

About Derivatives II: Examples

TABLE OF CONTENTS

Product Rule.....	1
Quotient Rule.....	1 - 2
Chain Rule.....	2 - 3
Exponential and Logarithmic Functions.....	4
Examples.....	5- 6

Product Rule

- Give the following function, find its derivative.

$$f(x) = (3x + 7)(4x^2 - 5x)$$

$$g(x) = 3x + 7 \quad g'(x) = 3$$

$$h(x) = 4x^2 - 5x \quad h'(x) = 8x - 5$$

$$f'(x) = g'(x) \cdot h(x) + g(x) \cdot h'(x)$$

$$f'(x) = 3(4x^2 - 5x) + (3x + 7)(8x - 5)$$

$$f'(x) = 12x^2 - 15x + 24x^2 - 15x + 56x - 35$$

$$f'(x) = 36x^2 + 26x - 35$$

- Consider the following function:

$$f(x) = (4x - 2)(x^2 + 5)$$

$$g(x) = 4x - 2$$

$$h(x) = x^2 + 5$$

$$g'(x) = 4$$

$$h'(x) = 2x$$

$$f'(x) = 4(x^2 + 5) + (4x - 2)(2x)$$

$$f'(x) = 4x^2 + 20 + 8x^2 - 4x$$

$$f'(x) = 12x^2 - 4x + 20$$

If we multiplied it out.

$$f(x) = 4x^3 + 20x - 2x^2 - 10$$

$$f'(x) = 12x^2 - 4x + 20$$

Quotient Rule

- Find the derivative of $f(x) = x/(x^3 + 5)$ using the quotient rule.
- Let $T(x) = x$ and $B(x) = x^3 + 5$, so $T'(x) = 1$ and $B'(x) = 3x^2$. Using the Quotient Rule.

$$f'(x) = \frac{B(x)T'(x) - T(x)B'(x)}{B(x)^2}$$

$$f'(x) = \frac{(x^3 + 5) \cdot 1 - x \cdot 3x^2}{(x^3 + 5)^2}$$

$$f'(x) = \frac{x^3 + 5 - 3x^3}{(x^3 + 5)^2}$$

$$f'(x) = \frac{5 - 2x^3}{(x^3 + 5)^2}$$

- Find the derivative of $f(x) = (x^3 - 2)/(5x + 3)$
- Let $T(x) = x^3 - 2$ and $B(x) = 5x + 3$, $T'(x) = 3x^2$ and $B'(x) = 5$. Using the Quotient Rule.

$$f'(x) = \frac{(5x + 3)(3x^2) - (x^3 - 2)(5)}{(5x + 3)^2}$$

$$f'(x) = \frac{15x^3 + 9x^2 - (5x^3 - 10)}{(5x + 3)^2}$$

$$f'(x) = \frac{10x^3 + 9x^2 + 10}{(5x + 3)^2}$$

Chain Rule

- Find the derivative of $y = (x^2 - 3x + 10)^4$

$$f(x) = x^2 - 3x + 10$$

$$n = 4$$

$$f'(x) = 2x - 3$$

Therefore:

$$y' = 4(x^2 - 3x + 10)^3(2x - 3)$$

- Find $D_x(6x-11x^2)^{1/2}$

$$y = (6x - 11x^2)^{1/2}$$

$$f(x) = 6x - 11x^2$$

$$n = 1/2$$

$$f'(x) = 6 - 22x$$

$$y' = (1/2)(6x - 11x^2)^{-1/2}(6 - 22x)$$

$$y' = \frac{6 - 22x}{2(6x - 11x^2)^{1/2}}$$

- Find the derivative of $f(x) = \frac{5}{(2x - 7)^6}$

- Quotient Rule: $T(x) = 5$, $B(x) = (2x - 7)^6$
 $T'(x) = 0$, $B'(x) = 6(2x - 7)^5$

$$f'(x) = \frac{(2x - 7)^6 \cdot 0 - 6(2x - 7)^5 \cdot 5 \cdot 2}{((2x - 7)^6)^2}$$

$$f'(x) = \frac{-60(2x - 7)^5}{(2x - 7)^{12}} = \frac{-60}{(2x - 7)^7}$$

- Chain Rule: (We bring the denominator up and give the exponent a negative sign)

$$y = 5(2x - 7)^{-6} \quad f(x) = 2x - 7, \quad n = -6$$

$$f'(x) = 2$$

$$y' = 5 \cdot (-6) \cdot (2x - 7)^{-7} \cdot 2 = \frac{-60}{(2x - 7)^7}$$

Which is the result we want.

Logarithmic and Exponential Functions

Examples: Find $f'(x)$ for each of the following functions.

$$1) f(x) = e^{3x} - 2$$

$$\begin{aligned} f'(x) &= e^{3x} - 2 D_x(3x-2) \\ &= e^{3x} - 2 * (3) = 3e^{3x} - 2 \end{aligned}$$

$$2) f(x) = x^3 e^x$$

$$\begin{aligned} f'(x) &= 3x^2 e^x + x^3 e^x \\ f'(x) &= e^x x^2 (3 + x) \end{aligned}$$

$$3) f(x) = e^{7x}$$

$$\begin{aligned} f'(x) &= e^{7x} * 7 \\ f'(x) &= 7e^{7x} \end{aligned}$$

$$4) f(x) = 3.14 * \ln x$$

$$f'(x) = 3.14/x$$

$$5) f(x) = x^3 + \ln x$$

$$f'(x) = 3x^2 + 1/x$$

$$6) f(x) = \ln(5x)$$

$$f'(x) = 1/5x * 5$$

$$f'(x) = 1/x$$

$$7) f(x) = 3x^2 * \ln(x + 5)$$

$$f'(x) = 6x * \ln(x + 5) + 3x^2 * (1/x + 5) * 1$$

$$f'(x) = 6x \ln(x + 5) + \frac{3x^2}{x + 5}$$

$$8) f(x) = \log_8(3x - 2)$$

$$f'(x) = \frac{3}{(\ln 8)(3x - 2)}$$

Examples

$$1) f(x) = (-2x + 7)(x^5 + 4x)$$

$$f'(x) = (-2)(x^5 + 4x) + (-2x + 7)(5x^4 + 4)$$

$$f'(x) = -2x^5 - 8x + (-10x^5 - 8x + 35x^4 + 28)$$

$$f'(x) = -12x^5 + 35x^4 - 16x + 28$$

$$2) f(x) = (3x^2 - 5x^{-2})^{10}$$

$$f'(x) = 10(3x^2 - 5x^{-2})^9(6x + 10x^{-3})$$

$$f'(x) = (60x + 100x^{-3})(3x^2 - 5x^{-2})^{10}$$

$$3) f(x) = \frac{3x^3}{(2x + 5)^4}$$

$$f'(x) = \frac{(2x + 5)^4(9x^2) - (3x^3)(4)(2x + 5)^3(2)}{(2x + 5)^8}$$

$$4) f(x) = (3^x - 2)^4$$

$$f'(x) = 4(3^x - 2)^3(\ln 3)3^x$$

$$5) f(x) = x^2 \cdot 2^x$$

$$f'(x) = x \cdot 2^x[(\ln 2)x + 2]$$

$$6) f(x) = \log_8 x$$

$$f'(x) = \frac{1}{(\ln 8)x}$$

$$7) f(x) = \ln(2x^2 - 5)$$

$$f'(x) = \frac{4x}{2x^2 - 5}$$

$$8) f(x) = (\ln 4x)(5x^2 + 7x - 4)$$

$$f'(x) = (4/x)(5x^2 + 7x - 4) + (\ln 4x)(10x + 7)$$

$$\begin{aligned} 9) f(x) &= e^x(3x + 2)^3 \\ f'(x) &= e^x(3x + 2)^3 + 3e^x(3x + 2)^2 \cdot 3 \\ f'(x) &= e^x(3x + 2)^3 + 9e^x(3x + 2)^2 \end{aligned}$$

$$\begin{aligned} 10) f(x) &= x^4 e^{5x+4} \\ f'(x) &= 4x^3 e^{5x+4} + 5x^4 e^{5x+4} \end{aligned}$$

$$\begin{aligned} 11) f(x) &= (7^x)/x \\ f'(x) &= \frac{x \cdot 7^x \ln 7 - 7^x}{x^2} = \frac{7^x(x \cdot \ln 7 - 1)}{x^2} \end{aligned}$$