**High-Precision Branching Ratio Measurement for the Superaligned β^+
Emitter ^{74}Rb**

by

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ABSTRACT

High-Precision Branching Ratio Measurement for the Superaligned β^+ Emitter ^{74}Rb

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Precision measurements of superallowed Fermi β decay allow for stringent tests of the magnitude of isospin-symmetry-breaking effects in nuclei, the validity of the conserved-vector-current hypothesis, and the unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing matrix. A high-precision measurement of the branching ratio for the superallowed β^+ -decay of ^{74}Rb has been performed at the Isotope Separator and ACcelerator (ISAC) facility at TRIUMF. The 8π spectrometer, an array of 20 HPGe detectors, was used to detect γ rays emitted following the Gamow-Teller and non-analogue Fermi β -decays of ^{74}Rb . PACES, an array of 5 Si(Li) detectors, was used to detect emitted conversion electrons, while half of SCEPTAR, a close-packed array of 10 plastic scintillators, was used to detect emitted β particles. In this experiment, 23 excited states were identified in ^{74}Kr which were populated following the β -decay of ^{74}Rb . A total of 58 γ -rays were identified following the $8.241(4) \times 10^8$ detected β -particles. An observed non-superaligned branching ratio of 0.396(7)% was determined from the intensity of the identified γ -rays, while the unobserved non-superaligned intensity was calculated to be 0.05(5)%, leading to a superaligned branching ratio of 99.55(5)%. The superaligned branching ratio is now the most precise experimental quantity in the determination of the superaligned ft -value of ^{74}Rb . Combining the half-life and Q -value with the superaligned branching

ratio measured in this work leads to a superallowed ft -value of $3082.6(66)$ s. Finally, comparisons between the superallowed ft -value, the world average $\overline{\mathcal{F}t}$ -value, and the non-analogue Fermi branching ratio are made to provide future guidance in the refinement of the theoretical models required to describe the crucial isospin-symmetry-breaking term in superallowed β -decay.

To My Family and Loved Ones

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Chapter 1

Introduction

1.1 The Fermi Theory of Nuclear β Decay

Nuclear β -decay is a weak interaction process in which a more energetically stable nucleus is created via the decay of a neutron(proton) to a proton(neutron). A β -decay involves the emission of a $W^\pm$ boson, which subsequently decays to an electron-neutrino pair. A β^- decay is defined as the decay of a neutron to a proton with the emission of an electron and electron anti-neutrino:

$${}^A_ZX_N \rightarrow {}^A_{Z+1}Y_{N-1} + e^- + \bar{\nu}_e, \quad (1.1)$$

while β^+ decay is the decay of a proton to a neutron with the emission of a positron and electron neutrino:

$${}^A_ZX_N \rightarrow {}^A_{Z-1}Y_{N+1} + e^+ + \nu_e. \quad (1.2)$$

A different time-ordering of β^+ decay can also occur in which an electron is captured from an atomic shell by a proton in the nucleus and an electron neutrino is emitted:

$${}^A_Z\text{X}_N + e^- \rightarrow {}^A_{Z-1}\text{Y}_{N+1} + \nu_e. \quad (1.3)$$

The difference in the mass-energy (Q -value) of the initial and final states of the nuclei involved in the β -decay process is shared among the final state particles. Neglecting the small binding energy difference contributed by the electron orbitals, as well as the neutrino mass, the Q -values of the different types of β -decay are as follows:

$$\begin{aligned} Q_{\beta^-} &= m({}^A_Z\text{X}_N)c^2 - m({}^A_{Z+1}\text{Y}_{N-1})c^2, \\ Q_{\beta^+} &= m({}^A_Z\text{X}_N)c^2 - m({}^A_{Z-1}\text{Y}_{N+1})c^2 - 2m_e c^2, \\ Q_{EC} &= m({}^A_Z\text{X}_N)c^2 - m({}^A_{Z-1}\text{Y}_{N+1})c^2, \end{aligned} \quad (1.4)$$

where the masses, m , are the atomic masses of the corresponding neutral atoms. The transition rate λ between an initial state i and final state f for a time-dependent quantum-mechanical process is given by Fermi's Golden Rule:

$$\lambda_{i \rightarrow f} = \frac{2\pi}{\hbar} |M_{f,i}|^2 \frac{dN_e dN_\nu}{dE_f}, \quad (1.5)$$

where M is the matrix element connecting the initial and final states, N is the number of accessible quantum states, and E_f is the energy of the final state wave function.

Due to the much larger mass of the nucleus compared to the masses of the electron and neutrino, the kinetic energy of the recoiling nucleus is generally small, so that:

$$T_\beta + T_{\nu_e} = E_f \approx Q. \quad (1.6)$$

Taking the initial momentum of the decaying nucleus to be zero, momentum conservation forces the condition:

$$0 = \vec{p}_D + \vec{p}_\beta + \vec{p}_\nu. \quad (1.7)$$

For clarity, future use of p will denote $|\vec{p}|$. To determine the number of accessible quantum states, an integral over the accessible phase-space is required. For each particle the total number of states is given by:

$$\begin{aligned} N_\beta N_\nu &= \frac{1}{(2\pi\hbar)^6} \int d^3r_\beta \int d^3p_\beta \int d^3r_\nu \int d^3p_\nu, \\ &= \frac{V^2}{(2\pi\hbar)^6} \int p_\beta^2 dp_\beta \int d\Omega_\beta \int p_\nu^2 dp_\nu \int d\Omega_\nu, \end{aligned} \quad (1.8)$$

where V is the volume in which the outgoing particle wave functions are normalized in. The density of final states becomes:

$$\frac{dN_e dN_\nu}{dE_f} = \frac{V^2}{(2\pi\hbar)^6} \frac{d}{dE_f} \int p_\beta^2 dp_\beta \int d\Omega_\beta \int p_\nu^2 dp_\nu \int d\Omega_\nu. \quad (1.9)$$

By calculating the probability for emission of the β particle with energy in the interval E_β to $E_\beta + dE_\beta$, the neutrino phase-space for a given β energy is constrained to:

$$\begin{aligned} \frac{d}{dp_\beta} \frac{dN_e dN_\nu}{dE_f} &= \frac{V^2(4\pi)^2}{(2\pi\hbar)^6} p_\beta^2 \frac{d}{dE_f} \int p_\nu^2 dp_\nu, \\ &= \frac{V^2(4\pi)^2}{(2\pi\hbar)^6} p_\beta^2 p_\nu^2 \frac{dp_\nu}{dE_\nu}. \end{aligned} \quad (1.10)$$

Assuming the emitted β -particle and neutrino are free particles, the total energy of the particles can be related to their momenta:

$$E_\nu^2 = p_\nu^2 c^2 + (m_\nu c^2)^2. \quad (1.11)$$

Since $m_\nu \approx 0$, Equation 1.11 can be simplified:

$$p_\nu^2 = \frac{E_\nu^2}{c^2}, \quad \frac{dp_\nu}{dE_\nu} = \frac{1}{c}. \quad (1.12)$$

Equation 1.10 can then be rewritten using Equation 1.6:

$$\frac{d}{dp_\beta} \frac{dN_e dN_\nu}{dE_f} = \frac{V^2 (4\pi)^2}{(2\pi\hbar)^6} p_\beta^2 \frac{(Q - E_\beta)^2}{c^3}. \quad (1.13)$$

The differential decay constant $d\lambda$ for β particles emitted with momenta between p and $p + dp$ is then obtained:

$$d\lambda = \frac{2\pi}{\hbar} |M_{f,i}|^2 \frac{V^2}{4\pi^4 \hbar^6 c^3} p_\beta^2 (Q - E_\beta)^2 dp_\beta. \quad (1.14)$$

However, the newly created β particle is not completely free and experiences the Coulomb field from the protons of the nucleus. The wave function of the charged β particle is distorted since β^+ particles are repelled by the nuclear Coulomb field, while β^- particles are attracted by the nucleus. The wave function of the neutrino can be well approximated by a plane-wave solution:

$$\psi_\nu(\vec{r}_\nu) = \frac{1}{\sqrt{V}} e^{\frac{i\vec{p}_\nu \cdot \vec{r}_\nu}{\hbar}}, \quad (1.15)$$

while the β -particle is approximated by a distorted plane-wave solution:

$$\psi_\beta(\vec{r}_\beta) = \frac{1}{\sqrt{V}} e^{\frac{i\vec{p}_\beta \cdot \vec{r}_\beta}{\hbar}} [F(Z, E_\beta)]^{1/2}. \quad (1.16)$$

$F(Z, E_\beta)$ is the relativistic Fermi function which describes the Coulomb distortion of

the wave function for the charged β -particles [1]:

$$F(Z, E_\beta) \approx 2L(Z, E_\beta)(1 + \gamma) \left(\frac{2pR}{\hbar} \right)^{2\gamma-2} e^{\pi\eta} \frac{|\Gamma(\gamma + i\eta)|^2}{|\Gamma(2\gamma + 1)|^2}. \quad (1.17)$$

where $\gamma = \sqrt{1 - (\alpha Z)^2}$, $\eta = \mp Z\alpha E/pc$ for $\beta^\pm$ decay, α is the fine structure constant, c is the speed of light, and Z and R are the charge and radius of the daughter nucleus respectively. The function $L(Z, E_\beta)$ is the part of the matrix element which accounts for the finite nuclear charge distribution that the β particle interacts with. Additional higher-order corrections to $F(Z, E_\beta)$ to account for the realistic nuclear charge distribution, screening of the nuclear charge by the atomic electron cloud, and recoil of the nucleus are included in high-precision superallowed β -decay studies[2]. Explicitly removing the Coulomb effects from the matrix element gives the new differential decay constant:

$$d\lambda = \frac{|M'_{f,i}|^2}{2\pi^3 \hbar^7 c^3} F(Z, E_\beta) p_\beta^2 (Q - E_\beta)^2 dp_\beta, \quad (1.18)$$

where $|M'_{f,i}|$ indicates that the Coulomb distortion as well as the volume normalization of the β -particle and neutrino wave functions have been accounted for independently from the nuclear matrix element.

In general, to get the final decay rate λ , Equation 1.14 must be integrated numerically. For this reason, a change of variables is performed to make the Equation dimensionless,

$$W = \frac{E}{m_\beta c^2} = \frac{Q}{m_\beta c^2} + 1, \quad \rho = \frac{p_\beta}{m_\beta c}. \quad (1.19)$$

These new variables W and ρ can be related by Equation. 1.11:

$$W = \sqrt{\rho^2 + 1}, \quad WdW = \rho d\rho. \quad (1.20)$$

If there is no transfer of orbital angular momentum to the neutrino and β particle, i.e. $L_\beta = 0$ and $\Delta J = 0, 1$ corresponding to the β particle and neutrino intrinsic spins coupled anti-parallel ($S = 0$) or parallel ($S = 1$), the matrix element $|M_{f,i}|$ of Equation 1.14 is independent of the β -particle momentum. $d\lambda$ can then be integrated using the new variables of Equation 1.19:

$$\lambda_{i \rightarrow f} = \frac{m_\beta^5 c^4}{2\pi^3 \hbar^7} |M'_{f,i}| \int_1^{W_0} F(Z, W) W \sqrt{W^2 - 1} (W_0 - W)^2 dW, \quad (1.21)$$

where the integration limit $W_0 = Q/m_\beta c^2$ corresponds to the maximum β -particle energy and the lower limit of 1 corresponds to a β -particle with no kinetic energy.

The integral in Equation 1.21 is called the *statistical rate function*:

$$f = \int_1^{W_0} F(Z, W) W \sqrt{W^2 - 1} (W_0 - W)^2 dW. \quad (1.22)$$

Typically the decay constant is given in terms of a more conceptually accessible observable; the partial half-life t of the transition $f \rightarrow i$ is given by:

$$\lambda = \frac{\ln 2}{t}, \quad t = \frac{T_{1/2}(1 + P_{EC})}{BR}, \quad (1.23)$$

where P_{EC} is the electron capture probability and BR is the β branching ratio of the transition. Finally, the coupling strength parameter of the weak interaction g is contained within the matrix element $|M'_{f,i}|$. The coupling parameter g can be

explicitly removed from $|M'_{f,i}|$ to give the reduced matrix element $|\overline{M}'_{f,i}|$. The β -decay ft -value can then be calculated by substituting Equations 1.22 and 1.23 into Equation 1.21 to obtain:

$$ft = \frac{K}{g^2 |\overline{M}'_{f,i}|}, \quad (1.24)$$

where K is a grouping of the fundamental constants such that [3]:

$$\frac{K}{(\hbar c)^6} = \frac{2\pi^3 \hbar \ln 2}{(m_e c^2)^5} = (8120.271 \pm 0.012) \times 10^{-10} \text{ GeV}^{-4} \text{ s}. \quad (1.25)$$

From Equation 1.24, the weak coupling strength g can be determined once f (or the Q -value), $T_{1/2}$, BR and the reduced matrix element $|\overline{M}'_{f,i}|$ are known. However, to determine the nuclear matrix element, the initial and final state wave functions must be known very precisely. For β -decay in general, the theoretical nuclear models used to calculate these matrix elements cannot deliver sufficient precision for a determination of the fundamental strength parameter g . In order to determine g , a special type of β -decay in which an analytical form of the matrix element can be derived is required.

1.2 Superaligned Fermi β -Decay

When a β particle and neutrino are emitted during nuclear β -decay, they can carry orbital angular momentum, L_β in units of $\hbar$, as long as conservation of angular momentum ($\vec{J}_P = \vec{J}_D + \vec{J}_\beta$) and parity ($\Pi_P = \Pi_D(-1)^{L_\beta}$) are obeyed where $J_{P,D}$ and $\Pi_{P,D}$ represent the total angular momentum and parity of the parent (P) and daughter (D) nuclear states. However, for each increase in L_β by 1, the decay rate $\lambda_{i \rightarrow f}$ decreases by roughly a factor of 10^4 . The suppression of the decay rate at

larger orbital angular momentum transfer gives a natural hierarchy of β -decay classes (Table 1.1).

Table 1.1: Classification of β -decay based on the orbital angular momentum of the emitted β -particle and neutrino.

L_β	Classification	Typical $\log(ft)$
0	“allowed” transitions	2.9-6.0
1	“1st forbidden” transitions	6-10
2	“2nd forbidden” transitions	10-13
3	“3rd forbidden” transitions	15-21
4	“4th forbidden” transitions	≥ 21

As stated above, “forbidden” transitions are not strictly forbidden in the proper sense of the word. However, β -decays to states which are the “least forbidden” (or are consistent with the lowest value of L_β) dominate the β -decay process.

Since every initial and final state particle involved in nuclear β -decay is a spin-1/2 fermion, when a proton or neutron undergoes β decay, the emitted β -particle and neutrino can be in one of two possible spin states; the spin-0 singlet or the spin-1 triplet,

$$\begin{aligned}
S = 0 & \quad \frac{1}{\sqrt{2}} [|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle] & (n \leftrightarrow p, \text{ no spin flip}), \\
S = 1 & \quad |\uparrow\uparrow\rangle, |\downarrow\downarrow\rangle, \frac{1}{\sqrt{2}} [|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle] & (n \leftrightarrow p, \text{ with spin flip}).
\end{aligned} \tag{1.26}$$

Specifically, a Fermi β -decay is a decay in which the spin of the neutron or proton remains unchanged, whereas a Gamow-Teller β -decay is a decay in which the neutron or proton spin is flipped after the β -decay. Experiments have shown that the strength of the weak interaction is different for Fermi β -decay than it is for Gamow-Teller β -decay. Accordingly, the weak interaction strength g is broken into two different

components:

$$g^2 |\overline{M}'_{f,i}|^2 = g_V^2 |\overline{M}'_{f,i}(F)|^2 + g_A^2 |\overline{M}'_{f,i}(GT)|^2, \quad (1.27)$$

where g_V is the weak-interaction vector coupling strength, and g_A is the weak-interaction axial-vector coupling strength. For the special case of $J_P = 0^+ \rightarrow J_D = 0^+$, angular momentum and parity conservation requires that $L_\beta = 0$ and $S_\beta = 0$; an allowed Fermi transition. Neglecting higher-order radiative corrections for the moment, there is no Gamow-Teller contribution to such a decay in leading order and as such, the strength of the axial-vector part of the weak interaction does not contribute to the β -decay, allowing for direct measurements of the weak-vector coupling strength,

$$g^2 |\overline{M}'_{f,i}|^2 = g_V^2 |\overline{M}'_{f,i}(F)|^2. \quad (1.28)$$

To determine the matrix element of the Fermi interaction, an even more special case of allowed Fermi transitions can be used which requires the concept of isospin symmetry. Protons and neutrons appear to be the same particles differing in nothing but electric charge and subsidiary properties. Due to this, the proton and neutron can be treated as different quantum states of the same particle [4]. These states, so called isobaric spin (isospin) states can be used to calculate many properties of the nucleon. Isospin t shares the spin symmetry $SU(2)$, giving it the same additive properties and commutation relations as spin angular momentum. The “3rd component”, or projection t_3 of isospin is defined as being $+\frac{1}{2}$ for the neutron while being $-\frac{1}{2}$ for the

proton. The isospin part of the wave function can then be described using:

$$|t, t_3\rangle = \begin{cases} |\frac{1}{2}, +\frac{1}{2}\rangle & \text{for } |n\rangle \\ |\frac{1}{2}, -\frac{1}{2}\rangle & \text{for } |p\rangle \end{cases} \quad (1.29)$$

For the nucleus as a whole, $T_3 = \frac{1}{2}(N_n - N_p)$, while the total isospin of each nuclear state is given by the vector addition of the nucleonic isospin $t = \frac{1}{2}$ and can, in principle, take any value in the range:

$$\frac{1}{2}|N_n - N_p| \leq T \leq \frac{1}{2}|N_n + N_p|. \quad (1.30)$$

Symmetry energy contributions to the nucleon-nucleon interaction, however, favour the lowest allowed values of T , with states of higher T typically appearing at high excitation energy. The operator responsible for transforming a proton into a neutron, while leaving all other properties of the nucleon unchanged, and vice versa is an isospin ladder operator $\hat{\tau}^\pm$ while the operator responsible for changing the total projection of isospin of a nuclear wave function is $\hat{T}^\pm$, each defined by:

$$\begin{aligned} \hat{\tau}^\pm |t, t_3\rangle &= \sqrt{(t \mp t_3)(t \pm t_3 + 1)} |t, t_3 \pm 1\rangle, \\ \hat{T}^\pm |T, T_3\rangle &= \sqrt{(T \mp T_3)(T \pm T_3 + 1)} |T, T_3 \pm 1\rangle. \end{aligned} \quad (1.31)$$

The operator $\hat{T}^\pm$ connects states with the same total isospin T and otherwise identical nuclear wave functions, called isobaric analogue states, but raises or lowers T_3 by ± 1 . A $J_P = 0^+ \rightarrow J_D = 0^+$ decay between isobaric analogue states is called a *superallowed Fermi β -decay*. During superallowed Fermi β -decay T_3 is altered by ± 1 while the rest of the nuclear wave function remains unaltered. Using Equation 1.31,

the Fermi matrix element can be calculated analytically,

$$\begin{aligned} |\overline{M}'_{f,i}(F)|^2 &= |\langle T, T_3 \pm 1 | \hat{T}^\pm | T, T_3 \rangle|^2, \\ &= (T \mp T_3)(T \pm T_3 + 1). \end{aligned} \quad (1.32)$$

Using Equation 1.32 for a superallowed β -decay within a $T = 1$ multiplet, the Fermi matrix element becomes:

$$|\overline{M}'_{f,i}(F)|^2 = \begin{cases} (1+1)(1-1+1) = 2 & \text{for } T_3 = -1 \rightarrow 0 \\ (1-0)(1+0+1) = 2 & \text{for } T_3 = 0 \rightarrow 1 \end{cases} \quad (1.33)$$

Thus, for a pure Fermi transition between $T = 1$ isobaric analogue states, the ft -value can be rewritten simply as:

$$ft = \frac{K}{2g_V^2}. \quad (1.34)$$

Equation 1.34 states that the ft -value for a $T = 1$ superallowed β -decay depends only on g_V . Experimentally, f can be determined from the mass difference between the parent and daughter nuclei and t can be determined via precise half-life and branching ratio measurements, allowing a precise determination of the weak-vector coupling strength g_V .

Many fundamental interactions of particles within bound systems appear to differ from free particle interactions. There do exist exceptions such as electromagnetic charge, which is independent of composite particle wave functions. Based on the electroweak theory of Weinberg and Salam, superallowed β -decay offers a unique opportunity to probe the coupling properties of the weak interaction. The theory predicts that for the decay of a nucleus between isobaric analogue states, the matrix

element corresponding to the vector contribution is equal to Equation 1.32, while all higher order matrix elements (axial, induced tensor, induced scalar, etc.) have nuclear multipole moment terms which are subject to the nuclear environment, i.e. the strength of the axial coupling is renormalized by the spin degrees of freedom of the constituent particles[5]. In fact, as a consequence of this theory, the conservation of electromagnetic charge directly implies the conservation of the weak vector current. This symmetry between the electromagnetic current and the weak vector-current has prompted the Conserved Vector Current (CVC) hypothesis which claims that the strength of the vector part of the weak interaction remains constant for all weakly interacting particles ($g_V \rightarrow G_V$) [6]. Observations to the contrary would be

Figure 1.1: Superaligned ft -values for the 13 most precisely determined $T = 1$ superallowed β -decays[2, 7].

evidence for physics beyond the electroweak theory of the Standard Model. Superaligned Fermi β -decay provides a unique test for the CVC hypothesis as the right side of Equation 1.34 should be constant for all superallowed β -decays of a given isospin. Figure 1.1 shows the ft values for the 13 most precisely determined $T = 1$ superallowed β -decays. Since β -decay ft values generally span more than 20 orders of magnitude (Table 1.1), the $T = 1$ superallowed β decays show strong evidence for the CVC hypothesis. However, Equation 1.34 is not strictly correct as it assumes perfect isospin symmetry, and also neglects the higher-order radiative corrections. Clearly, to truly test the nucleus-independence of the superallowed ft -values, and thus, provide the most stringent test of the CVC hypothesis, these corrections must be carefully accounted for.

1.3 Theoretical Corrections and $\mathcal{F}t$ -values

When the corrections to 1.34 are applied, the ft -value becomes[8]:

$$\mathcal{F}t = ft(1 + \delta'_R)(1 + \delta_{NS} - \delta_C) = \frac{K}{2G_V^2(1 + \Delta_R^V)}. \quad (1.35)$$

The CVC hypothesis predicts that the bare vector coupling strength of the weak interaction between up and down quarks G_V is a constant, however higher-order radiative terms (QCD and QED loop diagrams) *effectively* re-normalize the coupling strength. Although the “inner” radiative correction ($\Delta_R^V=2.361(38)\%$) is a nucleus-independent correction which does not affect the test of CVC constancy, it is important to include in Equation 1.35 to determine the overall weak-vector coupling strength [9]. The transition-dependent corrections due to the interaction of the emitted β particle with the Coulomb field of the nucleus are given by the “outer radiative” corrections δ'_R

which depend only on the number of protons in the daughter nucleus and the mean decay energy of the particular transition and are independent of the details of nuclear structure [10]. The nuclear-structure dependent radiative correction δ_{NS} contains contributions from both the quenched single-nucleon and inter-nucleon axial-vector interactions involving Gamow-Teller β decay followed by a photon exchange with the outgoing β particle, which flips the nucleon spin back. The outer radiative corrections are determined using QED calculations, and are generally considered to be well understood.

Finally, the assumption of perfect isospin symmetry clearly needs to be adjusted for two reasons: (i) protons and neutrons have slightly different masses, and (ii) a proton replacing a neutron within a nucleus will have a slightly different radial wave function due to Coulombic effects and charge-dependent nuclear forces. Without the broken symmetry, the mixing of the isospin part of the parent nucleus' 0^+ wave function across many of the 0^+ states in the daughter nucleus would not occur and 100% of the isobaric analogue state would be populated directly following a superallowed β -decay.

The *isospin-symmetry-breaking correction* δ_C can be broken into two parts [11]:

$$\delta_C = \delta_{C_1} + \delta_{C_2}. \tag{1.36}$$

The δ_{C_1} correction is due to configuration mixing in a truncated shell-model calculation with charge-dependent interactions. The δ_{C_2} correction deals with the imperfect radial wave function overlap between the proton and neutron involved in the decay due to the differing Coulomb interaction experienced by the two particles and differences in their separation energies.

Figure 1.2: A plot of the calculated δ_C values by Towner and Hardy for each of the 13 high-precision $T = 1$ superallowed β -decays. (TH08WS)[12], (TH09HF)[2].

Once these corrections have been input into Equation 1.35, a test of CVC is established. Currently two sets of calculations [2, 12] for the isospin-symmetry-breaking term, both of which were performed by Towner and Hardy, are considered in the evaluation of the superallowed data (Figure 1.2). The calculated correction factors differ only in the determination of the δ_{C_2} term, using radial wave functions in either a Woods-Saxon or Hartree-Fock potential. The two different approaches to the δ_C values determined by Towner and Hardy share very similar trends as a function of Z which does not affect the validation of the CVC hypothesis. However, the absolute value of the Woods-Saxon δ_{C_2} corrections are slightly larger than the Hartree-Fock corrections in most cases, becoming an important systematic difference for the determination of the Standard Model constants G_V and V_{ud} (Section 1.4). Figure 1.3

Figure 1.3: Superallowed $\mathcal{F}t$ -values for the 13 most precisely determined $T = 1$ superallowed β -decays using either Woods-Saxon or Hartree-Fock radial wave functions for the δ_{C_2} calculations. The average of the $\mathcal{F}t$ values determined using the Woods-Saxon correction is $\overline{\mathcal{F}t} = 3072.38(75)$ s while using the Hartree-Fock corrections is $\overline{\mathcal{F}t} = 3071.59(87)$ s. Data is taken from [2, 7].

shows the $\mathcal{F}t$ values determined using the different δ_C values calculated by Towner and Hardy, which validate the CVC hypothesis to 1.3 parts in 10^4 . To determine the weak vector coupling constant, a systematic uncertainty is assigned to account for the slightly different $\overline{\mathcal{F}t}$ values determined using the two different theoretical approaches. The current superallowed $\overline{\mathcal{F}t}$ -value is taken to be the average of the two $\overline{\mathcal{F}t}$ -values (3071.99(75)) with a systematic uncertainty equal to half the difference[7]:

$$\begin{aligned}
 \overline{\mathcal{F}t} &= 3071.99 \pm 0.75_{\text{stat}} \pm 0.40_{\text{syst}} \text{ s}, \\
 &= 3071.99 \pm 0.85 \text{ s}.
 \end{aligned}
 \tag{1.37}$$

When combined with Equation 1.35 and the value of $\Delta_R^V = 2.361(38)\%$ [9], the vector

coupling constant G_V becomes:

$$\begin{aligned}
G_V^2 &= \frac{K}{2\overline{\mathcal{F}t}(1 + \Delta_R^V)}, \\
G_V^2/(\hbar c)^6 &= 1.29118(34)_{exp+\delta'_R+\delta_C}(17)_{\delta_{C_{\text{syst.}}}}(50)_{\Delta_R^V} \times 10^{-10} \text{ GeV}^{-4}, \\
G_V/(\hbar c)^3 &= 1.13630(15)_{exp+\delta'_R+\delta_C}(08)_{\delta_{C_{\text{syst.}}}}(22)_{\Delta_R^V} \times 10^{-5} \text{ GeV}^{-2}, \\
&= 1.13630(28) \times 10^{-5} \text{ GeV}^{-2},
\end{aligned} \tag{1.38}$$

where the three uncertainties are from the experimental ft -values plus the outer radiative corrections δ'_R and δ_{NS} and the quoted uncertainties on the individual δ_C values, the systematic δ_C uncertainty arising from the model-dependent difference in the $\overline{\mathcal{F}t}$ values calculated by from the Hartree-Fock and Woods-Saxon approaches, and the inner radiative correction Δ_R^V , respectively.

1.4 The Cabibbo-Kobayashi-Maskawa (CKM)

Matrix

According to the Standard Model, there exists a broken symmetry, which is allowed but not required by the Higgs mechanism, between quark mass eigenstates and their weak interaction eigenstates. The quark mass eigenstates are thus mixtures of the weak interaction eigenstates allowing for the weak coupling of quarks across different generations. The magnitude of the mixing of the weak and mass eigenstates

is defined by the Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing matrix[13]:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}, \quad (1.39)$$

where d, s , and b are the mass eigenstates of the down, strange and bottom quarks while d', s' , and b' are the respective weak-interaction eigenstates. The CKM matrix is a unitary transformation between the mass and weak-interaction eigenstates, meaning that the sum of squares of any row or column of the CKM matrix must equal unity. Any deviation from this would be evidence for physics beyond the Standard Model. Currently, the individual matrix elements of the CKM matrix cannot be described by theory and must be measured experimentally.

Due to the very high precision of the V_{us} and V_{ud} matrix elements, and the statistical insignificance of V_{ub} , the sum-of-squares of the top row elements of the CKM matrix currently gives the most stringent test of the CKM matrix unitarity. The magnitude of V_{ud} is extracted using the value of G_V determined by superallowed β -decay and the very precise Fermi coupling constant G_F measured in purely leptonic decay of the muon, where $G_F/(\hbar c)^3 = 1.166364(5) \times 10^{-5} \text{ GeV}^{-2}$ [3]. Taking the value of G_V and dividing by the Fermi constant G_F , the CKM matrix element $|V_{ud}|$ is calculated to be:

$$\begin{aligned} |V_{ud}| &= \frac{G_V}{G_F}, \\ &= 0.97422(13)_{exp+\delta'_R+\delta_C} (06)_{\delta_{C_{\text{sys}}}} (19)_{\Delta_R}, \\ &= 0.97422(24). \end{aligned} \quad (1.40)$$

The matrix element $|V_{us}| = 0.2252 \pm 0.0009$ is determined using kaon decays while $|V_{ub}| = 0.00389 \pm 0.00044$ is determined using various B decays [3]. Combining the squares of the top-row matrix elements gives:

$$\begin{aligned} |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 &= 0.9998(5)_{V_{ud}}(4)_{V_{us}}, \\ &= 0.9998(6). \end{aligned} \tag{1.41}$$

Unitarity is currently satisfied, but increasing the precision may shed light on new physics. However, to push the currently obtainable precision, we must continue to scrutinize and reevaluate the uncertainty-dominating theoretical corrections described in Section 1.3. One correction that needs to be addressed to provide stricter tests on the CVC hypothesis and CKM unitarity is the isospin-symmetry-breaking correction δ_C .

1.5 Tests of Isospin-Symmetry Breaking

Currently, the Δ_R^V correction dominates the overall uncertainty in the determination of V_{ud} but has no effect on the CVC hypothesis. Recently, the uncertainty in Δ_R^V was improved by a factor of 2[9] and may be improved even further in the near future[14]. The small systematic uncertainty due to δ_C in Equation 1.40 is potentially misleading as it is only half the difference in the most recent Hartree-Fock and Woods-Saxon calculations. The quoted theoretical uncertainties in the δ_C corrections leading to the individual $\mathcal{F}t$ -values are included in the “statistical” uncertainties in Equations 1.38, 1.40 and 1.41. Thus, to lower the uncertainty in the average $\overline{\mathcal{F}t}$ -value, a decrease in the uncertainty in the δ_C corrections will be required.

Furthermore, both δ_{C_1} and δ_{C_2} have been calculated using various nuclear models [15] and seem to be the least understood corrections input into the superallowed $\mathcal{F}t$ -values. Many calculations have been performed using different models which do not agree for many of the superallowed β -decays (Figure 1.4), specifically in the $A \geq 62$ nuclei where the isospin symmetry breaking is predicted to be the largest, making them prime candidates to test the validity of current theoretical calculations. However, both of the Towner and Hardy Woods-Saxon and Hartree-Fock sets of calculations use the same δ_{C_1} component calculated using various shell models. The δ_{C_1} calculations require reliable shell-model descriptions of the nuclei involved, which currently do not exist in the $A \geq 62$ region, and are currently constrained by limited independent experimental data such as the isobaric multiplet mass equation (IMME). Recent studies have, however, suggested that the δ_{C_1} term may be overestimated in the $A \geq 62$ superallowed decays [16].

A very good approximation can be used to determine the total δ_{C_1} correction from the configuration mixing into each non-analogue 0^+ state:

$$\delta_{C_1} = \sum_{n=1}^{\infty} \delta_{C_1}^{(n)}, \quad (1.42)$$

where $\delta_{C_1}^{(n)}$ is the component of isospin-symmetry breaking due to configuration mixing of the isobaric analogue of the parent 0^+ state into the n^{th} non-analogue 0^+ state of the daughter nucleus. Each of the $\delta_{C_1}^{(n)}$ components can be calculated theoretically, but they can also be determined experimentally which provides a good touch-stone

Figure 1.4: A plot of the calculated δ_C values from different models for each of the 13 high-precision $T = 1$ superallowed β -decays. (TH08WS)[12], (TH02WS)[17], (TH09HF)[2], (OB95HF)[18], (LVM09PK, LVM09DD)[19], (CGS09PR)[20]. (Courtesy of G.F. Grinyer[21]).

for theoretical output. To experimentally determine the contribution of the n^{th} non-analogue 0^+ state to δ_{C_1} :

$$\frac{\delta_{C_1}^{(n)}}{(1 - \delta_C)} \approx \left(\frac{f_0}{f_n} \right) \frac{B_n}{B_0}, \quad (1.43)$$

where f_n is the statistical-rate function and B_n is the β -branching ratio to the n^{th} non-analogue 0^+ state respectively. Equation 1.5 can be well approximated with $\delta_C \ll 1$ and $B_0 \approx 1$ as:

$$\delta_{C_1}^{(n)} \approx \left(\frac{f_0}{f_n} \right) B_n, \quad (1.44)$$

which gives a stringent test of the individual δ_{C_1} components provided by the current theoretical approaches. Constraining the isospin-symmetry-breaking corrections for different nuclei, will lead to a deeper understanding of the required shell-model spaces, and hence a systematic reduction in the uncertainty of these theoretical terms for the $A \geq 62$ nuclei.

As can be seen from Figures 1.2 and 1.4, the reliability of the isospin-symmetry-breaking corrections for ^{74}Rb is questionable at best and the testing and constraint of these corrections forms the primary motivation of this thesis.

1.6 The β -Decay of ^{74}Rb

The β -decay of ^{74}Rb is a superallowed β -decay between $T = 1$ analogue states. Approximately 99.5% of the β -decay directly feeds the 0^+ analogue ground-state in ^{74}Kr , while the remaining intensity is distributed between the excited 1^+ and non-analogue 0^+ states. However, ^{74}Rb is far from stability and the Q value for this β decay is large (10.4 MeV) leading to a large energy window of excited 0^+ and 1^+ states that can be fed by non-analogue Fermi and Gamow-Teller β -decay, respectively. Furthermore, in this nuclear region the density of excited 1^+ states in nuclei is also large. The combined effect of the increased phase space and increased level density leads to the so-called “Pandemonium” effect[8, 22].

The Pandemonium effect occurs when a significant combined γ -ray intensity is fragmented across many excited states causing individually very weak γ -ray transitions which, based on the sensitivity of the experiment, can go unobserved[22]. In a high-precision branching ratio measurement, such as this work, the Pandemonium effect limits the achievable precision on the determination of the non-analogue branch,

Figure 1.5: An example of the “Pandemonium” effect occurring in the decay of ^{74}Rb . Dashed arrows represent unobserved γ -ray intensity, while the solid arrows represent observed intensity. The unobserved γ -ray intensity from the >400 excited 1^+ states can be estimated using the collected intensity of the low-lying 2^+ states.

and hence, the superallowed branching ratio. Figure 1.5 shows a simplified decay scheme for the ^{74}Rb decay exhibiting the Pandemonium effect.

However, by exploiting the low-lying 2^+ states, as *collector states*, which are populated entirely due to γ -ray feeding from above due to the 2nd forbidden nature of their direct β feeding from ^{74}Rb decay, one can estimate the total unobserved γ -ray intensity to the ground state. The excess γ -ray intensity *out* of the 2^+ states compared to the observed γ -ray intensity *into* these states is a direct measurement of the unobserved γ -ray intensity feeding these levels. By using detailed shell-model calculations to estimate the average γ -ray branching ratio for the decay of the non-analogue states between the 2^+ excited states and the 0^+ ground state, the unobserved 2^+ feeding can be used to estimate the unobserved ground-state feeding, which can be combined with the observed γ decay to the ground state to determine the total non-superallowed branching ratio.

Through the detailed γ -ray spectroscopy presented in this thesis, combined with

the input on average branching ratios from theory, we have been able to largely overcome the Pandemonium effect and achieve, for the first time, a superallowed branching ratio for ^{74}Rb with sufficient precision (0.05%) for inclusion in Standard Model tests (Chapter 6).

Chapter 2

Experimental Facilities and Apparatus

2.1 Tri-University Meson Facility

The Tri-University Meson Facility (TRIUMF), located in Vancouver, B.C., is Canada's National Laboratory for Nuclear and Particle Physics. At the heart of the TRIUMF experimental program is the world's largest cyclotron. The TRIUMF cyclotron can accelerate protons to energies of up to 500 MeV before they are delivered to the various experimental halls to be used directly or to make secondary beams.

2.1.1 ISAC and Surface Ionization Source

Radioactive-ion beams are created at the Isotope Separator and ACcelerator (ISAC) using the high-energy protons delivered from the main cyclotron. In principle, spallation reactions on a stationary target caused by the high-energy high-intensity

proton beam form all nuclei with neutron and proton number less than or equal to the target material. For the study of ^{74}Rb , the products of the spallation reactions were surface ionized by resistively heating the target to temperatures up to 2200 °C [23]. Atoms with low ionization potential such as: Al, Ga, In, Tl, the rare earths and the alkalis, are prime candidates for surface ionization. The atoms with low ionization potential, are thermally desorbed leaving an electron on the surface of the high work function target [24]. The positive ions are then extracted using an electrode held at a negative potential with respect to the target.

ISAC I and ISAC II

Figure 2.1: A schematic representation of the ISAC experimental hall including the target/ion source, mass separator and 8π spectrometer.

To selectively deliver one of the surfaced ionized products, an on-line mass separator is used. The mass separator uses a constant magnetic field to steer the ions

through a 45-degree bend according to,

$$R = \frac{1}{B} \sqrt{\frac{2m\Delta V}{q}}, \quad (2.1)$$

where R is the radius of the circular orbit, B is magnitude of the magnetic field, m is the mass of the ion, ΔV is the voltage difference between the ion source and the mass separator and q is the charge of the ion. Adjustable slits at the end of the separator are used to finely tune the mass-to-charge ratio of ions leaving the mass separator with a resolving power of $\Delta m/m \approx 1/1000$. However, as in the case of the present work, isobaric contaminants can be present in significant quantities and must be paid the appropriate attention.

2.2 Interactions of Particles in Radiation Detectors

2.2.1 Electron Interactions in Radiation Detectors

The low-energy electrons and positrons involved in nuclear decay can undergo the physical interaction processes of *bremsstrahlung radiation*, *collision loss*, and *backscattering* in materials which can affect the amount of energy imparted to a radiation detector.

Collision Loss

The exchange of energy between an incoming charged particle and the atomic electrons of a detector is the main process used for detection. Depending on the energy, an incoming e^- can scatter multiple times off of different atoms imparting partial to full energy. The *stopping power* dE/dx is given by the Bethe-Bloch formula,

Figure 2.2: Plot of the stopping powers for collisional loss and radiative loss in both silicon (Si) and tungsten (W).

which must be modified for electrons due to their indistinguishability with the atomic electrons[25]. The stopping power scales roughly linearly with the atomic charge Z of the detector and inversely with the atomic number A . As shown in Figure 2.2, to a very good approximation, the stopping power of electrons in a detector is constant for electrons above a few hundred keV, thereby making the geometrical length of the active region for a given material the limiting factor for detecting the full electron energy.

Bremsstrahlung Radiation

As previously mentioned, low-energy electrons and positrons lose the majority of their kinetic energy by imparting it to atoms through collisions while passing through matter. However, as the energy of the electron is increased, electromagnetic radiation can be emitted in the form of bremsstrahlung. Classically, bremsstrahlung is the electromagnetic radiation emitted when a charged particle is accelerated by an electromagnetic field. The energy at which the stopping power due to collisions and bremsstrahlung is equivalent is called the *critical energy* and can be approximated

as[25]:

$$E_c \simeq \frac{800 \text{ MeV}}{Z + 1.2}, \quad (2.2)$$

while the ratio of collisional energy loss to radiative energy loss at low energies is given as[26]:

$$\frac{(dE/dx)_{Brem.}}{(dE/dx)_{Col.}} \simeq \frac{EZ}{700} \quad (2.3)$$

where Z is the atomic charge of the material and E is given in units of MeV. Equation 2.2 shows that the energy losses of electrons produced in nuclear decay processes, which typically have $E \lesssim 10$ MeV, are dominated by collisional losses in low Z materials (such as Si). For low-energy electrons, if bremsstrahlung radiation does occur, the low-energy photon produced is often absorbed by photoelectron production (Section 2.2.2) leading to no loss of energy from the detector.

Backscattering

Although technically not a unique physical process being a subprocess of collisional loss, backscattering is an important phenomenon when trying to understand the efficiency of charged-particle detectors. The deflections of an electron losing energy by collisional loss can be large enough to actually cause the electron to leave the surface in which it entered. An electron which backscatters out of a detector will not deposit all of its energy. The effect is most pronounced for low-energy electrons as well as high Z materials, which kinematically, are easier to have large-angle scattering.

2.2.2 Gamma-Ray Interactions in Radiation Detectors

The most common physical processes associated with γ rays in materials include: *photoelectric absorption*, *Compton scattering*, and *pair production*. These processes are essential for the understanding of γ -ray detection and the backgrounds encountered.

Figure 2.3: Plot of the linear attenuation coefficients for photoelectric absorption, Compton scattering and pair production in both high-purity germanium (HPGe) and bismuth germanate (BGO).

Photoelectric Absorption

Of the processes outlined in this section, photoelectric absorption is the dominant interaction process for low-energy photons ($0 < E < 100$ keV in HPGe). Photoelectric absorption is the interaction of a photon with an atom, where a complete transfer of energy occurs. The energy of the photon produces a *photoelectron* in its place whose energy is equal to the energy of the photon minus the binding energy of the previously bound electron. The hole created by the electron is typically filled by the capture of a free electron from a nearby atom and/or the deexcitation of electrons from other shells in the atom. Both of these processes emit characteristic low-energy

Auger electrons and/or X-rays which are absorbed nearby in the detector[26]. As this process depends on the atomic shell structure of the absorbing material, there is no single analytic formula which describes the probability of photoelectric absorption but a rough approximation is given by:

$$\tau \propto \frac{Z^n}{E_\gamma^{3.5}}, \quad (2.4)$$

where n varies between 4 and 5 for a given γ -ray energy [27]. The strong Z dependence in Equation 2.4 is the main reason high- Z materials, such as lead or tungsten, are used for γ -ray shielding.

Compton Scattering

Compton scattering is the dominant interaction for the typical γ -ray energies experienced in low-energy nuclear studies ($0.1 < E < 10$ MeV in HPGe). It is the interaction between a photon and an electron in the absorbing material which causes the photon to scatter at some angle θ relative to the incoming photon direction [25]. A photon can Compton scatter multiple times within a detector before losing enough energy to be photoelectrically absorbed.

If a photon exits the absorbing material following one or more Compton scatters, a fraction of the incident photon energy is left in the detector giving rise to a *Compton-continuum* of background below the full energy peak or *photopeak*. Compton suppressors are used to detect Compton-scattered photons in order to veto Compton-scattered events. The probability of Compton scattering is directly proportional to the available number of electrons in the absorbing material for a photon to scatter from, giving a linear Z dependence. Compton suppressors are thus typically made

from a high- Z high-density material in order to maximize the probability of interaction between the scattered photon and suppressor either by photoelectric absorption, or Compton scattering.

Pair Production

Pair production becomes the dominant γ -ray interaction at higher energies ($E > 5$ MeV). A γ ray with an energy above two times the rest mass of an electron (or 1.022 MeV) can interact with the Coulomb field of a nucleus in the absorbing material, producing an electron-positron pair. The energy of the incoming photon is split between the electron-positron pair. Within a material such as HPGe, both of the created electron and positron give up energy to the absorbing material through collisional losses (Section 2.2.1). The positron comes to rest in the material via collisions or bremsstrahlung before annihilating with an electron in the material creating two γ rays at 511 keV each. If one or both of these γ rays escape the detector, a γ ray will be recorded by the detector at the incident energy minus 511 or 1022 keV, called *single-* and *double-escape* peaks respectively.

Single- and double-escape peaks can lead to incorrect γ -ray energy and intensity determinations when there are real photopeaks of the same energy as the escape peaks. Luckily, Compton-suppression shields not only detect Compton-scattered photons, but can also detect the 511 keV γ -rays, significantly reducing the area of the single- and double-escape peaks.

2.3 The 8π γ -ray Spectrometer

High-resolution γ -ray spectroscopy offers unique advantages in low-energy nuclear physics research. With the ability to measure very-precise intensities and energies, important information on some nuclear properties can be gained in a way that is, at this time, unmatched by any other method. Currently installed at the ISAC-I hall at TRIUMF, the Canadian 8π spectrometer has been a powerful tool for nuclear decay studies at TRIUMF[28]. The 8π is an array of 20 high-purity germanium (HPGe) detectors sharing a mutual centre at a radioactive ion beam (RIB) implantation site. As shown in Figure 2.4, the HPGe detectors are situated in the hexagonal positions of a close-packed truncated icosahedron geometry (a “soccer ball”). Situated in the 10 pentagonal positions of the truncated icosahedron which are not used for the beam line, are barium fluoride and lanthanum bromide scintillators (DANTE) for the fast timing measurements required by many of the other nuclear structure experiments performed at the 8π facility.

The various components of an 8π γ -ray detector are shown in Figure 2.5. An 8π γ -ray detector consists of a cylindrical HPGe crystal within an aluminium housing. Surrounding the aluminium housing is a bismuth germanate (BGO) Compton-suppression shield. BGO is a relatively slow inorganic scintillator whose real advantages are its high density (7.13 g/cm^3) and large atomic Z which gives a large γ -ray interaction cross-section, making it an ideal material for tagging Compton-scattered γ rays leaving the HPGe crystal. To veto Compton-scattered events, the HPGe and BGO detectors are operated in anti-coincidence mode.

To prevent the BGO detectors from detecting γ rays which do not Compton scatter out of the HPGe crystal, 2.54 cm thick Heavimet collimators (denoted “HM” in Figure 2.5) are placed in front of the side BGO Compton-suppression shields. Heavimet

Figure 2.4: Photograph of one hemisphere of the 8π showing the geometry of the array. The HPGe detectors are covered by Delrin absorbers to both shield the HPGe crystals and limit bremsstrahlung background caused by high energy β particles. Housed between the HPGe detectors is the DANTE array used for fast-timing measurements. The free volume inside the array of HPGe detectors is used to house auxiliary detectors such as SCEPTAR and PACES. (Courtesy of TRIUMF).

is extremely good at shielding γ rays due to its very-high density ($17\text{--}19\text{ g/cm}^3$), and being primarily ($\approx 90\%$) composed of tungsten ($Z = 74$). However, as explained in Section 2.2.1, high- Z materials stop high-energy charged particles more abruptly than their lower Z counterparts, leading to a greater probability of bremsstrahlung. For the high-energy β -particles emitted from the decay of ^{74}Rb , a significant fraction of the energy loss in the Heavimet collimator will be due to bremsstrahlung. To limit the bremsstrahlung background in the HPGe crystals caused by high-energy β -particles striking the Heavimet collimators, a 1.27 cm thick Delrin absorber is placed at the front of each HPGe detector. Delrin, a low- Z plastic material, is weakly absorbing for γ rays while causing charged particles to lose energy primarily through collisional

Figure 2.5: Schematic diagram of a single 8π HPGe detector (courtesy of G. F. Grinyer).

loss. However, very-high-energy β -particles will still emit bremsstrahlung regardless of the added Delrin layer, requiring further methods to veto bremsstrahlung events (Section 2.6).

2.3.1 p-n Junction Semiconducting Diode Detectors

The HPGe detectors used in the 8π are examples of *semiconducting diode detectors* which take advantage of a reverse biased p-n junction formed by the combination of a *p-type* with an *n-type* material.

A *p-type* semiconductor is a material with trace amounts of an impurity which,

as a neutral atom, has one less valence electron than the bulk of the semiconducting material (boron in HPGe). If the impurity occupies a substitutional site within the bulk material lattice, one of the covalent bonds between the lattice and the substituted impurity is left unsatisfied. The missing electrons behave as “holes”, which act as positive charge carriers under the application of a bias.

An *n-type* semiconductor is the opposite of a p-type semiconductor; the impurity contains one more valence electron rather than one less (lithium in HPGe). This leads to an unbonded electron after the formation of the covalent bonds within the crystal lattice. The “free” electrons act as negative charge carriers when bias is applied to the crystal.

A *p-n junction* is formed when a p-type semiconductor is brought into contact with an n-type semiconductor. Due to the differing electron densities between the p and n-type materials, a diffusion of the electrons takes place from the n-type to the p-type, and holes from the p-type to the n-type materials. The build up of negative charge within the p-type, and positive charge in the n-type, creates an electric field gradient the so-called *contact potential* at the junction of the two materials, halting the diffusion process.

The region of changing electric potential is called the *depletion region*, as this zone is free of any mobile charge carriers. Any electrons or holes created in the depletion region are quickly swept out by the electric field causing a current to flow, making the p-n junction extremely useful for radiation detection. When ionizing radiation with enough energy strikes the depletion region of a p-n junction, electron-hole pairs are created, causing an energy-dependent current to flow through the semiconductor.

Unlike scintillators (Section 2.5.1), which require an average of ~ 100 eV to create a single information carrier, the average amount of energy required to form an

electron-hole pair is extremely small; on the order of 3 eV in HPGe[26]. For the typical energies encountered in low-energy nuclear physics studies (1 keV–10 MeV), a very large number of electron-hole pairs are created, and due to the relatively small statistical fluctuations to the number of collected charge carriers, a very-high energy resolution is obtained.

An unbiased p-n junction has a very small depletion region making the active volume of the detector very small. The unbiased contact potential is only on the order of 1 V making the movement of the charge carriers very slow. Furthermore, the high capacitance of the junction causes poor noise properties when connected to a preamplifier. To remove these issues, a bias is placed in the “reverse direction” as to not readily conduct current (i.e. a negative bias on the p-type material). The *reverse bias junction* has an electric field across the depletion region which is much larger than the contact voltage, causing rapid movement of the mobile charge carriers created by ionizing radiation. Moreover, the increase in potential across the p-n junction caused by the reverse bias greatly increases the thickness of the depletion region leading to larger active detector volumes. With a reverse bias applied the length of the depletion region of the detector, and thus the active detector thickness, is given by [26]:

$$d = \left(\frac{2\epsilon V}{eN} \right), \quad (2.5)$$

where V is the voltage applied across the junction, ϵ is the dielectric constant of the material, e is the electronic charge, and N is the net impurity concentration in the bulk semiconductor material. However, when a sufficient potential is applied across a semiconductor, the insulating properties of the material break down, and the semiconductor becomes a conductor. Even at a bias just below the *breakdown voltage*,

typical semiconductor-grade germanium has a depletion depth of only 1–3 mm. To increase the depletion depth further, the net impurity N of the germanium must be reduced. Techniques have been developed to refine the impurities in germanium to less than 10^9 atoms/cm³ or 1 part in 10^{12} . With these high-purity germanium crystals, a reverse bias voltage of only 3000–4000 V can deplete up to 5 cm, making HPGe semiconducting diode detectors powerful tools in high-resolution γ -ray spectroscopy.

2.4 The Pentagonal Array for Conversion Electron Spectroscopy (PACES)

The PACES detector system is an array of 5 lithium-drifted silicon (Si(Li)) diode detectors each with 5 mm thickness and an area of 250 mm² symmetrically placed within the upstream half of the target chamber. Each detector is placed approximately 3 cm from the beam spot and provides information on emitted conversion electrons.

2.4.1 Lithium-Drifted Silicon Detectors

Silicon, a low- Z semiconducting material, is advantageous in the detection of electrons since the probability of backscattered electrons is lower compared to most other semiconducting materials. Using a typical reverse-biased p-n junction semiconducting diode from Section 2.3.1, Equation 2.5 limits the size of the depletion depth based on the maximum bias and purity that can be physically achieved in the detector. Using the highest purity of silicon available, a reversed biased p-n junction silicon diode can obtain depletion depths of only 1–2 mm [26]. To compensate for the large silicon impurity, a technique called *lithium drifting* is used.

Silicon of the highest purity tends to be slightly p-type due to acceptor impurities.

Figure 2.6: Photograph of the PACES array. The normal 8π configuration was altered by replacing the upstream half of the SCEPTAR array with the single hemisphere of PACES.

By adding lithium — an electron donor — to the impure silicon, the net impurity can be reduced. To create the lithium-drifted silicon detector, the lithium is diffused through one end of the silicon creating an n-type part of the crystal. Reverse biasing the detector and raising the temperature to approximately 40°C , the lithium drifts towards the p-type region exactly cancelling out the acceptor impurities in the *intrinsic* or *compensated* region of the crystal which can be as large as 5–10 mm [26].

Figure 2.7 shows a schematic representation of a Si(Li) detector.

The intrinsic region of the *p-i-n* configuration serves as the active detector volume when a reversed-bias voltage is placed across the silicon crystal. When ionizing radiation strikes the intrinsic region, electron-hole pairs are created which migrate due to the electric field much like a typical p-n junction producing a measurable current directly proportional to the energy of the ionizing radiation.

Figure 2.7: Schematic representation of a Si(Li) detector showing the p-i-n configuration.

2.5 The SCEPTAR Plastic Scintillator Array

The Scintillating Electron-Positron Tagging Array (SCEPTAR) is an array of 20 thin (1.6 mm) plastic scintillating detectors placed within the volume subtended by the HPGe detectors of the 8π spectrometer. For the experiment described in this thesis, 10 of the detectors, or half of the array, were placed in the “downstream” half of the 8π array while the other half was removed to include PACES. SCEPTAR is used to tag the β particles emitted during a β -decay process. Each detector is placed in a closed-packed configuration approximately 4 cm from the beam implantation site allowing for 45% of the total solid angle coverage and very good β -particle detection efficiency.

2.5.1 Organic Scintillators

Organic scintillators are commonly made of very low- Z materials (BC-404 in SCEPTAR) making them essentially transparent to γ radiation. They are made of compounds which contain molecules with free valence electrons occupying π -*molecular orbitals*. When ionizing radiation interacts with one of these molecules, it is put into both an excited vibrational and excited π -electronic state. The vibrational state

Figure 2.8: Cutaway drawing showing approximately one quarter of the SCEPTAR array (courtesy of TRIUMF).

quickly relaxes through collisional processes to the ground state of the excited π -electronic state. The excited electronic state then decays to an excited vibrational state in the ground electronic state via *fluorescence*, with the whole process taking place on the order of a few nanoseconds [26]. Due to the smaller energy of the fluoresced light compared to the absorbed light required to excite a ground-state molecule, called the *Stokes shift*, the plastic scintillator is transparent to the fluoresced light allowing it to be collected by a photomultiplier tube and turned into a usable signal.

2.6 The Complete Array

Due to the low activity in this experiment with an exotic radioactive-ion beam, if a γ ray and electron are measured in a very short time interval ($\approx 1 \mu s$), the events are likely to have come from the same nuclear β -decay and are said to be in “coincidence”. The high efficiency of the SCEPTAR array ($\approx 40\%$) allows for very efficient $\beta - \gamma$

and $\beta - e^-$ coincidence detection. $\beta - e^-$ coincidences remove a significant amount of background in the β -sensitive PACES array by suppressing the β -singles background within the PACES data.

$\beta - \gamma$ coincidences can be used to remove the “room background”; γ rays from the decays of ^{40}K , the U-Th decay chains, various inelastic neutron scattering, etc. Furthermore, each SCEPTAR detector has a one-to-one correspondence with a HPGe detector; the SCEPTAR detector is placed directly in front of the HPGe detector’s Delrin absorber (Figure 2.9). As mentioned in Section 2.9, high-energy β -particles incident on one of the HPGe detectors can emit bremsstrahlung radiation. In this one-to-one configuration, a SCEPTAR detector which tags a β particle can veto the HPGe directly behind it highly suppressing bremsstrahlung background which becomes very important for the extremely weak γ -ray intensities involved in superallowed β -decay.

2.7 The Tape Transport System

The tape-transport system consists of a 13 mm wide, 50 μm thick, continuous aluminized Mylar tape. The tape is positioned such that the beam can be implanted at the mutual centre of the SCEPTAR and 8π arrays. The tape can be moved manually or by setting a Jorway timing and sequencing controller to move the tape automatically at a set time allowing the experiment to operate continuously. The utility of the tape-transport system is its ability to remove any long-lived contaminants built up at the beam spot to behind a thick lead wall located 2 m away from the 8π array (Figure 2.10). In doing so, the effects of in-beam and daughter contamination are minimized.

Figure 2.9: Photograph of SCEPTAR within the 8π array. The one-to-one positioning of SCEPTAR in front of the HPGe detectors makes it possible to veto bremsstrahlung caused by high-energy β -particles.

Figure 2.10: Photograph of the tape-transport system. The tape is collected behind a thick lead wall to shield the experimental apparatus from exposure to long-lived in-beam or daughter contaminants.

Chapter 3

Analysis of SCEPTAR data

The experiment to determine the superallowed branching ratio of ^{74}Rb was performed in November 2010 at TRIUMF in Vancouver, B.C., using the 8π and PACES spectrometers, as well as the “downstream” half of the SCEPTAR scintillator array (Chapter 2). TRIUMF’s main cyclotron was used to produce $98\ \mu\text{A}$ of 500 MeV protons which were sent to the ISAC facility and impinged on a stack of electrically heated $^{\text{nat}}\text{Nb}$ foils. Products of the spallation reactions on the Nb target were surface ionized and extracted as singly-charged positive ions. ^{74}Rb ions ($T_{1/2} = 64.776\ \text{ms}$)[29] were selected using the ISAC on-line mass separator and sent to the 8π spectrometer at a rate of $6.5 \times 10^3\ \text{ions/s}$. The largest beam contaminant was ^{74}Ga ($T_{1/2} = 8.16\ \text{min}$)[30, 31] which was delivered at $1.2 \times 10^3\ \text{ions/s}$.

The experiment was run in *cycles mode* to limit the interference of in-beam contaminants and daughter activities with the determination of the various γ -ray intensities following the decay of ^{74}Rb . The cycle consisted of counting background for 1 s before the radioactive-ion beam (RIB) was implanted on the aluminized Mylar tape at the mutual centre of the SCEPTAR and HPGe arrays. After counting background, the RIB was implanted on the Mylar tape for 20 s. Afterwards, a beam kicker after

the mass separator deflected the radioactive beam and the activity on the tape was counted for a further 2 s, before the tape was moved to remove the beam spot into the lead-shielded collector box, thereby removing the long-lived contaminants and allowing the cycle to be repeated.

3.1 β Gates

A SCEPTAR event was categorized by the detector ID, energy, and time flags. Gates were applied to the energy and time stamps during sorting of the raw SCEPTAR data to keep “good” events and remove “bad” events (or noise).

3.1.1 β Energy Gates

Low-energy threshold gates were set on each of the SCEPTAR paddles to discriminate between low-energy noise from the scintillators and real β -particle events. By removing low-energy noise, not only were false $\beta-\gamma$ coincidences removed, but potential false bremsstrahlung vetoes of the HPGe detectors were also avoided. Since the Q -value for the ^{74}Rb β -decay is large ($Q_{EC} = 10.4168(3.9)$ MeV[32]), the low-energy threshold is expected to introduce negligible bias in the β -detection efficiency for the various non-analogue β -decay branches; even those to the relatively high-energy (5-6 MeV) excited states in the daughter ^{74}Kr [33]. Figure 3.1 shows an example of where the SCEPTAR low-energy threshold gate was set.

3.1.2 β Time Gates

The timing of SCEPTAR events was extremely important in the determination of the superallowed branching ratio. β -correlated events are used to remove background,

Figure 3.1: A typical SCEPTAR energy deposition spectrum with the low-energy software gate (Red) to remove low-energy noise form the analysis.

but if the allowed timing separation of the β and β -delayed events is too small, real γ rays and conversion electrons will not be correlated with a β decay, falsely increasing the superallowed branching ratio. If the timing separation is too large, unnecessary time-random background will be added to the $\beta - \gamma$ and $\beta - e^-$ events as well as false bremsstrahlung suppression.

The high-precision timing information was recorded using multi-hit time-to-digital converters (TDC). The SCEPTAR TDC converted the fast signal output by the plastic scintillators into a digital bin representing the time the scintillator fired with respect to the master trigger with 0.5 ns precision. The master trigger for the SCEPTAR array was set to β -singles. Time gates were placed on the prompt-timing peak in the SCEPTAR TDC data as shown in Figure 3.2, such that a SCEPTAR hit with a TDC time outside of this window was rejected to remove false coincidences and

Figure 3.2: A typical SCEPTAR TDC spectrum (0.5 ns/bin) contracted to (4 ns/channel) with the prompt-timing gate (Red) to remove time-random β coincidences. The prompt time gate had a width of 88 ns.

false bremsstrahlung vetoing of the HPGe detectors.

To determine the absolute time that an event occurred relative to the start of the cycle, a Lecroy 2367 Universal Logic Module (ULM) was used. The ULM consisted of three 32-bit latching scalars, one of which counts pulses from the master clock; a Stanford Research Systems high-precision $10 \text{ MHz} \pm 0.1 \text{ Hz}$ temperature-stabilized oscillator, providing an absolute time stamp with respect to the start of a cycle with a precision of 100 ns. Comparing the ULM times between the master triggers of the different detector systems, coincidences between the detector streams can be reconstructed. As shown in Figure 3.3, detected β and γ events with ULM times within $2.4 \mu\text{s}$ were considered to be real $\beta - \gamma$ coincidences, while detected β and electron events with ULM times within $1.2 \mu\text{s}$ were considered to be real $\beta - e^-$ coincidences. Using the gates outlined above, the SCEPTAR data was then analyzed.

Figure 3.3: ULM time difference spectra (100 ns/channel) between (Left) $\beta - \gamma$ master triggers and (Right) $\beta - e^-$ master triggers with time gates (Red).

3.2 Cycle Selection

During the experimental run the RIB was interrupted for various technical reasons. This lead to cycles where there were no radioactive ions implanted on the tape. Since the data are dead-time affected, an assumption that the total β -events in a cycle was roughly the same for all cycles was required. Consequently, in order to fit the summed SCEPTAR data, cycles with much less than the typical number of β -events were rejected (Fig. 3.4).

To quantify the amount of each radioactive ion implanted on the tape, a fit to the activities of the radioactive ions in the beam was performed on a histogram of the β ULM times. However, the fit assumed a constant beam intensity, so a further cycle rejection was preformed to remove any cycles where the beam intensity fluctuated significantly (Figure 3.5). Since there were just less than 18000 cycles performed over

Figure 3.4: Total β events in each cycle for run 6408. Cycles with less β events than the rejection threshold (Red) were discarded from the experimental analysis.

the course of the experiment, statistical methods were employed to flag any bad cycles. Fitting the individual cycles with the fitting function to be explained in Section 3.3.1, a cycle with $\chi^2/\text{d.o.f} \gtrsim 1.2$ was flagged and upon review, if it was observed that the beam intensity fluctuated significantly, it was removed from the analysis.

After rejection, 16857 out of a total of 17863 cycles were kept for the analysis, which included $\approx 96\%$ of the total events recorded by the SCEPTAR detectors.

3.3 β -Counting

For this low-rate experiment, it was much more probable that a β particle was detected in one SCEPTAR detector and then scattered into a different SCEPTAR detector than it was for two different β particles to be detected in coincidence. Because of this, only one β decay was assumed per master trigger. Accordingly, to determine

Table 3.1: Summary of the runs used in the ^{74}Rb β -decay analysis.

Run	#of cycles used	Total cycles	Run	#of cycles used	Total cycles
6408	259	289	6453	269	285
6409	263	275	6454	327	334
6411	230	272	6455	334	349
6413	264	271	6456	346	358
6414	54	62	6457	281	299
6415	251	272	6458	346	381
6416	269	289	6459	152	157
6417	281	283	6460	337	360
6418	273	296	6461	144	162
6419	268	280	6462	64	67
6420	266	274	6463	254	256
6421	270	277	6464	188	195
6422	272	283	6465	28	28
6424	273	281	6467	344	350
6425	244	283	6468	280	350
6426	260	286	6469	0	1
6427	282	286	6470	344	346
6428	99	103	6471	344	352
6429	282	289	6477	65	66
6430	242	302	6478	346	349
6431	270	286	6480	335	344
6432	282	289	6481	235	238
6433	77	78	6482	105	105
6435	288	303	6486	221	238
6436	292	296	6487	231	241
6437	2	3	6488	217	222
6438	289	293	6489	211	224
6439	285	294	6490	127	129
6440	284	297	6491	24	30
6441	284	303	6492	234	247
6442	228	231	6494	245	252
6444	264	270	6495	248	256
6446	8	10	6496	2	3
6448	84	110	6497	251	261
6449	278	284	6498	254	261
6450	285	290	6499	230	261
6451	266	332	6500	169	179
6452	58	99	Sum:	16857	17863

Figure 3.5: A histogram of β ULM times (25 ms/channel) for run 6411 cycle #142. This is an example of a cycle with a fluctuating beam intensity and was therefore rejected from the analysis.

the total number of β events, the number of master triggers was counted. Due to contaminants in the beam, the β particles associated with each species could not be identified on an event-by-event basis, but were deconvoluted based on the implanted β -activity profile. The detected β events which passed the time and energy gates were thus histogrammed based on the ULM time and fitted by an activity function.

3.3.1 Fitting the β -Activity Curve

The histogram of SCEPTAR ULM times was fit using a maximum likelihood fitting algorithm[34]. The activity was fit assuming that ^{74}Rb , ^{74}Ga , and room background were the only contributions to the measured β activity. The activity was also fitted with a ^{74}Kr ($T_{1/2} = 11.50$ min)[35] component, which is the daughter of ^{74}Rb . In this case, the fitted intensity of the ^{74}Ga contaminant was found to be negative to

compensate for the amount of ^{74}Kr that there should have been as a result of the ^{74}Rb decays. The lack of detected ^{74}Kr decays was believed to be caused by the diffusion of the ^{74}Kr noble gas out of the aluminized mylar tape on a time scale shorter than its half-life. The effect of any ^{74}Kr decays on the determined number of ^{74}Rb decays was found to be negligible due to the relatively long half-life (11.50 min) compared to ^{74}Rb (64.776 ms), and the fact that its effectively linear grow-in during the 20 s implantation time was incorporated in the fitted intensity of the much more prominent ^{74}Ga contaminant.

The Fitting Function

The fit was performed using a maximum-likelihood method [34] using a function which contained the activities of each of the radioactive nuclei on the tape during each of the different periods of the tape cycle.

Before beam on: ($0s \leq t < 1s$)

$$A(t) = I_B. \tag{3.1}$$

During beam on: ($1s \leq t < 21s$)

$$A(t) = I_B + I_{\text{Rb}} \left(1 - e^{-\lambda_{\text{Rb}}(t-t_{\text{on}})}\right) + I_{\text{Ga}} \left(1 - e^{-\lambda_{\text{Ga}}(t-t_{\text{on}})}\right). \tag{3.2}$$

During beam off: ($21s \leq t < 23s$)

$$A(t) = I_B + I_{\text{Rb}} \left(1 - e^{-\lambda_{\text{Rb}}(t_{\text{off}} - t_{\text{on}})}\right) e^{-\lambda_{\text{Rb}}(t - t_{\text{off}})} \quad (3.3)$$

$$+ I_{\text{Ga}} \left(1 - e^{-\lambda_{\text{Ga}}(t_{\text{off}} - t_{\text{on}})}\right) e^{-\lambda_{\text{Ga}}(t - t_{\text{off}})},$$

where λ represents the decay constants of the respective nuclei. The data recorded were not continuous, but were collected within discrete time bins. Thus, the fit needed to be done to the integrated activity for each time bin,

$$y_i = \int_{t_i}^{t_{i+1}} A(t) dt. \quad (3.4)$$

During run 6461, the automatic liquid nitrogen fill for HPGe #10 did not work properly and this detector was brought to room temperature for the remainder of the ^{74}Rb data collection. In order to sum the entirety of the γ -ray and conversion electron data over the entirety of the experiment, the efficiency curves discussed in Section 4.3 were weighted by the number of β particles detected when all of the detectors were active compared to when HPGe #10 was removed. Therefore, both the β -activity curve from the entire experiment as well as the β -activity curve before the removal of HPGe #10 were fitted separately.

Dead-time Correction

The form of the β -activity fitting function assumes that the detection system is constantly sensitive to particle detection. However, when SCEPTAR is triggered, the data acquisition needs time to process the event. During this time, the array is “dead” and cannot be retrigged. Since the number of missed events due to the array being dead is rate-dependent, a correction for the detector dead-time was included

into the fitting function to account for the missing events. Since the experiment was performed in singles mode, the SCEPTAR dead-time is independent of the other detector systems, such as the HPGe detectors. Typically, data are dead-time corrected in the following way:

$$N' = \frac{N}{1 - N(\frac{d_t}{t_{bin}})}, \quad (3.5)$$

where N is the number of counts in a bin, d_t is the average dead-time per event, and t_{bin} is the time size of a bin. Since the data being fit are the sum of many cycles, Equation 3.5 was modified. The number of counts in a given bin was divided by the number of cycles (N_c), giving the average number of counts in that bin per cycle. Rather than modifying the experimental data, Equation 3.5 can be inverted, allowing the fitting function to be dead-time affected,

$$y' = \frac{y_i}{1 + \frac{y_i}{N_c}(\frac{d_t}{t_{bin}})}. \quad (3.6)$$

where y_i is the number of counts with no dead-time. The average SCEPTAR dead-time per event during the experiment was $22.2 \mu s$ as shown in Figure 3.6. The total experiment included 16857 cycles while 11245 cycles were completed before HPGe #10 was removed. Once the dead-time corrections were applied, the β -activity curves were fitted (Figure 3.7). The maximum dead-time correction on a single bin in the activity curve was 5.5%. All of the γ -ray efficiency curves that were dependent on the number of active detectors were then weighted using,

$$\epsilon' = \frac{\beta_{<6461}\epsilon_{all} + (\beta_{all} - \beta_{<6461})\epsilon_{no10}}{\beta_{all}}, \quad (3.7)$$

Figure 3.6: SCEPTAR dead-time per event over the course of the experiment. The average dead-time was taken and used to affect the fit to the β -activity measured by the ULM clock.

where $\beta_{<6461}$ is the number of ^{74}Rb β -particles measured before detector 10 was removed, β_{all} is the total number of β -particles measured over the course of the experiment, ϵ_{all} is the combined efficiency of all detectors, and ϵ_{no10} is the combined efficiency of the remaining detectors with #10 removed. To account for possible systematic uncertainties in the fit of the β -activity curve, the data were re-fit considering two different experimental scenarios. The data were fitted with the ^{74}Rb half-life fixed to 65.227 ± 0.078 ms with half the difference in the number of ^{74}Rb β -particles from the fits adopted as a systematic uncertainty. To explore sensitivity to the ^{74}Kr daughter decay, two more fits were performed: one where the contaminant was assumed to be entirely the longer lived ^{74}Kr with half-life = $11.50 + 0.11$ min, and another where the contaminant was the shorter lived ^{74}Ga with half-life = $8.16 - 0.12$ min. Half of the difference in the number of ^{74}Rb β -decays extracted from the fits was taken

Figure 3.7: (Top) β -activity curve for the cycles before HPGc #10 was removed. (Bottom) β -activity curve over the course of the entire experiment. The γ -ray efficiency curves (Section 4.3) in the two scenarios were weighted by the number of ^{74}Rb β -particles detected during each scenario.

as an additional systematic uncertainty. These very small systematic uncertainties shown in Table 3.3 were then combined in quadrature with the uncertainty in the ^{74}Rb intensity.

During the “beam-on analysis window” of 1 s to 21.43 s, a total of $8.241(3)\times 10^8$ and $5.6064(22)\times 10^8$ β -particles were detected during the whole experiment and before HPGe #10 was removed, respectively. Inflating the uncertainties by the $\sqrt{\chi^2/\nu}$ of the individual fits, the adopted number of detected β particles were $8.241(4)\times 10^8$ and $5.6064(25)\times 10^8$ during the whole experiment and before HPGe #10 was removed, respectively.

Table 3.2: Summary of results from the β -activity curve fit. The fit considered the activity produced from the β decay of ^{74}Rb , ^{74}Ga , as well as room background. The analysis time window was set from 1.00 s to 21.43 s.

Counts in time window				
	with HPGe 10	whole expt.	$T_{1/2}$	I_{avg} ion/s
^{74}Rb	$5.6064(22)\times 10^8$	$8.241(3)\times 10^8$	64.776 ms	6442
^{74}Ga	$1.550(4)\times 10^6$	$2.284(5)\times 10^6$	8.16 min	1212
Background	$8.16(4)\times 10^5$	$1.176(5)\times 10^6$	n.a.	9
χ^2/ν	1.285	1.117		
Adopted ^{74}Rb :	$5.6064(25)\times 10^8$	$8.241(4)\times 10^8$		
t_{on}	0.99990(3)	0.99997(3)		
t_{off}	20.00114(6)	20.00104(7)		

Table 3.3: Summary of the uncertainty contributions to the number of detected ^{74}Rb β particles. The uncertainties in the ^{74}Rb and contaminant half-lives, as well as the actual contaminants present were considered. The analysis time window was set from 1.00 s to 21.43 s.

Systematic Uncertainty	Adopted Uncertainty
^{74}Rb Half-life	1197
$^{74}\text{Ga}/^{74}\text{Kr}$ Interchange	2238
^{74}Rb Intensity	33260
Final Adopted Uncertainty:	33357

Chapter 4

Analysis of the 8π γ -ray Data

4.1 Data Selection for the γ -ray Analysis

4.1.1 Rejected Detectors

To measure the most precise and accurate branching ratios, statistics must be maximized while minimizing systematic uncertainties. Unfortunately, two of the detectors in the 8π array had technical issues which forced their removal from the data analysis. HPGe #3 had an electronics issue where two energy peaks were created in place of one (Figure 4.1). As the extra peaks could lead to false γ -ray identification as well as incorrect peak area determination, detector #3 was removed from the analysis.

Detector #11 was removed from the analysis since it was extremely noisy with very poor resolution during the γ -ray calibration data. Since it is unknown whether the efficiency of the detector was the same during the calibration as during the experiment, it was also discarded from the analysis. This left 18 out of the possible 20 HPGe detectors for most of the experiment, and 17 out of 20 after HPGe #10 had a liquid

Figure 4.1: (Top) Energy spectrum of a normal HPGe detector. (Bottom) “Double peaking” in HPGe #3 caused by electronics issues. This detector was discarded from the analysis.

nitrogen failure mid-way through the experiment.

4.1.2 γ -ray Time Gates

To select the data that would be used for the γ -ray analysis, the timing data of the HPGe and BGO detectors provided by multi-hit TDCs were used. Software gates were placed on the prompt times provided by the HPGe detector TDCs to eliminate time-random $\gamma - \gamma$ coincidences (Figure 4.2(a)). Likewise, gates were placed on the BGO TDC times to ensure that Compton-suppressed events were not time-random, but actually from Compton scattered events (Figure 4.2(b)). Due to an improper set-up for one of the BGO suppression shields, the BGO TDC on HPGe #16 was not correlated with HPGe #16, i.e. it did not measure Compton scattering when the HPGe was triggered. Instead, it was correlated with both HPGe #17 and #18

Figure 4.2: Typical TDC spectra with software gates (Red) for (a) the HPGe detectors and (b) the BGO suppression shields.

(Figure 4.3). To avoid vetoing good γ rays in detector # 16, Compton suppression was not performed on events where the BGO on detector #16 fired.

4.2 γ -ray Energy Calibration

Gain-Matching

When an 8π HPGe detector detects an event, an electronic signal generated by a preamplifier connected to the detector is sent to a spectroscopy amplifier which integrates and smooths the voltage pulse. The output from the spectroscopy amplifier, which is related to the total charge collected from the detector, is sent to an analog-to-digital converter (ADC). The ADC determines the “height” of the peak of the signal sent to it and converts this into a digital channel. The gain that is set on the spectroscopy amplifier determines the magnitude of the signal that is sent to the ADC, and thus, the bin that the event gets placed into. Before the experiment, each spectroscopy amplifier was roughly set to the same gain. In order to sum the data

Figure 4.3: The correlated hits between the HPGe detectors and BGO suppression shields. BGO 16 was not correlated with HPGe 16 meaning that to avoid false vetoing of Compton scattering, BGO 16 had to be rejected from the analysis.

from all available HPGe detectors, a set of strong photopeaks from each detector was aligned. To do this, a linear transformation was used to align the 276 keV ^{133}Ba and 4295 keV ^{66}Ga lines. Gain drift in a spectroscopic amplifier was corrected by aligning the centroids of the 456 keV and 1461 keV transitions of ^{74}Rb and ^{40}K respectively, in the gain-drifted detector with peaks in the summed spectrum of the well behaved detectors on a run-by-run basis.

However, the alignment of the two photopeaks was done using a linear transformation while it is well understood that the ADC response is nonlinear. To obtain a precision measurement for the γ -ray energies, a non-linear energy calibration was performed.

Energy Calibration

To a very good approximation, the digital channels output by the ADC are linear with respect to the number of charge carriers collected by the HPGe detector. However, the ADC modules do have a slight non-linearity that requires an energy calibration to extract the correct γ -ray energy. Typically, each ADC is well behaved and gives the equivalent non-linear output. The energy of a γ ray as a function of channel can be described by:

$$E(x) = M_l x + B_l + \Delta E(x), \quad (4.1)$$

where x is the channel number, and $\Delta E(x)$ is the non-linear term given by

$$\Delta E(x) = \begin{cases} A_1 x + B_1 & \text{if } x \leq \text{turnover}, \\ A_2 x^2 + B_2 x + C_2 & \text{if } x > \text{turnover}. \end{cases}$$

After gain-matching, it was observed that detector #14 was not well aligned at the intermediate energies between the photopeaks chosen for gain-matching (Fig. 4.4). Since many high-energy transitions were expected, it was extremely important to try to have the best possible gain matching in order to maximize energy resolution in the summed spectrum. To correct the unique output of this particular ADC a separate energy calibration was performed, and a simple transformation was taken from the detector #14 energy calibration to the energy function of the rest of the detectors, to ensure gain matching of all detectors at all energies.

Afterwards, when all of the detectors had been aligned a proper energy calibration was performed. Since the γ -ray analysis software RadWare allows only 4096×4096

Figure 4.4: Plot of Non-linearity of the ADC on detector 14 compared to the non-linearity of the other ADC modules.

matrices, the γ -ray spectra had to be contracted to fit into the available analysis window[36]. For γ rays below 4096 keV, a 1 keV/channel contraction of the energy spectrum was sufficient, while γ rays above 4096 keV required a 2 keV/channel contraction. Since contraction affects binning, the energy calibration was performed at both 1 and 2 keV/channel. For both binning contraction factors, the accepted γ -ray channel vs. energy was determined using ^{133}Ba , ^{152}Eu , and ^{66}Ga sources (Table 4.1). A linear fit was performed (Figure 4.5) on the channel vs. energy data and the residuals extracted. Using the residuals, the $\Delta E(x)$ term was fit using a linear function for channels below 1122 for 1 keV/channel and 560 for 2 keV/channel. Beyond the linear part of the residuals, a quadratic function was fit (Figure 4.5).

A systematic uncertainty is present in the ^{74}Rb γ -ray energy determination from small gain drifts in the spectroscopic amplifiers over the course of the experiment. Figure 4.7 shows the centroid of the 456 keV transition determined in each of the ^{74}Rb

Table 4.1: Data used for γ -ray calibrations. ^{133}Ba and ^{152}Eu data were taken from the Evaluated Nuclear Structure Data Files[37]. ^{66}Ga data were taken from [38]. The “Summing Corrections” are specific to the 8π and are described in Section 4.3.2.

Source	Energy (keV)	Relative Intensity	Summing Correction
^{133}Ba	160.6109(17)	645(8)	1.008
	223.2373(14)	450(4)	1.013
	276.3997(13)	7164(22)	1.017
	302.8510(6)	18330(60)	1.008
	356.0134(6)	62050(190)	1.007
	383.8480(12)	8940(30)	1.004
^{152}Eu	121.7817(3)	28670(150)	1.011
	244.6975(8)	7610(40)	1.014
	295.9392(17)	448(6)	1.016
	344.2785(12)	26600(500)	1.005
	367.7887(16)	862(10)	1.013
	411.1163(11)	2237(25)	1.011
	443.965(3)	3158(30)	1.012
	488.6792(20)	420(4)	1.016
	563.99(70)	491(7)	1.012
	586.2648(25)	460(7)	1.010
	678.623(5)	472(7)	1.013
	688.6700(50)	859(9)	1.008
	778.9040(18)	12960(140)	1.007
	867.373(3)	4260(30)	1.013
	964.079(18)	14650(70)	1.008
	1089.7371(5)	1730(20)	1.006
	1112.069(3)	13690(70)	1.007
	1212.948(11)	1426(9)	1.013
	1299.140(9)	1625(17)	1.007
	1408.006(3)	21070(100)	1.007
^{66}Ga	833.5324(21)	15930(50)	1.015
	1039.220(3)	100000(300)	1.009
	1333.1121(50)	3175(12)	1.012
	1918.329(5)	5368(21)	1.014
	2189.616(6)	14420(50)	1.007
	2422.5249(70)	5085(22)	1.015
	2751.835(5)	61350(230)	1.008
	3228.800(6)	4082(19)	0.998
	3380.8501(60)	3960(19)	1.000
	4085.853(9)	3445(18)	1.001
	4295.187(10)	10300(80)	1.001
	4461.2021(90)	2260(30)	1.000
	4806.0068(90)	5030(30)	1.001

Figure 4.5: (Top) Linear fit of the accepted γ -ray energy versus channel for the 1 keV/channel energy calibration. (Bottom) ΔE fit of residuals for 1 keV/channel energy calibration.

Figure 4.6: (Top) Linear fit of the accepted γ -ray energy versus channel for the 2 keV/channel energy calibration. (Bottom) ΔE fit of residuals for 2 keV/channel energy calibration.

Table 4.2: Energy calibration parameters for both of the 1 keV/channel and 2 keV/channel binning contractions. The turnover point between the linear and quadratic regions of ΔE occurs at channel 1122 for 1 keV/channel and 560 for 2 keV/channel.

	1 keV/chn	2 keV/chn
Linear Fit: $E(x) = M_l x + B_l$		
M_l	1.0002344(25)	2.0008381(73)
B_l	0.7915(30)	1.3271(50)
Linear ΔE Fit: $\Delta E(x) = A_1 x + B_1$		
A_1	0.002031(11)	0.003656(34)
B_1	-1.1534(61)	-1.171(10)
Quadratic ΔE Fit: $\Delta E(x) = A_2 x^2 + B_2 x + C_2$		
A_2	$4.20(52) \times 10^{-8}$	$1.06(32) \times 10^{-7}$
B_2	-0.001033(29)	-0.002350(84)
C_2	2.227(34)	2.164(47)

runs. There was no significant systematic gain shift, but if only a small fraction of the detectors drifted the extracted energy will be incorrect. To quantify the systematic uncertainty in the energy calibration, the high-precision energies of the transitions of the room background decay of ^{40}K as well as a weak ^{207}Bi source present in the 8π chamber were used. Figure 4.8 shows the difference between the measured energy and the accepted high-precision energies of the background lines during the ^{74}Rb decay experiment. An additional systematic uncertainty of 0.035 keV makes the measured energies of the ^{40}K and ^{207}Bi transitions consistent with the accepted energies of these transitions. Therefore a systematic uncertainty of 0.035 keV was added in quadrature to the energies determined for all of the transitions following the β -decay of ^{74}Rb .

Figure 4.7: A plot of the centroid of the 456 keV transition from ^{74}Rb decay determined in each run over the course of the experiment. There was no systematic gain drift observed during the course of the experiment.

4.3 γ -ray Efficiency Calibration

Due to the cryogenic liquid nitrogen filling issue that occurred with HPGe #10 during run 6461, the detector was removed for the remainder of the experiment. To keep the γ -ray data collected before the removal of the detector, the efficiency data had to be re-weighted by the relative number of detected β -events before and after the removal of this detector.

4.3.1 γ -ray Single-Escape Efficiency

In order to determine the absolute photopeak efficiency for the HPGe detectors at various energies, precisely known intensities of various γ -ray calibration sources were required. Due to the high Q-value of decay for ^{74}Rb , high-energy excited states in

Figure 4.8: A plot of the difference of the determined energy and accepted energy of the 570 and 1064 keV transitions in ^{207}Bi and the 1461 keV transition in ^{40}K . A systematic uncertainty of 0.035 keV was added to make the measured energies consistent with the accepted high-precision values.

^{74}Kr were populated followed by their deexcitation via high-energy γ -rays. In order to precisely determine the intensities of these high-energy γ -rays, high-energy γ -ray calibration sources were required, such as ^{66}Ga .

In principle, γ rays which undergo pair production followed by the escape of one or both of the 511 keV photons should be suppressed by the BGOs in the same manner as Compton scattered events. However, the BGOs are not 100% efficient at removing scattered γ rays — the removal of BGO 16 certainly did not improve this efficiency — and single escape peaks were observed. The *single-escape efficiency* is the probability that, as a function of energy, a single escape peak would be measured in the 8π data.

The 4806 keV photopeak in ^{66}Ga was observed to have a single-escape peak that interferes with the 4295 keV transition. To obtain the proper peak area of the 4295 keV

transition, the single-escape peak contribution was required to be quantified and subtracted. Furthermore, the single-escape efficiency was important since ^{74}Kr has an excited 0^+ state at 509 keV. To determine the experimental branching ratios, as well as identify any previously unobserved γ rays, it was extremely important that γ rays feeding the 509 keV state were separated from single-escape peaks created by the high-energy decays to the ground state.

To measure the single-escape peak efficiency, the ratio of the area of the single-escape peak to the area of the full energy photopeak was measured using the decay of ^{66}Ga . The ratio of the areas as a function of the full-energy peak was then fit to $R(E_F) = Ax^2 + B$ (Fig. 4.9) using a maximum likelihood fitting technique.

Table 4.3: The single-escape efficiency fit parameters from the fitting function $R(E_F) = Ax^2 + B$.

	no 10	with 10	Final(Weighted)
A	$1.0731(39) \times 10^{-8}$	$1.067(12) \times 10^{-8}$	$1.0690(21) \times 10^{-8}$
B	$-0.02629(28)$	$-0.02638(79)$	$-0.02635(63)$
$\chi^2/d.o.f$	0.99934	0.99968	

4.3.2 Relative Efficiency Calibration

A relative efficiency calibration was performed using the accepted γ -ray data from Table 4.1. To determine the relative efficiency, the areas of the photopeaks from the γ -rays emitted from the standard sources were measured, and divided by the intensities of the corresponding γ -ray transitions. The relative activities of the three sources were normalized to minimize the χ^2 fit and give a continuous efficiency function. A correction to the 4295 peak of ^{66}Ga was performed using the single-escape efficiency from section 4.3.1.

Figure 4.9: (Top) Single-escape efficiency for the 8π with HPGc #10 removed. (Bottom) Single-escape efficiency for the 8π with HPGc #10 included.

Summing Corrections

Emitted γ rays following a β -decay can strike the same HPGe detector causing what are called *summing effects*. For example, when more than one γ ray hits a detector, one of three possible scenarios can occur: (i) Both γ rays have their full energy recorded by the detector causing the signal from the HPGe detector to appear as though one γ ray was detected at an energy equal to the sum of the two distinct γ -rays, (ii) One of the γ -rays is fully absorbed but the other γ ray Compton scatters out of the detector causing a signal from the HPGe detector that does not correspond to the energy of either γ ray, or (iii) both γ rays Compton scatter out of the detector. Summing effects are dependent on the γ -ray multiplicity of a decay, and are thus transition dependent. Events of type (iii) are not of interest as neither γ ray would have been detected at its full energy independent of the other γ ray. Events of type (i) and (ii) effectively *lower* the measured efficiency for detecting each individual γ -ray and must be corrected for. Events of type (i) are also of particular importance if two lower energy γ rays, γ_1 and γ_2 , are in parallel with a third γ ray, γ_3 , such that $E_{\gamma_1} + E_{\gamma_2} = E_{\gamma_3}$. In this case summing of γ_1 and γ_2 in a single detector effectively *increases* the measured intensity for γ_3 and must also be corrected for.

In order to correct for summing effects *out of* the photopeaks (both of type (i) and (ii)), $\gamma - \gamma$ coincidences between *different* HPGe detectors were analyzed and it was assumed that the probability of a second γ ray hitting the same detector would be $1/(N - 1)$ times the probability of the measured γ -ray coincidence involving different detectors. A matrix of γ rays passing the BGO-suppression gate vs. γ rays that had no BGO-suppression was thus created. Projecting the matrix onto the γ -rays passing the BGO-suppression gate and fitting the peak areas, the estimated probability that

a good γ -ray event in a single detector was ruined by summing is given as:

$$P_{Ruin} = \frac{1}{A_{singles}} \frac{A_{\gamma-BGO}}{(N-1)}, \quad (4.2)$$

where $A_{singles}$ is the area of the peak of interest when no other HPGe fires, $A_{\gamma-BGO}$ is the area of the peak of interest in the BGO-suppressed projection of the above-mentioned $\gamma - \gamma$ matrix, and N is the number of active HPGe detectors.

To correct for summing effects *into* a photopeak, the probability of each γ -ray cascade, and the probability of measuring the sum of any 2 or more γ -rays appearing as γ_{tot} in a given HPGe detector was calculated from the product of the individual γ -ray efficiencies. A Monte-Carlo simulation generated γ rays following a β decay based on the input β -decay branching ratios and subsequent γ decay scheme, and simulated whether a given γ ray would be detected within a single detector based on the individual detector efficiencies. Once the number of summed γ -rays *into* a photopeak was determined, the area of the experimental photopeak was corrected using:

$$A' = A(1 - \frac{\text{summed}}{\text{total}}), \quad (4.3)$$

where A is the uncorrected experimental photopeak area, ‘summed’ is the simulated number of γ -rays which were summed *into* the peak, and ‘total’ was the simulated total number of γ -rays detected at the energy of the photopeak. A list of the summing corrections for each γ -ray calibration source is given in Table 4.1, and the summing corrections were applied to each γ -ray area before fitting the relative efficiencies.

A fit of the relative efficiency data was then performed using a maximum-likelihood

fitting algorithm. The relative efficiency was fit using:

$$\ln(\epsilon) = \sum_{i=0}^8 a_i [\ln(E)]^i, \quad (4.4)$$

where a_i are the fitted parameters and E is the measured γ -ray energy in MeV. Figure 4.10 shows the fitted γ -ray efficiency curve.

Table 4.4: Parameters for the γ -ray relative efficiency fit in Equation 4.4. The parameters with an uncertainty larger than their absolute value were fixed to zero.

	no 10	with 10
a_0	4.7329(14)	4.7991(16)
a_1	-0.6165(45)	-0.6318(28)
a_2	-0.0536(21)	-0.0823(54)
a_3	-0.0896(14)	-0.0451(27)
a_4	0	0
a_5	0.0159(21)	0
a_6	-0.0034(14)	-0.0132(16)
a_7	0	0
a_8	0	0
$\chi^2/d.o.f$	2.38	1.79

4.3.3 Absolute Efficiency

To obtain the absolute efficiency for the HPGe detectors, a ^{60}Co ($T_{1/2} = 1925.28$ days)[37] source with a precise activity was used. The activity of the source at the time of the experiment was $0.6581(68) \mu\text{Ci}$. By calculating the number of γ rays that were emitted following the decay of ^{60}Co (N_γ), and comparing that to the number of γ rays that were measured (A_γ), the absolute efficiency was determined.

$$\epsilon = \frac{A_\gamma}{N_\gamma}. \quad (4.5)$$

Figure 4.10: (Top) Relative γ -ray efficiency for all detectors not including detector 10. (Bottom) Relative γ -ray efficiency for all detectors.

To determine the number of γ rays that were emitted, the activity, the branching ratio of the γ rays measured, and the time that the nuclear β -decay was observed was required. The number of emitted γ -rays was:

$$N_{\gamma} = t_{live} \times I_{\gamma} \times A, \quad (4.6)$$

where $A = 0.6581(68) \mu\text{Ci}$, $t_{live} = 6378.392 \text{ s}$, and the intensity I_{γ} for the 1173 keV and 1332 keV γ rays per β decay are 99.85(3)% and 99.9826(6)%[37] respectively. The absolute efficiencies determined from the 1173 and 1332 keV γ -rays following the decay of ^{60}Co had to be corrected for various systematic effects. Before the absolute efficiency was applied to the relative efficiency curve, summing corrections (which were explained in Sec. 4.3.2) as well as pile-up corrections, were applied to the γ rays.

Pile-up Corrections

Pile-up occurs when at least two γ rays from different nuclear decay events strike the same detector before the ADC can integrate the total charge collected, but also on a long enough time scale as to be considered two distinguishable events and not summed. Since piled-up events do not give the correct energy of the detected γ rays, and pile-up probabilities are transition independent, any event flagged as piled-up by the spectroscopy amplifier circuitry was given a TDC time and rejected in the offline analysis. The number of events lost due to pile-up was corrected using[39]:

$$A'_{\gamma} = \frac{A_{\gamma}}{1 - \rho}, \quad (4.7)$$

where A_γ is the number of counts in the photopeak being corrected and ρ is the probability of piling-up given by:

$$\rho = \frac{PU}{PU + NP}, \quad (4.8)$$

where PU is the number of events with a good HPGe TDC time that have at least 1 pile-up time associated with them, and NP is the number of events with a good HPGe TDC time and no pile-up time associated with them, both without any condition on the HPGe energy. The pile-up probability for the ^{60}Co run was 0.289% for all detectors, and 0.255% when detector 10 was not included. After the pile-up and summing corrections were applied, the absolute efficiencies were determined to be $8.08(8) \times 10^{-3}$ and $7.46(8) \times 10^{-3}$ for the 1173 keV and 1332 keV γ rays respectively.

Table 4.5: Corrections to the absolute γ -ray efficiency for all detectors

Energy	ϵ	Pile-up Corr.	Summing Corr.	ϵ'
1173 keV	$8.03(8) \times 10^{-3}$	1.0029	1.0035	$8.08(8) \times 10^{-3}$
1332 keV	$7.41(8) \times 10^{-3}$	1.0029	1.0034	$7.46(8) \times 10^{-3}$

Table 4.6: Corrections to the absolute γ -ray efficiency without detector 10

Energy	ϵ	Pile-up Corr.	Summing Corr.	ϵ'
1173 keV	$7.66(8) \times 10^{-3}$	1.0026	1.0035	$7.70(8) \times 10^{-3}$
1332 keV	$7.06(7) \times 10^{-3}$	1.0026	1.0034	$7.11(7) \times 10^{-3}$

The absolute efficiencies gave normalization factors for the relative efficiency curves shown in Tables 4.7 and 4.8. Figure 4.11 shows the absolute efficiency curves for all of the detectors as well as when detector 10 was excluded.

Table 4.7: Normalization factors for the γ -ray efficiency for all detectors.

Energy	ϵ_{abs}	ϵ_{rel}	Norm.
1173 keV	$8.08(8) \times 10^{-3}$	109.50(18)	$7.38(8) \times 10^{-5}$
1332 keV	$7.46(8) \times 10^{-3}$	100.50(17)	$7.42(8) \times 10^{-5}$
Average			$7.40(8) \times 10^{-5}$

Table 4.8: Normalization factors for the γ -ray efficiency without detector 10.

Energy	ϵ_{abs}	ϵ_{rel}	Norm.
1173 keV	$7.70(8) \times 10^{-3}$	102.78(17)	$7.50(8) \times 10^{-5}$
1332 keV	$7.11(7) \times 10^{-3}$	94.57(16)	$7.51(8) \times 10^{-5}$
Average			$7.50(8) \times 10^{-5}$

4.4 γ -ray Analysis for the Decay of ^{74}Rb

4.4.1 Constraining Peak Widths

The photopeak width of the γ -ray detectors is energy dependent. In order to produce reasonable Gaussian fits to peaks with very low statistics, as well as identify possible doublets, the widths of the high-statistics photopeaks from the ^{74}Rb decay data were measured as a function of energy, and then fit to the function:

$$FWHM = \sqrt{A^2 + B^2(x/1000) + C^2(x/1000)^2}, \quad (4.9)$$

where $FWHM$ is the full width at half maximum of the peak and x is the channel number. The fit was performed for both 1 keV/channel and 2 keV/channel spectra (Figure 4.12). The $FWHM$ was then fixed to the energy-dependent value given by this expression when fitting the ^{74}Rb decay peak areas.

Figure 4.11: (Top) Absolute γ -ray efficiency for all detectors not including detector 10. (Bottom) Absolute γ -ray efficiency for all detectors. The ^{60}Co data points are displayed in green.

Table 4.9: Results used for determining the width constraint parameters for 1 keV/channel data.

Channel	Width(Chn)	Source
455.151	1.779(24)	^{74}Rb
568.731	1.878(14)	^{207}Bi
1061.667	2.2110(74)	^{207}Bi
1762.767	2.754(19)	^{214}Bi
2613.36	3.490(20)	^{208}Tl
A	B	C
1.563(40)	1.154(86)	0.955(36)

Table 4.10: Results used for determining the width constraint parameters for 2 keV/channel data.

Channel	Width(Chn)	Source
530.580	1.2350(95)	^{207}Bi
1175.845	1.762(25)	^{74}Ga
1306.426	1.849(11)	^{208}Tl
2466.800	2.80(30)	$^{28}\text{Si}(\text{n},\gamma)$
2809.150	3.05(18)	$^{58}\text{Ni}(\text{n},\gamma)$
A	B	C
0.80(13)	1.16(23)	0.77(18)

4.4.2 γ -ray Identification

To determine which γ rays were emitted following the decay of ^{74}Rb , a matrix was created which contained the energy of a measured γ -ray in coincidence with a β -particle on one axis and the ULM clock time in the cycle on the other axis. By placing a time gate on the beam on portion of the cycle, and comparing the γ -ray spectrum to the spectrum with a gate on the beam off portion, it was possible to distinguish γ rays emitted following the decay of ^{74}Rb from long-lived contaminants

Figure 4.12: Fit to constrain the γ -ray peak-width for (Top) 1 keV/channel data, (Bottom) 2 keV/channel data.

or room background (Figures 4.13 to 4.20).

4.4.3 γ -ray Intensities

The γ -ray intensity can be determined from the photopeak area, the efficiency, and the number of β -particles detected viz.,

$$I_{\gamma} = \frac{A_{\beta\gamma}}{\epsilon_{\gamma}\epsilon_{\beta}} \frac{\epsilon_{\beta}}{N_{\beta}} = \frac{A_{\beta\gamma}}{\epsilon_{\gamma}N_{\beta}}, \quad (4.10)$$

where $A_{\beta\gamma}$ is the area of the photopeak in $\beta - \gamma$ coincidence, ϵ_{γ} and ϵ_{β} are the efficiencies of the HPGe array and SCEPTAR respectively, and N_{β} is the number of β particles detected in SCEPTAR. However, this equation must be corrected for various systematic effects.

Dead-time Correction

Even at the low HPGe rates encountered during the decay of ^{74}Rb , the effects of dead-time (Section 3.3.1) must be accounted for. The determined dead-time correction for the HPGe detectors during the beam-on portion of the experiment was 1.011. No correction for the dead-time in the SCEPTAR data stream is required as it affects ϵ_{β} for $\beta - \gamma$ coincidences and ϵ_{β} for the number of detected β 's N_{β} in the same way and cancels in the ratio of Equation 4.10.

Summing Correction

The areas of the photopeaks following the β -decay of ^{74}Rb were corrected for summing effects as described in Section 4.3.2. The 456 keV transition, which was observed to have the largest peak area, had a summing correction of 1.0075 caused

Figure 4.13: Portions of the γ -ray spectra gated on the time windows defined in Table 3.2 between 0 and 500 keV. The top panel is the raw γ -ray spectrum, whereas the bottom two panels are bremsstrahlung suppressed $\beta - \gamma$ coincidence spectra.

Figure 4.14: Portions of the γ -ray spectra gated on the time windows defined in Table 3.2 between 500 and 1000 keV. The top panel is the raw γ -ray spectrum, whereas the bottom two panels are bremsstrahlung suppressed $\beta - \gamma$ coincidence spectra.

Figure 4.15: Portions of the γ -ray spectra gated on the time windows defined in Table 3.2 between 1000 and 2000 keV. The top panel is the raw γ -ray spectrum, whereas the bottom two panels are bremsstrahlung suppressed $\beta - \gamma$ coincidence spectra.

Figure 4.16: Portions of the γ -ray spectra gated on the time windows defined in Table 3.2 between 2000 and 3000 keV. The top panel is the raw γ -ray spectrum, whereas the bottom two panels are bremsstrahlung suppressed β - γ coincidence spectra.

Figure 4.17: Portions of the γ -ray spectra gated on the time windows defined in Table 3.2 between 3000 and 4000 keV. The top panel is the raw γ -ray spectrum, whereas the bottom two panels are bremsstrahlung suppressed β - γ coincidence spectra.

Figure 4.18: Portions of the γ -ray spectra gated on the time windows defined in Table 3.2 between 4000 and 5000 keV. The top panel is the raw γ -ray spectrum, whereas the bottom two panels are bremsstrahlung suppressed $\beta - \gamma$ coincidence spectra.

Figure 4.19: Portions of the γ -ray spectra gated on the time windows defined in Table 3.2 between 5000 and 6000 keV. The top panel is the raw γ -ray spectrum, whereas the bottom two panels are bremsstrahlung suppressed $\beta - \gamma$ coincidence spectra.

Figure 4.20: Portions of the γ -ray spectra gated on the time windows defined in Table 3.2 between 6000 and 7000 keV. The top panel is the raw γ -ray spectrum, whereas the bottom two panels are bremsstrahlung suppressed $\beta - \gamma$ coincidence spectra.

by γ -ray feeding from above while the 4245 keV to ground state transition had a summing correction of 1.0018. As these summing corrections were determined from the experimental $\gamma-\gamma$ coincidence data, they included the effects of 511 keV annihilation photons from the ^{74}Rb β^+ particles summing with the γ -rays of interest.

Correction for Bremsstrahlung Suppression

A correction had to be applied to the measured photopeaks because of bremsstrahlung suppression. When a SCEPTAR detector is triggered by a good β event, the corresponding HPGe detector is vetoed to remove possible bremsstrahlung radiation background caused by the β particle interacting with the Heavimet collimators. However, a γ ray could have been detected by the vetoed HPGe and the missing γ -ray efficiency must be accounted for. This effect did not occur during the absolute efficiency calibration which used a closed source. Since β particles can scatter off more than one SCEPTAR detector, thereby removing more than one HPGe detector from the event, the average number of SCEPTAR detectors struck needs to be accounted for. Also, SCEPTAR detector #3 did not have a corresponding HPGe detector behind it, so anything that hit SCEPTAR detector #3 did not have any vetoed γ -rays associated with it. The correction factor was given by:

$$\delta_{Brem.} = \frac{N_{HPGe}}{N_{HPGe} - \langle SC_{mult} \rangle}, \quad (4.11)$$

where N_{HPGe} was the number of active HPGe detectors in the array, and $\langle SC_{mult} \rangle$ was the average number of SCEPTAR detectors hit which were in front of active HPGe detectors. Over the course of the experiment, the average number of SCEPTAR detectors struck that were in front of an active HPGe detector was 0.9185 per detected

β event, giving $\delta_{Brem.} = 1.055$ when averaged over the data with $N_{HPGe} = 18$ and $N_{HPGe} = 17$.

SCEPTAR Efficiency

Typically ϵ_β can be divided out of Equation 4.10. However, the 495 keV conversion electron emitted following the decay of the 509 keV excited 0^+ state of ^{74}Kr following ^{74}Rb β decay was also detected by SCEPTAR with the same efficiency as a β particle, leading to an effective increase in the “ β ”-tagging efficiency for these events in which the 495 keV conversion electron is also emitted. The correction due to this increased “ β ”-tagging efficiency for γ -ray transitions which proceed through the 509 keV state is represented as $\frac{\epsilon_\beta}{\epsilon'_\beta}$ where ϵ'_β is the corrected efficiency. The true efficiency of the SCEPTAR array for detecting β particles was determined by taking the ratio $\frac{A_{\beta\gamma}}{A_\gamma}$ of the 456 keV γ -ray transition in γ -ray singles and β - γ coincidence, as this transition is *not* in coincidence with the 495 keV conversion electrons. This yielded $\epsilon_\beta = 38.3(4)\%$. The bremsstrahlung suppression of a HPGe detector due to the detection of a 495 keV conversion electron was also corrected. The probability that both the β particle and the 495 keV conversion electron hit different SCEPTAR detectors for an event proceeding through the 509 keV level is given as:

$$H_{\beta+e} = \frac{9}{10} \frac{I_{509}}{I_{509} + I_{53}} \epsilon_\beta^2, \quad (4.12)$$

where I is the intensity of the 509 and 53 keV transitions from the 509 keV level. It is assumed that the 39 keV conversion electrons from the 53 keV branch fall below the SCEPTAR detector threshold and are not detected as “ β ” particles. The probability that neither the β particle nor the 495 keV conversion electron hits a SCEPTAR

detector is:

$$H_{none} = \frac{I_{509}}{I_{53} + I_{509}}(1 - \epsilon_{\beta})^2. \quad (4.13)$$

Finally, the probability that the SCEPTAR array only registers one hit which includes the case where both the β particle and conversion electron hit the same detector is:

$$H_{\beta||e} = 1 - H_{\beta+e} - H_{none}, \quad (4.14)$$

while the corrected SCEPTAR efficiency is:

$$\epsilon'_{\beta} = H_{\beta||e} + H_{\beta+e}, \quad (4.15)$$

and the average number of SCEPTAR hits for these events is:

$$\langle H \rangle = H_{\beta||e} + (2 \times H_{\beta+e}), \quad (4.16)$$

or

$$\langle SC'_{mult} \rangle = \langle H \rangle \times \langle SC_{mult} \rangle, \quad (4.17)$$

which provides a correction to the bremsstrahlung suppression for transitions through the 509 keV level. To determine the SCEPTAR efficiency for transitions which must take more than one step to get to the 509 keV level, I_{509} was multiplied by the branching fraction of the γ ray to go to the 509 keV level in Equations 4.12 and 4.13. The results of the SCEPTAR efficiency corrections are given in Table 4.11.

Table 4.11: SCEPTAR efficiency and bremsstrahlung correction factors for transitions in ^{74}Kr based on the level the γ -ray transition feeds into.

Level Fed (keV)	509 Branching Fraction (%)	ϵ'_β (%)	$\delta_{Brem.}$	$\delta_{Brem.} \frac{\epsilon_\beta}{\epsilon'_\beta}$
0	0	38.3	1.055	1.055
456	0	38.3	1.055	1.055
509	100	51.1	1.063	0.797
1013	0	38.3	1.055	1.055
1203	14.0	40.1	1.056	1.008
1654	0	38.3	1.055	1.055
1742	64.7	46.6	1.060	0.872
2437	9.66	39.6	1.056	1.023

Internal Conversion

A correction to the transition intensity for electron conversion was performed for the transitions below 1.3 MeV using $A' = A(1 + \alpha)$. The internal conversion coefficients for the corrected transitions are shown in Table 4.12.

Table 4.12: Calculated conversion coefficients α for transitions in ^{74}Kr with $E < 1.3$ MeV. A correction due to electron conversion was performed using $A' = A(1 + \alpha)$.

Energy (keV)	$\alpha(\times 10^{-3})$
399.88(15)	3.62
455.84(4)	4.25
557.5(4)	2.31
694.11(14)	1.24
728.27(21)	1.09
747.67(7)	0.86
1197.94(7)	0.34
1203.40(7)	0.34
1233.21(8)	0.32
1286.44(13)	0.32

Contaminant Corrections

To correct for beam contaminants, a dedicated set of runs tuned to the mass of ^{74}Ga were used. The cycles consisted of counting for 60 s during implantation of the radioactive beam followed by 33 min of counting while the beam was turned off. The trigger mode was set to singles with a β scale-down factor of 5. Figure 4.21 shows the summed $\beta - \gamma$ coincidence spectrum for the ^{74}Ga runs. The most intense photopeak following the decay of ^{74}Ga is at 596 keV. By taking the ratio of the areas of other peaks in the spectrum to the 596 keV peak, the amount of ^{74}Ga contamination in a photopeak from the ^{74}Rb decay data could be accounted for:

$$A' = A_\gamma - A_{595}^{(^{74}\text{Rb})} \left(\frac{A_\gamma}{A_{595}} \right)^{(^{74}\text{Ga})}, \quad (4.18)$$

where A_γ is the area of the photopeak (including the contaminant) being corrected, $A_{596}^{(^{74}\text{Rb})}$ is the area of the 596 keV transition during the ^{74}Rb runs and $\left(\frac{A_\gamma}{A_{596}} \right)^{(^{74}\text{Ga})}$ is the ratio of the contaminating γ -ray to the 596 keV γ -ray during the dedicated ^{74}Ga runs. A similar correction was performed for the ^{74}Br contaminant using the 634 keV transition and the beam-off portion of the ^{74}Rb data. A summary of the contributed photopeak area caused by the beam contaminants and long-lived daughter decays are listed in Table 4.13.

Figure 4.21: Bremsstrahlung suppressed $\beta - \gamma$ coincidence spectra for the ^{74}Ga data. The largest intensity photopeak, at 596 keV, was used to correct for in-beam ^{74}Ga contamination during the ^{74}Rb experiment.

Table 4.13: Corrections to the γ rays in ^{74}Kr for beam contaminants and long-lived daughter decays. A dedicated run was used for ^{74}Ga corrections, while beam-off intensities were used for ^{74}Br . The 634 keV transition was used as a reference for ^{74}Br while the 596 keV transition was used as a reference for ^{74}Ga .

Energy (keV)	Contaminant	Ratio of Area to Reference (%)	Contributed Area
728.27(21) ^a	^{74}Br	2.0(3)	20(3)
1203.40(6)	^{74}Ga	5.217(20)	439(6)
1987.7(3) ^a		0.557(19)	46.9(17)
2437.29(13)		0.116(8)	9.8(6)
3639.1(6)		0.0158(25)	1.33(21)

a: Not placed in level scheme.

Single-Escape Peak Corrections

Single-escape peak corrections were performed using the single-escape efficiencies listed in Table 4.3. The contributed photopeak areas of irresolvable single-escape peaks are listed in Table 4.14.

Table 4.15 shows the intensities for the γ rays following the decay of ^{74}Rb , with the raw peak area corrected for summing effects, single escape, contaminants, SCEPTAR efficiency, internal conversion and dead-time.

Table 4.14: Single-escape corrections to ^{74}Kr γ rays that have been placed in the level scheme. The single-escape efficiency is given in Section 4.3.

Energy (keV)	Single Escape Parent (keV)	Raw Escape Parent Area	Single Escape Efficiency (%)	Contributed Area
1203.40(6)	1717	667(37)	0.608	4.06(9)
1233.21(7)	1745 ^a	240(30)	0.713	1.71(21)
1580.3(3)	2088 ^b	81(23)	2.122	1.7(5)
1927.4(4)	2437	468(28)	3.815	17.8(11)
2532.8(3)	3042	199(20)	7.363	14.7(15)
2827.10(20)	3337	244(21)	9.378	22.9(20)
2932.1(3)	3442	43(13)	10.01	4.4(13)
3225.65(20)	3735 ^c	352(23)	12.39	36.8(29)
3278.06(24)	3789	105(15)	12.83	13.5(19)
3303.81(27)	3813	88(14)	13.03	11.5(18)
3639.1(6)	4151	56(12)	15.91	8.8(19)
3735 ^c	4245	327(21) ^d	16.75	55(4)
3882.4(9)	4392	52(11)	18.11	9.4(20)
4245.06(28)	4759 ^b	47(10)	21.71	10.2(22)
4783.7(8)	5293	62(9)	21.96	17.0(25)
5175.1(9)	5683	26(7)	32.06	8.3(22)

a: From contaminant.

b: Not placed in level scheme.

c: Includes contributions of 3735 and 3734 keV γ rays.

d: Escape Parent corrected for single escape as well.

Table 4.15: Part 1: Intensities of the γ rays that have been placed in the level scheme of ^{74}Kr following the decay of ^{74}Rb .

Energy (keV)	Raw Peak Area	Corrected Area	Efficiency (%)	Efficiency Corrected Area	I (ppm)
399.88(15)	1286(184)	1342(192)	1.556(18)	86256(12383)	105(15)
455.84(4)	28273(249)	30512(269)	1.423(16)	2134427(30435)	2590(37)
557.5(4)	419(61)	455(66)	1.260(14)	36070(5266)	44(6)
694.12(14)	751(59)	612(48)	1.104(12)	55416(4393)	67(5)
728.27(21) ^b	262(50)	262(54)	1.072(11)	24400(5069)	30(6)
747.67(7)	1137(58)	1224(62)	1.055(11)	116012(042)	141(7)
1197.94(7)	3147(68)	3374(73)	0.787(8)	428839(10295)	520(12)
1203.40(7) ^{ab}	2216(59)	1904(64)	0.784(8)	242727(8505)	295(10)
1233.21(8) ^a	1530(52)	1242(42)	0.772(8)	160942(5729)	195(7)
1286.44(13)	477(38)	514(41)	0.750(8)	68512(5505)	83(7)
1580.3(3) ^a	152(29)	122(24)	0.650(7)	18739(3622)	23(4)
1594.6(3)	169(28)	151(25)	0.646(7)	23349(3876)	28(5)
1633.41(19)	246(30)	265(32)	0.635(7)	41802(5117)	51(6)
1716.73(11)	667(37)	716(40)	0.612(7)	117096(6615)	142(8)
1795.9(3)	120(27)	98(22)	0.591(6)	16616(3743)	20(5)
1849.37(26)	123(26)	132(28)	0.578(6)	22886(4844)	28(6)
1927.4(4) ^a	117(25)	81(20)	0.560(6)	14410(3640)	17(4)
1981.53(25)	181(28)	195(30)	0.548(6)	35516(5508)	43(7)
2081.30(12)	572(32)	614(34)	0.527(6)	116436(6640)	141(8)
2437.29(14) ^b	468(28)	492(30)	0.463(5)	106135(6604)	129(8)
2532.8(3) ^a	160(24)	149(25)	0.448(5)	33287(5521)	40(7)
2827.10(20) ^a	296(24)	221(20)	0.407(5)	54418(4848)	66(6)
2838.2(4)	96(18)	100(19)	0.405(5)	24748(4651)	30(6)
2863.72(21)	241(22)	214(20)	0.402(5)	53216(4906)	65(7)
2880.7(3)	139(19)	149(20)	0.400(5)	37355(5129)	45(6)
2932.1(3) ^a	110(19)	86(15)	0.393(5)	21824(3945)	26(5)
2985.6(3)	145(20)	155(21)	0.387(5)	40127(5561)	49(7)
3041.84(27)	199(20)	204(21)	0.380(5)	53738(5450)	65(7)
3225.65(20) ^a	393(25)	288(20)	0.359(5)	80098(5777)	97(7)
3278.06(24) ^a	216(20)	217(22)	0.354(5)	61400(6159)	75(7)
3303.81(27) ^a	127(17)	93(14)	0.351(5)	26564(3951)	32(5)
3336.83(22)	244(21)	261(22)	0.347(5)	75154(6568)	91(8)
3357.2(3)	106(17)	114(18)	0.345(5)	32877(5297)	40(6)
3403.2(6)	63(15)	68(16)	0.341(5)	19898(4748)	24(6)
3442.3(7)	43(13)	46(14)	0.337(5)	13619(4123)	17(5)
3531.4(4)	112(16)	91(13)	0.328(5)	27597(3969)	33(5)
3585.2(4)	88(15)	95(16)	0.323(5)	29371(5031)	36(6)

a: Centroid and area corrected for single-escape peak.

b: Area and centroid corrected for contaminant.

c: Area inferred from coincidences.

Table 4.16: Part 2: Intensities of the γ rays that have been placed in the level scheme of ^{74}Kr following the decay of ^{74}Rb .

Energy (keV)	Raw Peak Area	Corrected Area	Efficiency (%)	Efficiency Corrected Area	I (ppm)
3639.1(6) ^{ab}	55(14)	48(15)	0.318(6)	15053(4759)	18(6)
3734(1) ^a	243(75)	164(76)	0.309(6)	53116(24536)	64(30)
3736(1)	108(71) ^c	109(72)	0.309(6)	35256(23289)	43(28)
3789.2(3)	105(15)	113(16)	0.304(6)	37075(5342)	45(6)
3813.4(5)	88(14)	94(15)	0.302(6)	31047(4975)	38(6)
3882.4(9) ^a	33(12)	19(10)	0.296(6)	6466(3338)	8(4)
3935.5(5)	58(19)	63(21)	0.292(6)	21492(7054)	26(9)
4150.8(6)	56(12)	61(13)	0.275(6)	22036(4749)	27(6)
4245.06(28) ^a	337(21)	349(23)	0.267(6)	130583(9008)	158(11)
4391.7(6)	52(11)	56(12)	0.257(7)	21969(4683)	27(6)
4605.6(8)	36(10)	38(11)	0.242(7)	15893(4440)	19(5)
4674.1(12)	19(8)	19(8)	0.237(7)	8179(3453)	10(4)
4783.7(8) ^a	29(8)	10(7)	0.230(8)	4201(2940)	5(4)
5175.1(9) ^a	32(8)	25(14)	0.206(8)	1238(6824)	15(8)
5228.1(16)	20(6)	21(11)	0.203(9)	10397(5217)	13(6)
5292.9(5)	62(9)	66(10)	0.199(9)	33280(5045)	40(6)
5368.4(12)	17(6)	14(5)	0.195(9)	7039(2505)	9(3)
5421.6(7)	33(8)	36(9)	0.192(9)	18558(4582)	23(6)
5683.7(9)	26(7)	26(12)	0.178(10)	14789(6984)	18(9)

a: Centroid and area corrected for single-escape peak.

b: Area and centroid corrected for contaminant.

c: Area inferred from coincidences.

Table 4.17: Intensities of the γ rays that have not been placed in the level scheme of ^{74}Kr following the decay of ^{74}Rb . The uncertainty in the corrected area takes into account the unknown corrected SCEPTAR efficiency for these transitions.

Energy (keV)	Raw Peak Area	Corrected Area	Efficiency (%)	Efficiency Corrected Area	I (ppm)
950.0(4)	137(42)	139(7)	0.912(10)	15185(7694)	18(9)
956.9(3)	182(43)	184(80)	0.908(9)	20264(8843)	25(11)
1148.1(3)	161(38)	164(72)	0.809(8)	20276(8843)	25(11)
1408.08(6)	93(31)	94(50)	0.705(7)	13387(7141)	16(9)
1987.7(3) ^b	155(25)	117(51)	0.547(6)	21417(9250)	26(11)
2088.1(5)	81(23)	82(40)	0.526(6)	15658(7529)	19(9)
2222.3(3)	129(23)	131(49)	0.500(6)	26137(9892)	32(12)
2279.0(4)	111(23)	113(46)	0.490(6)	23106(9412)	28(11)
2429.6(3)	141(23)	143(52)	0.464(5)	30869(11215)	37(14)
2812.7(5)	70(18)	72(33)	0.409(5)	17664(8078)	21(10)
2937.9(7)	53(18)	54(29)	0.393(5)	13683(7386)	17(9)
2969.8(10)	32(16)	33(23)	0.389(5)	8418(5894)	10(7)
3872.5(7)	38(12)	39(20)	0.297(6)	13105(6764)	16(8)
4581.6(7)	58(11)	59(23)	0.243(7)	24115(9423)	29(11)
4758.7(7)	47(10)	48(20)	0.231(8)	20565(8515)	25(10)
5800.4(14)	14(6)	14(9)	0.172(1)	8238(5199)	10(6)
5820.0(7)	46(8)	47(18)	0.171(1)	27468(10391)	33(13)

a: Centroid and area corrected for single-escape peak.

b: Area and centroid corrected for contaminant.

c: Area inferred from coincidences.

Chapter 5

Analysis of the PACES Conversion Electron Data

5.1 Data Selection for the Conversion Electron Analysis

5.1.1 Conversion Electron Time Gates

Much like the analysis for the HPGe detectors, TDC time gates are generally placed in order to reduce false coincidences in the conversion electron data. During the setup of the DAQ electronics, a two-prong connector into the ULM was reversed flipping the polarity of the logic signal of the PACES trigger, affecting the TDC time relative to the master trigger. This produced PACES TDC times that were not simple to understand. In order to avoid rejecting “good” PACES events, the TDC gate was left wide open (Figure 5.1).

Figure 5.1: Typical TDC spectra for the PACES detectors. The time gate for the PACES array was left wide open due to an electronics issue.

5.2 Conversion e^- Efficiency Calibration

5.2.1 Relative Efficiency Calibration

The relative efficiency of the Si(Li) detectors was simulated using GEANT4[40]. The simulation consisted of a cylindrical piece of silicon with a radius of 7.979 mm, a thickness of 5 mm and positioned 3.0 cm from a point source. Electrons of different energies were emitted from the point source in random directions within the solid angle subtended by the silicon detector. As can be seen from Figure 5.2, appreciable loss of efficiency due to punch-through and bremsstrahlung are not present until approximately 1 MeV. Using this result, the efficiency of the PACES detectors was taken to be approximately constant across the range of energies for the conversion electrons in this experiment (≤ 600 keV).

Figure 5.2: The simulated intrinsic efficiency for detecting electrons of a PACES-like detector at various energies. The approximately constant efficiency of the detector below 600 keV was used when determining the absolute efficiency of PACES for the conversion electrons in this analysis.

5.2.2 Absolute Efficiency Calibration

To obtain an absolute efficiency for the PACES detectors, a RIB of ^{80}Rb was used. Following the decay of ^{80}Rb to ^{80}Kr , the first excited 2^+ decays by a 617 keV E2 transition to the ground state, with an intensity of 21.6(26)%. At this energy, there is a non-negligible probability for internal conversion. The absolute efficiency for each PACES detector was measured via the internal conversion coefficient, viz.,

$$\epsilon_e = \frac{A_e \epsilon_\gamma}{\alpha_K A_\gamma}, \quad (5.1)$$

where A_γ and A_e are the areas of the γ -ray and K-shell conversion electron peaks of the transition respectively, ϵ_γ is the efficiency of the 8π at the γ -ray energy

Figure 5.3: (Top) The bremsstrahlung suppressed $\beta - \gamma$ coincidence energy spectrum showing the 617 keV γ -ray following the β decay of ^{80}Rb . (Bottom) The $\beta - e$ coincidence energy spectrum showing the converted 617 keV transition.

(1.131(12)%) and α_K is the K-shell electron conversion coefficient which was calculated to be $1.527(22) \times 10^{-3}$ from a relativistic self-consistent Dirac-Fock method[41]. Before correcting the areas in Equation 5.1 for rate and transition dependent effects, the efficiency of the PACES array was found to be 3.03(6)% at an electron energy of 603 keV.

5.2.3 Corrections to Absolute Efficiency

Pile-Up

Due to the large activity of ^{80}Rb decay, the HPGe detectors as well as each of the Si(Li) detectors had to be corrected for pile-up using the same method as Section 4.3.3. The pile-up probability for the HPGe detectors given by Equation 4.8 was 0.878%,

while the β sensitive PACES array had a pile-up probability of 3.28%.

Dead-Time

The high rate of decay also caused a significant amount of dead-time in both the HPGe data stream as well as the Si(Li) data stream. The correction from Equation 3.5 was used to correct the dead-time of the Si(Li) array. The average dead-time for the PACES array during the ^{80}Rb portion of the experiment was $22.07\ \mu\text{s}$, giving a dead-time correction of 1.407 when applied to the PACES ULM data. A similar correction was performed for the γ -ray data. The average dead-time for the HPGe array was $29.10\ \mu\text{s}$ with a dead-time correction of 1.424.

Summing Corrections

During the decay of ^{80}Rb , there is a small but non-zero probability that two particles from the same decay will strike a single detector. A very small percentage of these events (on the order of 0.01% depending on the detector) will add together and give a sum peak, for example a K-conversion electron and a subsequent X-ray summing together in the same PACES detector to give an energy equal to the L or M conversion electrons. However, most of the time the partial energy detection of one or more of the particles will result in only a contribution to the background. In Section 4.3.2 the summing correction for HPGe detectors was discussed. PACES, being sensitive to β -particles, conversion electrons, X-rays, Auger electrons, and γ -rays, was simulated via Monte Carlo to account for summing effects. Since ^{80}Rb and ^{74}Rb both β decay to a krypton isotope, the atomic decay properties of the daughter nuclei are the same and in each case a pair of 511 keV photons is present in each event that could interact with a Si(Li) detector. Both ^{74}Rb and ^{80}Rb have very-low decay multiplicities for

conversion electrons and low multiplicities of coincident low-energy γ rays, so both decays are expected to have very similar “summing-out” corrections. Furthermore, fitting only the K-conversion electron guarantees no two electrons or X-rays can sum *into* the peak of interest based on the energy spacings of the atomic transitions. Since the summing *out* as well as the summing *in* corrections for conversion electrons were expected to be very similar in ^{74}Rb and ^{80}Rb , no summing corrections were applied to the conversion electrons emitted following the ^{80}Rb β -decay.

By estimating the number of coincident γ -rays hitting the same detector using $\gamma - \gamma$ coincidences (Section 4.3.2), the correction for the γ -ray summing *out* of the full energy peak was 0.288% with no probability of being summed into. A summary of all of the corrections for ^{80}Rb is given in Table 5.1.

Table 5.1: Area of the 617 keV transition, pile-up, summing and dead-time corrections for each detector involved in the PACES calibration.

Detector ID	Area ($\times 10^3$)	Pileup	Dead-time Correction	Summing
8π	7046(27)	1.009	1.424	1.003
0	6.86(13)	1.035	1.407	-
1	7.48(14)	1.040	1.407	-
2	4.78(13)	1.034	1.407	-
3	5.11(12)	1.031	1.407	-
4	5.87(13)	1.031	1.407	-
Sum	29.72(27)	1.034	1.407	-

Missing ADC Data

PACES was found to be missing a different fraction of the ADC data in the ^{80}Rb runs compared to the ^{74}Rb runs (Figure 5.4). To correct for the missing ADC data, the probability of losing an ADC event was determined by comparing the number of

Figure 5.4: The ADC loss fraction of the Si(Li) detectors during both the ^{80}Rb and ^{74}Rb portions of the experiment with and without β -coincidence conditions.

good TDC times with no ADC information to the total number of good TDC times. This ADC loss fraction was also determined using both the PACES singles data and PACES events that were in coincidence with a good β event in SCEPTAR. Using the loss fraction, the corrected number of electrons was:

$$N'_e = \frac{N_e}{1 - P_{ADC}}, \quad (5.2)$$

where N_e is the number of electrons in the peak of interest and P_{ADC} is the probability of a lost ADC or the *ADC loss fraction*. The ADC loss fraction is not completely understood, so to take this into account, the value of P_{ADC} was taken to be the β -coincident value as $\beta-e$ coincidences were used in the analysis of the ^{74}Rb data, while the difference between the singles and β -coincident ADC loss fractions was taken as

an additional systematic uncertainty. The absolute efficiencies calculated using each correction are given in Table 5.2.

Table 5.2: A summary of the absolute efficiency of the PACES detectors determined using the ^{80}Rb data and both sets of ADC correction factors.

Detector ID	β -Coinc. ADC loss(%)	Absolute Efficiency (%)	Singles ADC loss(%)	Absolute Efficiency (%)
0	3.04	0.757(19)	3.49	0.760(19)
1	12.5	0.919(23)	16.1	0.959(24)
2	8.89	0.526(17)	8.11	0.521(17)
3	3.48	0.565(17)	3.68	0.566(17)
4	1.65	0.636(17)	2.06	0.639(17)
Sum	6.28	3.339(68)	6.98	3.418(69)

To be sure that the very different detector rates between the ^{80}Rb and ^{74}Rb runs were not causing further unforeseen complications, the γ -ray information from Chapter 4 was used to perform an internal calibration of the PACES detector efficiencies. The internal calibration eliminates all pile-up, dead-time, summing and ADC effects when applied to the data, with a cost in statistical uncertainty.

5.3 PACES Efficiency Check

To internally calibrate the PACES efficiency, gates were set on the β -coincident γ -rays feeding into the 509 keV state. Each of these γ -rays must have been followed by a 53 or 509 keV transition. Assuming that the efficiency of PACES is constant as a function of energy, the efficiency of PACES was determined using:

$$\epsilon_{Si} = \frac{\sum A_e^{(\gamma-e)}}{\sum A_\gamma}, \quad (5.3)$$

where A_γ is the area of a γ ray feeding the 509 keV state in $\beta - \gamma$ coincidence and $A_e^{(\gamma-e)}$ is the *total* projected area of the conversion electron peaks in triple coincidence with these γ -rays as well as a β particle in SCEPTAR.

The effective SCEPTAR efficiency for γ rays feeding the 509 keV state was increased due to the 54% $E0$ branch of the 509 keV state to decay by a 495 keV conversion electron which was detectable by SCEPTAR. Using the measured SCEPTAR efficiency of 38.3% for transitions not feeding the 509 keV level, the increased SCEPTAR efficiency was calculated to be 51.1%, with a bremsstrahlung veto rate of 1.063/event. Table 5.3 shows the corrected areas of each of the transitions used in internally determining the PACES efficiency.

Table 5.3: A summary of the peak areas used in internally calibrating the PACES efficiency. The γ -ray areas are in $\beta - \gamma$ coincidence and have been corrected for the effective increase in SCEPTAR efficiency due to the detection of the 495 keV conversion electron in SCEPTAR. The conversion electron areas are in triple coincidence with β particles and a sum of gates on all of the γ -rays feeding the 509 keV level.

γ -rays		Conversion e^-	
Energy (keV)	Area	Energy (keV)	Area
695	601(47)	53	39(7)
1233	1221(43)	509	55(10)
1580	120(23)		
1796	96(22)		
1927	79(20)		
2828	218(19)		
2933	84(15)		
3226	284(20)		
3304	92(14)		
3532	89(13)		
3883	19(10)		
5369	14(5)		
Sum	2917(354)	Sum	94(12)
Efficiency		3.21(55)%	

The efficiency from this method was determined to be 3.21(55)% in excellent agreement with the external ^{80}Rb calibration value of 3.339(68)%.

5.4 Conversion Electrons From ^{74}Rb Decay

Two internally converted transitions were observed following the β -decay of ^{74}Rb . Both transitions are from the deexcitation of the first excited 0^+ level at 509.40(17) keV. Figure 5.5 shows both the heavily converted 53.23(14) keV $E2$ transition to the first excited 2^+ state and the 509.40(17) keV $E0$ transition to the ground state. Figure 5.6 shows the $\beta - \gamma$ -coincident PACES spectrum for all γ -ray energies above 511 keV. In this spectrum the 53 keV L-shell conversion electron is more pronounced than in the β -coincident case due to the suppression of background by requiring the additional γ -ray coincidence. The energy of the transitions was determined with combinations of

Figure 5.5: β -coincident PACES energy spectrum for ^{74}Rb .

Figure 5.6: β – γ -coincident PACES energy spectrum for ^{74}Rb . The coincident γ -rays had the condition that $E_\gamma > 511$ keV.

the measured γ -ray energies of Section 4.4. The K-conversion electron for both transitions was fit and corrected using the same corrections described in Section 5.2.3. The ratio of 509-K/53-K was determined to be 1.27(9) in the β -coincident spectrum shown in Figure 5.5 and 1.49(23) when the conversion electrons were in coincidence with both a β particle in SCEPTAR and a γ -ray with energy greater than 511 keV as shown in Figure 5.6.

To correct for dead-time, the average PACES dead-time of $22.27 \mu\text{s}$ was used. This gave a dead-time correction factor of 1.012 for the PACES data during the ^{74}Rb β -decay experiment.

Table 5.4: A summary of corrections for the ^{74}Rb conversion electron data.

Detector ID	Pileup (%)	Dead-time Correction (%)	β -Coinc. ADC loss(%)	Singles ADC loss(%)
0	0.077	1.2	12.1	4.90
1	0.098	1.2	4.03	8.72
2	0.085	1.2	19.8	10.9
3	0.067	1.2	10.9	5.74
4	0.064	1.2	5.36	1.95
Sum	0.078	1.2	10.7	6.64

The 53 keV Transition

For the 53 keV $E2$ transition, the total electron intensity of the internally converted transition could be calculated using $\frac{\alpha_{tot}}{\alpha_K}$. The calculated value for $\frac{\alpha_{tot}}{\alpha_K}$ from a relativistic self-consistent Dirac–Fock method using the “Frozen Orbital” approximation was 1.268(31) [41]. The total intensity I_{tot} , including the γ -ray intensity, could be calculated from the electron intensity I_e using:

$$I_{tot} = \left(\frac{\alpha_{tot} + 1}{\alpha_{tot}} \right) I_e, \quad (5.4)$$

which gave a value for $\frac{I_{tot}}{I_K} = 1.406(33)$. The calculated ratio was then applied to the K-shell converted electron intensity to get a total intensity of $38(3) \times 10^{-5}$ per ^{74}Rb β decay (Table 5.5). The intensity determined using the internal calibration was $35(8) \times 10^{-5}$.

Table 5.5: The intensity of the 53 keV transition following the decay of ^{74}Rb . The value of I_{tot} was derived from “ I_K Coinc.”, which used the β -coincidence ADC loss fraction from Table 5.4 while the difference in the intensity with “ I_K Singles”, which used the PACES singles ADC loss fractions from Table 5.4, was taken as an additional systematic uncertainty.

Detector ID	Raw Area K-Shell	I_K Singles ($\times 10^{-5}$)	I_K Coinc. ($\times 10^{-5}$)	I_{tot} ($\times 10^{-5}$)
0	1498(290)	25.4(50)	27.7(54)	39(8)
1	1638(178)	23.0(26)	22.8(25)	32(4)
2	885(176)	23.4(48)	25.8(52)	36(7)
3	1026(171)	23.6(40)	25.0(42)	35(6)
4	1397(171)	27.4(34)	28.5(36)	40(5)
Sum	6582(458)	25.3(18)	26.7(19)	38(3)
	Internal Calibration Intensity:			35(8)

The 509 keV Transition

For the 509 keV $E0$ transition, the total intensity had to be calculated from the ratio of electronic factors Ω of each of the electron subshells viz.,

$$\frac{\sum_j \Omega_j}{\Omega_K}, \quad (5.5)$$

where Ω_j is the electronic factor for the j^{th} subshell.

To date, no calculation of Ω_K has been performed for a transition in ^{74}Kr below 511 keV, but due to the linearity of Ω_K in this region, the values calculated by Passoja *et al.* [42] were extrapolated down to 509 keV. The L_1 and L_2 values were taken from Bell *et al.* [43]. For an $E0$ transition to occur, there must be significant overlap of the electron and nuclear wavefunctions. Strictly speaking, the converted electrons should come predominantly from the $S_{1/2}$ shells and approximately in the ratio $K:L1:M1 = 1 : \frac{1}{8} : \frac{1}{27}$ [44]. When calculating the ratio of the K-shell conversion

Figure 5.7: The intensity of the 53 keV transition for each of the 5 PACES detectors as well as the sum of the detectors (blue) and the intensity via the internal calibration method (green).

electrons to the total intensity of the 509 transition, the approximation $\Omega_{M_1} \approx \frac{8}{27}\Omega_{L_1}$ was used.

By itself, Ω is not an experimentally measurable quantity, with no assigned uncertainties in the calculated values. However, the ratio in Equation 5.5 can be determined experimentally and has found to vary up to 5% from the calculated value in some cases [45]. Thus, the conservative uncertainty of 5% was adopted for the calculated ratio of electronic factors for the 509 keV transition in ^{74}Kr .

Table 5.6: The calculated electronic factors used for the determination of I_{tot}/I_K . Subshells not listed were taken to be ~ 0 .

Subshell	Calculated Value	Reference
K	2.632×10^8	Passoja[42]
L ₁	2.859×10^7	Bell[43]
L ₂	1.124×10^5	Bell[43]
M ₁	8.471×10^6	Church[44]
Ratio	1.141(57)	

The calculated ratio of $\frac{\Omega_{L+}}{\Omega_K} = 0.141$, while using the data shown in Figure 5.5, the experimentally determined ratio was 0.141(7), in excellent agreement with the calculated result.

Equation 5.5 was applied to the intensity of the K-shell conversion electron to determine a total 509 intensity of $44(3) \times 10^{-5}$ per ^{74}Rb β decay (Table 5.7). The intensity determined by using the internal calibration was $41(7) \times 10^{-5}$.

The branching ratio of the 509 keV level decaying via the ground-state transition was determined to be $BR(509) = 54(4)\%$ while the decay of the 509 keV level via the transition to the first excited 2_1^+ state was determined to be $BR(53) = 46(4)\%$.

Table 5.7: The intensity of the 509 keV transition following the decay of ^{74}Rb . The value of I_{tot} was derived from “ I_K Coinc.”, which used the β -coincidence ADC loss fraction from Table 5.4 while the difference in the intensity with “ I_K Singles”, which used the PACES singles ADC loss fractions from Table 5.4, was taken as an additional systematic uncertainty.

Detector ID	Raw Area K-Shell	I_K Singles ($\times 10^{-5}$)	I_K Coinc. ($\times 10^{-5}$)	I_{tot} ($\times 10^{-5}$)
0	2133(53)	36.2(13)	39.4(14)	45(5)
1	2970(63)	41.7(14)	41.4(14)	47.2(29)
2	1588(50)	42.0(19)	46.3(21)	53(6)
3	1399(46)	32.2(14)	34.2(15)	39(3)
4	1385(45)	27.2(17)	28.3(12)	32.2(25)
Sum	8378(111)	36.8(9)	38.7(9)	44(3)
Internal Calibration Intensity:				41(7)

Figure 5.8: The intensity of the 509 keV transition for each of the 5 PACES detectors as well as the sum of the detectors (blue) and the intensity via the internal calibration method (green).

Chapter 6

Discussion and Conclusions

6.1 ^{74}Rb Decay Scheme

The decay scheme of ^{74}Kr populated in ^{74}Rb decay, as determined in this work is shown in Figure 6.1. Most of the γ rays in the level scheme were confirmed using γ – γ coincidences. The previously known β -decay level scheme contained the 456, 509, 1204 and 1742 keV levels with tentative 1654 and 4245 keV levels. Previously known transitions included the 53, 456, 509, 695, 748, 1198, 1233 and 1286 keV transitions and a tentative 4245 keV transition. To find new transitions and levels, coincidence gates were placed on the previously observed 456 keV transition (Figure 6.3), the converted 509 keV transition (Figure 6.3) as well as the 748, 694 and 1204 keV γ -rays following the decay of the 1204 keV level. These gates confirmed the location of the 1654 and 4245 keV levels. Once new transitions and levels were placed using this method, gates on new γ -rays could be made to further expand the level scheme. For instance, gating on the γ -rays emitted following the deexcitation of the 2437 keV level (1927, 1982, 2437 keV) a γ -ray at 400 keV was observed corresponding to the transition between the 2837 and 2437 keV levels in ^{74}Kr .

Figure 6.1: Decay scheme of ^{74}Kr populated through β -decay of ^{74}Rb . The thickness of the arrows represents the relative intensity of the transitions.

Figure 6.2: (Top) γ -ray energy spectrum in $\beta - \gamma$ coincidence. (Bottom) γ rays between 525 and 2500 keV in coincidence with the 456-keV γ ray following the decay of ^{74}Rb .

Once all possible coincidences had been exhausted, γ rays without observed coincidences were placed between two levels in the ^{74}Kr level scheme whose energy difference corresponded to the energy of the placed transition. Note that some γ rays could not be conclusively placed in the level scheme and are listed in Table 4.17.

6.1.1 Assignment of spin-parities

Typically, to determine the spin-parity of an excited state using γ -ray spectroscopy, the angular correlation of γ -rays is used. However, the vast number of weakly populated excited states in ^{74}Kr makes the detection of transitions feeding high-energy states in ^{74}Kr difficult, especially when coupled with the low efficiency

Figure 6.3: (Top) γ -ray energy spectrum in $\beta - \gamma$ coincidence. (Bottom) γ rays between 525 and 3000 keV in coincidence with the 53 and 509 keV conversion electrons as well as the 13 keV K_{α} X-rays detected in PACES following the decay of ^{74}Rb .

of the HPGe detectors at high energies. Sufficient $\gamma - \gamma$ coincidence statistics for angular correlation analysis were thus not available in this experiment. Rather than looking for full energy peaks for high-energy γ -rays where the efficiencies are on the order of $\approx 0.2 - 0.3\%$, the Compton scattered events can be used to essentially increase the ‘efficiency’ of the detectors ≈ 50 -fold to a significant fraction of their combined solid angle ($\approx 13\%$). To do so, a coincidence matrix was constructed between good HPGe events that were Compton-suppressed vs. good HPGe events that had no Compton-suppression applied. To remove false Compton- γ coincidence events caused by positron annihilation, a gate on all γ rays above 511 keV was set on the Compton-suppressed events. Bremsstrahlung suppression was also required to veto events

that were actually bremsstrahlung background instead of Compton background.

The ratio of the areas of the γ rays in $\beta - \text{Compton} - \gamma$ coincidence to the $\beta - \gamma$ coincidence which directly fed the ground and first excited states were taken when statistically permitted. This ratio is related to the average multiplicity of γ rays above 511 keV emitted in coincidence with the γ ray of interest. For the remainder of this work the ratio will be called the *Compton-multiplicity*. In cases where the area of the $\beta - \text{Compton} - \gamma$ peak was 0, a bremsstrahlung-suppressed Compton - γ coincidence was taken and corrected for the SCEPTAR efficiency to obtain an upper limit. However, this upper limit was slightly overestimated since only half of the SCEPTAR array was included allowing only half of the HPGe array to be bremsstrahlung suppressed.

This method exploits the fact that the Compton-multiplicity should be different for levels differing in spin due to the β decay selection rules. The Compton-multiplicity for γ -rays originating from 2^+ levels should be comparatively large as these levels must be fed by γ rays from higher-lying 0^+ and 1^+ states with direct β -feeding being 2nd forbidden and thus totally negligible. Next, 0^+ states have lower average Compton-multiplicity since these states can be fed by γ decay from 1^+ and 2^+ states but also potentially have some weak non-analogue β -feeding. Finally, γ -rays decaying from the 1^+ states directly to the ground or 2_1^+ state should have next to no Compton-multiplicity as these states are largely populated directly by Gamow-Teller β -decay rather than γ decay from above. Since these statements are only general guidelines, as transition strength is also strongly dependent on the nuclear wavefunctions involved, the assignment of the spin-parities using this method are clearly tentative at best.

The ratio of $\beta - \text{Compton} - \gamma$ to $\beta - \gamma$ -coincidences of transitions to the ground and first excited states is shown in Figure 6.4. The previously assigned spin-parities

Figure 6.4: The ratio of the area of β -Compton- γ coincidences to the area of β - γ coincidences for (Top) ground-state transitions and (Bottom) transitions to the first excited state at 456 keV. Data points with energy labels in blue were estimated using Compton- γ coincidences and correcting for the $\approx 40\%$ efficiency of SCEPTAR.

of levels are from references Piechaczek[46] and Korten[47].

2⁺ Levels

The previously known 2⁺ states display similar average Compton multiplicities clustered around 0.05. The relatively large Compton-multiplicities are consistent with states that are not accessible by direct β -decay but are always in coincidence with at least 1 other γ -ray feeding from above. Each of the 456, 1203 and 1742 keV states which were previously assigned as 2⁺ states give ratios of approximately 5%. The Compton multiplicity of the 1013 keV 4₁⁺ state is approximately twice this value,

or 0.10, as this level must be fed by the higher-lying 2^+ states and is expected to be fed from above by a cascade of at least 2 coincident γ -rays.

The levels at 2089 and 2305 keV are consistent with excited 2^+ states since they have high Compton-multiplicities and transitions to both the 0_1^+ and 2_2^+ states.

The 2437 keV state appears to have no Compton-multiplicity. However, this state is fed from above by the 400 keV γ -ray transition — from the 2837 keV level — which is removed by the energy gate on the Compton background above 511 keV. The 2437 keV level decays to the 0_{gs}^+ , 0_1^+ and 2_1^+ levels, while the 2837 keV level decays to the 0_{gs}^+ and (2_6^+) states, consistent with the 2437 and 2837 keV levels both having a spin-parity of 2^+ .

The 3442 and 4392 keV levels also have Compton-multiplicities consistent with 2^+ states and decay to the 0_{gs}^+ , 2_1^+ and 0_1^+ levels.

0^+ Levels

Perhaps most interesting of all, is the possible appearance of new 0^+ states which allow direct tests of the δ_{C_1} isospin-symmetry-breaking correction components. At lower, but generally non-zero Compton-multiplicity, are the 0^+ states. The Compton-multiplicities consistent with 0^+ states are at 1654, 2173, and 2537 keV. The 0^+ state at 1654 keV was previously seen in Coulex[47], and the similar Compton-multiplicities of the 2173 and 2537 keV levels provide tentative evidence that these are also 0^+ states. The levels at 1654, 2173 and 2537 keV are also not observed to decay to any 0^+ states as these would be γ -forbidden $E0$ transitions, further strengthening the assignment of these states as excited 0^+ levels.

1⁺ Levels

Finally, the remaining levels that have coincident γ -ray multiplicities consistent with 0, or do not have sufficient statistics for this method, are assigned as tentative (1⁺) states. Each of these levels is observed to decay to at least one 0⁺ state removing the possibility for a 0⁺ assignment. However, these states may be weakly fed by low-energy $M1$ or $E2$ transitions from above which do not satisfy the energy gate above 511 keV. Given the high-excitation energy of the levels involved, feeding by low-energy γ -ray transitions from above appears less likely than direct Gamow-Teller β -feeding of 1⁺ states, warranting a tentative spin-parity assignment of (1⁺) for these states. The first excited (1⁺) level is observed to be at 3337 keV, although this level is not populated as strongly as some of the more highly excited (1⁺) levels.

6.2 Superaligned Branching Ratio for ⁷⁴Rb

For every excited state that was observed to be populated in ⁷⁴Kr following the β -decay of ⁷⁴Rb, the total feeding and emitted γ -ray peak areas are presented in Tables 6.1 through 6.4. The superallowed β -decay of ⁷⁴Rb leaves the daughter nucleus ⁷⁴Kr in the ground state. The non-superaligned β -decay of ⁷⁴Rb leaves ⁷⁴Kr in an excited state which must decay by the emission of internal conversion electrons or γ -ray transitions. This means that any γ -ray or electron intensity directly feeding the ground state must be from non-superaligned β -decay. The total observed γ -ray and conversion electron intensity feeding the ground-state was:

$$I_{gs} = 0.396(7)\%. \tag{6.1}$$

Table 6.1: Part 1: Level Energies, and transition intensities for levels > 3 MeV. The final column represents the sum of direct β plus unobserved γ -ray feeding. Note that the uncertainty in the $I_{out} - I_{in}$ is not the individual I_γ uncertainties added in quadrature due to common uncertainties.

Level Energy (keV)	J^π	Feeding Transition (keV)	Intensity (ppm)	Emitted Transition (keV)	Intensity (ppm)	$I_{out} - I_{in}$ (ppm)
5877.4(7)	(1_9^+)	None Observed	—	4674 5368 5422	10(4) 9(3) 23(6)	41(7)
5683.7(9)	(1_8^+)	None Observed	—	5175 5228 5684	15(8) 13(6) 18(9)	46(8)
5292.9(5)	(1_7^+)	None Observed	—	3639 4784 5293	18(6) 5(4) 40(6)	64(9)
4605.6(8)	(1_6^+)	None Observed	—	2864 3403 4151 4606	65(7) 24(6) 27(6) 19(5)	135(11)
4391.7(6)	(2_9^+)	None Observed	—	3882 3936 4392	8(4) 26(9) 27(6)	61(11)
4245.06(28)	(1_5^+)	None Observed	—	3042 3736 3789 4245	65(7) 43(28) 45(6) 158(11)	311(32)
4041.0(4)	(1_4^+)	None Observed	—	3531 3585	33(5) 36(6)	69(8)
3813.4(5)	(1_3^+)	None Observed	—	3304 3357 3813	32(5) 40(6) 38(6)	110(10)
3733.91(24)	(1_2^+)	None Observed	—	2533 3226 3278 3734	40(7) 97(7) 75(7) 64(30)	277(32)
3442.3(7)	(2_8^+)	None Observed	—	2932 2986 3442	26(5) 49(7) 17(5)	92(10)
3336.83(22)	(1_1^+)	None Observed	—	1595 2827 2881 3337	28(5) 66(6) 45(6) 91(8)	231(13)

Table 6.2: Part 2: Level Energies, and transition intensities for levels < 3 MeV and > 0.5 MeV. The final column represents the sum of direct β plus unobserved γ -ray feeding. Note that the uncertainty in the $I_{out} - I_{in}$ is not the individual I_γ uncertainties added in quadrature due to common uncertainties.

Level Energy (keV)	J^π	Feeding Transition (keV)	Intensity (ppm)	Emitted Transition (keV)	Intensity (ppm)	$I_{out} - I_{in}$ (ppm)
2837.17(29)	(2_7^+)	None	—	400	105(15)	135(16)
		Observed		2838	30(6)	
2537.14(12)	(0_4^+)	None	—	2081	141(8)	141(8)
		Observed				
2437.29(14)	(2_6^+)	400	105(15)	1927	17(4)	85(19)
				1982	43(7)	
				2437	129(8)	
2305.21(26)	(2_5^+)	None	—	1796	20(5)	48(7)
		Observed		1849	28(6)	
2172.58(11)	(0_3^+)	None	—	1717	142(8)	142(8)
		Observed				
2089.25(19)	(2_4^+)	None	—	1580	23(4)	73(8)
		Observed		1633	51(6)	
1742.29(13)	2_3^+	1595	28(5)	728	30(6)	215(14)
		2864	65(7)	1233	195(7)	
				1286	83(7)	
1653.78(7)	0_2^+	3639	18(6)	1198	520(12)	502(14)
1203.40(7)	2_2^+	2533	40(7)	694	67(5)	363(18)
		3042	65(7)	748	141(7)	
		3403	24(6)	1203	295(10)	
		4674	10(4)			
1013.3(4)	4_1^+	728	30(6)	557	44(6)	14(9)
509.40(17)	0_1^+	694	67(5)	53	375(34)	160(58)
		1233	195(7)	509	442(33)	
		1580	23(4)			
		1796	20(5)			
		1927	17(4)			
		2827	66(6)			
		2932	26(5)			
		3226	97(7)			
		3304	32(5)			
		3531	33(5)			
		3736	43(28)			
		3882	8(4)			
		4784	5(4)			
		5175	15(8)			
		5368	9(3)			

Table 6.3: Part 3: Level Energy, and transition intensities for 456 keV level. The final column represents the unobserved γ -ray feeding. Note that the uncertainty in the $I_{out} - I_{in}$ is not the individual I_γ uncertainties added in quadrature due to common uncertainties.

Level Energy (keV)	J^π	Feeding Transition (keV)	Intensity (ppm)	Emitted Transition (keV)	Intensity (ppm)	$I_{out} - I_{in}$ (ppm)
455.84(4)	2_1^+	53	375(34)	456	2590(37)	645(53)
		557	44(6)			
		748	141(7)			
		1198	520(12)			
		1286	83(7)			
		1633	51(6)			
		1717	142(8)			
		1849	28(6)			
		1982	43(7)			
		2081	141(8)			
		2881	45(6)			
		2986	49(7)			
		3278	75(7)			
		3357	40(6)			
		3585	36(6)			
		3789	45(6)			
		3936	26(9)			
		4151	27(6)			
		5228	13(6)			
		5422	23(6)			

This however, is a lower limit to the non-superaligned β -decay intensity as it does not account for unobserved ground-state feeding by weak γ -rays not detected in this experiment. Based on the very-large Q -value (10.4 MeV) of the ^{74}Rb decay, it has been predicted that there exists over 400 β -decay accessible 1^+ states in the ^{74}Kr daughter nucleus[8]. The very-weak β -decay branching ratio to each of these excited 1^+ states may give rise to γ -ray intensities which are below the current experimental sensitivity, but which combine to become significant; the so-called ‘‘Pandemonium’’ effect[22].

Table 6.4: Transition intensities to the ground state. The final column represents the total observed γ -ray feeding. Note that the uncertainty in the I_{in} is not the individual I_γ uncertainties added in quadrature due to common uncertainties.

J^π	Feeding Transition (keV)	Intensity (ppm)	I_{in} (ppm)
0_{gs}^+	456	2590(37)	3960(70)
	509	442(33)	
	1203	295(10)	
	2437	129(8)	
	2838	30(6)	
	3337	91(8)	
	3442	17(5)	
	3734	64(9)	
	3813	38(6)	
	4245	158(11)	
	4392	27(6)	
	4606	19(5)	
	5293	40(6)	
	5684	18(9)	

To estimate the amount of unobserved feeding to the ground state I'_{gs} , we consider the low-lying 2_1^+ *collector state*. Since the direct β -feeding of a 2^+ level is a 2nd forbidden process, the entire detected population of a 2^+ state must be from γ -ray feeding from above. The difference between the “collected” intensity emitted following the deexcitation of the 2^+ level and the observed intensity feeding the collector state is a direct measurement of the total unobserved feeding to the collector state. The unobserved γ -ray feeding to the first excited 2_1^+ state in this experiment was:

$$I'_{2_1^+} = 0.0645(53)\%. \quad (6.2)$$

The unobserved γ -ray intensity to the ground state I'_{gs} of interest and the unobserved feeding of the 2_1^+ collector state $I'_{2_1^+}$ can be related by defining the branching

ratio[16, 48]

$$\overline{B}_{gs} = \frac{I'_{gs}}{I'_{gs} + I'_{2^+}}. \quad (6.3)$$

Rearranging Equation 6.3, the unobserved ground-state intensity can be determined viz.,

$$I'_{gs} = \frac{I'_{2^+} \overline{B}_{gs}}{1 - \overline{B}_{gs}}. \quad (6.4)$$

Table 6.5: The predicted γ -ray and direct nonsuperaligned feeding to the six lowest 0^+ and 2^+ states in ^{74}Kr per β -decay[46].

Level	Expt. $GT + F$ (ppm)	Theory			
		$GT^a + F$ (ppm)	$GT^b + F$ (ppm)	GT^a (%)	GT^b (%)
g.s., 0_{gs}^+	631(37)	1030	2590	26.4	28.2
456, 2_1^+	1471(48)	1450	3240	37.3	35.2
509, 0_1^+	555(49)	600+360	1580+360	15.3	17.2
1203, 2_2^+	503(14)	410	870	10.6	9.4
1654, 0_2^+	520(13)	200+230	460+230	5.0	5.0
1742, 2_3^+	278(10)	210	460	5.4	5.0
Sum	3958(70)	4490	9790	100	100

a: Lowest 1^+ level at 3.6 MeV, $g_A = 0.8$.

b: Lowest 1^+ level at 3.2 MeV, $g_A = 1.0$.

Estimate of $\overline{B}_{gs}$ using the 2_1^+ state

A theoretical estimate of $\overline{B}_{gs}$ was obtained using a shell-model calculation[46] with the effective Gamow-Teller strength quenched to $g_A = 0.8$. The γ -ray feeding following Gamow-Teller β decay to excited 1^+ states in ^{74}Kr was calculated for the 6 lowest identified 0^+ and 2^+ states (Table 6.5). The energy of the first excited 1^+ level in the calculation was 3.6 MeV in reasonable agreement with the tentative assignment

of the first (1^+) level at 3.337 MeV in this experiment. The calculated value of $\overline{B}_{gs} = 0.414$ is obtained from the average ratio of the γ -ray feeding intensity of many excited 1^+ states to the ground state and the first excited 2_1^+ state weighted by the amount of β -feeding to the excited 1^+ levels. This gave an unobserved ground-state feeding intensity of $I'_{gs} = 456(37)$ ppm. Combining this with the observed ground-state intensity $I_{gs} = 0.396(7)\%$ gives a non-superalowed branching ratio of $0.442(8)\%$. The combined non-superalowed-Fermi and Gamow-Teller feeding predicted by the shell-model calculation with $g_A = 0.8$ and the first 1^+ state at 3.6 MeV was 0.449% , which is in very good agreement with the $0.442(8)\%$ determined here. Furthermore, the $g_A = 0.8$ calculation predicts an intensity of 1030 ppm for the ground-state Gamow-Teller feeding from excited states above 1742 keV in excellent agreement with the combined observed plus unobserved intensity of 1087(52).

However, in order to investigate systematics in the theoretical branching ratio, the same calculation was performed with the full strength weak axial-vector coupling of $g_A = 1.0$ and the first excited 1^+ state at 3.2 MeV, which gave a value of $\overline{B}_{gs} = 0.445$. The unobserved ground state feeding in this case was $I'_{gs} = 517(42)$ ppm. Combining this with the observed ground-state intensity gives a non-superalowed branching ratio of $0.448(8)\%$, while the superallowed-Fermi and Gamow-Teller feeding predicted by the shell-model calculation with $g_A = 1.0$ and the first 1^+ state at 3.2 MeV was 0.979% , overestimating the non-superalowed branching ratio by a factor of 2. Furthermore, the $g_A = 1.0$ calculation predicts an intensity of 2590 ppm for the ground-state Gamow-Teller feeding from excited states above 1742 keV which is also roughly a factor of 2 larger than the combined observed plus unobserved intensity of 1148(56).

The shell-model calculation with $g_A = 0.8$ and the first 1^+ state at 3.6 MeV reproduces the experimental γ plus non-analogue β feeding of the first 5 excited states

of ^{74}Kr very well (Table 6.5) making it the best available calculation to estimate the unobserved branching ratio. The same calculation with $g_A = 1.0$ and the first excited 1^+ state at 3.2 MeV was not consistent with the absolute feeding intensity, but yields very similar γ -decay branching ratios to the ground state and low-lying 2^+ states, and hence a very similar prediction for $\overline{B}_{gs}$. Taking 50% of the value of $\overline{B}_{gs}$ obtained using these two calculations, a generous uncertainty in the result of $\overline{B}_{gs}$, we adopt the value of $\overline{B}_{gs} = 0.4(2)$, and obtain a range for the possible values of $I'_{gs} = 161 - 967$ ppm.

Estimate of $\overline{B}_{gs}$ using the 2_1^+ , 2_2^+ and 2_3^+ states

Another way to estimate the unobserved ground-state γ -ray feeding, is by using the γ -ray feeding branching ratios to multiple collector states. The unobserved feeding to the first three excited 2^+ states was:

$$\begin{aligned}
 I'_{2^+} &= I'_{2_1^+} + I'_{2_2^+} + I'_{2_3^+} \\
 &= 645(53) + 363(18) + 215(14) \text{ ppm} \\
 &= 1223(57) \text{ ppm}.
 \end{aligned} \tag{6.5}$$

The same calculations as mentioned previously are employed, but now also incorporating the 2_2^+ and 2_3^+ states. Using the shell-model calculation with $g_A = 0.8$, the value of $\overline{B}_{gs} = 0.33$ was calculated, and we thereby obtain $I'_{gs} = 602(28)$ ppm, giving a non-superaligned intensity of 0.456(7)%. Similarly, using the shell-model calculation with $g_A = 1.0$, the value of $\overline{B}_{gs} = 0.36$ was calculated, and we obtain $I'_{gs} = 688(32)$ ppm with a total non-superaligned intensity of 0.465(8)%. The value of $\overline{B}_{gs}$ using this method is arguably more precise, as we have averaged the γ -ray feeding from

above across more 2^+ states, removing undue sensitivity to the specific wavefunction of the 2_1^+ state. In this case we now consider a 33% relative uncertainty to the value of $\overline{B}_{gs}$ obtained from the calculation with $g_A = 0.8$.

Adopting the value of $\overline{B}_{gs} = 0.33(11)$, we obtain a range for the possible values of $I'_{gs} = 344 - 961$ ppm.

Adopted value for I'_{gs}

The final adopted value for I'_{gs} was obtained by taking the average of the $g_A = 0.8$ calculations using the first 2_1^+ excited state, and the first three 2^+ states. A generous uncertainty was then adopted to cover the entire range of unobserved ground-state feeding intensities obtained using both shell-model calculations for $\overline{B}_{gs}$. We thus adopt the unobserved γ -ray feeding to the ground state of:

$$I'_{gs} = 529(500) \text{ ppm} = 0.05(5)\%. \quad (6.6)$$

Combining this with the observed ground-state intensity $I_{gs} = 0.396(7)\%$ gives a non-superaligned branching ratio of $0.45(5)\%$ and a superaligned branching ratio of $99.55(5)\%$.

It should be noted that the experimental γ -ray branching ratios to the first excited 2_1^+ state and ground state based only on the transitions which were *observed* following the decay of ^{74}Rb were 23.5% and 33%, respectively. These γ -ray branching ratios give a value for $\overline{B}_{gs} = 0.584$, which is arguably slightly larger than the average over all of the 1^+ states since the observed 1^+ states are likely the lowest energy 1^+ states, and hence have stronger energy weighting factors in favour of decay to the ground state. Nevertheless, the experimentally determined value of $\overline{B}_{gs}$ is covered by the

adopted branching ratio of 0.4(2). To further emphasize that the uncertainty in I'_{gs} is clearly conservative, and indeed covers the entire range of possible values, the unobserved γ -ray intensity into the ground state from levels above the 1742 keV level was determined based entirely on experimentally measured γ -ray intensities:

$$I'_{gs} = \left(\frac{I_{gs}}{I_{2_1^+}} \right) I'_{2_1^+} = 493(52) \text{ ppm}, \quad (6.7)$$

yielding a value in excellent agreement with the adopted value of 500 ppm for the unobserved γ -ray intensity to the ground-state.

The superallowed branching ratio from this work, 99.55(5)% is in good agreement with, but is a factor of 2 more precise than the only previous precision measurement of 99.5(1)%[46]. Although these two measurements, both performed at ISAC, employed different experimental apparatus and are statistically independent, both used the same theoretical calculation of $\overline{B}_{gs}$ in order to estimate the unobserved ground-state intensity I'_{gs} . As it is the theoretical uncertainty in $\overline{B}_{gs}$ that dominates the final uncertainty in the superallowed branching ratio, we do not average the above results, but rather supersede the earlier work of [46] with the higher statistics and more precise branching ratio measurement reported here. With the large number of new γ rays and levels observed in this experiment, the ^{74}Rb superallowed branching ratio of 99.55(5)% has now achieved the 0.05% precision benchmark required as input to precision Standard Model tests through superallowed Fermi β -decay $\mathcal{F}t$ -values.

6.3 ^{74}Rb superallowed ft and $\mathcal{F}t$ -Values

Recent experiments performed at TRIUMF aiming to reduce the uncertainty in the superallowed $\mathcal{F}t$ -value for ^{74}Rb decay have lead to new measurements of the charge

radius and mass of ^{74}Rb in addition to the superallowed branching ratio reported here. For the superallowed ft -value, a recent mass measurement at TRIUMF[32] combined with a previous measurement at ISOLTRAP[49, 50] leads to an average Q_{EC} -value of 10416.8(3.9) keV, which is a 12% improvement over the previously adopted value, resulting in a statistical rate function of 47 283(94)[32]. Combined with the ^{74}Rb half-life of $T_{1/2} = 64.776(43)$ ms[2] which is the average of the measurements of [29] and [51], the electron capture fraction $P_{EC} = 0.194\%$ [2] and the new superallowed branching ratio of $BR = 99.55(5)\%$ determined in this work, the superallowed ft -value for ^{74}Rb becomes:

$$\begin{aligned} ft = \frac{fT_{1/2}(1 + P_{EC})}{BR} &= 3082.6(61)_f(20)_{T_{1/2}}(15)_{BR} \text{ s} \\ &= 3082.6(66) \text{ s}, \end{aligned} \tag{6.8}$$

where the statistical rate function f , determined from the Q -value, dominates the uncertainty. However, it is predicted that the uncertainty in the Q -value will be decreased by a factor of approximately 8 to 0.5 keV in the very near future[32] dramatically lowering the uncertainty in the ft -value and fully exploiting the new high-precision branching ratio measurement presented here. The ^{74}Rb superallowed ft precision will then become dominated by the uncertainty in the half-life, currently precise to 0.07%[2].

A recent measurement of the charge radius of ^{74}Rb has also been performed at TRIUMF reducing the uncertainty in the Woods-Saxon δ_{C_2} value by $\sim 20\%$ [52] from $\delta_{C_2} = 1.50(30)\%$ to $\delta_{C_2} = 1.50(23)\%$. Using the corrections $\delta'_R = 1.50(12)\%$, $\delta_{NS} = -0.075(30)\%$ and $\delta_{C_1} = 0.130(60)\%$ [12], the superallowed $\mathcal{F}t$ -value of ^{74}Rb

Figure 6.5: A comparison of the superallowed ft -value for ^{74}Rb obtained in this work with the 12 other most precisely determined superallowed ft -values. The current limit on the precision in the superallowed ft -value of ^{74}Rb is the uncertainty in the Q -value, but this is expected to be improved significantly in the near future[32].

becomes:

$$\begin{aligned}
\mathcal{F}t_{WS} &= ft(1 + \delta'_R)(1 + \delta_{NS} - \delta_C) \\
&= 3075.4(66)_{ft}(36)_{\delta'_R}(9)_{\delta_{NS}}(74)_{\delta_C} \text{ s}, \\
&= 3075(11) \text{ s}.
\end{aligned} \tag{6.9}$$

While the superallowed $\mathcal{F}t$ value using the Hartree-Fock $\delta_{C_2} = 1.29(16)[2]$ correction is:

$$\begin{aligned}
\mathcal{F}t_{HF} &= 3082.1(66)_{ft}(36)_{\delta'_R}(9)_{\delta_{NS}}(53)_{\delta_C} \text{ s}, \\
&= 3082(9) \text{ s}.
\end{aligned} \tag{6.10}$$

Figure 6.6: A comparison of the relative uncertainties in the quantities required for the determination of the superallowed ft and $\mathcal{F}t$ -values for ^{74}Rb . The superallowed branching ratio is now the most precisely measured quantity.

In Table 6.7, a summary of the most recent ft and $\mathcal{F}t$ -values are given from references [2, 7, 12, 32, 52] as well as the present work. As can be seen from Figure 6.7 and Table 6.7, the superallowed $\mathcal{F}t$ -values from $Z = 6$ to $Z = 37$ continue to strongly support the CVC hypothesis. Thus, if we assume that the CVC hypothesis is valid, we can perform an internal consistency check on the δ_C corrections. The “experimental” isospin-symmetry-breaking correction for ^{74}Rb was thus determined by enforcing CVC with the average $\overline{\mathcal{F}t}_{WS}$ determined from the other 12 precision

Figure 6.7: A comparison of the superallowed $\mathcal{F}t$ -value for ^{74}Rb obtained in this work with the 12 other most precisely determined superallowed $\mathcal{F}t$ -values using Woods-Saxon shell-model calculations for δ_{C_2} (upper panel) and the Hartree-Fock δ_{C_2} values (lower panel).

cases via:

$$\begin{aligned}
 \delta_C^{exp} &= 1 + \delta_{NS} - \frac{\overline{\mathcal{F}t}_{WS}}{ft(1 + \delta'_R)}, \\
 &= 1.73(2)_{\overline{\mathcal{F}t}(21)}_{ft(12)}\delta'_R(3)_{\delta_{NS}}\%, \\
 &= 1.73(24)\%,
 \end{aligned} \tag{6.11}$$

while the value of δ_C^{exp} obtained using the average $\overline{\mathcal{F}t}_{HF}$ from the other 12 precision cases with Hartree-Fock corrections was:

$$\begin{aligned}
 \delta_C^{exp} &= 1.76(2)_{\overline{\mathcal{F}t}(21)}_{ft(12)}\delta'_R(3)_{\delta_{NS}}\%, \\
 &= 1.76(24)\%,
 \end{aligned} \tag{6.12}$$

Table 6.6: A summary of the transition dependent radiative corrections and the isospin-symmetry-breaking corrections for the 13 most precise superallowed β -decays from [2, 52].

Nucleus	δ'_R (%)	δ_{NS} (%)	δ_{C_1} (%)	$\delta_{C_2}^{HF}$ (%)	$\delta_{C_2}^{WS}$ (%)
^{10}C	1.679(4)	-0.345(35)	0.010(10)	0.215(35)	0.165(15)
^{14}O	1.543(8)	-0.245(50)	0.055(20)	0.255(30)	0.275(15)
^{22}Mg	1.466(17)	-0.225(20)	0.010(10)	0.250(55)	0.370(20)
^{26m}Al	1.478(20)	+0.005(20)	0.030(10)	0.410(50)	0.280(15)
^{34}Cl	1.443(32)	-0.085(15)	0.100(10)	0.595(55)	0.550(45)
^{34}Ar	1.412(35)	-0.180(15)	0.030(10)	0.510(60)	0.635(55)
^{38m}K	1.440(39)	-0.100(15)	0.105(20)	0.640(60)	0.550(55)
^{42}Sc	1.453(47)	+0.035(20)	0.020(10)	0.620(55)	0.645(55)
^{46}V	1.445(34)	-0.035(10)	0.075(30)	0.525(55)	0.545(55)
^{50}Mn	1.444(62)	-0.040(10)	0.045(20)	0.575(55)	0.610(50)
^{54}Co	1.443(71)	-0.035(10)	0.050(30)	0.635(55)	0.720(60)
^{62}Ga	1.459(87)	-0.045(20)	0.275(55)	0.93(16)	1.20(20)
^{74}Rb	1.50(12)	-0.075(30)	0.130(60)	1.29(16)	1.50(23)

These values are to be compared with the theoretical values $\delta_C^{WS} = 1.63(24)\%$ and $\delta_C^{HF} = 1.42(17)\%$, showing excellent self-consistency within the Woods-Saxon calculations, but less so in the Hartree-Fock calculations for ^{74}Rb .

6.3.1 The Isospin-Symmetry-Breaking Terms $\delta_{C_1}^{(1)}$, $\delta_{C_1}^{(2)}$, $\delta_{C_1}^{(3)}$, $\delta_{C_1}^{(4)}$

An additional test of the theoretical isospin-symmetry-corrections was performed using the non-analogue Fermi branching ratio measurements, thereby constraining the isospin configuration mixing components δ_{C_1} . The shell model calculations described in section 6.2 predict that 80% of the non-analogue Fermi decay will be shared between the first two excited 0^+ states, 0_1^+ and 0_2^+ , with branching ratios of 360 ppm and 230 ppm respectively[46]. Experimentally, the β plus γ -feeding intensity of the 0_1^+ level was determined to be 160(58) ppm while the β plus γ -ray feeding intensity of the 0_2^+ level was measured to be 502(14) ppm. Due to unobserved γ -ray feeding

Table 6.7: A comparison of the superallowed $\mathcal{F}t$ values for the 13 most precisely determined superallowed β -decays. Data are from [2, 7, 12, 32, 52] as well as the current work.

Nucleus	ft (s)	HF $\mathcal{F}t$ (s)	WS $\mathcal{F}t$ (s)
^{10}C	3041.7(43)	3075.1(47)	3076.7(45)
^{14}O	3042.3(27)	3072.1(34)	3071.5(33)
^{22}Mg	3052.0(72)	3081.7(75)	3078.0(74)
^{26m}Al	3037.58(60)	3069.0(19)	3073.1(12)
^{34}Cl	3049.4(11)	3069.2(23)	3070.6(21)
^{34}Ar	3052.7(82)	3073.5(86)	3069.6(85)
^{38m}K	3051.9(10)	3069.7(25)	3072.5(24)
^{42}Sc	3047.6(14)	3073.1(27)	3072.4(27)
^{46}V	3049.5(9)	3073.9(27)	3073.3(27)
^{50}Mn	3048.4(12)	3072.0(29)	3070.9(28)
^{54}Co	3050.8(13)	3072.5(31)	3069.9(32)
^{62}Ga	3074.1(15)	3080.0(61)	3071.5(72)
^{74}Rb	3082.6(66)	3082.1(92)	3075.4(106)
$\mathcal{F}t$ (s)		3071.59(87)	3072.35(75)
χ^2/ν		0.85	0.31

from above, only upper limits could be set on the non-analogue Fermi β -feeding, of ≤ 218 ppm and ≤ 516 ppm for the 0_1^+ and 0_2^+ states, respectively.

Using Equation 1.43, a direct test of the $\delta_{C_1}^{(n)}$ term was performed. Combining the ratio of the phase space integrals $(f_0/f_1) = 1.3$ and the upper limit of the non-analogue Fermi β -decay branch of 160(58) ppm for the first excited 0_1^+ state gave an upper limit on the $\delta_{C_1}^{(1)}$ correction of:

$$\delta_{C_1}^{(1)} \leq 0.021(8)\%, \quad (6.13)$$

which is consistent with the Towner and Hardy adopted value of 0.050(30)%[17]. However, it should be noted that the shell-model calculation predicts that approximately 15% of the γ -ray intensity from the highly excited 1^+ states populated in Gamow-Teller β decays feeds the 0_1^+ state. Using the method for calculating the

unobserved ground-state γ -ray feeding intensity, the $g_A = 0.8$ shell-model calculation described above provided a γ -ray branching ratio between the first 2_1^+ state and the first excited 0_1^+ state of $\overline{B}_{0_1^+} = 0.29(15)$ where the 50% uncertainty discussed in Section 6.2 was adopted. The calculated unobserved γ -ray feeding to 0_1^+ using this branching ratio was $I'_{0_1^+} = 263$ ppm with a possible range of values of $105 - 507$ ppm. We adopt the unobserved feeding of $260(260)$ ppm, obtaining an upper limit on the non-analogue Fermi β -decay branch of ≤ 166 ppm which is a factor of 2 smaller than the predicted Fermi β -feeding to the first excited 0_1^+ state of 360 ppm. Combining this upper limit of 166 ppm with the ratio of phase space factors, the upper limit on the isospin-symmetry-breaking correction is lowered to $\delta_{C_1}^{(1)} \leq 0.022\%$.

The second excited 0_2^+ state has a ratio of statistical rate functions of $(f_0/f_2) = 2.5$ and an upper limit on the non-analogue β -branching ratio of $502(14)$ ppm. Combining these values to determine the isospin-symmetry-breaking component gives:

$$\delta_{C_1}^{(2)} \leq 0.129(4)\%. \quad (6.14)$$

This upper limit is consistent with the Towner and Hardy value of $0.060(36)\%$ [46] where for the uncertainty we have assumed the same relative uncertainty in $\delta_{C_1}^{(2)}$ as was published for $\delta_{C_1}^{(1)}$ in [17]. However, theory predicts a γ -ray feeding of 5% of the decay of the high-lying 1^+ states populated in Gamow-Teller β decays. Repeating the unobserved intensity arguments used for the $\delta_{C_1}^{(1)}$ component, we calculate a γ -ray feeding branching ratio to the 0_2^+ state relative to the 2_1^+ state of $\overline{B}_{0_2^+} = 0.12(6)$. Combining with $I'_{2_1^+}$, the unobserved γ -ray intensity feeding the 0_2^+ state is estimated to be $I'_{0_2^+} = 88(88)$ ppm, providing a non-analogue Fermi decay to the 0_2^+ state of $414(89)$ ppm, almost a factor of 2 larger than the predicted Fermi β -feeding of

230 ppm. Using the value of 414(89) ppm for the β -feeding to the 0_2^+ state, we find that the isospin-symmetry-breaking component $\delta_{C_1}^{(2)} = 0.104(22)\%$ which is consistent with the Towner-Hardy value of 0.060(36)%[46] listed above.

The third tentative excited (0_3^+) state at 2173 keV has a ratio of statistical rate functions of $(f_0/f_3) = 3.47$ and an upper limit on the non-analogue β -branching ratio of 142(8) ppm. Combining these values to determine the isospin-symmetry-breaking component gives:

$$\delta_{C_1}^{(3)} \leq 0.049(3)\%, \quad (6.15)$$

while the fourth tentative excited (0_4^+) state at 2537 keV has an upper limit on the non-analogue β -branching ratio of 141(8) ppm with $(f_0/f_4) = 4.43$ giving an isospin-symmetry-breaking component of:

$$\delta_{C_1}^{(4)} \leq 0.062(4)\%. \quad (6.16)$$

6.4 Conclusions

Superallowed β -decay is currently one of the most powerful tools for testing the electroweak theory of the Standard Model. Using superallowed β -decay, the CVC hypothesis has been confirmed to 1.3×10^{-4} and the CKM matrix element V_{ud} has been determined to 2.5 parts in 10^4 , with unitarity verified at the 0.06% level via the top row sum-of-squares. The limits of the precision on the Standard Model tests are due almost exclusively to uncertainties in the theoretical corrections required to

Table 6.8: A summary of the constraints set on the isospin-symmetry-breaking terms $\delta_{C_1}^{(n)}$ from configuration mixing of the ^{74}Rb ground-state with the excited 0^+ states in ^{74}Kr . The $\delta_{C_1}^{(n)}$ terms were obtained both with and without estimating the unobserved γ -ray feeding from above. The relative uncertainty on the Towner-Hardy adopted value of $\delta_{C_1}^{(2)}$ is assumed to be the same as $\delta_{C_1}^{(1)}$ [17].

Level Energy (keV)	Fermi β BR (ppm)	Theor. BR (ppm)	$\left(\frac{f_0}{f_n}\right)$	$\delta_{C_1}^{(n)}$ (excl. unobs.) (%)	$\delta_{C_1}^{(n)}$ (incl. unobs.) (%)	Adopted Theor. Value (%) [2]
509	$\leq 160(58)$	360	1.30	$\leq 0.021(8)$	≤ 0.022	0.050(30) [2]
1654	$\leq 502(14)$	230	2.50	$\leq 0.129(4)$	0.104(22)	0.060(36)[46]
2173	$\leq 142(8)$	—	3.47	$\leq 0.049(3)$	—	—
2537	$\leq 141(8)$	—	4.43	$\leq 0.062(4)$	—	—
Sum	$\leq 945(98)$			$\leq 0.261(10)$		

extract the superallowed $\mathcal{F}t$ values. To strengthen these tests on the Standard Model and ultimately reach the limits required to reveal physics beyond the Standard Model, will require continued scrutiny and reevaluation of these theoretical corrections.

A high-precision branching ratio measurement was performed for the superallowed β -decay of ^{74}Rb in November 2010 using the 8π spectrometer at TRIUMF. In this experiment, 23 excited states were identified in ^{74}Kr which were populated following the β -decay of ^{74}Rb . A total of 58 γ -rays were identified following the $8.241(4) \times 10^8$ detected β -particles. An observed non-superallowed branch of 0.396(7)% was determined from the intensity of the identified γ -rays, while the unobserved non-superallowed intensity was calculated to be 0.05(5)% leading to a superallowed branching ratio of 99.55(5)% making the superallowed branching ratio the most precise experimental quantity in the determination of the superallowed ft -value of ^{74}Rb . Combining the half-life and Q -value with the measured superallowed branching ratio of this work leads to a superallowed ft -value of 3082.6(66) s, with an uncertainty that is expected to be greatly reduced in the near future by an order of magnitude reduction of the

uncertainty in the Q -value[32].

Tests of the model-dependent isospin-symmetry-breaking correction δ_C were performed by comparing the ft -value of this work with the average $\overline{\mathcal{F}t}$ -value. Furthermore, stringent tests of the isospin-symmetry-breaking components $\delta_{C_1}^{(1)}$, $\delta_{C_1}^{(2)}$, $\delta_{C_1}^{(3)}$ and $\delta_{C_1}^{(4)}$ were performed which will guide future calculations of the configuration mixing component of the isospin-symmetry-breaking correction.

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