firm: $e_L > \frac{2e_H}{\mu + \beta}$

* Ex: $\frac{2e_H}{\mu + \beta}$

Figure 12: Firm choice for formation of teams #3
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$E_n \begin{cases} \frac{2(\delta_{e_H} + \delta_{e_L})}{\gamma_{H} + \gamma_{L}} & \text{Homogeneous} \\ \frac{2\gamma_{H}}{\gamma_{H} + \gamma_{L}} & \text{Heterogeneous} \end{cases}$

$E_n \begin{cases} \frac{2\gamma_{H}}{\gamma_{H} + \gamma_{L}} & \text{Homogeneous} \\ \frac{2\gamma_{H}}{\gamma_{H} + \gamma_{L}} & \text{Heterogeneous} \end{cases}$

Figure 13: Firm choice for formation of teams #4
\[\begin{array}{|c|c|c|c|c|c|}
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\hline
H(e_H) & e_H & e_H & 0 & 0 & 0 \\
\hline
L(0) & e_L & e_L & e_L & e_L & 0 \\
\hline
L(e_L) & e_L & 0 & 0 & 0 & 0 \\
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Figure 14: Firm choice for formation of teams #5
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Figure 15: Firm choice for formation of teams #6
Chapter 3

3 The Impact of Managerial Wage Premiums on Overtime Participation

3.1 Introduction

The goal of this article is to analyze the impact of financial incentives from a promotion on an individual’s overtime participation. The results indicate that there is a positive and significant impact for individuals on the lower end of the hourly wage distribution. A $1 per hour increase in the hourly wage of an individual in the lower end of the wage distribution increases the probability the individual works overtime hours by approximately 4% or 84 hours above the regular work hours per year.

In many organizations or firms, there exist incentive schemes that are designed to induce effort which, in turn, increases performance or profits. The design of these incentive schemes or contracts has become a major research field in economics. Economists have tackled this problem with sophisticated models where the aim is to find the optimal contract to induce effort that is unobservable. Problems of moral hazard and adverse selection have been thoroughly analyzed with many different methods. Starting with the work of James Mirrlees (1975), subsequent researchers have expanded his work to apply to many different scenarios. In reality, these theoretical models may be too hard or impractical to implement.

For an individual to exert effort for an organization, he must be incentivized to choose

this costly action. If the effort is unobserved or unverifiable and the incentives are not fully understood, the contract or incentives may be flawed. The goal of this article is to investigate a specific incentive scheme of organizations to induce effort that is not directly related to an individual’s contract; namely promotions. In an organizational structure, assessing an individual’s potential for a promotion is often based on their actions in the current job. Is the financial incentives from a promotion for an individual enough to incentivize greater effort?\footnote{“Promotions serve two roles in an organization. First, they help assign people to the roles where they can best contribute to the organization’s performance. Second, promotions serve as incentives and rewards.” (Milgrom and Roberts (1992, p.364))}

According to the Panel Study of Income Dynamics survey, approximately 72\% of individuals in managerial positions between the years 1999 to 2013 reported that they worked more than the regulated normal work hours in a previous non-managerial position. This suggests that a common characteristic for individuals to be in a managerial position is to participate in overtime. Overtime participation potentially increases an individual’s probability of a promotion.

There has been research in the past that have found a positive correlation between an individual’s overtime hours and the probability that he is promoted. Anger (2005) and Pfeifer (2010) found that more overtime was positively correlated with a higher probability of promotion in Germany. Pannenberg (2002) found that individuals in Germany with overtime experience, on average, had a 10 percentage point increase in real labor earnings. Bell and Freeman (2001) concluded that a similar correlation exists in the United States; there is a positive correlation between the average hours worked and the worker’s perceived likelihood of promotion. Therefore, if participation in overtime does have a positive impact on probability of promotion and the promotion includes a financial incentive, a promotion should act as an incentive for workers to exert more effort. The analysis in this article looks at the financial incentives from a promotion and its impact on the individual’s choice for overtime participation and the magnitude of that participation (i.e., probability of participation and number of overtime hours).\footnote{Chapter 1 of the thesis in which this article is a part of uses promotions as an incentive scheme for its workers to exert effort and produces better quality output for a firm. In addition to providing an incentive to its workers to exert costly effort, the promotion also acts as an imperfect screening mechanism for the firm to only promote high skilled individuals into managerial roles.}
There exists a small empirical literature looking at promotions as an incentive for worker effort but they do not focus on the exact returns those incentives provide. However, in the context of their respective articles, they do document a positive correlation between promotions and effort. Booth et al. (2002b) examined whether temporary contracts are “stepping stones” to permanent positions. The authors found that, in the United Kingdom, a large proportion of workers on temporary contracts move to permanent contracts with higher wages and better benefits. They showed that high effort among those temporary workers is positively correlated with the probability of career advancement. Similarly, Bell and Freeman (2001) found that, in Germany and the United States, there is a positive correlation between the average hours worked and the worker’s perceived likelihood of a promotion. The articles support the assumption that exerted effort is positively correlated to the probability of promotion. This analysis looks at the impact of promotions on effort and how much effort, in terms of overtime participation, can be induced from an increase in the financial incentive of the promotion. A $1 per hour increase in the hourly wage of an individual in the lower end of the wage distribution increases the probability the individual works overtime hours by approximately 4% or 84 hours above the regular work hours per year. On average, a promotion increases an individual’s hourly wage by $9.22 which increases an individual’s likelihood to work overtime by 37% or 756 additional hours above regular work hours per year (approximately 15 hours per week).

Consider an individual working at a firm or organization with a hierarchical structure with managers and workers. The individual works as a worker; a low level position in the firm. Managers are high level positions in a firm and is expected to command a higher income compared to a worker. The individual is assumed to be motivated by money to exert effort. He sees the potential increase in income from a promotion by his managers’ income. The monetary incentive creates the motivation for the individual to exert effort,

\footnote{Engellandt and Riphahn (2005) studied the effects of temporary and fixed term contracts on the probability an employee works unpaid overtime. The results suggest that temporary workers provide significantly more effort. They have a 60 percent higher probability of working unpaid overtime than employees with permanent contracts.}

\footnote{At the time of this article, to the best of the author’s knowledge, there are not studies investigating the impact of an expected increase in hourly wage from a promotion on overtime participation.}

\footnote{According to the summary statistics of this article, Table 4, the average manager can make approximately double the hourly wage of non-managerial workers. This statistic was calculated using data provided by the Bureau of Labor Statistics (BLS).}
such as overtime participation, and increase his probability of a promotion.

The panel dataset used in this analysis is the PSID (Panel Study of Income Dynamics) and it follows a given sample of individuals over time. It contains a wide variety of information covering topics such as employment, income, wealth, education, and other demographic characteristics that are important to this analysis. Common problems when employing panel datasets relating to labour statistics is the endogeneity problems that arise, especially when estimating labour supply with hours worked and wages. The analysis uses an expected increase in income as an incentive to participate in overtime. As there is a potential that overtime participation might conversely impact expected future income, a reverse causality problem arises. In most organizations, there already exists an incentive scheme for individuals to participate in overtime; overtime pay schedules. Employees are given a percentage, usually 1.5 or double, of their hourly wage when overtime hours are performed. This increase in income is an incentive for individuals to participate in overtime and increase productivity. Participating in overtime, exerting effort, potentially increases the probability that an individual is promoted, which increases the expected payoffs from a promotion. This creates a relationship that implies overtime participation impacts the expected payoff of individuals from a promotion; opposite of the relationship this analysis is searching for. The causality problem between overtime participation and expected payoff from a promotion is introduced. There also exists individual unobserved characteristics (i.e. ability) that might play an important role in the decision of overtime participation. Finally, there might exist a tendency for individuals to participate in overtime regularly. To account for this, lagged variables of overtime are used might not be strictly an exogenous variable.

Several approaches have been developed to account for these issues; causality problem between overtime participation and expected payoff, autocorrelation of variables, and unobserved heterogeneity of individuals. In this article, the AB estimator is employed because it accounts for issues apparent in this dataset. It is a generalized method of moments estimator used to estimate panel data models.

76 According to the Fair Labor Standards Act of the United States, they establish minimum wage, overtime pay, record keeping, and youth employment standards affecting employees in the private sector and in Federal, State, and local government. Overtime pay at a rate not less than one and one-half times the regular rate of pay is required after 40 hours of work in a workweek. United States Department of Labor, Wage and Hour Division (WHD). [https://www.dol.gov/whd/flsa/].
There are several measurements of effort. The measurements used in this article are related to overtime participation, specifically, the likelihood of participating in overtime and the number of hours worked in overtime. It is assumed that if an individual put in overtime compared to someone who does not, the individual participating in overtime is exerting more effort than the individual who does not.

The incentives of a promotion, in this article, is financial. Assume a worker wants to be promoted to manager. The incentives that attract the worker to a promotion is the increase in hourly wages. A good expectation of the wage he receives as a manager is the wage of his direct manager or managers in the same industry. Therefore, the expectation of his hourly wage, if he was promoted, is the average managerial wage of all the managers in his job’s industry. The incentives of a promotion is measured as the difference in the average managerial wage in the worker’s industry and his current wage; the expected increase in hourly wage earned from a promotion. As this difference increases, financial incentives from a promotion increases impacting the overtime participation decision of the worker.

3.2 Data

The analysis uses the Panel Study of Income Dynamics (PSID). The study began in 1968 with a nationally representative sample of over 18,000 individuals living in 5,000 families in the United States. The dataset is the longest running longitudinal household survey in the world. It contains information on families that span many years covering topics such as employment, income, wealth, education, and other demographic characteristics that are important to the analysis in this article. The survey is usually conducted every other year asking questions regarding many different topics for the previous year. A table of all

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77 Many different measures of effort have been used. Borjas (1980) used weekly hours of work. Engellandt and Riphahn (2003) used unpaid overtime hours. Altonji (2005) and Fakih (2014) used work hours and vacation leave. Bell and Freeman (2001) used work hours in Germany and the United States.

78 The work experience obtained from a specific position can be applied to other jobs in a similar industry or occupation. i.e., A worker can apply for a managerial position in a different firm. A worker in a car manufacturing company can apply to be a manager in any mechanical manufacturing firm.

79 On average, non-manager individuals reported total hours per year of 1956 and managers reported 2360. The average hourly wage of non-managers and managers is $11.09 and $19.70, respectively. The analysis is done using hourly wage and does not account for the difficulty of tasks between non-managers and managers.

80 Panel Study of Income Dynamics, public use dataset. Produced and distributed by the Survey Research Center, Institute for Social Research, University of Michigan, Ann Arbor, MI (2015).

81 i.e., The survey conducted in 2001, the wage data is collected from survey question ER20425 which is the individual’s income in 2000. All income is self reported.
the variables and their descriptions used in this analysis is presented in Table 3.

The dataset used in this article contains surveys from 1999 to 2013. There are 8 years of survey data. Subjects responded to surveys every other year and not all subjects responded every year.

The choice to use the PSID is because it contains detailed personal information on individuals (job related questions and demographics) that are important to this analysis. Many questions regarding the individual’s employment were asked which are very useful. Questions relating to the number of hours worked or their wages were asked each survey year. The questions regarding wages is also important as it is used in the construction of the variable related to promotions. The other detailed demographic variables will help control for any effects that are not related to promotion specifically.

3.2.1 Effort

The variables for effort chosen for this analysis are related to overtime hours worked. The first variable is in regards to whether the individual participated overtime during the year, $OT$. It is assumed that if a worker puts in overtime, he is exerting effort compared to someone that does not. Overtime may potentially be a criteria for promotion. The PSID survey asked the individual if they worked any overtime hours. The variable, $OT$, is a dummy variable that takes on a value of 1 if the individual reported that they participated in overtime in the survey year and 0 if they did not participate in overtime. If the individual reported that they did not know or did not work in the survey year, they were dropped from the sample. The results of the analysis with the variable $OT$ indicates the likelihood the individual will participate in overtime if the financial incentive of becoming manager was increased. Table 4 reports the average response of individuals for this variable. App-

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82The time period chosen is due to the availability of some variables in the dataset which is needed for this analysis. i.e., The availability of data for individual’s regarding their pensions and wage information was only provided starting 1997 and 1999, respectively.

83Most individuals provided responses to a subset of the 8 years.

84The PSID survey is conducted with household heads which are mostly males. Future analysis could be performed with spousal information which would incorporate more women into the sample.

85The variables used for effort have been used by other researchers in other studies such as Bell and Freeman (2001), Engellandt and Riphahn (2003), Antonji (2005), and Fakih (2014) and many more.

86Individuals that reported they are self employed are also dropped from the sample because there are no promotions for self employed individuals. Therefore, they are not impacted by a financial incentive from a promotion to participate in overtime.
approximately 22% to 24% of the sample reported they participated in overtime in the survey year.\textsuperscript{87}

A second variable for effort is the number of overtime hours performed during the year, $OTHOURS$. The PSID survey asked the individual the total number of overtime hours worked reported separately from regular hours. The value of this variable is the number the individual reported in the survey for the given year. Any individual who reported they did not know or did not work in the survey year were dropped from the sample.\textsuperscript{88} Table 4 reports the average statistics for the variable $OTHOURS$. The average number of annual hours of overtime per individual is approximately 25 to 43 hours depending on the year. If the average is calculated conditional on the individual participating in overtime, the average increase to approximately 121 to 231 annual hours.\textsuperscript{89}

In addition to the variables described above, the analysis includes estimates of the variables for $OT$ and $OTHOURS$ called $OTEST$ and $OTHOURSEST$, respectively. The first variable, $OTEST$, is constructed similarly to the original $OT$ variable. The variable takes on the value of 1 if they worked overtime and 0 if they have not worked overtime during the year. The criteria to determine the value is different. Under the Fair Labor Standards Act (FLSA), unless exempt, employees covered by the Act must receive overtime pay for hours worked over 40 in a workweek.\textsuperscript{90} Therefore, any individual reporting total hours greater than 2000 are assumed to be putting in overtime that year.\textsuperscript{91} On average, the value of $OTEST$ is between 54% and 73%. This is much greater than $OT$ where the highest average is approximately 43%. This shows that, under the criteria of overtime from FLSA, individuals underestimate the number of overtime hours they perform. It may also be possible that individuals have a different definition for overtime compared to the legal definition. An individual working hours beyond the typical work day might not consider

\textsuperscript{87}In the year 2009, there was an unusually high number if individuals that reported they participated in overtime compared to the other years. Given that 2009 was the year of the global recession, it can be assumed more people participated in overtime in 2009 to earn extra income due to the suffering economy.

\textsuperscript{88}Any missing values were assigned by PSID.

\textsuperscript{89}This statistic is not shown in Table 4. The averages reported in Table 3 are the unconditional averages for $OTHOURS$.

\textsuperscript{90}United States Department of Labor Wage and Hour Division (WHD), [http://www.dol.gov/whd/overtime_pay.htm].

\textsuperscript{91}The 2000 annual work hours is calculated by taking a 40 hour work week and multiplying 50 work weeks. It is also assumed a work year contains 50 work weeks with 2 vacation weeks.
the time as overtime because the task they were assigned is incomplete. He considers his job as task oriented rather than time oriented ("typical 9 to 5 job"). For completeness, an analysis will be conducted on the estimate variable.\textsuperscript{92}

The variable $\text{OTHOURSEST}$ is constructed by taking the difference between total hours reported by the individual and 2000 (the estimated annual work hours). The resulting value is $\text{OTHOURSEST}$. This number is an estimate of the total number of overtime hours worked by the individual in their place of work according to the criteria for overtime from the FLSA. The statistics of this variable is presented in Table 4. Again, similar to the scenario above with $\text{OT}$ and $\text{OTEST}$, the value of this estimate is much greater than the value that is self reported by the individuals. The average value of this variable is approximately between 226 and 337 overtime hours per year depending on the year. This is much higher than the self reported values with the highest average being approximately 43 hours per year. The estimated averages for overtime hours per year translates to approximately 4.5 to 6.75 hours per week assuming a year contains 50 work weeks.

### 3.2.2 Promotion

The variable $P$ is the variable for the expected increase in hourly wage an individual expects to earn if they are promoted into a managerial position in the same industry as his current job. This variable is constructed by taking the difference between average managerial wage in the individual’s current industry and their hourly wage.

$$P_{i,t} = \overline{\text{MWage}_{i,IND,t}} - \text{Wage}_{i,t}$$ \hfill (123)

The expected managerial wage was obtained from the United States Department of Labor, Bureau of Labor Statistics (BLS).\textsuperscript{93} In the survey for PSID, each individual was asked for an industry code of their current place of work. This code corresponded with the industry codes used by BLS. In the BLS dataset, the average wage of a manager in each industry was reported each year. The reported average manager wage is $\overline{\text{MWage}_{IND,t}}$

\textsuperscript{92}There exists a possibility that employers do not comply with the FLSA. This cannot be proven in the current analysis.


96
for each individual in each year in equation (123). The individual’s hourly wage for his current place of work was obtained from PSID where the survey asked each individual every year. The value for the individual’s current wage obtained from PSID is $Wage_t$ in equation (123). The difference was taken to obtain the variable $P_{i,t}$ for each individual in each year.

The average statistics for $Mwage$ and $Wage$ are reported in Table 4. All wages (average managerial and current) are in real values. The average real wage of an individual in the sample is $11.76 to $12.58 per hour depending on the year. The average of the average managerial wage in the same industry is $16.98 to $22.85 per hour. Therefore, an individual can expect an increase of approximately $5 to $12 dollars per hour if they are promoted to a managerial position. This translates to approximately an increase in annual income of $10,000 to $24,000. This significant increase in real values may be a big incentive to work harder to achieve a promotion.

Table 6 looks at the same statistics for the variables in Table 4 but the sample is split between individuals that make an hourly wage or “Salary & Other”. This table illustrates that the overtime participation statistics are larger for individuals with an hourly wage which can be a sign of under reporting of overtime hours for individuals making a salary. An individual with specific times that he needs to be at work can state any hours beyond the requirement as overtime hours. An individual with a salary might have a more task oriented occupation. His regular work hours are not finished until the task is finished. Therefore, overtime hours considered by hourly wage individuals are not overtime hours.

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94 Prior to 2001, the 3-digit industry code from 1970 Census of Population; Alphabetical Index of Industries and Occupations issued June 1971 by the U.S. Department of Commerce and the Bureau of Census was used for this variable. Please refer to Appendix V2, Wave XIV (1981) documentation, for complete listings. After 2001, the 3-digit industry code from 2000 CENSUS OF POPULATION AND HOUSING: ALPHABETICAL INDEX OF INDUSTRIES AND OCCUPATIONS issued by the U.S. Department of Commerce and the Bureau of the Census was used for this variable. Please refer to www.census.gov/hhes/www/ioindex/ioindex.html for complete listings.

95 Any missing values were assigned by PSID.

96 Due to availability of data from BLS, the sample used in this analysis only runs from 1999 to 2013.

97 The average managerial wages obtained from BLS are much higher compared to other findings. It is possible that the BLS measure reflects factors that are not related to a promotion. Baker, Gibbs, and Holmstrom found that in the financial firm in their study, the average increase in wages from a promotion is much smaller than the difference calculated in this paper. On average, they found that a promotion means an immediate 6% increase in pay. In this paper’s dataset, that translates to approximately an increase of $0.75 per hour. It is possible that managers are under reporting work hours and therefore lead to higher average managerial wages. Other factors such as tenure in the job, education level, and unobserved ability are likely to drive the average differential in wages between the manager’s wage and current wage.
for a salary individual. Also, it is illustrated that the difference in the averages between the current wage, \( W_{\text{age}} \), and average managerial wage, \( M_{W_{\text{age}}} \) is much greater for hourly compared to salary individuals. It is possible that the substitution effect is dominant for hourly wage individuals while income effect is dominant for salary wage individuals.

The analysis assumes that individuals are working legally and are paid accordingly. Any evidence to suggest otherwise, the individual is dropped from the sample. Throughout the duration of the sample used in this analysis, the minimum wage in the United States changed several times. Any individual who reported a nominal wage (or had an assigned wage from PSID) lower than the federal minimum wage was dropped from the sample. On October 1, 1996, the minimum wage increased from $4.25 to $4.75. In 1997, the minimum wage increased to $5.15. The minimum wage increased to $5.85, $6.55, and $7.25 in 2007, 2008, and 2009, respectively. For an individual who is not working legally (i.e., being paid below minimum wage), there exists other possible reasons for working overtime. Also, the organization that employs the individual is possibly not complying with FLSA regulations. To minimize the distortions of these explanations, individuals assumed to be working illegally are dropped from the analysis.

The variables described so far only apply to individuals who reported an occupation that does not fall into the category of management occupations. Since the analysis looks at the effect of a promotion to manager, the individuals who already have managerial positions do not apply.

Table 5 provides promotion statistics regarding individuals in the sample. The first column illustrates that working more hours in a previous non-managerial position is a common characteristic among individuals in a managerial position. Therefore, working more than 2000 hours a year increases an individual’s probability of being promoted to a managerial role. On average, approximately 73% of managers, according to PSID, reported that in a previous period, if they were not in a manager occupation, put in more than 2000 hours of work a year. The second column looks at the percentage of managers that reported overtime in a previous non-managerial occupation in the previous period. The

\[98\text{The historical federal minimum wage under the FLSA was obtained from the United States Department of Labor, Wage and Hour Division. [http://www.dol.gov/whd/minwage/chart.htm].}\]
last two columns look at the percentage of individuals in non-managerial occupations that participated in overtime and are promoted in the next period.\footnote{A promotion in the data is assumed to be a change from a non-managerial occupation in one period to a managerial occupation next period.}

### 3.3 Estimation Methodologies

To investigate the impact of a promotion on individual overtime participation, the Arellano-Bond (AB) estimator is used for the model presented in the following equation (124),

\[
Y_{i,t} = \alpha_{i,t} + \gamma Y_{i,t-1} + \beta_1 P_{i,t} + \beta_2 W_{i,t} + \beta_3 X_{i,t} + \rho_i + \epsilon_{i,t} \tag{124}
\]

The variable \( Y \) denotes the measures of effort (\( OT, OTEST, OTHOURS, \) or \( OTHOURSEST \)); \( P \) is the expected financial incentive of a promotion; \( W \) is the current overtime pay schedule of the individual; \( X \) is a collection of individual characteristics such as demographic information; \( \rho \) is an individual fixed effect; and \( \epsilon \) accounts for other unobservables. The focus of this analysis is on \( \beta_1 \) that measures the impact of the financial incentive associated with a promotion on effort. The coefficient \( \beta_1 \) can be interpreted as the increase in likelihood of participating in overtime or increase in the number of overtime hours for each $1 increase in the differential between average managerial wage and current wage. It can be expected that individuals respond positively with effort if there is an increase in wage from a promotion relative to their current wage; the coefficient \( \beta_1 \) should be positive.

Several problems may arise with the estimation of equation (124) which the AB estimator is designed to cope with. The variable, \( P \), for the financial incentives of a promotion may be endogenous because the causality may run in both directions. Many individuals working for firms or organizations have an overtime pay schedule which dictates the percentage increase in hourly wages or income for overtime hours performed. This creates a direct monetary incentive to participate in overtime. If the participation in overtime impacts the probability of a promotion, it also impacts the expectation the individual has for the expected increase in hourly wages from a promotion. This explains a causality from overtime participation to expected financial incentives from a promotion. The analysis in this article focuses on
the impact of the expected financial incentives from a promotion on overtime participation. Assuming an increase in the hourly wage from a promotion, an individual should exert effort, overtime participation, to increase his likelihood of a promotion. Therefore, the causality analyzed in this article is the impact of \( P_{i,t} \) on \( Y_{i,t} \).

Unobserved time-invariant individual characteristics, such as ability, are represented by \( \rho \). It is assumed the time-invariant unobserved component is related to the individual characteristics. When unobservables and observables are correlated, there exists an endogeneity problem.

The inclusion of \( Y_{i,t-1} \) captures the persistence on overtime participation (i.e., the tendency for individuals to participate in the same amount of overtime every year) but creates another endogeneity problem. The variable \( Y_{i,t-1} \) is correlated with \( \epsilon_{i,t-1} \) by construction. This means that \( Y_{i,t-1} \) is not a strictly and sequentially exogenous explanatory variable. Estimation without accounting for this results in an inconsistent estimate, \( \beta_1 \).

The AB estimator is employed to cope with the endogeneity problems described previously. It uses first-differencing to transform equation (124). By transforming the regressors, the fixed individual specific effects are controlled for.

\[
\Delta Y_{i,t} = \Delta \alpha_{i,t} + \gamma \Delta Y_{i,t-1} + \beta_1 \Delta P_{i,t} + \beta_2 \Delta W_{i,t} + \beta_3 \Delta X_{i,t} + \Delta \epsilon_{i,t} \quad (125)
\]

The AB estimator is preferred over fixed effect model because it also accounts for the endogeneity in \( Y_{i,t-1} \) and \( P_{i,t} \). After taking the first-difference, equation (125), the AB estimator uses the lagged levels of \( Y_{i,t-1} \) and \( P_{i,t} \) as instruments for \( \Delta Y_{i,t-1} \) and \( \Delta P_{i,t} \), respectively. The methodology allows one to use all further lags as instruments. The second lags are used in this analysis because of the short time dimension in the panel.\(^\text{100}\)

Finally, the AB estimator is designed for dynamic panel data sets with “small \( T \) and large \( N \)”. The data set used in this article contains eight years of surveys \( (T = 8) \) and approximately 20,000 individuals \( (N > 20,000) \).

\(^{100}\)In the absence of autocorrelation in the original residual, \( \epsilon_{i,t} \), the second lag of the independent variables, \( Y_{i,t-1} \) and \( P_{i,t-2} \), are uncorrelated with the differenced error term. Under the assumption of no further autocorrelation in \( \epsilon_{i,t} \), all further lags of \( Y_{i,t-1} \) and \( P_{i,t} \) are uncorrelated with \( \Delta \epsilon_{i,t} \). Therefore, the second lags and all further lags are suitable instruments for the differenced values in the differenced equation to obtain consistent estimates.
3.4 Results

3.4.1 Arellano-Bond (AB) Estimator

The results from the Arellano-Bond estimator are reported in Tables 7, 8, and 9. The results indicate that, overall, there is no impact of promotions on an individual’s overtime participation but it plays the largest role in an individual’s overtime participation decisions if they are in the lower third of the hourly wage distribution.

Table 7 reports the results of the study sample. The coefficients on $P$ indicate that there is no significant impact of promotions on overtime participation decisions using individual reported values or estimated values. There is a small impact of the increase in expected financial incentives from a promotion on the reported value of the probability of overtime participation, $OT$. On average, a $1 increase in the wage differential per hour between their current wage and the expected managerial wage increases an individual’s likelihood of participating in overtime by 0.2%. Overtime participation seems to be persistent, on average, individuals participate in more overtime compared to the previous period. An individual is 6% to 13% more likely to participate in overtime compared to the previous period if they participated in overtime in the previous period and participate in 5 to 17 more hours for every 100 hours of overtime in the previous period.

Tables 8 and 9 report the results from an analysis of the study sample by tertiles of hourly wage. The study sample is separated into three tertiles by hourly wage. Q1 is the bottom third of the distribution and Q3 is the upper third. The results indicate, for every dollar increase in the wage differential for a promotion, individuals at the bottom third of the hourly wage distribution have the largest impact. Except for the coefficient on $P$ when the dependent variable is $OT$, there is a positive and significant impact of the financial incentives from a promotion on overtime participation for the lowest hourly wage individuals. In column 4 of Table 8, using the estimated value of overtime, an individual in the lowest tertile of the wage distribution are 4% more likely to participate in overtime if there is a $1 increase in the expected hourly wage from a promotion. Looking at the other coefficients of $P$ from Table 9, the coefficients are 9.21 and 84.45 for the dependent variable $OTHOURS$ and $OTHOURSEST$, respectively. Using reported values of overtime.
hours from individuals, it is estimated that, on average, an individual increases his annual number of overtime hours by 9.2 for a $1 increase in the wage differential between expected managerial wage and current income. Using estimated values, the coefficient increases to approximately 84.5 hours. That translates to approximately two additional hours of work per week.

The difference in coefficients using reported and estimated values is also a signal that individuals are under reporting overtime hours. An explanation could be that individuals believe the work day ends when the task is finished rather than follow a hourly schedule. Therefore, if an individual did not finish their assigned task within a typical 8 hour workday, he does not report the extra hours as overtime because the task is incomplete.

The persistence of overtime participation is also positive and significant among all wage groups.

Approximately, 60% of individuals in the low hourly wage tertile are paid an hourly wage. On the other hand, approximately 79% of individuals in the highest hourly wage tertile are paid a yearly salary or other methods. An hourly job usually has a predetermined hour limit in a day; a typical “9 to 5” job. Any hours beyond the required hours is considered overtime. Any promotions is probably to another hourly wage job but with a higher hourly wage. An individual with a salary job might consider the work day as finished when the assigned task is complete. If promoted, the individual is most likely promoted to another salary job. If the managerial position requires more effort and he believes the compensation is not high enough, a promotion is not an incentive to work harder. Therefore, promotions do not play a role in the decision regarding overtime participation.101

The low income hourly wage individuals, on average, have an expected increase in income from a promotion of $15 per hour while high income individuals have an expected increase of $7 per hour (assuming the wage differential is positive). An individual from the bottom of the hourly wage distribution has more incentive to get a promotion because of the wage increase. An increase of approximately $15 per hour translates to $30,000 per year

101 Individuals in the low hourly wage tertile are in more physically demanding occupations (Sales, Office and Administrative Support, Transportation and Material Moving) while individuals in the high income tertile are in more intellectually demanding occupations (Education, Training, and Library, Health care Practitioners and Technical, and Architecture and Engineering).
(assuming 8 hour work days, 5 work days per week, and 50 work weeks per year). This $30,000 increase income for a low wage individual is very large relative to expenses compared to a high income individual that can increase his income by $14,000 per year. This provides a possible explanation in the significance observed in the results between the hourly wage tertiles.

3.5 Sensitivity Analysis

3.5.1 Fixed Effect (FE)

Table 10 reports the results from the Fixed Effects (FE) model. The results are not similar to the results reported using the AB estimator. The positive and significant coefficients appear using the estimated values of overtime participation while the reported values are not significant. This indicates that the possibility of endogeneity created from the causality between expected increase in financial incentives from a promotion and overtime participation and the lagged value of overtime participation does impact the results of the variable of interest, $P$.

The difference from the FE model also appears in the lagged variable of overtime participation, $Y_{t-1}$. The coefficients are negative and significant for all measures of overtime participation used in this analysis compared to positive and significant reported using the AB estimator. The endogeneity of the financial incentives of overtime might have an impact in estimating $Y_{t-1}$ on $Y_t$.

3.5.2 Ordinary Least Squares (OLS)

Table 11 reports the results from the OLS regression. The results indicate there is a positive persistence in overtime participation which is similar to the results from the AB estimator although the magnitude is approximately two and a half times of the AB results. If an individual participated in overtime in the previous period, there is a positive likelihood the individual continues to participate in the same amount of overtime.

The difference appears in the significance of the variable $P$'s impact on overtime participation. There is a positive and significant impact from all the measures of overtime.
participation. The magnitudes are small but positive and significant. From the reported value of overtime participation $OT$, the results show positive and significance similar to the AB estimator but with a much smaller magnitude. The results also indicate that the endogeneity issues do have an impact on the estimation of promotional incentives on overtime participation.

### 3.6 Conclusion

The goal of the article is to examine the effects of a specific incentive scheme, promotions, on individual efforts in the workplace. It examines the effects of an expected increase in income from a promotion on overtime participation. The incentive is an increase in expected hourly wage from a promotion. It is measured by taking the difference between the average managerial wage in the individual’s industry and his current hourly wage. The measure of effort is the overtime participation which is measured by the likelihood of individuals participating in overtime or the total number of overtime hours individuals perform. It is hypothesized that, the expected increase in income should have a positive impact on an individual’s overtime participation.

This analysis found that, on average, there is no impact of promotions on overtime participation among individuals. When the analysis is performed by different wage groups, the results indicate that there is a positive impact of financial incentives from a promotion on the lowest tertile of the wage distribution. With low hourly wage individuals being in hourly wage occupations and having larger expected wage differentials, there are more incentives for low wage individuals to participate in overtime (60% of individuals in the lower third tertile of the wage distribution earn an hourly wage). Low wage individuals are also in occupations where service or physicality is required. Therefore, a promotion may be to a less physically demanding occupation with a higher hourly wage. On the other hand, high hourly wage individuals can see the increase in income as insignificant compared to their current income. Therefore, they will choose not to work more hours to increase their chances of being promoted. The extra effort for a promotion is less enticing. High hourly wage individuals are mostly earning yearly salaries or other forms of payments. Therefore, their occupations may be more task oriented compared to time oriented (78% of individuals
in the highest tertile of the wage distribution earns a salary or other form of payment). The hours worked beyond a “typical 9 to 5” may not be reported. Their hours could be more understated compared to low hourly wage individuals. Finally, salaried individuals may not prefer the promotion because the promotion could possibly mean more work and the increase in hourly wage is not enough to compensate for the extra effort.

Also, the fact that the financial incentives from a promotion are only important to low wage individuals suggests that the main driver of overtime participation is the substitution effect. The large potential increase in income attracts the low wage individuals to exert more effort to increase their chances of a promotion. For high wage individuals, the extra income does not compensate the cost they incur from exerting more effort. Therefore, for high wage individuals, the income effect is dominant.

The analysis provides a different perspective for firms and organizations to increase employee productivity by overtime participation through the possibility of promotions. The results indicate that this could be a viable option but only on low wage individuals.
### 3.7 Appendix

Table 3: List of variables and descriptions.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>OT</td>
<td>Whether the individual participated in overtime at the place of work (self reported).</td>
</tr>
<tr>
<td>OTHOURS</td>
<td>The annual overtime hours worked at the reported occupation separate from regular work hours (self reported).</td>
</tr>
<tr>
<td>OTEST</td>
<td>Whether the individual participated overtime at the place of work (estimated). If the TOThOURS exceeded 2000, the individual is assumed to have worked overtime during the year.</td>
</tr>
<tr>
<td>OTHOURSEST</td>
<td>The annual overtime hours worked at the reported occupation separate from regular work hours (estimated). The total hours exceeding 2000 is assumed to be overtime hours.</td>
</tr>
<tr>
<td>Wage</td>
<td>The hourly wage of the individual. All missing data were assigned by PSID.</td>
</tr>
<tr>
<td>MWage</td>
<td>The average hourly wage of managers in the same industry as the individual.*</td>
</tr>
<tr>
<td>AGE</td>
<td>The age of the individual.</td>
</tr>
<tr>
<td>SEX</td>
<td>The gender of the individual.</td>
</tr>
<tr>
<td>CHILDREN</td>
<td>The number of persons in the household under 18 years of age.</td>
</tr>
<tr>
<td>MARITAL</td>
<td>The marital status of the individual. Married or Other.**</td>
</tr>
<tr>
<td>UNION</td>
<td>Whether the current place of work of the individual is part of a union.</td>
</tr>
<tr>
<td>OCC</td>
<td>The occupation of the individual’s place of work.</td>
</tr>
<tr>
<td>IND</td>
<td>The industry of the individual’s place of work.</td>
</tr>
<tr>
<td>SALARYWAGE</td>
<td>Whether the individual earns a salary or an hourly wage at their place of work.</td>
</tr>
<tr>
<td>OTHOURLY</td>
<td>If the individual earns an hourly wage, the overtime pay schedule of the individual.***</td>
</tr>
<tr>
<td>OTSALARY</td>
<td>If the individual earns a salary, the overtime pay schedule of the individual.***</td>
</tr>
<tr>
<td>RETIRE</td>
<td>Whether the individual’s place of work is eligible for pension or retirement plan.</td>
</tr>
<tr>
<td>RACE</td>
<td>The race of the individual. White or Other.†</td>
</tr>
<tr>
<td>COLLEGE</td>
<td>Whether the individual received a college degree.</td>
</tr>
<tr>
<td>REGION</td>
<td>The region the survey took place.‡</td>
</tr>
<tr>
<td>EDU</td>
<td>The highest level of education attained by the individual.</td>
</tr>
</tbody>
</table>


**Other includes Never Married, Widowed, Divorced/Annulled, and/or Separated.

***The overtime pay schedule is \( \times 1 \) (same pay), \( \times 1.5 \), or \( \geq \times 2 \).

†Other includes Black, American Indian, Aleut, Eskimo, Asian Pacific Islander, Latino, and mentions color other than black or white.

‡Northeast, North Central, South, West, and Other (Alaska, Hawaii).
Table 4: Descriptive statistics for individuals within the sample by year.

<table>
<thead>
<tr>
<th>Year</th>
<th>1999</th>
<th>2001</th>
<th>2003</th>
<th>2005</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Std. Dev.</td>
<td>Mean</td>
<td>Std. Dev.</td>
</tr>
<tr>
<td>OT</td>
<td>0.22</td>
<td>0.42</td>
<td>0.23</td>
<td>0.42</td>
</tr>
<tr>
<td>OTHOURS</td>
<td>25.46</td>
<td>88.43</td>
<td>30.00</td>
<td>94.94</td>
</tr>
<tr>
<td>OTEST</td>
<td>0.73</td>
<td>0.44</td>
<td>0.67</td>
<td>0.47</td>
</tr>
<tr>
<td>OTHOURSEST</td>
<td>337.07</td>
<td>442.92</td>
<td>298.24</td>
<td>437.97</td>
</tr>
<tr>
<td>Wage</td>
<td>11.76</td>
<td>10.80</td>
<td>12.34</td>
<td>14.02</td>
</tr>
<tr>
<td>MWage</td>
<td>16.98</td>
<td>2.96</td>
<td>19.18</td>
<td>3.22</td>
</tr>
<tr>
<td>AGE</td>
<td>40.15</td>
<td>11.12</td>
<td>40.66</td>
<td>11.37</td>
</tr>
<tr>
<td>CHILDREN</td>
<td>1.05</td>
<td>1.17</td>
<td>0.98</td>
<td>1.15</td>
</tr>
<tr>
<td>EDU</td>
<td>13.15</td>
<td>2.65</td>
<td>13.18</td>
<td>2.59</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Year</th>
<th>2007</th>
<th>2009</th>
<th>2011</th>
<th>2013</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Std. Dev.</td>
<td>Mean</td>
<td>Std. Dev.</td>
</tr>
<tr>
<td>OT</td>
<td>0.24</td>
<td>0.43</td>
<td>0.49</td>
<td>0.50</td>
</tr>
<tr>
<td>OTHOURS</td>
<td>39.28</td>
<td>111.48</td>
<td>126.42</td>
<td>214.78</td>
</tr>
<tr>
<td>OTEST</td>
<td>0.62</td>
<td>0.48</td>
<td>0.55</td>
<td>0.50</td>
</tr>
<tr>
<td>OTHOURSEST</td>
<td>306.61</td>
<td>465.56</td>
<td>226.81</td>
<td>398.32</td>
</tr>
<tr>
<td>Wage</td>
<td>12.19</td>
<td>19.22</td>
<td>12.26</td>
<td>16.61</td>
</tr>
<tr>
<td>MWage</td>
<td>21.58</td>
<td>3.72</td>
<td>21.85</td>
<td>3.97</td>
</tr>
<tr>
<td>AGE</td>
<td>41.99</td>
<td>12.71</td>
<td>41.80</td>
<td>12.90</td>
</tr>
<tr>
<td>CHILDREN</td>
<td>0.92</td>
<td>1.18</td>
<td>0.89</td>
<td>1.16</td>
</tr>
<tr>
<td>EDU</td>
<td>13.31</td>
<td>2.47</td>
<td>13.53</td>
<td>2.50</td>
</tr>
</tbody>
</table>
Table 5: Promotion statistics from PSID.

<table>
<thead>
<tr>
<th>Year</th>
<th>Promoted individuals that worked &gt; 2000 hours in Period $t - 1$ ($OT_{TEST}$)</th>
<th>Promoted individuals that participated in overtime in Period $t - 1$ ($OT$)</th>
<th>Individuals that worked &gt; 2000 hours and promoted in Period $t + 1$ ($OT_{TEST}$)</th>
<th>Individuals that participated in overtime and promoted in Period $t + 1$ ($OT$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1999</td>
<td>0.73</td>
<td>0.19</td>
<td>0.08</td>
<td>0.06</td>
</tr>
<tr>
<td>2001</td>
<td>0.77</td>
<td>0.09</td>
<td>0.07</td>
<td>0.02</td>
</tr>
<tr>
<td>2003</td>
<td>0.70</td>
<td>0.13</td>
<td>0.05</td>
<td>0.03</td>
</tr>
<tr>
<td>2005</td>
<td>0.74</td>
<td>0.15</td>
<td>0.06</td>
<td>0.03</td>
</tr>
<tr>
<td>2007</td>
<td>0.72</td>
<td>0.17</td>
<td>0.05</td>
<td>0.04</td>
</tr>
<tr>
<td>2009</td>
<td>0.72</td>
<td>0.14</td>
<td>0.03</td>
<td>0.02</td>
</tr>
<tr>
<td>2011</td>
<td>0.57</td>
<td>0.16</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The number represents the percentage of individuals in the PSID survey for a given year.
Table 6: Descriptive statistics comparing individuals with salaries and hourly wages.

<table>
<thead>
<tr>
<th>Year</th>
<th>1999</th>
<th>2001</th>
<th>2003</th>
<th>2005</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Salaries &amp; Other</td>
<td>Mean</td>
<td>Std. Dev</td>
<td>Mean</td>
</tr>
<tr>
<td>OT</td>
<td>0.13</td>
<td>0.33</td>
<td>0.13</td>
<td>0.33</td>
</tr>
<tr>
<td>OTEST</td>
<td>0.27</td>
<td>0.45</td>
<td>0.27</td>
<td>0.44</td>
</tr>
<tr>
<td>OTHOURS</td>
<td>17.19</td>
<td>87.51</td>
<td>17.20</td>
<td>86.97</td>
</tr>
<tr>
<td>OTHOURSEST</td>
<td>155.45</td>
<td>395.21</td>
<td>152.60</td>
<td>393.59</td>
</tr>
<tr>
<td>Wage</td>
<td>12.47</td>
<td>17.58</td>
<td>12.82</td>
<td>18.34</td>
</tr>
<tr>
<td>Mwage</td>
<td>20.56</td>
<td>4.17</td>
<td>20.84</td>
<td>4.03</td>
</tr>
<tr>
<td>Percentage</td>
<td>34.88</td>
<td>35.77</td>
<td>39.52</td>
<td>39.11</td>
</tr>
<tr>
<td>Year</td>
<td>2007</td>
<td>2009</td>
<td>2011</td>
<td>2013</td>
</tr>
<tr>
<td></td>
<td>Salaries &amp; Other</td>
<td>Mean</td>
<td>Std. Dev</td>
<td>Mean</td>
</tr>
<tr>
<td>OT</td>
<td>0.13</td>
<td>0.34</td>
<td>0.16</td>
<td>0.37</td>
</tr>
<tr>
<td>OTEST</td>
<td>0.27</td>
<td>0.44</td>
<td>0.27</td>
<td>0.44</td>
</tr>
<tr>
<td>OTHOURS</td>
<td>17.93</td>
<td>88.81</td>
<td>26.24</td>
<td>111.27</td>
</tr>
<tr>
<td>OTHOURSEST</td>
<td>154.57</td>
<td>398.46</td>
<td>150.53</td>
<td>392.11</td>
</tr>
<tr>
<td>Percentage</td>
<td>37.33</td>
<td>40.91</td>
<td>41.11</td>
<td>39.77</td>
</tr>
<tr>
<td>Year</td>
<td>1999</td>
<td>2001</td>
<td>2003</td>
<td>2005</td>
</tr>
<tr>
<td></td>
<td>Hourly</td>
<td>Mean</td>
<td>Std. Dev</td>
<td>Mean</td>
</tr>
<tr>
<td>OT</td>
<td>0.29</td>
<td>0.46</td>
<td>0.28</td>
<td>0.45</td>
</tr>
<tr>
<td>OTEST</td>
<td>0.64</td>
<td>0.48</td>
<td>0.60</td>
<td>0.49</td>
</tr>
<tr>
<td>OTHOURS</td>
<td>31.24</td>
<td>93.95</td>
<td>37.22</td>
<td>103.20</td>
</tr>
<tr>
<td>OTHOURSEST</td>
<td>240.84</td>
<td>385.27</td>
<td>218.83</td>
<td>391.57</td>
</tr>
<tr>
<td>Wage</td>
<td>9.39</td>
<td>7.70</td>
<td>9.00</td>
<td>6.33</td>
</tr>
<tr>
<td>Percentage</td>
<td>65.12</td>
<td>64.23</td>
<td>60.48</td>
<td>60.89</td>
</tr>
<tr>
<td>Year</td>
<td>2007</td>
<td>2009</td>
<td>2011</td>
<td>2013</td>
</tr>
<tr>
<td></td>
<td>Hourly</td>
<td>Mean</td>
<td>Std. Dev</td>
<td>Mean</td>
</tr>
<tr>
<td>OT</td>
<td>0.33</td>
<td>0.47</td>
<td>0.56</td>
<td>0.50</td>
</tr>
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Table 7: Arellano Bond regression results by dependent variable.

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Region       Yes   Yes   Yes   Yes
Occupation   Yes   Yes   Yes   Yes
Year         Yes   Yes   Yes   Yes

N          11716 13643 13637 13643

Statistical significance: * = 10%, ** = 5%, and *** = 1%
Reference category for SALARYWAGE is salary, OTHOURLY AND OTSALARY is $\times 1$, $t − 1$ is $t$ for the respective variable, Not Married is Married, No Union is Union, No Retirement/Pension is Retirement/Pension, and No College is College.
Table 8: Arellano Bond regression results by dependent variable and quantiles by hourly wage.

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Statistical significance: * = 10%, ** = 5%, and *** = 1%
Reference category for **SALARYWAGE** is salary, **OTHOURLY** and **OTSALARY** is ×1, t − 1 is t for the respective variable, Not Married is Married, No Union is Union, No Retirement/Pension is Retirement/Pension, and No College is College.
Table 9: Arellano Bond regression results by dependent variable and quantiles by hourly wage.

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Statistical significance: * = 10%, ** = 5%, and *** = 1%
Reference category for **SALARYWAGE** is salary, **OTHOURLY** AND **OTSALARY** is ×1, $t - 1$ is $t$ for the respective variable, Not Married is Married, No Union is Union, No Retirement/Pension is Retirement/Pension, and No College is College.
Table 10: Fixed effect regression results by dependent variable.

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<th>OTHOURS</th>
<th>OTHOURSEST</th>
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<td>-0.0678***</td>
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Statistical significance: * = 10%, ** = 5%, and *** = 1%
Reference category for $SALARYWAGE$ is salary, $OTHOURLY$ AND $OTSALARY$ is $\times 1$, $t - 1$ is $t$ for the respective variable, Not Married is Married, No Union is Union, No Retirement/Pension is Retirement/Pension, and No College is College.
Table 11: OLS regression results by dependent variable.

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<th>OTHOURS</th>
<th>OTHOURSEST</th>
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<td>-0.0860***</td>
<td>15.90**</td>
<td>-63.57***</td>
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<td>(0.0190)</td>
<td>(5.752)</td>
<td>(16.41)</td>
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<td>15.90**</td>
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Region Yes Yes Yes Yes
Occupation Yes Yes Yes Yes
Year Yes Yes Yes Yes

N 19433 21926 21921 21926

Statistical significance: * = 10%, ** = 5%, and *** = 1%
Reference category for \textit{SALARYWAGE} is salary, \textit{OHOURLY} AND \textit{OTSALARY} is \( \times 1 \), \( t - 1 \) is \( t \) for the respective variable, Not Married is Married, No Union is Union, No Retirement/Pension is Retirement/Pension, and No College is College.
References


