University of Guelph Numeracy Project

About Derivatives II

UNIVERSITY of GUELPH

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About Derivatives II

WHAT IS a DERIVATIVE?

- The derivative is the mathematical process of evaluating limits to find an instantaneous rate of change or the slope of a curve at any point.

- However so far we've only taken the derivative of simple functions, this module will present three new rules to go with the "Power Rule"

Product Rule

- Before going on to discuss more properties of the derivative we'll present again the Power Rule for derivatives.

- Power Rule for Derivatives:
  If $f(x) = x^n$, then $f'(x) = D_x(x^n) = nx^{n-1}$, where n is any real number.

- When taking the derivative of a function that is the product of other functions, we must take the derivative in a special way. We present the Product Rule.

- Product Rule
  If $y = f(x)*g(x)$, then $y' = D_x[f(x)g(x)] = g(x)*f'(x) + f(x)*g'(x)$

Quotient Rule

- Quotient Rule: If $y = T(x)/B(x)$, then

$$y' = D_x\frac{T(x)}{B(x)} = \frac{B(x)T'(x) - T(x)B'(x)}{B(x)^2}$$

Chain Rule

- We move onto now the Chain Rule, which is really a more general Power Rule from before.

- General Power Rule (Chain Rule)
  If $y = [f(x)]^n$, then
  $y' = Dx[f(x)]^n = n[f(x)]^{n-1} * f'(x)$

Exponential and Logarithmic Functions

- We'll now look at the derivatives of exponential and logarithmic functions.

- If $f(x) = e^x$ then
  $f'(x) = e^x$

- Now, we'll need another version of the Chain Rule, but this one for Exponentials.
- Chain Rule: Exponential Form
  $D_x e^{f(x)} = e^{f(x)} * f'(x)$

- Now we'll look at the derivative of functions involving logarithmics.

If $f(x) = \ln x$ then
$f'(x) = 1/x$

- Chain Rule: Logarithmic Form
  $D_x \ln f(x) = 1/f(x) * f'(x) = f'(x)/f(x)$
  where $f(x) > 0$ and $f'(x)$ exists.

- Derivative of the General Exponential Function:
  $D_x a^{f(x)} = a^{f(x)} * \ln a * f'(x)$

- Derivative of the General Logarithmic Function:
  $D_x \log_a x = \frac{1}{(\ln a)x}$ where $x > 0$

- $D_x \log_a f(x) = \frac{f'(x)}{(\ln a) f(x)}$ where $f(x) > 0$

- To recap:

$D_x e^x = e^x$	$D_x e^{f(x)} = e^{f(x)} * f'(x)$
$D_x \ln x = 1/x$, where $x > 0$	$D_x \ln f(x) = f'(x)/f(x)$, where $f(x) > 0$
$D_x a^x = a^x * (\ln a)$	$D_x a^{f(x)} = a^{f(x)} * (\ln a) * f'(x)$
$D_x \log_a x = \frac{1}{(\ln a)x}$, where $x > 0$	$D_x \log_a f(x) = \frac{f'(x)}{(\ln a) f(x)}$, where $f(x) > 0$

Glossary

Chain Rule: A way of taking the derivative of a composite function (a function made up of the product of other functions).

Product Rule: A way of taking the derivative of a function that is the product of two other functions.

Quotient Rule: A way of taking the derivative of a rational function.

References

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